Controllable vs. Random: Renewable Generation Competition in a Local Energy Market

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***Abstract*—Renewable energy resources are playing an increasingly important role in serving consumers at the distribution level of power systems. This paper studies a duopoly two-settlement local renewable energy market, in which one energy supplier has controllable generations (with the help of energy storage) while the other supplier has random generations. In the day-ahead energy market, suppliers determine the bidding prices and quantities, and then consumers decide the energy quantity to purchase from each supplier. In the real-time energy market, a supplier gets penalized if he cannot deliver the amount of energy as committed in the day-ahead market. We formulate the interactions between suppliers and consumers in the day-ahead market as a two-stage problem. The two-dimensional bidding strategies (price and quantity) in the day-ahead market together with the penalty in the real-time market increase the complexity of the equilibrium analysis. To address such a challenge, we first derive weakly dominant bidding quantity strategies for both suppliers, and then characterize the corresponding pure and mixed price equilibrium. We demonstrate that the supplier with controllable generations can earn a much higher payoff than the supplier with random generations. In some cases, however, we show the perhaps counterintuitive result that a higher penalty or a higher variance of random generations may increase both suppliers' payoffs.**

I. Introduction

A. Background and motivation

Distributed renewable energy resources are playing an increasingly important role in power systems. The capacity of global renewable generation has been growing significantly over the past decade. For example, the capacity of globally installed solar panels has increased from 8 Gigawatts in 2007 to 402 Gigawatts in 2017 [1]. Different from traditional larger-scale generators, renewable energy resources are usually of smaller capacities and spatially distributed across the power system, e.g., at the distribution level near residential consumers [1]. To facilitate a local energy market at the distribution level, renewable energy platforms, e.g., Bulb [2] and EnergyLocal [3], have recently emerged to enable consumers to purchase locally generated renewable energy. However, so far there is still no systematic design for local energy markets at the distribution level [4].

This motivates us to *study a local energy market in which renewable suppliers compete to sell electricity* in this paper. One challenge faced by renewable suppliers is the intermittent and uncontrollable nature of renewable energy generation. Some suppliers may choose to stabilize their outputs by flexible resources, such as energy storage, which effectively makes their output controllable [5]. However, deploying energy storage incurs substantial investment costs. Thus, it is important to understand *the benefit that energy storage may bring to a renewable supplier in a competition market.*

B. Related work

The local energy market is usually smaller in scale compared with the traditional wholesale market and as a result has several distinct features. First, suppliers are more likely to have strong market power in a local energy market and can be strategic in their energy offer [4], [6]. Second, consumers can potentially have multiple choices in terms of from whom they can purchase energy from [4], [7]. These features naturally lead to a competition between suppliers (to attract consumers), which is one focus in our work.

Furthermore, local energy markets should ensure the reliability of power systems [8]. Inspired by the wholesale market practice, studies (e.g., [4], [7]) suggested a multi-settlement market framework for the local energy market. For example, in a two-settlement market, a day-ahead market is responsible for matching supply and demand 24 hours ahead of the operation; a real-time market is responsible for settling any possible mismatch (due to unpredictable variations) between supply and demand using reserve resources. Suppliers who cannot meet their day-ahead generation commitment get penalized for the shortfall in the real-time market [9], [10]. Our work will adopt such a two-settlement market for the local energy market.

The literature on the suppliers' strategic interactions in electricity markets can be divided into two categories: those focusing on the deterministic supply (e.g., [11], [12]), and those considering random generations (e.g., [10], [13], [14]). Our current study falls into the second category. Zhang *et al.* in [10] studied renewable suppliers competition based on the Cournot quantity competition model, which requires the complete information of the consumers' demand function that may be difficult to obtain. Works [13] and [14] studied Bertrand price competition among renewable suppliers in a single-settlement market, without considering the potential mismatch between supply and demand in the real-time market. Furthermore, neither of [13] or [14] considered the competition

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between a random renewable supplier and a renewable supplier who can stabilize his output. This is a scenario that becomes increasingly common in practice due to the deployment of energy storage [5], and is one focus of our paper. Furthermore, we study a two-settlement energy market, where renewable suppliers will bid both the price and the quantity in the day-ahead market. The potential penalty in the real-time market will also have a significant impact on the suppliers' decisions.

C. Main results

Our work focuses on a two-settlement local energy market, which consists of a day-ahead market and a real-time market. In the day-ahead market, renewable suppliers decide on their bidding prices and quantities, and consumers decide how much to purchase from each supplier. In the real-time market, a supplier gets penalized if his actual generation falls short of the scheduled generation. Since the decisions of suppliers and consumers are coupled, we formulate the interaction between suppliers and consumers as a two-stage sequential decision problem. Furthermore, we study the competition between two suppliers: one has controllable renewable generations (e.g., using energy storage) while the other has random generations (without any regulating resources). Their bidding decisions are coupled, which naturally leads to a game-theoretic model.

