IE479 - Distribution Logistics

Logistics Planning for Emergency Aid Distribution Centers

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Contents

Introduction

The aim of this project is to open local aid distribution centers in regions recently impacted by a natural disaster. In that regard, this report presents the effective positioning of aid distribution centers based on varying coverage radii for different objective measures. These centers are strategically placed in locations that maximize coverage for the affected population and minimize transportation distances by creating a mathematical model, which is then solved using Gurobi.

Executive Summary

As stated in the first question of the project, the modeling is based on opening emergency centers on districts and locating each opened center less than 3000 meters from at least one major road. As requested in the objective function of the first question, emergency centers were positioned to serve all districts. The results obtained were tested one by one with the radius values given in the question and the optimal results emerged as stated in the report. In the second question of the project, the distance of the emergency centers determined to be opened to the major roads was kept constant at 3000 meters. New decision variables were assigned depending on the objective function requested in the question, and the positioning of emergency centers to serve the maximum number of people was carried out in an optimal manner, taking into account the population parameters specified in the question. While doing that the maximum number of centers that could be opened were also considered as given in the question. In the third and last question of the project, the total distance of the emergency centers planned to be opened to the districts they will serve was tried to be minimized while covering all the given districts. In this question, the maximum distance between major roads and emergency centers is still assumed to be 3000 meters. Accordingly, the centers planned to be opened have been determined optimally. As in the first two questions, the optimal values in this question were calculated separately based on the desired service radius values. The results obtained using all these methodologies and assumptions will be evaluated one by one in the content of this report.

Parameters and Decision Variables Used in Project

Parameters:

M: Very big number. (Assumed to be 1000 in the model.)

R: Radius of covered area by a center (meter).

N: Maximum distance between a major road and a center which can be considered as near (meter).

p_j: Population of district j.
$j \in \{1, \ldots, 20\}$

d_{ij}: Distance between served customer i and distribution center j.
$i \in \{1, \ldots, 20\}, \quad j \in \{1, \ldots, 20\}$

md_{kj}: Distance between major road k and distribution center j.
$i \in \{1, \ldots, 25\}, \quad j \in \{1, \ldots, 20\}$

$$a_{ij} = \begin{cases} 1, & \text{if } d_{ij} \leq R \\ 0, & \text{otherwise.} \end{cases} \qquad i \in \{1, \ldots, 20\}, \quad j \in \{1, \ldots, 20\}$$

$$b_{kj} = \begin{cases} 1, & \text{if } md_{kj} \leq N \\ 0, & \text{otherwise.} \end{cases} \qquad k \in \{1, \ldots, 25\}, \quad j \in \{1, \ldots, 20\}$$

Decision Variables:

$$X_{ij} = \begin{cases} 1, & \text{if affected are } j \text{ is covered by center } i \\ & \qquad\qquad\qquad\qquad i \in \{1, \ldots, 20\}, \quad j \in \{1, \ldots, 20\} \\ 0, & \text{otherwise.} \end{cases}$$

$$Y_i = \begin{cases} 1, & \text{if the center is located to district } i. \\ & \qquad\qquad\qquad\qquad i \in \{1, \ldots, 20\} \\ 0, & \text{otherwise.} \end{cases}$$

$$Z_j = \begin{cases} 1, & \text{if affected area } j \text{ is covered by any center.} \\ & \qquad\qquad\qquad\qquad j \in \{1, \ldots, 20\} \\ 0, & \text{otherwise.} \end{cases}$$

Case 1

In this case, it is aimed to optimize the locations of aid distribution centers whilst covering every affected area within the designated radius. Also, the aid distribution centers must be located near a major road or transportation hub, this distance requirement is assumed as N = 3000 as mentioned above. This case is analyzed for the radius values of 2 km, 3 km, 4 km, 5 km, and 6 km. The model is given below.

