# On Arbitrating the Power-Performance Tradeoff in SaaS Clouds

Fangming Liu, *Member, IEEE,* Zhi Zhou, Hai Jin, *Senior Member, IEEE,* Bo Li, *Fellow, IEEE,* Baochun Li, *Senior Member, IEEE,* Hongbo Jiang, *Member, IEEE*

**Abstract**—In this paper, we present an analytical framework for characterizing and optimizing the power-performance tradeoff in Software-as-a-Service (SaaS) cloud platforms. Our objectives are two-folded: (1) we maximize the operating revenue when serving heterogeneous SaaS applications with unpredictable user requests, and (2) we minimize the power consumption when processing the user requests. To achieve these objectives, we construct a unified profit-maximizing objective to jointly consider revenue and cost in an economic view. An offline solution to maximize the supreme bound of the objective is first developed, in order to (1) justify the validity of our theoretical model, and (2) establish a benchmark to examine the effectiveness of other control solutions. As a highlight of our contributions, we take advantage of the Lyapunov optimization techniques to design and analyze an optimal yet practical control framework, which makes online decisions on request admission control, routing, and virtual machine (VMs) scheduling. Our control framework can accommodate a variety of design choices and operational requirements in a datacenter. Specifically, buffering facilities can be introduced to alleviate the bursty admitted requests and to improve the robustness of the system, and a power budget can be enforced to improve the datacenter performance (dollar) per watt. Our mathematical analyses and simulations have demonstrated both the optimality (in terms of the cost-effective power-performance tradeoff) and stability (in terms of robustness and adaptivity to time-varying and bursty user requests) achieved by our proposed control framework.

**Index Terms**—SaaS cloud, datacenter, power-performance tradeoff, Lyapunov optimization, online control.

## 1 Introduction

Software-as-a-Service (SaaS) cloud platforms, such as the Google Apps [1] and Salesforce.com [2], have quickly ascended to the spotlight in the realm of cloud computing platforms, surpassing Infrastructure-as-a-Service (IaaS). With SaaS cloud services, enterprise applications — as critical as customer relationship management (CRM) and as simple as online slide presentations — can be hosted in the cloud, with large-scale datacenters serving a wide range of applications.

With SaaS, users are typically charged for each transaction or request. For example, a popular cloud-based email marketing application, called the Campaign Monitor [3], charges users according to the number of processed campaigns and delivered recipients. By following the law of *diminishing marginal utility* [4] in economics, heavy users with more subscribers (*e.g.*, recipients of email newsletters) will receive a more significant discount on the per-request charge as the number of recipients increases[1]. From the perspective of the SaaS cloud, however, as requests from users arrive in an unpredictable and even bursty fashion, such a per-request charging model may lead to fluctuating revenues over time.

Let us consider a typical SaaS cloud platform in Fig. 1. Due to risks of going beyond the processing capacity, a front-end proxy is needed to admit requests. We also need a load dispatcher to redirect admitted requests

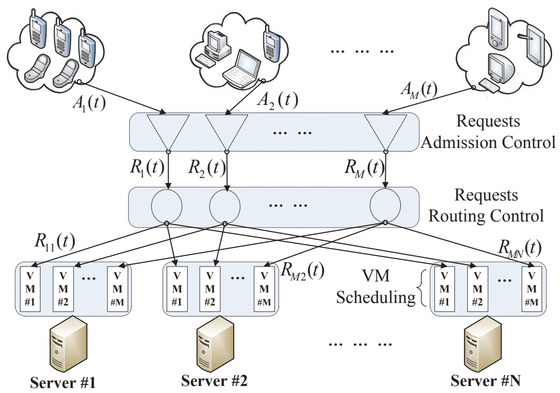

Fig. 1: A basic virtualized datacenter with three control decisions: (1) admission control that accepts (or denies) arrived requests from different applications, (2) routing control that dispatches admitted requests to VMs hosted on different servers in a datacenter, and (3) scheduling of VMs by switching between running and idle states.

