IEEE TRANSACTIONS ON
SMART GRID

IEEE Xplore® Digital Library
http://ieeexplore.ieee.org/Xplore

Virtual Energy Storage Sharing and Capacity Allocation

Dongwei Zhao, *Student Member, IEEE,* Hao Wang, *Member, IEEE,*
Jianwei Huang, *Fellow, IEEE,* and Xiaojun Lin, *Fellow, IEEE*

***Abstract*—Energy storage can play an important role in energy management of end users. To promote an efficient utilization of energy storage, we develop a novel business model to enable virtual storage sharing among a group of users. Specifically, a storage aggregator invests and operates the central physical storage unit, by virtualizing it into separable virtual capacities and selling to users. Each user purchases the virtual capacity, and utilize it to reduce the energy cost. We formulate the interaction between the aggregator and users as a two-stage optimization problem. In Stage 1, over the investment horizon, the aggregator determines the investment and pricing decisions. In Stage 2, in each operational horizon, each user decides the virtual capacity to purchase together with the operation of the virtual storage. We characterize a stepwise form of the optimal solution of Stage-2 Problem and a piecewise linear structure of the optimal profit of Stage-1 Problem, both with respect to the virtual capacity price. Based on the solution structure, we design an algorithm to attain the optimal solution of the two-stage problem. In our simulation results, the proposed storage virtualization model can reduce the physical energy storage investment of the aggregator by 54.3% and reduce the users' total costs by 34.7%, compared to the case where users acquire their own physical storage.**

***Index Terms*—Energy storage, storage virtualization, business model, two-stage optimization**

I. Introduction

A. Background and motivation

Energy storage is becoming a crucial element to ensure the stable and efficient operation of the new-generation of power systems. The benefits of the energy storage at the grid side have been well-recognized (e.g., for generation backup, transmission support, voltage control, and frequency regulation) [2]. Recently, there has also been an increasing interest in leveraging energy storage for end users (e.g., by harvesting distributed generations, and cutting electrical bill) [2]. However, deploying energy storage at the end-user side also faces challenges. On one hand, the current commercial storage products for end users often have high price tags.[1] Further, since a storage product lasts for years, it is challenging for a user to decide the storage size due to the uncertainty of future energy demand. In fact, the Tesla Powerwall only provides one or two choices of storage size for users. Both of these factors can discourage users from purchasing such storage products and enjoying the benefits. On the other hand, if many users invest in energy storage, it is possible for them to cooperate and share the benefits of storage due to complementary charge and discharge needs. The above considerations motivate us to study the following problem in the paper: *what would be a good business model that promotes users' more efficient use of energy storage?*

In our work, we develop a novel business model to virtualize and allocate central energy storage resources to end users through a pricing mechanism. This is analogous to the practice of cloud service providers, who set prices for virtualized computing resources shared by end users [4]. In the power system, we can also envision that a storage aggregator invests in a central physical storage unit and then virtualizes it into separable virtual storage capacities that are sold to end users at a suitable price. Users purchase the virtual storage to reduce the energy cost.

One key advantage of our storage virtualization framework is the ability to leverage users' complementary charge and discharge profiles. Note that the aggregator only cares about the net power flowing in and out the storage. As some users may choose to charge while others choose to discharge in the same time slot, some requests will cancel out at the aggregated level. This suggests that even if all the users are fully utilizing their virtual storage capacity, it is possible to support users' needs by using a smaller central storage comparing with the total virtual storage capacities sold to users. Such complementary charge and discharge profiles can arise in practice due to the diverse load and renewable generation profiles of end users. Specifically, as the most promising sources of clean and sustainable energy, solar and wind energy have both been increasingly adopted by households, commercial buildings, and residential communities [5] [6]. Studies in [7]–[9] showed that solar and wind energy exhibit diverse and locational-dependent generation profiles. Similarly, end users' load profiles can also be significantly diverse even in a

This work is supported by the Presidential Fund from the Chinese University of Hong Kong, Shenzhen, China, the Shenzhen Institute of Artificial Intelligence and Robotics for Society (AIRS), and in part by the NSF award ECCS-1509536. Part of the results have appeared in IEEE ICC 2017 [1].

Dongwei Zhao is with the Department of Information Engineering, The Chinese University of Hong Kong, Hong Kong, China (e-mail: zd015@ie.cuhk.edu.hk). Hao Wang is with the Department of Civil and Environmental Engineering and the Stanford Sustainable Systems Lab, Stanford University, CA 94305 USA (e-mail: hwang16@uw.edu). Jianwei Huang is with the School of Science and Engineering, The Chinese University of Hong Kong, Shenzhen, China, the Shenzhen Institute of Artificial Intelligence and Robotics for Society (AIRS), and the Department of Information Engineering, The Chinese University of Hong Kong, Hong Kong, China (e-mail: jianweihuang@cuhk.edu.cn). Xiaojun Lin is with the School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47907, USA (e-mail: linx@ecn.purdue.edu).

[1]A Tesla Powerwall storage with a capacity of 13.5 kWh costs $6200 [3].

Digital Object Identifier: 10.1109/TSG.2019.2932057

1949-3061 © 2019 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission.
See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

localized region [10].[2]

Another key advantage of storage virtualization is that a user can flexibly change the amount of virtual capacity to purchase over time based on his varying demand. Such flexibility is difficult to realize if the user owns physical storage by himself, and encourages the users to take advantage of the energy storage. The above key advantages can further increase users' demand for the storage and reduce the aggregator's investment cost, which can increase the aggregator's profit.

To rigorously study such benefits of storage virtualization, in this paper, we consider two possible types of aggregators. The first possibility is a profit-seeking aggregator. In a deregulated energy market, the profit-seeking storage aggregator can decide whether or not and how much storage capacity to invest in, so as to maximize her profits. Such deregulated markets can be found in the U.S. and many European countries, and third parties are encouraged to participate in the market to provide different services for the grid and end users [12] [13]. The second possibility is that the aggregator is regulated by the system operators or regulatory agents, which may have the goal of maximizing the benefit of end users subject to a nonnegative profit.

B. Main results and contributions

To the best of our knowledge, our paper is the first work that develops a pricing mechanism for the storage virtualization and sharing. In such a framework, a storage aggregator invests in a central physical storage unit and then virtualizes it into separable virtual storage capacities that are sold to end users at a suitable price. Users purchase the virtual storage to reduce the energy cost.

A new question for this storage virtualization model is how the aggregator's investment and pricing decisions are coupled with the users' purchase and storage operation decisions. To answer this question, we formulate a two-stage optimization problem for the interactions between the aggregator and users at two different horizons: the investment horizon divided into many operational horizons. Over the investment horizon (e.g., 15 years), the aggregator determines the size of the physical storage for virtualization and the price of the virtual storage. At the beginning of each operational horizon (e.g., one day), each user determines the virtual capacity to purchase as well as the charge and discharge decision. The aggregator chooses a price of the virtual storage to balance her profit and users' benefits. For a profit-seeking aggregator, we aim to find the optimal-profit price to maximize her profit. For an aggregator that is regulated by the system operator or regulatory agents, we aim to find the lowest-nonnegative-profit price, which can give the most benefits to users while maintaining a nonnegative profit for the aggregator. We demonstrate that such a virtualization leads to more efficient use of the physical energy storage, compared with the case where each user acquires his own physical storage.

The main contributions of this paper are as follows:

[2]We show the diversity of users' load profiles in the online Appendix O of the technical report [11] based on the data from [10].

- *Storage virtualization framework*: In Section II, we develop a storage virtualization and sharing framework. To the best of our knowledge, this is the first work that develops a pricing mechanism for storage virtualization and sharing.
- *Pricing-based virtual capacity allocation*: In Section III, we formulate a two-stage optimization problem between the aggregator and users. In Stage 1, the aggregator determines the pricing and investment of the storage. In Stage 2, each user decides his purchase decision and the storage schedule. We consider two pricing strategies for the aggregator: one maximizes the aggregator's profit while the other gives the highest benefits to users.
- *Threshold-based search algorithm*: In Section IV, we resolve a multi-optima issue of Stage 2 by introducing a penalty on users' charge and discharge power. As the penalty approaches zero, we characterize a stepwise structure of users' optimal solutions, and show a piecewise linear structure of the aggregator's optimal profit with respect to the virtual storage price. This structure then allows us to iteratively search for near-optimal investment and pricing strategies within an arbitrary precision.
- *Realistic-data simulations*: In Section V, we conduct the simulation using realistic load data from PG&E Corporation and meteorology data from Hong Kong Observatory. We show that our model enables the aggregator to save the physical storage investment cost by 54.3% and the users to reduce energy costs by 34.7%, compared with the case where users acquire their own physical storage.

C. Related works

There have been several studies on the deployment of energy storage at the end-user side [14]–[23]. In [14]–[17], each user only utilizes his own energy storage units for demand management without mutual sharing, which may lead to inefficient use of the storage. In contrast, the works in [18]–[20] considered user sharing of a central storage without considering the investment issue of the central storage and the potential impact on the users. In [21] and [22], end users share the energy storage with a third party. However, both works allocate the physical storage capacities (instead of virtual capacities) to the users, which does not take advantage of the complementariness of users' profiles.

The work that is most closely related to ours is [23], in which the authors proposed a business model to enable users to share the central storage. Our work differs from [23] in several crucial ways. First, in [23] the storage sizing decisions are not coordinated between the storage aggregator and users. More specifically, the aggregator needs to invest in a sufficiently large capacity to satisfy users' needs, which is not cost-efficient. In contrast, in our work, the aggregator can adjust the price of virtual storage to influence the demand for storage, and effectively coordinate the benefit sharing of virtual storage between the aggregator and users. Second, the model in [23] assumes that a user's purchased virtual storage cannot change on a daily basis. In contrast, our model allows a user to flexibly choose the amount of virtual storage to purchase every day, depending on his daily renewable generation and load demand. This additional level of flexibility further explores the potential of storage virtualization and reduces users' cost.