Model

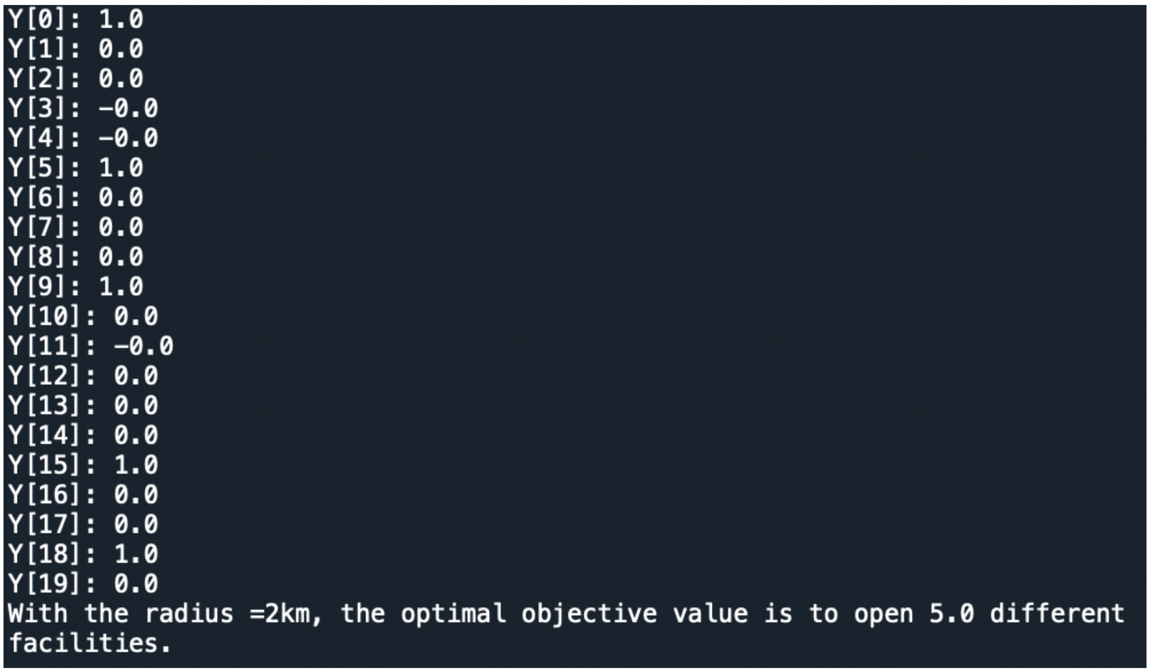
$$
\begin{aligned}
\text{min.} \quad & \sum_{i=1}^{20} Y_i & & (1)\\
\text{s.t.} \quad & \sum_{i=1}^{20} X_{ij} \geq 1 & \forall j \in \{1, \ldots, 20\} \quad & (2)\\
& X_{ij} \leq a_{ij} Y_i & \forall i, j \in \{1, \ldots, 20\} \quad & (3)\\
& \sum_{k=1}^{25} b_{ki} \geq Y_i & \forall i \in \{1, \ldots, 20\} \quad & (4)\\
& X_{ij}, Y_i, Z_j \in \{0, 1\} & \forall i, j \in \{1, \ldots, 20\} \quad & (5)
\end{aligned}
$$

Explanation of Objective Function and Constraints

1. Objective is to minimize the total number of centers which will be opened (see (1)).
2. Each district should be covered by at least 1 center (see (2)).
3. Logical constraint: A district should be covered if and only if there is a center that covers that district. (see (3)).
4. The center which will be opened should be close to a major road (see (4)).

Solutions for R = 2000, 3000, 4000, 5000, 6000

The optimal solution found for when R = 2000 meters is given below in Figure 1. It can be observed that the Y values are equal to 1 for 5 different indexes (facilities). The indexes 0,5,9,15,18, which correspond to the districts 1,6,10,16 and 19. This means that for this districts, there was a center located



```
Y[0]: 1.0
Y[1]: 0.0
Y[2]: 0.0
Y[3]: -0.0
Y[4]: -0.0
Y[5]: 1.0
Y[6]: 0.0
Y[7]: 0.0
Y[8]: 0.0
Y[9]: 1.0
Y[10]: 0.0
Y[11]: -0.0
Y[12]: 0.0
Y[13]: 0.0
Y[14]: 0.0
Y[15]: 1.0
Y[16]: 0.0
Y[17]: 0.0
Y[18]: 1.0
Y[19]: 0.0
With the radius =2km, the optimal objective value is to open 5.0 different
facilities.
```