The main contributions of this paper are listed as follows:

- *Local energy market*: In Section II, we study a two-settlement local energy market, where the renewable suppliers compete via price and quantity. To the best of our knowledge, this is the first work that studies such a two-dimensional competition between suppliers in a two-settlement local energy market.
- *Characterization of market equilibrium*: The two-dimensional bidding strategies (price and quantity) of suppliers in the day-ahead market combined with the penalty in the real-time market pose a challenge to the equilibrium analysis. In Section III, we first characterize the optimal purchase decision of consumers. Then, in Section IV, we characterize a weakly dominant strategy for suppliers' bidding quantities as well as an equilibrium for the bidding price.
- *Practical insights*: In Sections V, our simulation results show that the supplier with the controllable output can earn a much higher payoff than the supplier with the random output. This helps us quantify the economic benefit of using energy storage in such a market. We also show a counterintuitive result in the stimulation that a higher penalty or variance of random generation may increase both suppliers' payoffs.

II. System Model

We consider a local energy market for the renewable suppliers and consumers at the distribution level, as shown in Fig. 1. Consumers can purchase energy from both the main grid and multiple local renewable suppliers. As a result, the local renewable suppliers compete with each other, under the constraint that their prices cannot be larger than the main grid energy price. Next we will introduce the detailed models of suppliers and consumers, and characterize their interactions in the two-settlement local energy market.

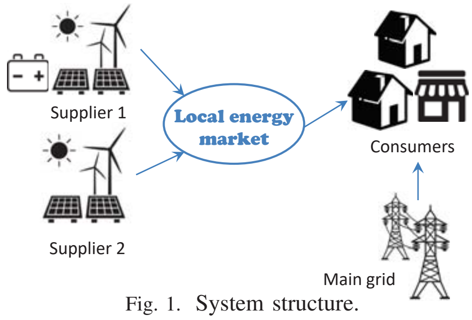


Fig. 1. System structure.

A. Suppliers and consumers

Since the number of energy suppliers is usually small in a local energy market [6], we will consider a simple model of two suppliers (duopoly) in this paper. We denote $\mathcal{I} = \{1, 2\}$ as the set of two renewable suppliers. One supplier can stabilize his random generation by utilizing the energy storage, while the other has the random generation without any regulating resources. More specifically, we assume that supplier 1 has a controllable output with the maximum generation $\bar{X}_1$ per unit time (say one hour). Supplier 2 has a random output X_2 per unit time, which is bounded in $[0, \bar{X}_2]$. We assume that the random generation X_2 has a continuous probability density function (PDF) f and a cumulative distribution function (CDF) F. As renewables usually have extremely low marginal production costs compared with traditional generators, we will not consider the production costs of the renewable suppliers.

We assume that the consumers have a total demand of D per unit time.[1] Consumers can fulfill their demand by either purchasing energy from the main grid (with a fixed unit price of P_g) or from the local renewable suppliers (with prices to be discussed later).

B. Two-settlement local energy market

We consider a two-settlement local energy market, which consists of a day-ahead market and a real-time market.

- In the day-ahead market, supplier $i \in \mathcal{I}$ decides the bidding price p_i and the bidding quantity y_i for each future time slot in the second day. Based on suppliers' bidding strategies, consumers decide to purchase energy quantity x_i $(\leq y_i)$ from supplier i. Supplier i will get the *ex-ante* revenue of of $p_i x_i$ by committing the delivery quantity x_i to consumers.
- In the real-time market, if supplier i's actual generation falls short of the committed quantity x_i, he will get an *ex-post* penalty proportional to the shortfall with a unit penalty price λ.

We assume that the operator of the local energy market will set a maximum allowable bidding price $\bar{p}$ such that $\bar{p} < P_g$. This is without loss of generality, as a supplier will not bid a price higher than P_g; otherwise, no consumer will purchase from him. Furthermore, we assume that the penalty price λ

[1] We consider a deterministic D is our current model and can generalize it by a random variable in the future work. We also assume that $\bar{X}_1$, $\bar{X}_2$, and D are public information known by the suppliers.

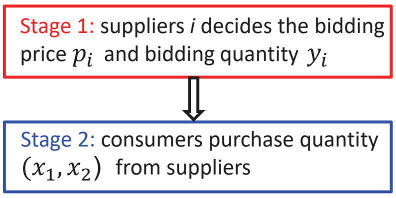


Fig. 2. Two-stage model.

is chosen by $\bar{p} < \lambda$, so that the penalty is high enough to discourage suppliers from bidding higher than they can deliver.

