**Title: Statistical Quality Control Analysis of Service Startup Times**

**1. Introduction**

In modern software systems, monitoring service startup times is essential to ensure system performance and detect early warnings of process instability. This report presents a statistical quality control (SQC) analysis using simulated data for the startup durations (in milliseconds) of services in a distributed environment.

The objective is to determine whether the process is in statistical control and to compare the performance of various control chart techniques in identifying assignable causes.

**2. Data Description**

The dataset contains 25 samples, each with 5 observations, representing service startup times in milliseconds.

**3. Control Charts Analysis**

**3.1 x̄-R Chart**  
This chart is used for subgroup monitoring. The sample mean (x̄) and range (R) of each sample were calculated.

* Overall mean of sample means (x̄̄): xˉˉ=125∑i=125xˉi\bar{\bar{x}} = \frac{1}{25} \sum\_{i=1}^{25} \bar{x}\_i
* Overall mean of ranges (R̄): Rˉ=125∑i=125Ri\bar{R} = \frac{1}{25} \sum\_{i=1}^{25} R\_i
* A2 constant (n=5): 0.577
* Control Limits for x̄ Chart:
  + UCL = xˉˉ+A2⋅Rˉ\bar{\bar{x}} + A\_2 \cdot \bar{R}
  + LCL = xˉˉ−A2⋅Rˉ\bar{\bar{x}} - A\_2 \cdot \bar{R}
* Control Limits for R Chart:
  + D3 = 0, D4 = 2.114
  + UCL = D4⋅RˉD\_4 \cdot \bar{R}
  + LCL = D3⋅Rˉ=0D\_3 \cdot \bar{R} = 0

Sample 10 clearly exceeds the UCL on the x̄ chart, indicating an assignable cause.

*Insert Figure 1: x̄ Chart*  
*Insert Figure 2: R Chart*

**3.2 Individual Chart (I-Chart)**  
The individual chart plots the means of each sample. The center line is the overall mean, and the control limits are calculated using the standard deviation (σ) of the means:

* UCL = xˉ+3σ\bar{x} + 3\sigma
* LCL = xˉ−3σ\bar{x} - 3\sigma

Sample 10 again exceeds the UCL, confirming the presence of an assignable cause.

*Insert Figure 3: Individual Chart*

**3.3 Moving Average Chart**  
A moving average with a window size of 3 was applied to the sample means. While no control limits are directly used, the chart helps highlight trends or shifts in process performance.

*Insert Figure 4: Moving Average Chart*

**3.4 Moving Range Chart**  
This chart shows the absolute difference between consecutive sample means:

* MRi=∣xˉi−xˉi−1∣MR\_i = |\bar{x}\_i - \bar{x}\_{i-1}|
* Average moving range: MRˉ\bar{MR}
* Estimate of standard deviation: σMR=MRˉ/d2\sigma\_{MR} = \bar{MR} / d\_2, with d2=1.128d\_2 = 1.128
* UCL ≈ 3⋅σMR3 \cdot \sigma\_{MR}

A significant jump is observed near Sample 10.

*Insert Figure 5: Moving Range Chart*

**3.5 EWMA Chart**  
The exponentially weighted moving average (EWMA) uses:

* Z0=xˉZ\_0 = \bar{x}, then Zt=α⋅xˉt+(1−α)⋅Zt−1Z\_t = \alpha \cdot \bar{x}\_t + (1 - \alpha) \cdot Z\_{t-1}
* α = 0.3
* Control limits:
  + UCL = xˉ+L⋅σα2−α\bar{x} + L \cdot \sigma \sqrt{ \frac{\alpha}{2 - \alpha} }
  + LCL = xˉ−L⋅σα2−α\bar{x} - L \cdot \sigma \sqrt{ \frac{\alpha}{2 - \alpha} }, with L = 3

The EWMA curve shows a notable increase at Sample 10.

*Insert Figure 6: EWMA Chart*

**3.6 CUSUM Chart**  
CUSUM charts use cumulative deviations from a target mean to detect small persistent shifts:

* Positive CUSUM: Ci+=max⁡(0,Ci−1++(xˉi−μ−k))C\_i^+ = \max(0, C\_{i-1}^+ + (\bar{x}\_i - \mu - k))
* Negative CUSUM: Ci−=min⁡(0,Ci−1−+(xˉi−μ+k))C\_i^- = \min(0, C\_{i-1}^- + (\bar{x}\_i - \mu + k))
* k = 0.5σ (reference value), h ≈ 5σ (decision interval)

An increasing trend is observed near Sample 10.

*Insert Figure 7: CUSUM Chart*

**4. Comparison of Control Charts**

| **Chart Type** | **Assignable Cause Detection** | **Sensitivity** | **Smoothness** | **Notes** |
| --- | --- | --- | --- | --- |
| x̄-R Chart | Yes | Moderate | Low | Best for subgroup analysis |
| Individual Chart | Yes | Moderate | Low | Simple, direct method |
| Moving Average | Partial | Low | High | Useful for noise reduction |
| Moving Range | Yes | Moderate | Moderate | Shows variability between means |
| EWMA | Yes | High | High | Excellent for small shifts |
| CUSUM | Yes | Very High | Cumulative | Best for persistent small shifts |

**5. Conclusion**

Through the application of six different statistical control charts, this study demonstrates that Sample 10 contains a detectable assignable cause. Charts like CUSUM and EWMA are particularly effective in early detection of small shifts, while traditional x̄-R and Individual charts are reliable for clear outliers. This analysis illustrates the importance of using multiple chart types to gain a comprehensive understanding of process behavior.

**Appendices**

Appendix A: Python Code Used for Data Generation and Charting  
Appendix B: Raw Data Tables  
Appendix C: Additional Chart Visualizations