Figure 1: R = 2000 m For Case 1

The optimal solution found for when R = 3000 meters is given below in Figure 2. It can be observed that the Y values are equal to 1 for 3 different indexes (facilities). The indexes 1,5,8, which correspond to the districts 2,6 and 9. This means that for this districts, there was a center located.

```
Y[0]: 0.0
Y[1]: 1.0
Y[2]: 0.0
Y[3]: 0.0
Y[4]: 0.0
Y[5]: 1.0
Y[6]: 0.0
Y[7]: 0.0
Y[8]: 1.0
Y[9]: 0.0
Y[10]: 0.0
Y[11]: 0.0
Y[12]: 0.0
Y[13]: 0.0
Y[14]: 0.0
Y[15]: 0.0
Y[16]: 0.0
Y[17]: 0.0
Y[18]: 0.0
Y[19]: 0.0
With the radius = 3km, the optimal objective value is to open 3.0 different
facilities.
```

Figure 2: R = 3000 m For Case 1

The optimal solution found for when R = 4000 meters is given below in Figure 3. It can be observed that the Y values are equal to 1 for 2 different indexes (facilities). The indexes 7,8, which correspond to the districts 8 and 9. This means that for this districts, there was a center located.

```
Y[0]: 0.0
Y[1]: 0.0
Y[2]: 0.0
Y[3]: 0.0
Y[4]: 0.0
Y[5]: 0.0
Y[6]: 0.0
Y[7]: 1.0
Y[8]: 1.0
Y[9]: 0.0
Y[10]: 0.0
Y[11]: 0.0
Y[12]: 0.0
Y[13]: 0.0
Y[14]: 0.0
Y[15]: 0.0
Y[16]: 0.0
Y[17]: 0.0
Y[18]: 0.0
Y[19]: 0.0
With the radius = 4km, the optimal objective value is to open 2.0 different
facilities.
```

Figure 3: R = 4000 m For Case 1

The optimal solution found for when R = 5000 meters is given below in Figure 4. It can be observed that the Y values are equal to 1 for 1 different indexes (facilities). The indexes 6, which correspond to the district 7. This means that for this districts, there was a center located.

```
Y[0]: 0.0
Y[1]: 0.0
Y[2]: 0.0
Y[3]: 0.0
Y[4]: 0.0
Y[5]: 0.0
Y[6]: 1.0
Y[7]: 0.0
Y[8]: 0.0
Y[9]: 0.0
Y[10]: 0.0
Y[11]: 0.0
Y[12]: 0.0
Y[13]: 0.0
Y[14]: 0.0
Y[15]: 0.0
Y[16]: 0.0
Y[17]: 0.0
Y[18]: 0.0
Y[19]: 0.0
With the radius = 5km, the optimal objective value is to open 1.0 different
facilities.
```

Figure 4: R = 5000 m For Case 1

The optimal solution found for when R = 6000 meters is given below in Figure 5. It can be observed that the Y values are equal to 1 for 1 different indexes (facilities). The indexes 4, which correspond to the district 5. This means that for this districts, there was a center located.

```
Y[0]: 0.0
Y[1]: 0.0
Y[2]: 0.0
Y[3]: 0.0
Y[4]: 1.0
Y[5]: 0.0
Y[6]: 0.0
Y[7]: 0.0
Y[8]: 0.0
Y[9]: 0.0
Y[10]: 0.0
Y[11]: 0.0
Y[12]: 0.0
Y[13]: 0.0
Y[14]: 0.0
Y[15]: 0.0
Y[16]: 0.0
Y[17]: 0.0
Y[18]: 0.0
Y[19]: 0.0
With the radius = 6km, the optimal objective value is to open 1.0 different
facilities.
```

Figure 5: R = 6000 m For Case 1

From the analysis of different radius levels, the effect of radius on the number of centers located can be observed. It can be seen that the number of centers decreases as the radius increases. This trend can be attributed to the expanding coverage radius, allowing each center to cover more districts as the radius increases. In the last two scenarios, where R = 5000 and 6000, it is seen that even though the number of centers located is 1 in both cases, the districts served was different where, district 7 is covered at R = 5000 and district 5 is covered at R = 6000. This indicates that beyond a certain radius, multiple optimal solutions may exist for the placement of centers in this case.