Our work models the virtual storage sharing framework as a two-stage optimization problem. Such multi-stage problems have been studied in smart grid systems (e.g., [7], [22], [24], [25]). The work [22] built a two-stage optimization problem for the sharing of a central storage unit between a distribution company and customers. The work [24] proposed a two-stage model for the energy pricing and dispatch problem of the electricity retailers. Both works [22] and [24] solved the two-stage problem by constructing a single optimization problem, which requires the operator to know all the users' private information. Compared with [22] and [24] that require complete network information, our work designs a distributed algorithm based on the information exchange between users and the aggregator.

The works [7] and [25] designed distributed algorithms based on the information exchange with an aggregator to coordinate the decisions among different users or microgrids. Such distributed algorithms can be used to realize the concept of transactive energy in the smart grid, which can achieve an equilibrium by exchanging value-based information [26]. In such a transactive energy framework, the agents of mid- or small-sized energy resources can automatically negotiate with each other as well as exchange information with the main grid through advanced energy management and control system. In our work, users can actively and automatically respond to the price signal from the aggregator, which is supported by the transactive energy framework. Compared with [7] and [25], our work focuses on the pricing mechanism of the aggregator who sells the virtual storage capacities to users and seeks the profits, while the works [7] and [25] focused on the coordination between users (or microgrids) under the help of the aggregator to reach the social optima or consensus.

II. System Overview

Figure 1 illustrates the system model, where a community of users are connected with a central storage unit and the main grid (and with each other) through power and communication infrastructures.[3] Each user has his load demand and may also own some local renewables. An aggregator invests and operates the central storage unit. Next, we introduce the models of the users and the aggregator in more details.

A. Users

We consider a set of users $\mathcal{I} = \{1, \ldots, I\}$ whose energy load profiles can be different. Users may own renewables of solar and wind energy. To satisfy the demand, a user can use the locally generated renewable energy, purchase energy from the main grid, or use the energy from the energy storage. Next, we first introduce the user's electricity bill, and then discuss how storage can be used to reduce the electricity bill.

[3]We assume that the grid constraints are not stringent, so that we can focus on how the aggregator sets the price of the virtual storage and how storage virtualization reduces the requirement of physical storage and the users' costs. We will consider the grid constraints in the future work.

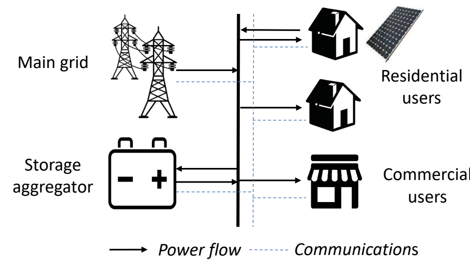

Fig. 1: System structure.

We adopt a peak-based demand charge tariff for the electricity bill. Peak-based demand charge has been widely adopted for commercial and industrial consumers in order to reduce the system peak and recover grid costs. Thanks to the increasingly more advanced metering infrastructure, such a demand charge scheme has also been offered to residential customers by some utilities in the United States [27] [28]. For a billing cycle $\mathcal{T} = \{1, 2, ..., T\}$ of T time slots, if user i consumes electricity $p_i^g[t]$ from the grid in time slot t, his electricity bill [29] in $\mathcal{T}$ is calculated by:

$$\pi_b \sum_{t \in \mathcal{T}} p_i^g[t] + \pi_p \max_{t \in \mathcal{T}} \ p_i^g[t], \tag{1}$$

where π_b is the unit energy price and π_p is the unit price for peak consumption in the billing cycle. To reduce users' peak demand, the utility usually sets π_p much higher than π_b [28].

Demand charge tariff in (1) provides a strong incentive for users to utilize energy storage to shave their peak loads. Specifically, users can proactively charge their storage using energy from the grid, and discharge to meet the peak load so as to reduce the electricity bill. Furthermore, if a user owns the renewables, he can store excessive renewable energy in the storage for later use. We assume that users can sell back renewable energy to the grid and the unit feed-in price π_s satisfies $\pi_s < \pi_b$, such that users prefer to first use the locally generated renewable energy to serve their loads rather than to directly sell to the grid.[4]

B. Storage aggregator

The aggregator invests and operates the central physical storage. She virtualizes the physical storage into separable virtual capacities and sells them to users. Since users can't control the central storage directly, they report their charge and discharge decisions to the aggregator, and the aggregator dispatches the central storage on behalf of users accordingly. Further, the aggregator can coordinate users' charge and discharge decisions by setting the price of virtual storage, which will ensure that users' charge and discharge decisions are well accommodated by the physical storage.

We assume that there is a billing arrangement among the utility, the storage aggregator and users such that when the utility calculates users' electricity bill, it will count both the physical load and the virtual storage charge/discharge. Thus, even though users don't own and operate their physical

[4]It is common that the renewable feed-in tariff is lower than the consumption tariff, for example, in Germany and some states of the U.S. [30].

storage, users can use the virtual storage to achieve a peak load reduction and reduce the electricity bill.

As we have discussed in Section I, our storage virtualization model can lead to a more efficient use of physical storage due to two reasons: (i) the complementarity of different users' charge and discharge decisions, and (ii) the flexibility in purchasing different amounts of virtual capacities on different days. The aggregator's investment in the physical storage will take advantage of these aspects while satisfying users' demand. However, occasionally there can be very high aggregate demand from users. Satisfying such demand with a fixed physical storage investment will lead to low efficiency due to either over-investment or over-pricing. Thus, we further generalize our model by allowing the aggregator to use additional energy resources other than the physical storage to meet users' demand. For example, the aggregator can contract with other generators (or consumers) to purchase additional energy (or sell surplus energy) to serve users' demand.[5]

III. Two-stage Formulation

Figure 2 illustrates two timescales of decision making in our model. Figure 3 illustrates a two-stage problem for the interactions between the aggregator and users. In Stage 1, at the beginning of an investment horizon $\mathcal{D}=\{1,2,...,D\}$ of D days (e.g., D corresponding to many years), the aggregator determines the size of the physical storage and the unit price of the virtual storage. The investment horizon is divided into many operational horizons, i.e., each $d \in \mathcal{D}$ corresponds to one operational horizon, which is further divided into many time slots $\mathcal{T}=\{1,2,...,T\}$ (e.g., 24 time slots corresponding to 24 hours). In Stage 2, at the beginning of each operational horizon, given the unit price of virtual storage, each user decides the optimal capacity to purchase and the corresponding charge and discharge profiles over the operational horizon, based on the prediction of their loads and renewable generations.[6] Then, the aggregator operates the physical storage by aggregating all the users' charge and discharge decisions. Note that we consider a daily operation of virtual storage sharing as well as the daily demand charge tariff for users' electricity bills, because users' electricity loads reflect their activities which are often periodic on a daily basis (see, e.g., extensive studies in [33] and [34]). Furthermore, users' load profiles can differ from one day to another (e.g., the differences between weekdays and weekends). The operation and billing cycle on a daily basis can leverage the diversity in users' loads and provide flexibility to users, such that users can purchase a different amount of virtual storage on different days to minimize their costs.

In order to solve the aggregator's investment and pricing problem over the investment horizon, the aggregator needs

[5]Such a generalization can be supported by the works [31] and [32], which propose a control framework for the general energy storage system by aggregating other energy resources, e.g., demand responses in addition to the physical energy storage.

[6]Since the focus of our work is on the design of the virtual storage sharing framework, we have initially chosen to assume that users can perfectly predict their renewable generations and loads. We include the discussions about the impact of uncertainties on the users' decisions in the online Appendix N [11].

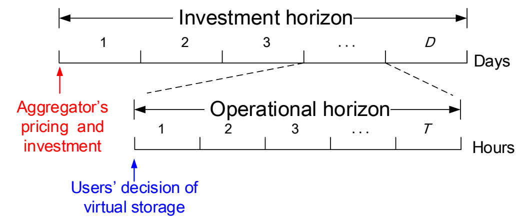

Fig. 2: Decision making over two timescales.

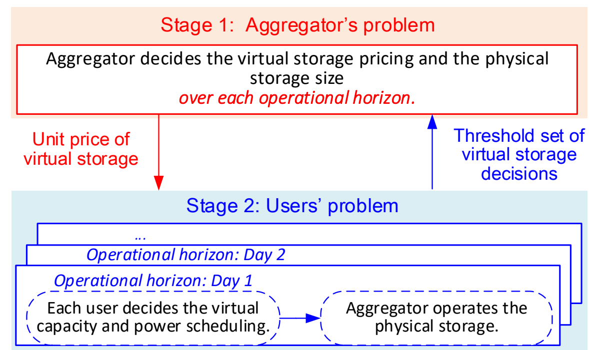

Fig. 3: Two-stage optimization.

to incorporate users' responses across different operational horizons in the entire investment horizon. Since users' responses depend on different operational conditions (e.g., local renewable generations and loads), we use historical data to build a set of *scenarios* Ω that empirically models the joint distribution of all users' load and renewable generation profiles. For each operational horizon d, scenario $\omega \in \Omega$ occurs with a probability ρ^{ω}. In scenario ω, we denote user i's load profile as $\boldsymbol{P}_i^{\omega,l}=\{P_i^{\omega,l}[t],\ \forall t \in \mathcal{T}\}$ and his renewable profile as $\boldsymbol{P}_i^{\omega,r}=\{P_i^{\omega,r}[t],\ \forall t \in \mathcal{T}\}$. Each user will report a set of the threshold decisions (explained in detail later in Algorithm 1) in each operational scenario to the aggregator. Based on users' reported information, the aggregator makes the investment and pricing decisions over the investment horizon by considering the expected profit over the scenarios.

Furthermore, note that the aggregator's decision and users' decisions in two stages are coupled. On the one hand, the aggregator's virtual storage pricing will affect the users' decisions of virtual storage, and the aggregator's invested physical storage size will constraint the aggregated charge and discharge decisions of users. On the other hand, the aggregated charge and discharge decisions of users will determine the aggregator's operation of the physical storage. Such a coupled two-stage problem needs to be solved through backward induction. Thus, in the next two subsections, we will first explain users' model in Stage 2 and then explain the aggregator's model in Stage 1.