Note that the suppliers and consumers make decisions only in the day-ahead market. There are no active decisions to be made in the real-time market: to minimize the penalty, each supplier just tries to deliver his committed quantity as much as possible, i.e., up to $\bar{X}_1$ for supplier 1 and up to X_2 for supplier 2. Next we introduce the two-stage model for the interaction between the suppliers and consumers in the day-ahead market.

C. The two-stage day-ahead market

Fig. 2 illustrates a two-stage model for the interaction between suppliers and consumers in the day-ahead market.

1) Stage 2: Given the bidding price (p_1, p_2) and bidding quantity (y_1, y_2) of both suppliers, consumers decide the electricity quantity (x_1, x_2) purchased from supplier 1 and supplier 2, respectively. The objective of consumers is to maximize the cost saving, $\pi_c(x_1, x_2) = (P_g - p_1)x_1 + (P_g - p_2)x_2$, compared with purchasing energy from the main grid only.[2] We present consumers' optimal purchase problem as follows.

Stage 2 Consumers' Cost Saving Maximization Problem

$$\max_{x_1, x_2} \ \pi_c(x_1, x_2) \triangleq (P_g - p_1)x_1 + (P_g - p_2)x_2 \tag{1a}$$

$$\text{s.t. } x_1 + x_2 \leq D, \tag{1b}$$

$$0 \leq x_i \leq y_i, i = 1, 2. \tag{1c}$$

Constraint (1b) implies that the total purchased quantity $x_1 + x_2$ is no greater than the demand D. Constraints (1c) indicates that the quantity purchased from supplier i should be no greater than his bidding quantity y_i. We denote the optimal solution of the Stage 2 problem as a function of the given values of $(\boldsymbol{p}, \boldsymbol{y})$, i.e., $x_i(\boldsymbol{p}, \boldsymbol{y}), \forall i = 1, 2$, where $\boldsymbol{p} = (p_1, p_2)$ and $\boldsymbol{y} = (y_1, y_2)$.

2) Stage 1: Both suppliers decide the bidding price $\boldsymbol{p}$ and quantity $\boldsymbol{y}$ to maximize their payoffs in Stage 1. The payoff of supplier i consists of two parts: the revenue $p_i x_i(\boldsymbol{p}, \boldsymbol{y})$ from committing the delivery quantity in the day-ahead market, and the penalty incurred in the real-time market. Supplier 1 is penalized if the commitment $x_1(\boldsymbol{p}, \boldsymbol{y})$ is larger than his maximum-capable generation $\bar{X}_1$ (as in (2a)). Supplier 2 is penalized if the commitment $x_2(\boldsymbol{p}, \boldsymbol{y})$ is larger than his actual generation X_2 (as shown in (2b)). Due to the randomness of X_2, we will compute the expected penalty cost for supplier 2.

Note that two suppliers' decisions are coupled with each other. If one supplier sets a higher price or a lower quantity, consumers are more likely to purchase more energy from the other supplier. We formulate the competition between the two suppliers in Stage 1 as a non-cooperative game as follows.

Stage 1 Game: renewable suppliers' competition

- Players: supplier $i = 1, 2$.
- Strategies: bidding quantity $y_i \geq 0$ and bidding price $p_i \in [0, \bar{p}]$ of each supplier i.
- Payoffs: supplier 1 has a payoff of

$$\pi_1(p_1, x_1(\boldsymbol{p}, \boldsymbol{y})) = p_1 x_1(\boldsymbol{p}, \boldsymbol{y}) - \lambda[x_1(\boldsymbol{p}, \boldsymbol{y}) - \bar{X}_1]^+, \tag{2a}$$

and supplier 2 has a payoff of:

$$\pi_2(p_2, x_2(\boldsymbol{p}, \boldsymbol{y})) = p_2 x_2(\boldsymbol{p}, \boldsymbol{y}) - \lambda\mathbb{E}[x_2(\boldsymbol{p}, \boldsymbol{y}) - X_2]^+, \tag{2b}$$

where $[x]^+ = \max(x, 0)$.

Next in Section III, we will first characterize consumers' optimal purchase solution in Stage 2 given suppliers' bidding strategies. Then, in Section IV, we will analyze the competition equilibrium between the suppliers in Stage 1.

III. Solution of Stage 2

In this section, we characterize consumers' optimal purchase solution to Problem (1) in Stage 2 . We use subscript $i \in \{1, 2\}$ to denote supplier i and $-i$ to denote the supplier (other than supplier i).