Case 2

In this case, the goal is to open a limited number of distribution centers (maximum of 4) while covering as many people as possible within a given radius. As in the previous case, this case is analyzed for the same radius values (2km, 3km, 4km, 5km, 6km). It is assumed that the aid distribution centers should be located near to a major road or transportation hub and this distance requirement is assumed as N = 3000. The model is given below.

Model

$$\text{max.} \quad \sum_{j=1}^{20} p_j Z_j \tag{6}$$

$$\text{s.t.} \quad \sum_{i=1}^{20} Y_i \leq 4 \tag{7}$$

$$\sum_{i=1}^{20} X_{ij} \leq M Z_j \qquad \forall j \in \{1, \ldots, 20\} \tag{8}$$

$$\sum_{i=1}^{20} X_{ij} \geq Z_j \qquad \forall j \in \{1, \ldots, 20\} \tag{9}$$

$$X_{ij} \leq a_{ij} Y_i \qquad \forall i, j \in \{1, \ldots, 20\} \tag{10}$$

$$\sum_{k=1}^{25} b_{ki} \geq Y_i \qquad \forall i \in \{1, \ldots, 20\} \tag{11}$$

$$X_{ij}, Y_i, Z_j \in \{0, 1\} \qquad \forall i, j \in \{1, \ldots, 20\} \tag{12}$$

Explanation of Objective Function and Constraints

1. Objective is to maximize the total number of people who are covered (see (6)).
2. Maximum of 4 centers can be opened (see (7)).
3. Logical constraint: If a district is covered by one center or multiple centers, affected districts are served equally (see (8)).

4. Logical constraint: If a district is covered by one center or multiple centers, affected districts are served equally (see (9)).

5. Logical constraint: A district should be covered if and only if there is a center that covers that district. (see (10)).

6. The center which will be opened should be close to a major road (see (11)).

Solutions for R = 2000, 3000, 4000, 5000, 6000

The optimal solution found for when R = 2000 meters is given below in Figure 6, which is covering 396733 people. It is seen by the Z values that every affected area is covered by at least one center. From the Y values that are equal to 1, it is seen that there are 4 centers located, which were the indexes 0,2,6,7, which correspond to districts 1,3,7, and 8.

```
Y[0]: 1.0
Y[1]: 0.0
Y[2]: 1.0
Y[3]: 0.0
Y[4]: 0.0
Y[5]: 0.0
Y[6]: 1.0
Y[7]: 1.0
Y[8]: 0.0
Y[9]: 0.0
Y[10]: 0.0
Y[11]: 0.0
Y[12]: 0.0
Y[13]: 0.0
Y[14]: 0.0
Y[15]: 0.0
Y[16]: 0.0
Y[17]: 0.0
Y[18]: 0.0
Y[19]: 0.0
Z[0]: 1.0
Z[1]: 1.0
Z[2]: 1.0
Z[3]: 1.0
Z[4]: 1.0
Z[5]: 1.0
Z[6]: 1.0
Z[7]: 1.0
Z[8]: 1.0
Z[9]: 1.0
Z[10]: 1.0
Z[11]: 1.0
Z[12]: 1.0
Z[13]: 1.0
Z[14]: 1.0
Z[15]: 1.0
Z[16]: 1.0
Z[17]: 1.0
Z[18]: 0.0
Z[19]: 1.0
With the radius = 2km, the optimal objective value is to cover 396533.0 people.
```

Figure 6: R = 2000 m For Case 2

The optimal solution found for when R = 3000 meters is given below in Figure 7, which is covering 426680 people. It is seen by the Z values that every affected area is covered by at least one center. From the Y values that are equal to 1, it is seen that there are 3 centers located, which were the indexes 0,1,2, which correspond to districts 1,2, and 3.

```
Y[0]: 1.0
Y[1]: 1.0
Y[2]: 1.0
Y[3]: -0.0
Y[4]: -0.0
Y[5]: -0.0
Y[6]: -0.0
Y[7]: -0.0
Y[8]: 0.0
Y[9]: -0.0
Y[10]: -0.0
Y[11]: -0.0
Y[12]: -0.0
Y[13]: -0.0
Y[14]: -0.0
Y[15]: -0.0
Y[16]: -0.0
Y[17]: -0.0
Y[18]: -0.0
Y[19]: -0.0
Z[0]: 1.0
Z[1]: 1.0
Z[2]: 1.0
Z[3]: 1.0
Z[4]: 1.0
Z[5]: 1.0
Z[6]: 1.0
Z[7]: 1.0
Z[8]: 1.0
Z[9]: 1.0
Z[10]: 1.0
Z[11]: 1.0
Z[12]: 1.0
Z[13]: 1.0
Z[14]: 1.0
Z[15]: 1.0
Z[16]: 1.0
Z[17]: 1.0
Z[18]: 1.0
Z[19]: 1.0
With the radius = 3km, the optimal objective value is to cover 426680.0 people.
```