A. Stage 2: User's model

1) User's power scheduling in each operational horizon: Given the price q of the virtual capacity, user i decides the virtual capacity x_i^{ω} and the corresponding power scheduling (as illustrated in Figure 4). We then explain the power scheduling in time slot t. Assume that user i has locally generated renewable energy $P_i^{w,r}[t]$. He decides the amount of self-used renewable energy $p_i^{\omega,r,u}[t]$ that will serve his load or charge into his virtual storage. He sells back the remaining renewable energy $P_i^{\omega,r}[t] - p_i^{\omega,r,u}[t]$ to the grid.[7] User i can purchase the amount of energy $p_i^{\omega,g}[t]$ from the grid. Part of energy $p_i^{\omega,g}[t]$ may serve his own load, and the remaining part will be charged into his virtual storage.[8] Finally, user i can discharge the amount of energy $p_i^{\omega,dis}[t]$ from his virtual storage to serve his load. To balance the power, we can express user i's energy purchase from the grid in time slot t as follows:

$$p_i^{\omega,g}[t] = P_i^{\omega,l}[t] - p_i^{\omega,r,u}[t] - p_i^{\omega,dis}[t] + p_i^{\omega,ch}[t]. \quad (2)$$

If a user has no renewables, the charged energy is only from the purchase from the grid, i.e., $p_i^{\omega,r,u}[t] = 0$ in (2). We denote $\boldsymbol{p}_i^{\omega,g} = \{p_i^{\omega,g}[t], \forall t \in \mathcal{T}\}$, $\boldsymbol{p}_i^{\omega,r,u} = \{p_i^{\omega,r,u}[t], \forall t \in \mathcal{T}\}$, $\boldsymbol{p}_i^{\omega,ch} = \{p_i^{\omega,ch}[t], \forall t \in \mathcal{T}\}$, and $\boldsymbol{p}_i^{\omega,dis} = \{p_i^{\omega,dis}[t], \forall t \in \mathcal{T}\}$.

User i's charge and discharge decision should satisfy the constraint of the virtual capacity:

$$e_i^{\omega}[t] = e_i^{\omega}[t-1] + \eta^c p_i^{\omega,ch}[t] - p_i^{\omega,dis}[t]/\eta^d, \ \forall t \in \mathcal{T}, \quad (3)$$
$$0 \leq e_i^{\omega}[t] \leq x_i^{\omega}, \ \forall t \in \mathcal{T}', \quad (4)$$
$$e_i^{\omega}[0] = e_i^{\omega}[T]. \quad (5)$$

We let $\boldsymbol{e}_i^{\omega} = \{e_i^{\omega}[t], \ \forall t \in \mathcal{T}'\}$ denote the energy level in the storage over the operational horizon, where $\mathcal{T}' = \{0\} \bigcup \mathcal{T}$ and $e_i^{\omega}[0]$ denotes the initial energy level. Since the user's storage is virtual, we allow user i to optimize $e_i^{\omega}[0]$ in each operational horizon. We let η^c and η^d denote the virtual charge and discharge efficiency rate respectively. We assume that the aggregator enforces the same virtual efficiency rate as the physical one. We model the charge and discharge efficiencies for the physical storage (e.g., Li-ion batteries) as constant values. Such an assumption has been widely used in the literature (e.g., [2] [14]) and can capture the key characteristics of energy loss during the charging and discharging process. Constraint (5) ensures the independent operation of the virtual storage across operational horizons [35]. Other power-related variables are constrained as follows:

$$p_i^{\omega,g}[t] \geq 0, \ p_i^{\omega,ch}[t] \geq 0, \ p_i^{\omega,dis}[t] \geq 0, \ \forall t \in \mathcal{T}, \quad (6)$$
$$0 \leq p_i^{\omega,r,u}[t] \leq P_i^{\omega,r}[t], \ \forall t \in \mathcal{T}. \quad (7)$$

2) User's net cost in each operational horizon: Each user minimizes his net cost in each operational horizon. The cost includes the payment for the virtual capacity and the electricity bill. The revenue is from selling back the renewable energy.

Specifically, given the virtual storage price q, over the operational horizon of scenario ω, user i's *payment to the aggregator* for purchasing the capacity x_i^{ω} is $C_i^s(x_i^{\omega}) = qx_i^{\omega}$. The *electricity bill* for the consumption from the grid is

$$C_i^e(\boldsymbol{p}_i^{\omega,g}) = \pi_b \sum_{t \in \mathcal{T}} p_i^{\omega,g}[t] + \pi_p \max_{t \in \mathcal{T}} p_i^{\omega,g}[t], \quad (8)$$

[7] We assume that users only feed in the unused locally-generated renewable energy to the grid. We do not consider the feed-in power from the storage to the grid, which may complicate users' decisions.

[8] We assume that users can have an extra meter and wires to connect the renewable generators to the grid. Hence, it is technically feasible for a user to sell back the renewable energy to the grid while consuming energy from the grid.

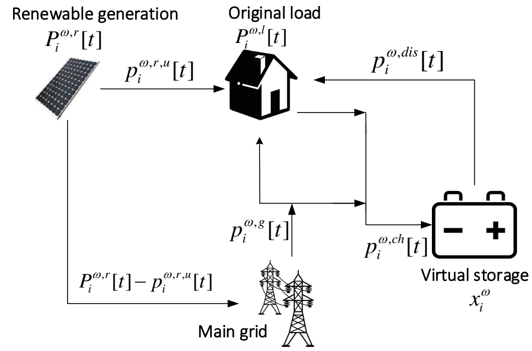

Fig. 4: Users' power scheduling.

and we can substitute the variable $p_i^{\omega,g}[t]$ from (2) and denote the electricity bill as $C_i^e(\boldsymbol{p}_i^{\omega,r,u}, \boldsymbol{p}_i^{\omega,ch}, \boldsymbol{p}_i^{\omega,dis})$. User i's *revenue of selling back renewable energy* is

$$R_i^r(\boldsymbol{p}_i^{\omega,r,u}) = \pi_s \sum_{t \in \mathcal{T}} (P_i^{\omega,r}[t] - p_i^{\omega,r,u}[t]). \quad (9)$$

Thus, the *net cost* in the operational horizon of scenario ω is

$$C_i^s(x_i^{\omega}) + C_i^e(\boldsymbol{p}_i^{\omega,r,u}, \boldsymbol{p}_i^{\omega,ch}, \boldsymbol{p}_i^{\omega,dis}) - R_i^r(\boldsymbol{p}_i^{\omega,r,u}). \quad (10)$$

We then formulate user i's Problem $\mathbf{UP}_i^{\omega}$ that minimizes the net cost in the operational horizon of scenario ω as follows.

Stage 2: User i optimization problem $\mathbf{UP}_i^{\omega}$

$$\begin{aligned} \min \quad & C_i^s(x_i^{\omega}) + C_i^e(\boldsymbol{p}_i^{\omega,r,u}, \boldsymbol{p}_i^{\omega,ch}, \boldsymbol{p}_i^{\omega,dis}) - R_i^r(\boldsymbol{p}_i^{\omega,r,u}) \\ \text{s.t.} \quad & (2), (3) - (7), \\ \text{var:} \quad & x_i^{\omega}, \boldsymbol{p}_i^{\omega,r,u}, \boldsymbol{p}_i^{\omega,ch}, \boldsymbol{p}_i^{\omega,dis}, \boldsymbol{e}_i^{\omega}, \end{aligned}$$

where the price q is determined by the aggregator in Stage 1. We denote the optimal solution to Problem $\mathbf{UP}_i^{\omega}$ by $\left(x_i^{\omega*}(q), \boldsymbol{p}_i^{\omega,r,u*}(q), \boldsymbol{p}_i^{\omega,ch*}(q), \boldsymbol{p}_i^{\omega,dis*}(q), \boldsymbol{e}_i^{\omega*}(q)\right)$. Note that the aggregator sets the same price for all users without any price discrimination, and each user i makes his own purchase decision to minimize his cost by solving Problem $\mathbf{UP}_i^{\omega}$. Therefore, if the virtual storage can bring higher revenues to some users (e.g., those who have more renewable energy), then these users have higher demands and purchase more virtual capacities (than the users who benefit less using the storage). In this sense, our business model is fair for all users.

B. Stage 1: Aggregator's model

In Stage 1, at the investment phase, the aggregator decides the unit price q of the virtual capacity, as well as the capacity X and power rating P of the physical storage.

1) Aggregator's power scheduling over each operational horizon: Each user decides the charge and discharge profiles (as in Stage 2) and then reports them to the aggregator. The aggregator aggregates these charge and discharge decisions and obtains the net charge $\boldsymbol{p}_a^{\omega,ch}(q) = \{p_a^{\omega,ch}[t],\ \forall t \in \mathcal{T}\}$ and discharge $\boldsymbol{p}_a^{\omega,dis}(q) = \{p_a^{\omega,dis}[t],\ \forall t \in \mathcal{T}\}$ as follows:

$$p_a^{\omega,ch}[t](q) = \left[\sum_{i\in\mathcal{I}} p_i^{\omega,ch*}[t](q) - \sum_{i\in\mathcal{I}} p_i^{\omega,dis*}[t](q)\right]^+, \quad (11)$$

$$p_a^{\omega,dis}[t](q) = \left[\sum_{i\in\mathcal{I}} p_i^{\omega,dis*}[t](q) - \sum_{i\in\mathcal{I}} p_i^{\omega,ch*}[t](q)\right]^+, \quad (12)$$

$\forall t \in \mathcal{T},\ \forall \omega \in \Omega$, where we define $[f]^+ = \max\{f, 0\}$. Note that (11) and (12) ensure that $p_a^{\omega,ch}[t](q)$ and $p_a^{\omega,dis}[t](q)$ cannot be positive at the same time, i.e., the physical storage cannot be charged and discharged simultaneously.