Given the bidding price $\boldsymbol{p}$ and bidding quantity $\boldsymbol{y}$ of both suppliers, we characterize consumers' optimal decision $\boldsymbol{x}(\boldsymbol{p}, \boldsymbol{y}) = (x_i(\boldsymbol{p}, \boldsymbol{y}), i = 1, 2)$ in Stage 2 in Proposition 1.

Proposition 1 (optimal purchase $\boldsymbol{x}(\boldsymbol{p}, \boldsymbol{y})$ in Stage 2)

- If $p_i < p_{-i}$, then $x_i(\boldsymbol{p}, \boldsymbol{y}) = \min(D, y_i)$ and $x_{-i}(\boldsymbol{p}, \boldsymbol{y}) = \min(D - \min(D, y_i), y_{-i})$.
- If $p_1 = p_2$, the optimal purchase solution[3] can be any element in the following set: $\mathcal{X}^{opt} = \{\boldsymbol{x}(\boldsymbol{p}, \boldsymbol{y}) : \sum_{i=1}^{2} x_i(\boldsymbol{p}, \boldsymbol{y}) = \min(D, \sum_{i=1}^{2} y_i), 0 \leq x_i \leq y_i, i = 1, 2\}$.

Proposition 1 shows that the consumers will first fulfill the demand from the supplier who sets a lower price. Then, they will fulfill the remaining demand (if any) from the other supplier. If consumers' demand cannot be completely satisfied by the two renewable suppliers, they will fulfill the remaining demand from the main grid. Next we incorporate the optimal purchase decisions $\boldsymbol{x}(\boldsymbol{p}, \boldsymbol{y})$ into Stage 1 and analyze the strategic bidding of suppliers.

IV. Solution of Stage 1

In Stage 1, we will characterize the bidding strategies of suppliers. We first derive the weakly dominant strategies for suppliers' bidding quantities, based on which we further derive the suppliers' bidding prices at the equilibrium.

A. Equilibrium quantity bidding strategy

We show that when the bidding price $\boldsymbol{p}$ has been determined, each supplier has a weakly dominant quantity bidding strategy that does not depend on the other supplier's quantity choice. Such a finding will significantly reduce the complexity of the analysis, and allow us to reduce the two-dimensional bidding process (involving both quantity and price) into a one-dimensional bidding process (involving only price). Deriving

[2]Note that consumers must satisfy their demand either from the local energy market or from the main grid. Thus, minimizing the total energy cost is equivalent to maximizing the cost savings in the local energy market.

[3]Note that there are multi-optima in this case. We assume that consumers will randomly allocate the demand to suppliers, uniformly within the region of the constraints.

the weakly dominant strategy is not straightforward, mainly due to the random generation of supplier 2.

We first present the definition of the weakly dominant strategy for the bidding quantity as follows, which allows a supplier to obtain a payoff at least as high as any other quantity, no matter how the other supplier acts.

Definition 1 (weakly dominant strategy): Given price $\boldsymbol{p}$, a bidding quantity y_i^* is a weakly dominant strategy for supplier i if $\quad \pi_i(p_i, x_i(\boldsymbol{p}, (y_i^*, y_{-i}))) \geq \pi_i(p_i, x_i(\boldsymbol{p}, (y_i, y_{-i}))),$
for any y_{-i} and $y_i \neq y_i^*$.

We characterize suppliers' weakly dominant strategy $\boldsymbol{y}^*(\boldsymbol{p})$ for the bidding quantity in Theorem 1.

Theorem 1 (weakly dominant strategy for the bidding quantity): The weakly dominant strategy $\boldsymbol{y^*(p)}$ is given by

$$y_1^* = \bar{X}_1,\ y_2^*(p_2) = F^{-1}\left(\frac{p_2}{\lambda}\right), \tag{3}$$

where F^{-1} is the inverse function of the CDF F of supplier 2's random generation.

Theorem 1 shows that supplier 1 can always bid the quantity at the maximum production level $\bar{X}_1$ (which does not depend on the price $\boldsymbol{p}$) so that he can attract the most demand without the penalty risk in the real-time market. For supplier 2, however, he needs to trade off between the bidding quantity and the shortage penalty due to the random generation. The weakly dominant strategy $y_2^*(p_2)$ depends on his own bidding price p_2, but does not depend on the supplier 1's bidding price p_1. Note that when $p_2 = 0$, the bidding quantity $y_2^*(0) = F^{-1}(0) = 0$. The bidding quantity $y_2^*(p_2)$ increases with price p_2, which implies that supplier 2 should bid more quantities with a higher bidding price. When $p_2 = \bar{p}$, the bidding quantity $y_2^*(\bar{p}) = F^{-1}\left(\frac{\bar{p}}{\lambda}\right) < \bar{X}_2$ since $\bar{p} < \lambda$.