Figure 7: R = 3000 m For Case 2

The optimal solution found for when R = 4000 meters is given below in Figure 8, which is covering 426680 people. It is seen by the Z values that every affected area is covered by at least one center. From the Y values that are equal to 1, it is seen that there are 2 centers located, which were the indexes 0,1, which correspond to districts 1 and 2.

```
Y[0]: 1.0
Y[1]: 1.0
Y[2]: 0.0
Y[3]: -0.0
Y[4]: -0.0
Y[5]: -0.0
Y[6]: -0.0
Y[7]: 0.0
Y[8]: -0.0
Y[9]: -0.0
Y[10]: -0.0
Y[11]: -0.0
Y[12]: -0.0
Y[13]: -0.0
Y[14]: -0.0
Y[15]: -0.0
Y[16]: -0.0
Y[17]: -0.0
Y[18]: 0.0
Y[19]: -0.0
Z[0]: 1.0
Z[1]: 1.0
Z[2]: 1.0
Z[3]: 1.0
Z[4]: 1.0
Z[5]: 1.0
Z[6]: 1.0
Z[7]: 1.0
Z[8]: 1.0
Z[9]: 1.0
Z[10]: 1.0
Z[11]: 1.0
Z[12]: 1.0
Z[13]: 1.0
Z[14]: 1.0
Z[15]: 1.0
Z[16]: 1.0
Z[17]: 1.0
Z[18]: 1.0
Z[19]: 1.0
With the radius = 4km, the optimal objective value is to cover 426680.0 people.
```

Figure 8: R = 4000 m For Case 2

The optimal solution found for when R = 5000 meters is given below in Figure 9, which is covering 426680 people. It is seen by the Z values that every affected area is covered by at least one center. From the Y values that are equal to 1, it is seen that there are 2 centers located, which were the indexes 1,3, which correspond to districts 2 and 4.

```
Y[0]: -0.0
Y[1]: 1.0
Y[2]: -0.0
Y[3]: 1.0
Y[4]: -0.0
Y[5]: -0.0
Y[6]: 0.0
Y[7]: -0.0
Y[8]: -0.0
Y[9]: -0.0
Y[10]: -0.0
Y[11]: -0.0
Y[12]: -0.0
Y[13]: -0.0
Y[14]: -0.0
Y[15]: -0.0
Y[16]: -0.0
Y[17]: -0.0
Y[18]: -0.0
Y[19]: 1.0
Z[0]: 1.0
Z[1]: 1.0
Z[2]: 1.0
Z[3]: 1.0
Z[4]: 1.0
Z[5]: 1.0
Z[6]: 1.0
Z[7]: 1.0
Z[8]: 1.0
Z[9]: 1.0
Z[10]: 1.0
Z[11]: 1.0
Z[12]: 1.0
Z[13]: 1.0
Z[14]: 1.0
Z[15]: 1.0
Z[16]: 1.0
Z[17]: 1.0
Z[18]: 1.0
Z[19]: 1.0
With the radius = 5km, the optimal objective value is to cover 426680.0 people.
```