The aggregator can use the physical storage and additional resources to satisfy users' requirement. We denote the charge and discharge requirement served by the physical storage as $\boldsymbol{p}_a^{\omega,ch,s} = \{p_a^{\omega,ch,s}[t], \forall t \in \mathcal{T}\}$, $\boldsymbol{p}_a^{\omega,dis,s} = \{p_a^{\omega,dis,s}[t], \forall t \in \mathcal{T}\}$. Similarly, we denote the charge discharge and requirement supported by the additional resources as $\boldsymbol{p}_a^{\omega,ch,a} = \{p_a^{\omega,ch,a}[t],\ \forall t \in \mathcal{T}\}$, $\boldsymbol{p}_a^{\omega,dis,a} = \{p_a^{\omega,dis,a}[t],\ \forall t \in \mathcal{T}\}$. They satisfy the following constraints for users' demand:

$$p_a^{\omega,ch,s}[t] + p_a^{\omega,ch,a}[t] = p_a^{\omega,ch}[t](q), \forall t \in \mathcal{T}, \forall \omega \in \Omega, \quad (13)$$
$$p_a^{\omega,dis,s}[t] + p_a^{\omega,dis,a}[t] = p_a^{\omega,dis}[t](q), \forall t \in \mathcal{T}, \forall \omega \in \Omega. \quad (14)$$

The aggregator's charge and discharge scheduling is constrained by the physical storage size as follows:

$$e_a^\omega[t] = e_a^\omega[t-1] + \eta_a^c p_a^{\omega,ch,s}[t] - p_a^{\omega,dis,s}[t]/\eta_a^d, \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega, \quad (15)$$
$$\gamma^{\min} X \le e_a^\omega[t] \le \gamma^{\max} X, \forall t \in \mathcal{T}', \forall \omega \in \Omega, \quad (16)$$
$$p_a^{\omega,ch,s}[t] \le P, p_a^{\omega,dis,s}[t] \le P, \forall t \in \mathcal{T}, \forall \omega \in \Omega, \quad (17)$$
$$e_a^\omega[0] = e_a^\omega[T], \forall \omega \in \Omega. \quad (18)$$

We let $\boldsymbol{e}_a^\omega = \{e_a^\omega[t], \forall t \in \mathcal{T}'\}$ denote the energy level in the physical storage. We let η_a^c and η_a^d denote the charge and discharge efficiency rate respectively. The fraction coefficients $\gamma^{\min}$ and $\gamma^{\max}$ correspond to the minimum and maximum energy levels that can be stored in the storage, respectively. Constraint (18) ensures the independent operation of storage in each operational horizon.[9]

2) Aggregator's profit over the investment phase (scaled in one operational horizon): Over the investment phase, the aggregator bears the storage capital cost which includes the capacity cost and power rating cost [2]. In each operational horizon, the aggregator obtains the revenue from selling the virtual capacity but also bears the operational cost of the physical storage and additional resources.

Specifically, the *capital cost* $C_a^{cap}(X, P)$ over the investment phase (scaled into one operational horizon) is:

$$C_a^{cap}(X, P) = \kappa c^X X + \kappa c^P P, \quad (19)$$

where the coefficient c^X and c^P are the unit cost of the capacity and power rating over the investment phase, respectively.[10] The coefficient κ is the scaling factor that is illustrated in the online Appendix L [11].

The expectation of the *revenue* $R_a^v(q)$ *of selling the virtual storage capacity* over all scenarios is

$$R_a^v(q) = \mathbb{E}_{\omega\in\Omega}[q \sum_{i\in\mathcal{I}} x_i^{\omega*}(q)] = \sum_{\omega\in\Omega} \rho^\omega q \sum_{i\in\mathcal{I}} x_i^{\omega*}(q). \quad (20)$$

The expectation of the *storage operational cost* $C_a^{op}(\boldsymbol{p}_a^{\omega,ch,s}, \boldsymbol{p}_a^{\omega,dis,s})$ over all scenarios is

$$\sum_{\omega\in\Omega} \rho^\omega c^s \sum_{t\in\mathcal{T}} (p_a^{\omega,ch,s}[t] + p_a^{\omega,dis,s}[t]), \quad (21)$$

where c^s is the unit cost of the charge and discharge, which models the cost of degradation of the storage. We adopt the linear cost model that is widely used in the literature [35] [36].

The expected *operational cost of additional resources* $C_a^{ad}(\boldsymbol{p}_a^{\omega,ch,a}, \boldsymbol{p}_a^{\omega,dis,a})$ over all scenarios is

$$\sum_{\omega\in\Omega} \rho^\omega \sum_{t\in\mathcal{T}} (c_a^c p_a^{\omega,ch,a}[t] + c_a^d p_a^{\omega,dis,a}[t]), \quad (22)$$

where c_a^c is the unit cost of absorbing users' charge demand and c_a^d is the unit cost of acquiring the energy to support users' discharge demand.

Thus, the aggregator's expected *profit* over the investment phase, scaled into one operational horizon, is

$$R_a^v(q) - C_a^{cap}(X, P) - C_a^{op}(\boldsymbol{p}_a^{\omega,ch,s}, \boldsymbol{p}_a^{\omega,dis,s}) - C_a^{ad}(\boldsymbol{p}_a^{\omega,ch,a}, \boldsymbol{p}_a^{\omega,dis,a}). \quad (23)$$

The aggregator's power scheduling is determined by price q and investment decisions X and P. Thus we denote the operational cost of storage and additional resources as functions of (q, X, P) respectively, i.e., $C_a^{op}(q, X, P)$ and $C_a^{ad}(q, X, P)$.

3) Aggregator's pricing: We consider two pricing strategies for the aggregator: the *optimal-profit price* (OP price) and the *lowest-nonnegative-profit price* (LNP price). The *OP price* maximizes the aggregator's profit, and the *LNP price* is the minimal price that keeps the aggregator's profit nonnegative. The latter is reasonable when the aggregator is regulated and required to provide the most benefit for users. We formulate Problem **AOP** to obtain the OP price $q^\star$ as follows. Due to the page limit, we enclose all the discussions about the problem of the LNP price q^l in the online Appendix J [11].

Stage 1: Aggregator's Optimal-profit Price Problem (AOP)

$$\max\ R_a^{pf} := R_a^v(q) - C_a^{cap}(X, P) - C_a^{op}(q, X, P) - C_a^{ad}(q, X, P)$$
s.t. $(11)-(14), (15)-(18)$,
var: q, X, P.

IV. Solving Two-stage Problem

The two-stage problem is challenging to solve due to its non-convex nature. To solve the problem, we first characterize the properties of each user's optimal solution (in Stage 2) under a fixed price q in Proposition 1 and Theorem 1, and then

[9] We assume that the aggregator can adjust the initial energy level of the storage by purchasing or selling the energy with the same price through an external market. Since the storage operational constraint in (18) restricts the terminal level to be equal to the initial level, the total adjustment and its cost is negligible in the long run. In addition, the case where the initial energy level is fixed can be viewed as a special case of our problem.

[10] Here we consider a single type of storage technology for the central storage. We can easily generalize this model to incorporate multiple types.

incorporate users' decisions into Stage 1 to characterize the properties of the aggregator's profit in Proposition 3. Based on the properties of Stage 2 (in Proposition 1 and Theorem 1) and Stage 1 (in Proposition 3), we propose Algorithm 1 to solve the two-stage problem, which determines the aggregator's optimal pricing and investment decisions.

A. Solution of Stage 2

For Stage 2, we first prove that the optimal capacity $x_i^{\omega*}(q)$ that user i purchases is stepwise in price q. Then we add a small penalty on the user's cost to solve the issue of multi-optima of the charge and discharge decision. We show that the user's optimal decision has a simple stepwise structure over price q as the penalty approaches zero.

1) Stepwise structure of $x_i^{\omega*}(q)$: The optimal capacity $x_i^{\omega*}(q)$ is stepwise over price q as shown in Proposition 1:

Proposition 1 (Stepwise property of virtual capacity)**.** *The optimal capacity $x_i^{\omega*}(q)$ of Problem $\textbf{UP}_i^\omega$ is a non-increasing and stepwise correspondence of the price q. Specifically, there exists the set of $K_i^\omega+1$ threshold prices $\mathcal{Q}_i^\omega = \{q_i^{\omega_0}, q_i^{\omega_1}, q_i^{\omega_2}, ..., q_i^{\omega_{K_i^\omega}}\}$ and $0 = q_i^{\omega_0} < q_i^{\omega_1} < \cdots < q_i^{\omega_{K_i^\omega}}$, such that $x_i^{\omega*}(q)$ is given by*

$$x_i^{\omega*}(q) = \begin{cases} x_i^{\omega_0}, q \in (q_i^{\omega_0}, q_i^{\omega_1}), \\ x_i^{\omega_1}, q \in (q_i^{\omega_1}, q_i^{\omega_2}), \\ \ldots \\ x_i^{\omega_{K_i^\omega}}, q \in (q_i^{\omega_{K_i^\omega}}, \infty), \end{cases} \tag{24}$$

where $x_i^{\omega_0} > x_i^{\omega_1} > \cdots > x_i^{\omega_{K_i^\omega}} = 0$. For any threshold price $q_i^{\omega_k} > 0$, $x_i^{\omega*}(q)$ can be any value in $[x_i^{\omega_{k-1}}, x_i^{\omega_k}]$. For $q_i^{\omega_0} = 0$, $x_i^{\omega*}(q)$ can achieve any value in $[x_i^{\omega_0}, \infty)$. We denote user i's optimal capacity set as $\mathcal{X}_i^\omega = \{x_i^{\omega_0}, x_i^{\omega_1}, ..., x_i^{\omega_{K_i^\omega}}\}$.

We illustrate Proposition 1 in Figure 5(a). As price q increases, the optimal capacity that a user purchases decreases. If the price is higher than the threshold $q_i^{\omega_{K_i^\omega}}$, the user will purchase none. Between two adjacent threshold prices, the optimal capacity remains the same. In the online Appendix C [11], we prove Proposition 1. In the Appendix D [11], we present Algorithm 3 for computing the sets $\mathcal{X}_i^\omega$ and $\mathcal{Q}_i^\omega$.