B. Equilibrium price bidding strategy

Based on the weakly dominant strategies for the bidding quantities, we further analyze the price equilibrium between the suppliers. We show that a pure price equilibrium exists when the demand D is larger than a threshold (characterized in the later analysis). However, when the demand D is smaller than the threshold, there exists no pure price equilibrium and we will characterize a mixed price equilibrium instead.

1) Pure price equilibrium: We first define the pure price equilibrium of suppliers, where no supplier can improve his payoff through unilateral price deviation.

Definition 2 (pure price equilibrium) A price vector $\boldsymbol{p}^*$ is a pure price equilibrium if for both $i = 1, 2$,

$$\pi_i\left(p_i^*, x_i(\boldsymbol{p}^*, \boldsymbol{y}^*(\boldsymbol{p}^*))\right) \geq \ \pi_i\left(p_i, x_i((p_i, p_{-i}^*), \boldsymbol{y}^*(p_i, p_{-i}^*)\right),$$

for all $0 \leq p_i \leq \bar{p}$, where $\boldsymbol{y}^*$ is the weakly dominant strategies derived in Theorem 1.

Then, we discuss the existence of the pure price equilibrium in Proposition 2.

Proposition 2 (existence of the pure price equilibrium):

- If $D \geq y_1^* + y_2^*(\bar{p})$, there exists a pure price equilibrium $p_1^* = p_2^* = \bar{p}$.
- If $0 < D < y_1^* + y_2^*(\bar{p})$, there is no pure price equilibrium.

Based on Proposition 2, when the demand D is higher than the summation of the suppliers' maximum weakly dominant bidding quantities (i.e., $D \geq y_1^* + y_2^*(\bar{p})$), both suppliers can bid the highest price $\bar{p}$. This is because both suppliers' bidding quantities will be fully sold out, and the highest price will lead to the highest payoff for each supplier. However, if the demand D is lower than the threshold (i.e., $0 < D < y_1^* + y_2^*(\bar{p})$), suppliers have to compete to attract the limited demand. In this case, there exists no pure price equilibrium.

2) Mixed price equilibrium: Next we define the mixed price equilibrium under the weakly dominant strategy $\boldsymbol{y}^*(\boldsymbol{p})$. We let μ denote a probability measure[4] of the price over $[0, \bar{p}]$ [12].

Definition 3 (mixed price equilibrium) A vector of probability measures (μ_1^*, μ_2^*) is a mixed price equilibrium if, for both $i = 1, 2$,

$$\int_{[0,\bar{p}]^2} \pi_i(p_i, \boldsymbol{x}((p_i, p_{-i}), \boldsymbol{y}^*(p_i, p_{-i})))d(\mu_i^*(p_i) \times \mu_{-i}^*(p_{-i}))$$
$$\geq \int_{[0,\bar{p}]^2} \pi_i(p_i, \boldsymbol{x}((p_i, p_{-i}), \boldsymbol{y}^*(p_i, p_{-i})))d(\mu_i(p_i) \times \mu_{-i}^*(p_{-i})).$$

Definition 3 indicates that the expected payoff of supplier i cannot be better off if he deviates from the mixed equilibrium price strategy μ_i^* unilaterally. Let F_i^e denote the cumulative distribution function of μ_i^*, i.e., $F_i^e(p_i) = \mu_i^*(\{p \leq p_i\})$. Let u_i and l_i denote the upper support and lower support of the mixed price equilibrium μ_i^*, respectively, i.e., $u_i = \inf\{p_i : F_i^e(p_i) = 1\}$ and $l_i = \sup\{p_i : F_i^e(p_i) = 0\}$. Then, we can characterize the mixed price equilibrium in Theorem 2, assuming that we know the lower support l. Later on in Theorem 3, we will describe how to compute l.

Theorem 2 (characterization of the mixed price equilibrium): When $0 < D < y_1^* + y_2^*(\bar{p})$, there exists a mixed price equilibrium that can be characterized as follows:

(i) Both suppliers have the same support:

$$l_1 = l_2 = l > 0,\ u_1 = u_2 = \bar{p}. \tag{4}$$

(ii) The expected payoffs satisfy:

$$\pi_1^E = l \cdot \min(D, \bar{X}_1),\ \pi_2^E = \pi_2(l, \min(D, y_2^*(l))). \tag{5}$$

(iii) Suppliers' mixed equilibrium price strategies are characterized by the following cumulative distribution functions:

$$F_1^e(p) = \frac{\pi_2\left(p, \min\{y_2^*(p), D\}\right) - \pi_2^E}{\pi_2(p, \min\{y_2^*(p), D\}) - \pi_2(p, [D - \bar{X}_1]^+)}, \tag{6a}$$

$$F_2^e(p) = \int_l^{\bar{p}} \frac{\pi_1^E}{p^2 \cdot \min\{y_2^*(p), D\} - p^2 \cdot [D - \bar{X}_1]^+} dp, \tag{6b}$$

for any $l \leq p < \bar{p}$.