Figure 9: R = 5000 m For Case 2

The optimal solution found for when R = 6000 meters is given below in Figure 10, which is covering 426680 people. It is seen by the Z values that every affected area is covered by at least one center. From the Y values that are equal to 1, it is seen that there are 4 centers located, which were the indexes 0,1,2,9, which correspond to districts 1, 2, 3, and 10.

```
Y[0]: 1.0
Y[1]: 1.0
Y[2]: 1.0
Y[3]: 0.0
Y[4]: 0.0
Y[5]: 0.0
Y[6]: 0.0
Y[7]: 0.0
Y[8]: 0.0
Y[9]: 1.0
Y[10]: 0.0
Y[11]: 0.0
Y[12]: 0.0
Y[13]: 0.0
Y[14]: 0.0
Y[15]: 0.0
Y[16]: 0.0
Y[17]: 0.0
Y[18]: 0.0
Y[19]: 0.0
Z[0]: 1.0
Z[1]: 1.0
Z[2]: 1.0
Z[3]: 1.0
Z[4]: 1.0
Z[5]: 1.0
Z[6]: 1.0
Z[7]: 1.0
Z[8]: 1.0
Z[9]: 1.0
Z[10]: 1.0
Z[11]: 1.0
Z[12]: 1.0
Z[13]: 1.0
Z[14]: 1.0
Z[15]: 1.0
Z[16]: 1.0
Z[17]: 1.0
Z[18]: 1.0
Z[19]: 1.0
With the radius = 6km, the optimal objective value is to cover 426680.0 people.
```

Figure 10: R = 6000 m For Case 2

It was observed that after R = 2000, although the number and selection of districts varied, the optimal number of people covered remained constant. This suggests that increasing the radius provides greater flexibility in center location choices without sacrificing coverage efficiency.

Case 3

In this case, the goal is to minimize the total distance between the affected districts and the aid distribution centers, with a maximum of 4 centers. This case is analyzed for the radius values of 3km, 4km, 5km, 6km. The aid distribution centers are also assumed to be located near a major road or transportation hub, with the distance requirement set to N = 3000 meters. The model is given below.

Model

$$
\begin{aligned}
\text{min.} \quad & \sum_{i=1}^{20}\sum_{j=1}^{20} d_{ij}X_{ij} && && (13)\\
\text{s.t.} \quad & \sum_{i=1}^{20} Y_i \leq 4 && && (14)\\
& \sum_{i=1}^{20} X_{ij} \leq MZ_j && \forall j \in \{1,\ldots,20\} && (15)\\
& \sum_{i=1}^{20} X_{ij} \geq Z_j && \forall j \in \{1,\ldots,20\} && (16)\\
& X_{ij} \leq a_{ij}Y_i && \forall i,j \in \{1,\ldots,20\} && (17)\\
& \sum_{i=1}^{20} X_{ij} \geq 1 && \forall j \in \{1,\ldots,20\} && (18)\\
& X_{ij}, Y_i, Z_j \in \{0,1\} && \forall i,j \in \{1,\ldots,20\} && (19)
\end{aligned}
$$

Explanation of Objective Function and Constraints

1. The objective is to minimize total distance between centers and affected districts (see (13)).

2. Maximum of 4 centers can be opened (see (14)).

3. Logical constraint: If a district is covered by one center or multiple centers, affected districts are served equally (see (15)).

4. Logical constraint: If a district is covered by one center or multiple centers, affected districts are served equally (see (16)).
5. Logical constraint: A district should be covered if and only if there is a center that covers that district. (see (17)).
6. Each district should be covered by at least 1 center (see (18)).

Solutions for R = 3000, 4000, 5000, 6000

For the following radius values, it was observed that each radius value had the same optimal value of the total distance that is approximately 22212 meters. The number and selection of districts were also the same for all of the values. This consistency across all radius values may indicate that the distribution centers were located in such a way at the radius of 3 km, that increasing the radius didn't lead to a different solution, as it was the optimal solution at the first place.

```
Y[0]: 1.0
Y[1]: 1.0
Y[2]: 0.0
Y[3]: 0.0
Y[4]: -0.0
Y[5]: 0.0
Y[6]: -0.0
Y[7]: -0.0
Y[8]: -0.0
Y[9]: -0.0
Y[10]: -0.0
Y[11]: 1.0
Y[12]: 0.0
Y[13]: -0.0
Y[14]: 1.0
Y[15]: -0.0
Y[16]: -0.0
Y[17]: -0.0
Y[18]: -0.0
Y[19]: -0.0
Z[0]: 1.0
Z[1]: 1.0
Z[2]: 1.0
Z[3]: 1.0
Z[4]: 1.0
Z[5]: 1.0
Z[6]: 1.0
Z[7]: 1.0
Z[8]: 1.0
Z[9]: 1.0
Z[10]: 1.0
Z[11]: 1.0
Z[12]: 1.0
Z[13]: 1.0
Z[14]: 1.0
Z[15]: 1.0
Z[16]: 1.0
Z[17]: 1.0
Z[18]: 1.0
Z[19]: 1.0
With the radius = 3km, the optimal objective value is the total distance of
22211.969999999998 meters.
```