2) Solving multi-optima problem: Although we have characterized a user i's optimal capacity decision in Proposition 1, we still face the difficulty of multi-optima. More specifically, even at a fixed optimal capacity $x_i^{\omega*}(q)$, there may still be multiple virtual charge and discharge solutions $\left(\boldsymbol{p}_i^{\omega,ch}, \boldsymbol{p}_i^{\omega,dis}\right)$ (the set of which is denoted as $\left(\boldsymbol{P}_i^{\omega,ch*}(q), \boldsymbol{P}_i^{\omega,dis*}(q)\right)$) that lead to the same net cost to the user. As a result, the aggregator's cost is not well-defined, because the cost is due to users' charge and discharge demand. To address this difficulty, we introduce a small positive penalty coefficient ε on the user's net cost in Problem $\textbf{UP}_i^\omega$ as follows:

$$C_i^q(\boldsymbol{p}_i^{\omega,ch}, \boldsymbol{p}_i^{\omega,dis}) = \varepsilon \sum_{t\in\mathcal{T}} ((p_i^{\omega,ch}[t])^2 + (p_i^{\omega,dis}[t])^2). \tag{25}$$

After including penalty (25) to the objective function of Problem $\textbf{UP}_i^\omega$, we obtain a new modified problem denoted as $\textbf{UPP}_i^\omega$, which is a quadratic programming problem. We denote the optimal solution to Problem $\textbf{UPP}_i^\omega$ as $\left(x_i^{\omega\star}(q,\varepsilon), \boldsymbol{p}_i^{\omega,r,u\star}(q,\varepsilon), \boldsymbol{p}_i^{\omega,ch\star}(q,\varepsilon), \boldsymbol{p}_i^{\omega,dis\star}(q,\varepsilon), \boldsymbol{e}_i^{\omega\star}(q,\varepsilon)\right)$ for any price $q > 0$ and penalty $\varepsilon > 0$. Proposition 2 below shows such a solution of Problem $\textbf{UPP}_i^\omega$ is unique, which is proved in Appendix E [11]. To ensure users' unique decisions, we let the aggregator choose $q > 0$ and $\varepsilon > 0$.[11]

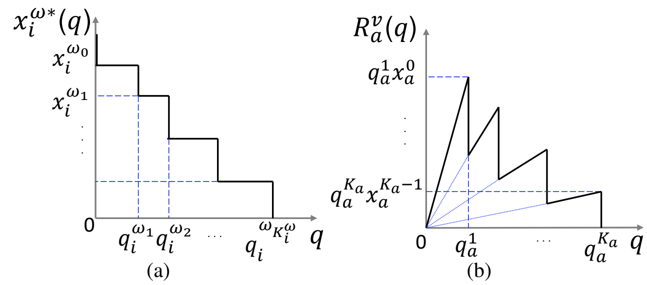

Fig. 5: (a) User i's optimal capacity $x_i^{\omega*}(q)$; (b) Revenue $R_a^v(q)$.

Proposition 2 (Uniqueness)**.** *For any $\varepsilon > 0$ and any $q > 0$, the optimal solution to Problem $\textbf{UPP}_i^\omega$ is unique.*

3) Asymptotic solution: Later in Section B-3, we will choose a small ε to obtain a near-optimal solution for Stage 1. Intuitively, when ε approaches zero, one would expect that the solution to Problem $\textbf{UPP}_i^\omega$ approaches the solution to Problem $\textbf{UP}_i^\omega$. Since the optimal charge and discharge decision of Problem $\textbf{UP}_i^\omega$ has multiple optima, it would seem that the same difficulty will persist as ε approaches zero. Surprisingly, we show below that the limit of the solution to Problem $\textbf{UPP}_i^\omega$ exists and can be uniquely determined as ε approaches zero.

Theorem 1 (Asymptotic solution)**.** *For any price $q \notin \mathcal{Q}_i^\omega$ (as defined in Proposition 1), as ε approaches zero,*
(a) there is a unique limit of the optimal charge and discharge decision $\left(\boldsymbol{p}_i^{\omega,ch\star}(q,\varepsilon), \boldsymbol{p}_i^{\omega,dis\star}(q,\varepsilon)\right)$. This limit belongs to the set $\left(\boldsymbol{P}_i^{\omega,ch*}(q), \boldsymbol{P}_i^{\omega,dis*}(q)\right)$, i.e.,

$$\lim_{\varepsilon\to 0^+} \left(\boldsymbol{p}_i^{\omega,ch\star}(q,\varepsilon), \boldsymbol{p}_i^{\omega,dis\star}(q,\varepsilon)\right) \in \left(\boldsymbol{P}_i^{\omega,ch*}(q), \boldsymbol{P}_i^{\omega,dis*}(q)\right),$$

and it remains constant for all prices within each threshold price interval $(q_i^{\omega_k}, q_i^{\omega_{k+1}})$, where $q_i^{\omega_k}, q_i^{\omega_{k+1}} \in \mathcal{Q}_i^\omega$;
(b) the optimal capacity $x_i^{\omega\star}(q,\varepsilon)$ approaches the optimal solution $x_i^{\omega*}(q)$ (given in Proposition 1) with $\varepsilon = 0$, i.e.,

$$\lim_{\varepsilon\to 0^+} x_i^{\omega\star}(q,\varepsilon) = x_i^{\omega*}(q).$$

Theorem 1(a) is highly non-trivial due to multiple optimal solutions $\left(\boldsymbol{p}_i^{\omega,ch*}(q), \boldsymbol{p}_i^{\omega,dis*}(q)\right)$ of Problem $\textbf{UP}_i^\omega$. We prove Theorem 1(a) based on the Maximum Theorem [37] by showing the uniqueness of the optimal objective value of Problem $\textbf{UP}_i^\omega$, and prove Theorem 1(b) by showing the uniqueness of $x_i^{\omega*}(q)$. We present the detailed proof in Appendix G [11].

Based on Theorem 1(b), we can compute the limit $\lim_{\varepsilon\to 0^+} x_i^{\omega\star}(q,\varepsilon)$ by the optimal solution $x_i^{\omega*}(q)$ of Problem $\textbf{UP}_i^\omega$, which takes discrete values from the set $\mathcal{X}_i^\omega$ defined in

[11]Note that when $q = 0$, even when we set $\varepsilon > 0$, users can purchase an arbitrarily large capacity beyond their minimum needs because they bear no capacity costs.

Proposition 1. Then, given each element in $\mathcal{X}_i^\omega$, we can solve optimization problems (by Algorithm 4 of Appendix H [11]) that minimize the penalty term to obtain the limiting optimal charge and discharge decision as ε approaches zero. In other words, the limiting optimal charge and discharge solution is a function of each element in $\mathcal{X}_i^\omega$. The above results show that as ε approaches zero, the user's solution in Stage 2 has a stepwise structure, which can be efficiently computed.

B. Solution of Stage 1

Based on the asymptotic structure of the users' decisions in stage 2 as shown in Theorem 1, we further analyze the structure of the aggregator's optimal profit as a function of price q as ε approaches zero. We propose an algorithm to derive the near-optimal profit price $\hat{q}^\star$ by considering ε that is small enough. We use a similar method to compute the LNP price and present it in Appendix J [11].

1) The optimal profit given (q,ε): Given (q,ε), the aggregator can vary the investment X and P to optimize her profit as follows. First, the aggregator can compute the revenue $R_a^v(q,\epsilon)$ as in (20) with $x_i^{\omega *}(q)$ replaced by $x_i^{\omega\star}(q,\varepsilon)$. Second, given users' optimal charge and discharge decision $\left(\boldsymbol{p}_i^{\omega,ch\star}(q,\varepsilon), \boldsymbol{p}_i^{\omega,dis\star}(q,\varepsilon)\right), \forall i,\omega$, the aggregator can solve Problem **CO** to compute her optimal cost $C_a(q,\varepsilon)$ as follows.

$$\textbf{CO:}\, C_a(q,\varepsilon) := \min C_a^{cap}(X,P) + C_a^{op}(q,X,P) + C_a^{ad}(q,X,P)$$
$$\text{s.t. } (11)-(14), (15)-(18),$$
$$\text{var: } X, P.$$

This is a linear programming problem, which can be solved efficiently using the simplex method [38]. Finally, we compute the optimal profit for any given (q,ε) by

$$R_a^{pf}(q,\varepsilon) = R_a^v(q,\epsilon) - C_a(q,\varepsilon). \tag{26}$$

2) The limit of the optimal profit as ε approaches zero: In Proposition 3 below, as ε approaches zero, we first show in part (a) that the revenue $R_a^v(q,\varepsilon)$ approaches a limit $R_a^v(q)$ as in (20) without ε. We further show in (b) that the optimal cost $C_a(q,\varepsilon)$ approaches a limit $C_a(q)$, which is stepwise over the threshold price set $\mathcal{Q}_a = \bigcup_{i,\omega} Q_i^\omega$. Finally, we show in (c) that the optimal profit $R_a^{pf}(q,\varepsilon)$ approaches $R_a^v(q) - C_a(q)$ denoted as $R_a^{pf}(q)$ as ε approaches zero. We present the proof of Proposition 3 in Appendix I [11].

Proposition 3 (Asymptotic profit)**.** *For any $q \notin \mathcal{Q}_a$, we have*

$$(a) \lim_{\varepsilon\to 0^+} R_a^v(q,\varepsilon) = R_a^v(q), \;\; (b) \lim_{\varepsilon\to 0^+} C_a(q,\varepsilon) = C_a(q),$$

*where $C_a(q)$ denotes the optimal value of Problem **CO** given the limit of each user i's optimal charge and discharge decision* $\left(\lim_{\varepsilon\to 0^+}(\boldsymbol{p}_i^{\omega,ch\star}(q,\varepsilon), \lim_{\varepsilon\to 0^+}\boldsymbol{p}_i^{\omega,dis\star}(q,\varepsilon)\right), \forall i,\omega,$

$$(c) \lim_{\varepsilon\to 0^+} R_a^{pf}(q,\varepsilon) = R_a^v(q) - C_a(q) \triangleq R_a^{pf}(q).$$

Based on Proposition 1, the revenue $R_a^v(q)$ is piecewise linear over the threshold price set $\mathcal{Q}_a$ as depicted in Figure 5(b): The revenue increases linearly from each threshold price and then decreases vertically at the next adjacent threshold price. The slopes of $R_a^v(q)$ over different price intervals are determined by $\sum_\omega \rho^\omega \sum_i x_i^{\omega *}(q)$. The limiting cost $C_a(q)$ is a stepwise function over the threshold price set $\mathcal{Q}_a$ as shown in Figure 6(a). Hence the limiting profit $R_a^{pf}(q)$ is a piecewise linear function of price q as shown in Figure 6(b) with multiple local optima.[12] Finally, based on each user i's limiting decision $\lim_{\varepsilon\to 0^+} x_i^{\omega\star}(q,\varepsilon)$ and $\lim_{\varepsilon\to 0^+}\left(\boldsymbol{p}_i^{\omega,ch\star}(q,\varepsilon), \boldsymbol{p}_i^{\omega,dis\star}(q,\varepsilon)\right)$ in Theorem 1, we can compute the limiting revenue $R_a^v(q)$, the limiting cost $C_a(q)$, and thus the limiting profit $R_a^{pf}(q)$ as in Lines 2-8 in Algorithm 1 below.