(iv) The left limit of F_i^e at $\bar{p}$ (denoted as $F_i^e(\bar{p}^-)$):
$F_i^e(\bar{p}^-) = 1$ is true for at least one of the suppliers.

Proof: We prove Theorem 2 based on the following properties shown in [12]: (a) the lower support $l_1 = l_2 = l > 0$, and the upper support $u_1 = u_2 = \bar{p}$; (b) F_1^e and F_2^e are strictly increasing over $[l, \bar{p}]$; (c) F_1^e and F_2^e both have no atoms[5] over $[l, \bar{p})$; (d) F_1^e and F_2^e cannot both have atoms at $\bar{p}$.

[4]A probability measure is a real-valued function that assigns a probability to each event in a probability space [15].

[5]An atom at p means $F_1^e(p^-) < F_1^e(p)$.

We obtain Theorem 2 (i) directly from Property (a) and obtain Theorem 2 (iv) from Property (d).

To prove Theorem 2 (ii) and (iii), besides using Property (b) and (c), we also utilize a basic property of a mix strategy equilibrium, where supplier i's expected payoff π_i under his equilibrium mixed strategy is equal to the expected payoff when he plays any pure strategy p_i in the support of the mixed strategy, i.e., $p_i \in [l, \bar{p}]$, against the mixed strategy μ_{-i}^* of the other suppliers at the equilibrium [16]. For supplier 1, the expected payoff π_1^E can be characterized by the expected payoff when he plays any pure strategy $p_1 \in [l, \bar{p})$ against the mixed strategy of supplier 2 (with CDF F_2^e and PDF f_2^e) at the equilibrium, i.e.,

$$\pi_1^E = \pi_1(p_1, \mu_2^*, \boldsymbol{x}(\cdot)) = \underbrace{p_1 \min(D, \bar{X}_1) \cdot (1 - F_2^e(p_1))}_{\text{when } p_1 \leq p_2} + \underbrace{p_1 \int_l^{p_1} \min\big(D - \min(y_2^*(p_2), D), \bar{X}_1\big) \cdot f_2^e(p_2) dp_2}_{\text{when } p_1 > p_2}. \quad (7)$$

First, we prove that in (7) we have $D - \min(y_2^*(p_2), D) \leq \bar{X}_1$ for any $p_2 \in [l, \bar{p}]$, which is equivalent to $D - \min(y_2^*(l), D) \leq \bar{X}_1$ since $y_2^*(p_2)$ is increasing in p_2. This can simplify the second part "when $p_1 > p_2$" in (7). We prove it by contradiction as follows. Based on (7), we choose $p_1 = l$ and have

$$\pi_1^E = \pi_1(l, \mu_2^*, \boldsymbol{x}(\cdot)) = l \cdot \min(D, \bar{X}_1). \quad (8)$$

If $D - \min(y_2^*(l), D) > \bar{X}_1$, then we can find a small $\varepsilon > 0$ such that $D - \min(y_2^*(l + \varepsilon), D) > \bar{X}_1$. Then we will choose $p_1 = l + \varepsilon$ and have

$$\pi_1^E = (l + \varepsilon) \cdot \min(D, \bar{X}_1)(1 - F_2^e(l + \varepsilon)) + (l+\varepsilon) \int_l^{l+\varepsilon} \bar{X}_1 \cdot f_2^e(p_2) dp_2 \geq (l + \varepsilon) \cdot \min(D, \bar{X}_1). \quad (9)$$

However, (8) and (9) contradict with each other, and thus $D - \min(y_2^*(p_2), D) \leq \bar{X}_1$ will hold for any $p_2 \in [l, \bar{p}]$.

Second, since π_1^E is a constant over $p_1 \in [l, \bar{p})$, the derivative of π_1^E with respect to p_1 is zero over $p_1 \in [l, \bar{p})$, i.e., $\frac{\partial \pi_1^E}{\partial p_1} = 0$. This fact, combined with (7), enables us to obtain (6b).

