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Informatics II
Exercise 4

March 15, 2021

Goal:

- Draw a tree that illustrates the divide and conquer approach.
- Practise divide-and-conquer algorithms.
- Draw a recursion tree, and estimate the asymptotic upper bound
- Calculate the symptotic tight bound of recurrences.
- Practise Master Theorem.

Divide and Conquer

Task 1. Given an array $A[...]$ of n integers sorted in ascending order and a target integer t, write a divide-and-conquer algorithm that finds the closest number to t in the array A. One integer a is closer to t than another integer b if $|a - t| < |b - t|$.

a) Draw a tree to illustrate the process of finding the closest number to $t = 20$ in the array $A =$ [2,3,5,7,9,13,15,27] according to your divide-and-conquer algorithm.

b) Implement your divide-and-conquer algorithm that takes an array A, n, and a target integer t as input, and returns the closest number to t in the array A. Use C code for your solution.

Task 2. Given an array $A[...]$ of n integers, write an algorithm to calculate the number of inversions in the array A. For array A, an *inversion* is a pair of positions (i, j) where $1 \leq i < j \leq n$ and $A[i] > A[j]$. Assume $A = \{3, 2, 1\}$, there are three inversions in A – $(1, 2), (1, 3)$ and $(2, 3)$. For example, $(1, 2)$ is an inversion because $A[1] > A[2]$.

a) Implement a solution with $O(n^2)$ time complexity in C.

b) Implement a divide-and-conquer solution in C. *Hint: think about merge sort, can you slightly modify and apply it here?*

c) Calculate the asymptotic tight bound in b).

Recurrences

Task 3. Consider the following recurrence:

$$T(n) = \begin{cases} 1 & \text{, if } n = 1 \\ T(n/3) + T(n/6) + T(n/9) + n & \text{, if } n > 1 \end{cases}$$

a) Draw a recursion tree and use it to estimate the asymptotic upper bound of $T(n)$. Demonstrate the tree-based computations that led to your estimate.

b) Use the substitution method to prove that your estimate in (a) is correct.

Task 4. Calculate the asymptotic tight bound of the following recurrences. If the Master Theorem can be used, write down a, b, $f(n)$ and the case (1-3).

1. $T(n) = 4T(\frac{n}{16}) + 16\sqrt{n}$
2. $T(n) = T(\sqrt{n}) + \log n$
3. $T(n) = 16T(n/8) + n^3$
4. $T(n) = 3T(n-2) + n$
5. $T(n) = \log n + T(\sqrt[3]{n})$