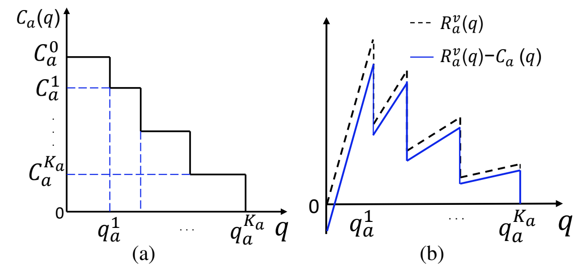

Fig. 6: (a) Cost $C_a(q)$; (b) Revenue $R_a^v(q)$ and profit $R_a^v(q) - C_a(q)$.

3) The algorithm to compute the near-OP price $\hat{q}^\star$: The above results demonstrate important solution structures of Stage 2 and Stage 1 as ε goes to zero. Unfortunately, the aggregator cannot choose $\varepsilon = 0$ (which has the multi-optima problem as mentioned in Section A). Thus, below we propose an iterative procedure in Algorithm 1 (i.e., Lines 9-16) for computing a sufficiently small ε so that (i) it is close enough to the above limit, and (ii) the near-OP price $\hat{q}^\star$ can be efficiently computed. Specifically, first, the aggregator computes the left-handed limit of profit $R_a^{pf}(q)$ at each threshold price $q_a \in \mathcal{Q}_a$, and selects the price q_a^m that achieves the global optimum (Line 9). Second, since profit $R_a^{pf}(q)$ is linearly increasing over each threshold price interval, the aggregator chooses a near-OP price $\hat{q}^\star$ slightly lower than q_a^m such that the profit $R_a^{pf}(\hat{q}^\star)$ approximates the maximum profit $R_a^{pf}(q_a^m)$ within the given accuracy err_1 (Lines 10-11). Finally, the aggregator chooses a small ε in an iterative fashion so that the profit $R_a^{pf}(\hat{q}^\star,\varepsilon)$ approximates $R_a^{pf}(\hat{q}^\star)$ within the given accuracy err_2 (Lines 12-16), where each user solves the quadratic programming problem $\textbf{UPP}_i^\omega$ in each iteration.[13] After computing the near-OP price $\hat{q}^\star$ and ε, we can obtain the near-optimal investment decision by Subroutine 1 accordingly.

Algorithm 1 is scalable in terms of the number of users and scenarios. On the users' side, they compute their decisions in parallel, which will not increase the execution time as the number of users increases. On the aggregator's side, she searches the threshold price set for the maximum profit, where the number of threshold prices is simply linear in the product of the number of users and the number of scenarios. Therefore, the execution time of Algorithm 1 is proportional to the number of users and scenarios, and thus scales well.

[12] Although in Proposition 3 these limiting values are only defined for $q \notin \mathcal{Q}_a$, we can use the left-handed limits of $R_a^{pf}(q)$ as the function values for $q \in \mathcal{Q}_a$. In this way, the function $R_a^{pf}(q)$ is well-defined for all q.

[13] The quadratic programming problem can be efficiently solved by the interior point method [39].

Algorithm 1 Search of the near-optimal-price $\hat{q}^\star$

initialization: set iteration index $k=0$, $\varepsilon^0>0$, relative error err_1 and err_2;
for user $i \in \mathcal{I}$ **in parallel do**
  Compute the set $\mathcal{Q}_i^\omega$ and $\mathcal{X}_i^\omega$ by Algorithm 3 of Appendix D [11], and compute the limiting charge/discharge decision by Algorithm 4 of Appendix H [11], $\forall \omega \in \Omega$;
  Report the computation results to the aggregator;
end for
for each $q_a \in \mathcal{Q}_a = \bigcup_{i,\omega} \mathcal{Q}_i^\omega$ **do**
  The aggregator computes the left-handed limit of profit $R_a^{pf}(q_a) = R_a^v(q_a) - C_a(q_a)$, with $R_a^v(q_a)$ computed by (20) and $C_a(q_a)$ computed as in Proposition 3(b);
end for
The aggregator searches the threshold price set $\mathcal{Q}_a$ and chooses the price $q_a^m \in \mathcal{Q}_a$ that maximizes $R_a^{pf}(q)$;
The aggregator calculates the slope $slp(q_a^m)$ of $R_a^{pf}(q)$ over the threshold price interval (q_a^{m-1}, q_a^m):

$$slp(q_a^m) = \sum_\omega \rho^\omega \sum_i x_i^{\omega *}\left(\frac{q_a^{m-1}+q_a^m}{2}\right);$$

The aggregator computes a price $\hat{q}^\star$ (lower than q_a^m) such that $R_a^{pf}(\hat{q}^\star)$ approximates $R_a^{pf}(q_a^m)$ within the relative error err_1:

$$\hat{q}^\star = q_a^m - \frac{R_a^{pf}(q_a^m)err_1}{slp(q_a^m)};$$

repeat
  $k := k+1$;
  $\varepsilon^k = \varepsilon^{k-1}/10$;
  $\big(R_a^{pf}(\hat{q}^\star,\varepsilon^k), X(\hat{q}^\star,\varepsilon^k), P(\hat{q}^\star,\varepsilon^k)\big) = CU(\hat{q}^\star,\varepsilon^k)$;
until

$$\frac{R_a^{pf}(\hat{q}^\star,\varepsilon^k) - R_a^{pf}(\hat{q}^\star)}{R_a^{pf}(\hat{q}^\star)} \le err_2;$$

output: $\big(\hat{q}^\star, \varepsilon^k, P(\hat{q}^\star,\varepsilon^k), X(\hat{q}^\star,\varepsilon^k)\big)$;

Subroutine 1 Communication unit $CU(q,\varepsilon)$

input: (q,ε) announced by the aggregator;
for each user $i \in \mathcal{I}$ **in parallel do**
  Solve Problem $\mathbf{UUP}_i^\omega$ given (q,ε), and reports to the aggregator the optimal decisions of $x_i^\omega(q,\varepsilon)$, $\boldsymbol{p}_i^{\omega,ch}(q,\varepsilon)$, $\boldsymbol{p}_i^{\omega,dis}(q,\varepsilon)$, for all $\omega \in \Omega$;
end for
The aggregator determines the investment decision $X(q,\varepsilon), P(q,\varepsilon)$ by solving Problem $\mathbf{CO}$, and computes the profit $R_a^{pf}(q,\varepsilon)$ by (26);
output: $(R_a^{pf}(q,\varepsilon), X(q,\varepsilon), P(q,\varepsilon))$;

V. Numerical Study

We simulate a system based on realistic data where users have diverse load and renewable profiles. We demonstrate that, as long as there are groups of users with diverse load and renewable profiles, our virtualization model can promote more efficient use of the physical storage, offer flexibility to users in the choices of virtual capacities, and reduce users' costs compared with the case where users acquire their own physical storage.

A. Simulation setup

1) ***Parameters:*** We consider the lithium-ion battery as the energy storage technology. We use realistic load data from PG&E Corporation in 2012 [40] to characterize users' load profiles, and we use wind speed and solar radiation data from Hong Kong Observatory [41] to characterize users' renewable generations. To illustrate how our storage virtualization model works, we simulate a system as in Figure 1 with three users of different types. Type-1 user owns the local wind generation, while Type-2 and Type-3 users own solar panels (with the same capacity).[14]

We obtain the one-year historical data, namely 366 scenarios for users' load and renewable profile jointly. As a large number of scenarios lead to a high computational complexity, we choose a smaller subset of 7 typical scenarios that can approximate the original scenario set as shown in Figure 7: Type-1 user has the peak load around noon (typical for some commercial users) and produces more wind power at night; Type-2 and Type-3 users have the peak load in the morning and evening (typical for residential users), and their solar power reaches peak supply around noon. For more details regarding the data of renewable generation and load, we include them in the online dataset [44]. For other parameters of electricity bill and energy storage, we present them in Appendix K [11]. Note that we do not make any assumptions on the correlation between users' load profiles and renewable generations; we just select an example with three types of users for illustration. Furthermore, since our framework for virtual energy storage sharing is general, interested readers can also use their own data as inputs to examine the performance of the framework.

2) ***Benchmark:*** To show the performance of our virtualization model, we consider a benchmark where each user invests in a physical storage product (e.g., Tesla Powerwall) by himself. He will optimize the (fixed) capacity of the physical storage, and can only use his own storage over the investment phase. Apart from the capacity cost, each user also bears the power rating cost and the operational cost by himself. Each user i' solves an optimization problem $\mathbf{BM}_i$ to minimize his

[14] Though the installation of wind turbines faces geographical restrictions, technological innovations (such as vertical axis wind turbines) have promoted the adoption of wind energy for households, commercial building, and residential communities [6] [42] [43]. Therefore, to capture a more complete picture of renewable energy deployment in practice, we consider both solar energy and wind energy in our simulation. We also conduct the simulations where Type-1 user has no renewable energy while Type-2 and Type-3 users have solar energy (with details in Appendix M [11]), which shows that our framework still works well for significantly reducing the invested physical storage capacity and the cost of users with solar energy.