For supplier 2, similarly, the expected payoff π_2^E can be characterized by the expected payoff when he plays any pure strategy $p_2 \in [l, \bar{p})$ against the mixed strategy of supplier 1 (with CDF F_1^e) at the equilibrium as follows

$$\pi_2^E = \underbrace{\pi_2(p_2, \min(D, y_2^*(p_2))) \cdot (1 - F_1^e(p_2))}_{\text{when } p_2 \leq p_1} + \underbrace{\pi_2\left(p_2, \min(D - \min(\bar{X}_1, D), y_2^*(p_2))\right) \cdot F_1^e(p_2)}_{\text{when } p_2 > p_1}. \quad (10)$$

Similarly, $D - \min(\bar{X}_1, D) \leq y_2^*(p_2)$ always holds for any $p_2 \in [l, \bar{p}]$. Furthermore, we obtain (6a) according to (10). This completes the proof of Theorem 2 (iii).

Finally, we choose $p_1 = l$ in (7) and $p_2 = l$ in (10), which will lead to (5). This proves Theorem 2 (ii). $\square$

Theorem 2 (i) shows that both suppliers have the same upper support $\bar{p}$ and the same (unknown at this moment) lower support l at the mixed price equilibrium. Theorem 2 (ii) and 2 (iii) characterize suppliers' expected payoffs and CDFs of the mixed price strategies given the lower support l.

Next we will explain the computation of the lower support l. Toward this end, in (6), we replace the equilibrium lower support l by a variable $l_i^\dagger$, and replace $F_i^e(p)$ by $F_i^e(p \mid l_i^\dagger)$ to emphasize the depending on $l_i^\dagger$. Thus, $F_i^e(\bar{p}^- \mid l_i^\dagger)$ is a function of $l_i^\dagger$. Theorem 2 (iv) implies that the equation $F_i^e(\bar{p}^- \mid l_i^\dagger) = 1$ has a feasible solution of $l_i^\dagger$ for at least one of the suppliers. Therefore, we can compute the lower support l in Theorem 3.

Theorem 3 (**computing the lower support** l) For $F_i^e(\bar{p}^- \mid l_i^\dagger) = 1$ with the variable $l_i^\dagger, \forall i = 1, 2$, we discuss a total of two cases and compute the lower support l as follows.

1) If $F_i^e(\bar{p}^- \mid l_i^\dagger) = 1$ has a feasible solution of $l_i^\dagger$ for both suppliers, then the common lower support $l = \max_i(l_i^\dagger)$.
2) If $F_i^e(\bar{p}^- \mid l_i^\dagger) = 1$ has a feasible solution of $l_i^\dagger$ for only one supplier i, we choose that unique feasible solution $l_i^\dagger$ as the common lower support l.

All the proofs of the theorems and propositions can be found in the future journal version of this paper.

V. Simulation Result

In this section, we first demonstrate the dominance of supplier 1 in the local energy market, whose payoff can be much higher than the payoff of supplier 2 with random generation. Furthermore, we will demonstrate a counterintuitive finding that a higher penalty λ or a higher variance of random generation of supplier 2 may increase supplier 2's payoff.

A. Simulation setup

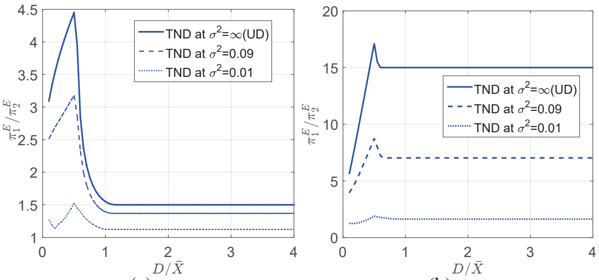
Let $\bar{p} = 1$ and $\bar{X} = 1$. Suppose that supplier 1 has the controllable generation with the maximum output $\bar{X}_1 = 0.5$. Suppose that supplier 2's random generation X_2 falls into the range of $[0, \bar{X}] = [0, 1]$. We let the mean $\mathbb{E}[X_2] = \bar{X}_1 = 0.5$ for a fair comparison. Besides the uniform distribution, we will also consider the truncated normal distribution for X_2 with a variance of σ^2. When σ^2 approaches infinity, the truncated normal distribution converges to the uniform distribution. When σ^2 goes to zero, the random variable X_2 approaches a deterministic value equal to the mean $\mathbb{E}[X_2] = 0.5$, which will be the same as supplier 1.

B. Dominance of supplier 1

We will show that in this market, supplier 1 with the controllable generation can earn a much higher payoff than supplier 2 with the random generation. This demonstrates the economic benefit of using energy storage to stabilize the renewable generation in a local energy market.