Figure 11: R = 3000 m For Case 3

```
Y[0]: 1.0
Y[1]: 1.0
Y[2]: 0.0
Y[3]: 0.0
Y[4]: -0.0
Y[5]: 0.0
Y[6]: -0.0
Y[7]: -0.0
Y[8]: -0.0
Y[9]: -0.0
Y[10]: -0.0
Y[11]: 1.0
Y[12]: 0.0
Y[13]: -0.0
Y[14]: 1.0
Y[15]: -0.0
Y[16]: -0.0
Y[17]: -0.0
Y[18]: -0.0
Y[19]: -0.0
Z[0]: 1.0
Z[1]: 1.0
Z[2]: 1.0
Z[3]: 1.0
Z[4]: 1.0
Z[5]: 1.0
Z[6]: 1.0
Z[7]: 1.0
Z[8]: 1.0
Z[9]: 1.0
Z[10]: 1.0
Z[11]: 1.0
Z[12]: 1.0
Z[13]: 1.0
Z[14]: 1.0
Z[15]: 1.0
Z[16]: 1.0
Z[17]: 1.0
Z[18]: 1.0
Z[19]: 1.0
With the radius = 4km, the optimal objective value is the total distance of
22211.969999999998 meters.
```

Figure 12: R = 4000 m For Case 3

```
Y[0]: 1.0
Y[1]: 1.0
Y[2]: 0.0
Y[3]: 0.0
Y[4]: -0.0
Y[5]: 0.0
Y[6]: -0.0
Y[7]: -0.0
Y[8]: -0.0
Y[9]: -0.0
Y[10]: -0.0
Y[11]: 1.0
Y[12]: 0.0
Y[13]: -0.0
Y[14]: 1.0
Y[15]: -0.0
Y[16]: -0.0
Y[17]: 0.0
Y[18]: -0.0
Y[19]: -0.0
Z[0]: 1.0
Z[1]: 1.0
Z[2]: 1.0
Z[3]: 1.0
Z[4]: 1.0
Z[5]: 1.0
Z[6]: 1.0
Z[7]: 1.0
Z[8]: 1.0
Z[9]: 1.0
Z[10]: 1.0
Z[11]: 1.0
Z[12]: 1.0
Z[13]: 1.0
Z[14]: 1.0
Z[15]: 1.0
Z[16]: 1.0
Z[17]: 1.0
Z[18]: 1.0
Z[19]: 1.0
With the radius = 5km, the optimal objective value is the total distance of
22211.969999999998 meters.
```

Figure 13: R = 5000 m For Case 3

```
Y[0]: 1.0
Y[1]: 1.0
Y[2]: 0.0
Y[3]: 0.0
Y[4]: -0.0
Y[5]: 0.0
Y[6]: -0.0
Y[7]: -0.0
Y[8]: -0.0
Y[9]: -0.0
Y[10]: -0.0
Y[11]: 1.0
Y[12]: 0.0
Y[13]: -0.0
Y[14]: 1.0
Y[15]: -0.0
Y[16]: -0.0
Y[17]: -0.0
Y[18]: -0.0
Y[19]: -0.0
Z[0]: 1.0
Z[1]: 1.0
Z[2]: 1.0
Z[3]: 1.0
Z[4]: 1.0
Z[5]: 1.0
Z[6]: 1.0
Z[7]: 1.0
Z[8]: 1.0
Z[9]: 1.0
Z[10]: 1.0
Z[11]: 1.0
Z[12]: 1.0
Z[13]: 1.0
Z[14]: 1.0
Z[15]: 1.0
Z[16]: 1.0
Z[17]: 1.0
Z[18]: 1.0
Z[19]: 1.0
With the radius = 6km, the optimal objective value is the total distance of
22211.969999999998 meters.
```

Figure 14: R = 6000 m For Case 3