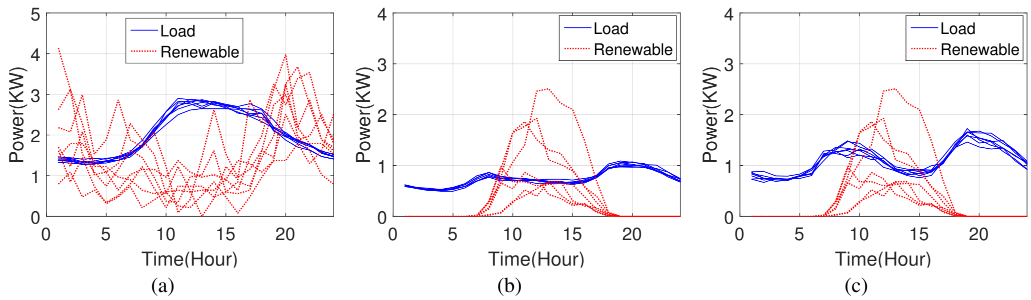

Fig. 7: 7 typical load and renewable generation scenarios for (a) Type-1 user; (b) Type-2 user; (c) Type-3 user.

cost and determines the storage size over the investment phase. We present the details of Problem $\mathbf{BM}_i$ in Appendix K [11].

*3) **Storage price**:* For the benchmark problem, we consider 2 different storage prices for users as follows:

- *Production cost* $\mathbf{c}^{\mathbf{P}}(c_u^x, c_u^p)$: Users pay the same storage production cost as the aggregator.
- *Retailer price* $\mathbf{c}^{\mathbf{r}}(c_u^x, c_u^p)$: Users pay a higher storage retailer price than the aggregator.

The *production cost* indicates the minimal cost for users to acquire the energy storage, and the *retailer price* is a more realistic price for users to purchase the storage on the market.

The simulation is implemented using MATLAB 2015 on a computer with an Intel Core i7 of 3.6 GHz and 8 GB memory. The execution time is about 13s.

B. Simulation results

We demonstrate several key benefits of our virtualization business model as follows.

*1) **More efficient use of storage**:* We show that our model can lead to more efficient use of energy storage, such that the aggregator can invest in a smaller physical storage capacity to support much larger virtual capacity allocation.

Figure 8(a) shows the comparison between the aggregator's invested physical capacity and the sold expected virtual capacity when the price varies from the LNP price q^l to the OP price $q^\star$. The sold virtual capacity is always much larger than the actual physical size. Specifically, compared with the virtual capacity, the physical capacity is reduced by 42.5% at the price q^l and 54.3% at the price $q^\star$. To understand this, in Figure 8(b), we show the virtual charge and discharge profiles of three types of users (represented by different colors) in seven scenarios (shown as the seven curves for each type of user). For each curve, the positive value part corresponds to charging the virtual storage, and the negative value part corresponds to discharging the virtual storage. We can see that that Type-1 user discharges the storage around noon to serve his peak load and charges the storage around morning and night. On the other hand, Type-2 and Type-3 users charge the storage around noon to store excessive solar energy and discharge around morning and night to serve their peak loads. Therefore, these users' charge and discharge profiles can partially cancel out at the aggregated level, which reduces the need of the physical storage. We also see from Figure 8(a) that the sold virtual capacity and invested physical capacity decrease with the price since a higher storage price generally reduces users' requirement for storage.

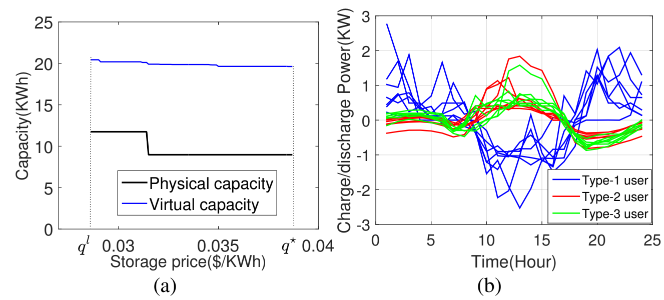

Fig. 8: (a) Physical capacity and virtual capacity from q^l to $q^\star$; (b) Users' charge and discharge decisions.

*2) **Purchasing flexible virtual capacity**:* We show that our model enables users to purchase flexible capacities over different operational horizons. To demonstrate this benefit, we compare users' cost under two cases as follows:

- Case 1: Users purchase flexible virtual capacities over different scenarios, as in the proposed framework
- Case 2: Users purchase capacities that cannot change in different scenarios.

Case 1 corresponds to the user's Problem $\mathbf{UP}_i^\omega$ in our virtualization model. Case 2 is similar to the user's benchmark problem $\mathbf{BM}_i$ except that in Case 2 users will only pay for the capacity without paying for the power rating cost and operational cost. This may make the comparison between Case 1 and Case 2 more fair. To illustrate the benefits of flexibility in choosing virtual capacities, we focus on the realistic load and renewable generation data of Type-1 user in one week (from 2012.10.1 to 2012.10.7), as shown in Figure 9(a). We compare the user's cost during this week in both cases under the same storage price, and show the cost reduction (in percentage) in Case 1 compared with Case 2 in Figure 9(b). We can see that when the storage price is very low, the cost reduction is small since the user pays little for energy storage in both cases. When the storage price is very high, the user will not purchase storage and the cost reduction will be zero. The gain can be as high as 17% when the price is medium.

*3) **Benefits of reducing users' cost**:* In Figure 10, we show how the users' cost and the aggregator's profit change from the LNP price q^l to the OP price $q^\star$. In Figure 8, we show users' cost reduction in our model compared with the benchmark

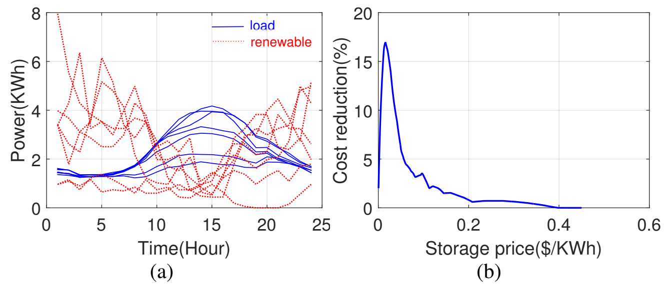

Fig. 9: (a) One-week load and renewable profiles of Type-1 user; (b) Cost reduction in Case 1 compared with Case 2.

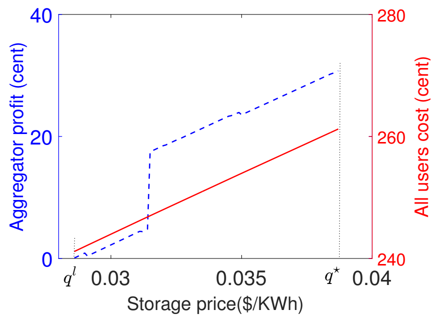

Fig. 10: Aggregator's profit and users' cost from q^l to $q^\star$.

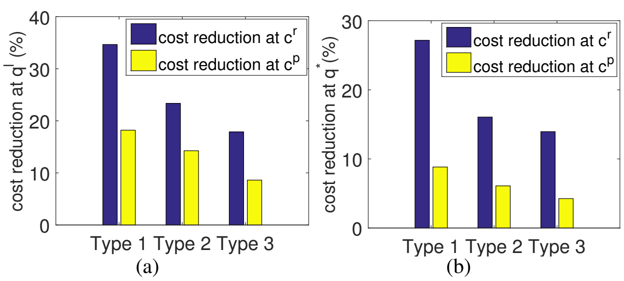

Fig. 11: (a) Cost reduction at q^l; (b) Cost reduction at $q^\star$.

under the price q^l and $q^\star$.[15]

In Figure 10, we see that both the aggregator's profit and the users' total cost increase as the price increases from q^l to $q^\star$. The LNP price q^l gives the aggregator a zero profit, meanwhile leaves the most benefits to users. The OP price $q^\star$ gives the aggregator maximum profit at the expense of the maximum cost of users.

We then show each user's cost reduction in our virtualization model compared with the benchmark in Figure 11. At the LNP price q^l in Figure 11(a), a user's cost can be reduced by up to 34.7% compared with the benchmark where the user affords retailer price $\mathbf{c^r}$, and the cost reduction can still be up to 18.2% if the user pays the production cost $\mathbf{c^p}$ in the benchmark. At the OP price $q^\star$ as shown in Figure 11(b), the user's cost reduction is not significant (up to 8.8%) if the user pays the production cost $\mathbf{c^p}$ in the benchmark. This result is natural, as the OP price maximizes the aggregator's profit at the expense of the users' benefit. Nonetheless, compared with the user's cost in the benchmark when user affords the retailer price $\mathbf{c^r}$, the user's cost can still be reduced by up to 27.2%. Furthermore, as shown in figures, Type-1 user obtains higher cost reduction than Type-2 and Type-3 users. The intuition is that Type-1 user has higher renewable penetration and high load, which increases the demand for the storage.

[15] We also numerically demonstrate in Appendix P [11] that our storage virtualization model not only helps users cut their electricity bill but also leads to a peak load reduction in the system.

VI. Conclusion

This paper proposed a pricing-based virtual storage sharing scheme among a group of users. An aggregator invests and operates the physical energy storage and virtualizes the physical storage into separable virtual capacities, which can be sold to serve different users. We formulated a two-stage optimization problem for the interaction between the aggregator and users. Simulation results showed that energy storage virtualization can save investment in physical energy storage by 54.3% and reduce users' costs by up to 34.7%, compared with the case where users utilize their own physical storage.

References

[1] D. Zhao, H. Wang, J. Huang, and X. Lin, "Pricing-based energy storage sharing and virtual capacity allocation," in *Communications (ICC), 2017 IEEE International Conference on*, May 2017, pp. 1–6.

[2] X. Luo, J. Wang, M. Dooner, and J. Clarke, "Overview of current development in electrical energy storage technologies and the application potential in power system operation," *Applied Energy*, vol. 137, pp. 511–536, 2015.

[3] "Tesla powerwall." [Online]. Available: http://www.energysage.com/solar/solar-energy-storage/tesla-powerwall-home-battery

[4] Q. Zhang, L. Cheng, and R. Boutaba, "Cloud computing: state-of-the-art and research challenges," *Journal of internet services and applications*, vol. 1, no. 1, pp. 7–18, 2010.

[5] E. Dickinson, *Solar Energy Technology Handbook: 0.* CRC Press, 2018.

[6] A. C. Orrell, N. A. Foster, J. S. Homer, and S. L. Morris, "2015 distributed wind market report," Pacific Northwest National Lab.(PNNL), Richland, WA (United States), Tech. Rep., 2016.