Fig. 3 shows two suppliers' equilibrium payoff ratio under different distributions of X_2. One is uniform distribution (UD) and the other two are the truncated normal distribution (TND) with $\sigma^2 = 0.01$ and 0.09. Fig. 3(a) and 3(b) correspond to a penalty price λ of 1.5 and 15, respectively.

We first focus on the payoff ratio under the uniform distribution (the solid curves in both subfigures). When $D/\bar{X} < 0.5$, $\pi_1^E/\pi_2^E > 2$ in Fig. 3(a) and 3(b). When $D/\bar{X} = 0.5$, $\pi_1^E/\pi_2^E > 4$ in Fig. 3(a) and $\pi_1^E/\pi_2^E > 16$ in Fig. 3(b). When



(a) (b)

Fig. 3. (a) π_1^E/π_2^E at $\lambda = 1.5$; (b) π_1^E/π_2^E at $\lambda = 15$.

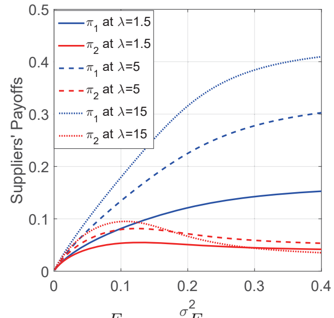


Fig. 4. π_1^E and π_2^E at $D = 0.5$.

the demand $D/\bar{X}$ is large enough, π_1^E/π_2^E approaches the value $\frac{\lambda}{\bar{p}}$ =1.5 and 15 in Fig. 3(a) and 3(b), respectively. These results show that supplier 1 can earn a much higher payoff than supplier 2.

Furthermore, we can see that the ratio π_1^E/π_2^E increases with the penalty λ (by comparing two curves under the same distribution across two subfigures) and the variance σ^2 (by comparing three curves within each subfigure). The intuition is that a higher λ or σ^2 increases the penalty cost on supplier 2 and gives more advantages to supplier 1.

C. Counterintuitive impact of λ and σ^2 on supplier 2's payoff

We further discuss the impact of the penalty λ and the variance σ^2 on each supplier's payoff. Although usually a higher λ or σ^2 will reduce supplier 2's payoff, counter-intuitively, we find that in some settings, a higher λ or σ^2 may increase supplier 2's payoff. This may happen especially when the demand is not high. The intuition is that a higher λ or σ^2 makes supplier 2 more disadvantageous compared with supplier 1, which reduces the level of market competition and may increase both suppliers' payoffs.

In Fig. 4, we consider the truncated normal distribution for X_2 at the demand D of 0.5 and show how the expected payoff of each supplier changes with the variance σ^2 of X_2, under the penalty $\lambda = 1.5$, $\lambda = 5$ and $\lambda = 15$, respectively.

In Fig. 4, we observe the counterintuitive finding that supplier 2's payoff can increase with σ^2 and λ. More specifically, the payoff π_2^E of supplier 2 (three red curves) first increases over $0 < \sigma^2 < 0.1$ and then decreases over $\sigma^2 > 0.1$. When comparing across the red curves, the payoff π_2^E can also increase with λ. For example, when $\sigma^2 = 0.1$, the payoff π_2 is larger under $\lambda = 5$ than under $\lambda = 1.5$. To explain this, first note that when σ^2 is close to zero, the random generation of supplier 2 will approach a deterministic value, which is the same as supplier 1.[6] In Fig. 4, when σ^2 approaches zero, both suppliers will get zero payoffs by bidding zero price because of the fierce competition for the limited demand. In this case, a higher σ^2 gives advantages to supplier 1 and reduces the market competition, which enables supplier 1 to bid a higher price (than zero) and in turn increases supplier 2's price and payoff. Note that although a larger σ^2 reduces the competition, it also increases the penalty cost on supplier 2. Thus, if σ^2 is too large, the payoff of supplier 2 may also decrease in σ^2. Similarly, a larger λ may also help reduce the competition and benefit supplier 2 especially when σ^2 is medium.

VI. Conclusion

In this paper, we study a two-settlement local energy market, where two renewable suppliers compete via price and quantity for serving consumers. One supplier has controllable generations through the use of energy storage, while the other one has random generations. We characterize the optimal purchase solution of consumers and analyze the competition equilibrium between suppliers. We demonstrate that the supplier with the controllable generation can earn a much higher payoff than the random one. We also show a counterintuitive result that, in some settings, a higher penalty or a higher variance of random generation may increase both suppliers' payoffs. For future work, we will analyze how the investment cost of the storage will affect the suppliers' decisions and the market competition.

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[6] We can compute suppliers' expected payoffs at the competition equilibrium between two deterministic suppliers according to [12].