[7] H. Wang and J. Huang, "Joint investment and operation of microgrid," *IEEE Trans. on Smart Grid*, vol. 8, no. 2, pp. 833–845, March 2017.

[8] A. M. Foley, P. G. Leahy, A. Marvuglia, and E. J. McKeogh, "Current methods and advances in forecasting of wind power generation," *Renewable Energy*, vol. 37, no. 1, pp. 1–8, 2012.

[9] R. H. Inman, H. T. Pedro, and C. F. Coimbra, "Solar forecasting methods for renewable energy integration," *Progress in energy and combustion science*, vol. 39, no. 6, pp. 535–576, 2013.

[10] "Pecan street grid demonstration program. final technology performance report," Pecan Street Project, Inc., Austin, TX (United States), Tech. Rep., 2015.

[11] D. Zhao, H. Wang, J. Huang, and X. Lin, "Technical report for "virtual energy storage sharing and capacity allocation" (2019)." [Online]. Available: https://arxiv.org/abs/1907.02005

[12] J. D. Craig, "Motivations for market restructuring: evidence from us electricity deregulation," *Energy Economics*, vol. 60, pp. 162–167, 2016.

[13] E. Parliament, "Understanding electricity markets in the eu," 2016.

[14] J. Jin and Y. Xu, "Optimal storage operation under demand charge," *IEEE Trans. on Power Systems*, vol. 32, no. 1, pp. 795–808, Jan 2017.

[15] C. O. Adika and L. Wang, "Non-cooperative decentralized charging of homogeneous households' batteries in a smart grid," *IEEE Trans. on Smart Grid*, vol. 5, no. 4, pp. 1855–1863, July 2014.

[16] I. Atzeni, L. G. Ordez, G. Scutari, D. P. Palomar, and J. R. Fonollosa, "Demand-side management via distributed energy generation and storage optimization," *IEEE Trans. on Smart Grid*, vol. 4, no. 2, pp. 866–876, June 2013.

[17] Z. Wang, C. Gu, F. Li, P. Bale, and H. Sun, "Active demand response using shared energy storage for household energy management," *IEEE Trans. on Smart Grid*, vol. 4, no. 4, pp. 1888–1897, Dec 2013.

[18] J. Yao and P. Venkitasubramaniam, "Stochastic games of end-user energy storage sharing," in *Proc.of IEEE CDC*, Dec 2016, pp. 4965–4972.

[19] ——, "Optimal end user energy storage sharing in demand response," in *IEEE SmartGridComm*, Nov 2015, pp. 175–180.

[20] C. P. Mediwaththe, E. R. Stephens, D. B. Smith, and A. Mahanti, "A dynamic game for electricity load management in neighborhood area networks," *IEEE Trans. on Smart Grid*, vol. 7, no. 3, pp. 1329–1336, May 2016.

[21] W. Tushar, B. Chai, C. Yuen, S. Huang, D. B. Smith, H. V. Poor, and Z. Yang, "Energy storage sharing in smart grid: A modified auction-based approach," *IEEE Transactions on Smart Grid*, vol. 7, no. 3, pp. 1462–1475, May 2016.

[22] H. Chen, Y. Yu, Z. Hu, H. Luo, C. Tan, and R. Rajagopal, "Energy storage sharing strategy in distribution networks using bi-level optimization approach," in *2017 IEEE Power Energy Society General Meeting*, July 2017, pp. 1–5.

[23] J. Liu, N. Zhang, C. Kang, D. S. Kirschen, and Q. Xia, "Decision-making models for the participants in cloud energy storage," *IEEE Trans. on Smart Grid*, vol. PP, no. 99, pp. 1–1, 2017.

[24] W. Wei, F. Liu, and S. Mei, "Energy pricing and dispatch for smart grid retailers under demand response and market price uncertainty," *IEEE Transactions on Smart Grid*, vol. 6, no. 3, pp. 1364–1374, May 2015.

[25] H. Wang and J. Huang, "Incentivizing energy trading for interconnected microgrids," *IEEE Transactions on Smart Grid*, vol. 9, no. 4, pp. 2647–2657, July 2018.

[26] F. Lezama, J. Soares, P. Hernandez-Leal, M. Kaisers, T. Pinto, and Z. M. A. do Vale, "Local energy markets: Paving the path towards fully transactive energy systems," *IEEE Transactions on Power Systems*, pp. 1–1, 2018.

[27] L. Wood, R. Hemphill, J. Howat, R. Cavanagh, S. Borenstein, J. Deason, and L. Schwartz, "Recovery of utility fixed costs: Utility, consumer, environmental and economist perspectives," Lawrence Berkeley National Lab, Berkeley, CA, Tech. Rep., 2016.

[28] R. Hledik, "Rediscovering residential demand charges," *The Electricity Journal*, vol. 27, no. 7, pp. 82–96, 2014.

[29] S. Zhao, X. Lin, and M. Chen, "Peak-minimizing online ev charging: Price-of-uncertainty and algorithm robustification," in *IEEE INFOCOM*, 2015, pp. 2335–2343.

[30] R. L. Fares and M. E. Webber, "The impacts of storing solar energy in the home to reduce reliance on the utility," *Nature Energy*, vol. 2, no. 2, p. 17001, 2017.

[31] M. Cheng, S. S. Sami, and J. Wu, "Virtual energy storage system for smart grids," *Energy Procedia*, vol. 88, pp. 436–442, 2016.

[32] ——, "Benefits of using virtual energy storage system for power system frequency response," *Applied energy*, vol. 194, pp. 376–385, 2017.

[33] J. Kwac, J. Flora, and R. Rajagopal, "Lifestyle segmentation based on energy consumption data," *IEEE Transactions on Smart Grid*, vol. 9, no. 4, pp. 2409–2418, July 2018.

[34] J. A. Jardini, C. M. V. Tahan, M. R. Gouvea, S. U. Ahn, and F. M. Figueiredo, "Daily load profiles for residential, commercial and industrial low voltage consumers," *IEEE Transactions on Power Delivery*, vol. 15, no. 1, pp. 375–380, Jan 2000.

[35] H. Wang and J. Huang, "Incentivizing energy trading for interconnected microgrids," *IEEE Trans. on Smart Grid*, vol. PP, no. 99, pp. 1–13, 2016.

[36] Y. Guo and Y. Fang, "Electricity cost saving strategy in data centers by using energy storage," *IEEE Trans. on Parallel and Distributed Systems*, vol. 24, no. 6, pp. 1149–1160, 2013.

[37] E. A. Ok, *Real analysis with economic applications*. Princeton University Press, 2007.

[38] G. Dantzig, *Linear programming and extensions*. Princeton university press, 2016.

[39] A. Altman and J. Gondzio, "Regularized symmetric indefinite systems in interior point methods for linear and quadratic optimization," *Optimization Methods and Software*, vol. 11, no. 1-4, pp. 275–302, 1999.

[40] "Understanding energy use and prices,." [Online]. Available: https://www.pge.com/tariffs/energy_use_prices.shtml

[41] "Summary of wind and solar meteorological data in hong kong." [Online]. Available: http://jianwei.ie.cuhk.edu.hk/energy.html

[42] U. D. of Energy, "The rise of community wind power in japan: Enhanced acceptance through social innovation," 2012.

[43] Y. Maruyama, M. Nishikido, and T. Iida, "Community wind benefits," *Energy Policy*, vol. 35, no. 5, pp. 2761–2769, 2007.

[44] "Data of vritual storage sharing." [Online]. Available: https://drive.google.com/open?id=1i5FCmfiwU33Sv0iogQWGASik0Bykzfjl

Dongwei Zhao Dongwei Zhao (S'16) received the B.S. degree from the College of Electrical Engineering, Zhejiang University, Hangzhou, China, in 2015. He is currently pursuing the Ph.D. degree with the Department of Information Engineering, The Chinese University of Hong Kong. His main research interests are in the optimization and game theory of power and energy systems.

Hao Wang Hao Wang (M'16) is a Postdoctoral Scholar at Stanford University. He received his Ph.D. from The Chinese University of Hong Kong and has been a Washington Research Foundation Innovation Fellow at University of Washington, Seattle. His main research interests are in the optimization, machine learning, and data analytics of power and energy systems. More information at https://web.stanford.edu/ haowang6/.

Jianwei Huang Jianwei Huang (F'16) is a Presidential Chair Professor and the Associate Dean of the School of Science and Engineering, The Chinese University of Hong Kong, Shenzhen. He is also a Professor in the Department of Information Engineering, The Chinese University of Hong Kong. He is the co-author of 9 Best Paper Awards, including IEEE Marconi Prize Paper Award in Wireless Communications 2011. He has co-authored six books, including the textbook on "Wireless Network Pricing". He has served as the Chair of IEEE ComSoc Cognitive Network Technical Committee and Multimedia Communications Technical Committee. He has been an IEEE Fellow, an IEEE ComSoc Distinguished Lecturer, and a Clarivate Analytics Highly Cited Researcher. More information at http://jianwei.ie.cuhk.edu.hk/.

Xiaojun Lin (S'02 M'05 SM'12 F'17) received his B.S. from Zhongshan University, Guangzhou, China, in 1994, and his M.S. and Ph.D. degrees from Purdue University, West Lafayette, IN, in 2000 and 2005, respectively. He is currently a Professor of Electrical and Computer Engineering at Purdue University.

Dr. Lin's research interests are in the analysis, control and optimization of large and complex networked systems, including both communication networks and power grid. He received the IEEE INFOCOM 2008 best paper and 2005 best paper of the year award from Journal of Communications and Networks. He received the NSF CAREER award in 2007. He was the Workshop co-chair for IEEE GLOBECOM 2007, the Panel co-chair for WICON 2008, the TPC co-chair for ACM MobiHoc 2009, the Mini-Conference co-chair for IEEE INFOCOM 2012, and General co-chair for ACM e-Energy 2019. He is currently serving as an Area Editor for (Elsevier) Computer Networks Journal, and has served as an Associate Editor for IEEE/ACM Transactions on Networking and a Guest Editor for (Elsevier) Ad Hoc Networks journal.