- F. Liu, Z. Zhou, and H. Jin are with the Services Computing Technology and System Lab, Cluster and Grid Computing Lab in the School of Computer Science and Technology, Huazhong University of Science and Technology, 1037 Luoyu Road, Wuhan 430074, China. E-mail: {fmliu, zhiz, hjin}@mail.hust.edu.cn.
- B. Li is with the Department of Computer Science and Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon 999077, Hong Kong. E-mail: bli@cse.ust.hk.
- B. Li is with the Department of Electrical and Computer Engineering, University of Toronto, 10 Kings College Road, Toronto, ON M5S 3G4, Canada. E-mail: bli@eecg.toronto.edu.
- H. Jiang is with the Department of Electronics and Information Engineering, Wuhan National Laboratory for Optoelectronics, Huazhong University of Science and Technology, 1037 Luoyu Road, Wuhan 430074, China. E-mail: hongbojiang2004@gmail.com.

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1. Even for long-term SaaS commitments with an upfront and flat payment, the law of diminishing marginal utility is still applicable in general, in the form of a discount.

across a large pool of servers with constrained power budgets [5], each of which is virtualized as multiple virtual machines (VMs) to process requests from different applications.

With the presence of *unpredictable* and *bursty* application demands, the objectives of such an SaaS cloud platform with a per-request charging model are two-folded: (1) to maximize its profit by accepting and processing as many application requests as possible (system throughput), and (2) to minimize the penalty from system congestion due to excessive requests and the resulted power consumption of servers. To balance such a *power-performance tradeoff*, three important control decisions need to be made in the SaaS cloud: (1) how many requests from diverse applications are to be admitted at any given time; (2) how to distribute the admitted requests from different applications across a large number of servers hosting the corresponding VMs; and (3) how to schedule each VM by switching between a *running* state for processing requests and an *idle* state for conserving the server power.

To address these challenges, this paper takes advantage of the *Lyapunov optimization* techniques [6], [7] to design and analyze a new *optimal online control framework*, designed to independently and concurrently make all three decisions in the SaaS cloud. Our framework may be used to design three simple yet effective strategies, corresponding to the three decisions that need to be made: (1) A threshold-based admission control strategy to improve system throughput while avoiding system congestion; (2) a "Join the Shortest Queue" request routing strategy for balancing the server load, reducing service delays of admitted requests; and (3) a decentralized greedy strategy to optimally schedule VMs, *i.e.,* which VMs are to process incoming requests, and which are to be kept idle for power conservation.

The upshot of our new framework is that it can be flexibly extended to explore various design choices and operational requirements of a realistic datacenter. From a design perspective, to improve the robustness of the datacenter system, some designers may introduce buffering facilities to mitigate bursty admitted requests. Through redefining the Lyapunov function to incorporate the buffering facilities, we show that our framework can be enhanced to accommodate such a design choice. From an operational perspective, as most real-world datacenters are operated within a certain power budget to improve the performance (dollar) per watt [5], we demonstrate how our model and framework can be extended to accomplish this goal, by achieving a desired performance level with a particular power budget.

With extensive simulations, we demonstrate that our new control framework can approach a time averaged profit that is arbitrarily close to the optimum, while still maintaining strong stability and low congestion. Further, it is able to quickly adapt to bursty and time-varying arrivals of application requests, without incurring overwhelming power consumption costs that may outweigh the benefit of aggressively admitting a large number of requests.

The rest of this paper is organized as follows. In Sec. 2, we survey the related works. In Sec. 3, we present the basic power-performance tradeoff model [8]. Different from and complementary to our preliminary work [8], Sec. 4 maximizes the supreme bound of the time averaged profit via an offline solution. This not only justifies the reliability of the power-performance tradeoff model, but also provides a profit benchmark for the subsequent proposed online control framework. We further construct a new optimal online control framework to approach a time averaged profit which is arbitrarily close to optimum in Sec. 5. We demonstrate that our framework can be extended to incorporate various design choices in the datacenter power-performance tradeoff, such as additional request buffering facilities in Sec. 6 to improve the system robustness as compared with our preliminary work [8], and an enforced power budget in Sec. 7. We evaluate our proposed framework in Sec. 8 with more comprehensive simulations than our preliminary work [8], and conclude the paper in Sec. 9.

## 2 Related Work

While recognizing the significance of many existing works (*e.g.,* [9]–[11]) on managing the two potentially conflicting objectives related to cloud application performance and datacenter power consumption, our study is different from and complementary to existing works.

*First*, a number of existing works heavily relied on prediction-based or statistical offline approaches. For example, Chen *et al.* [12] applied a multiplicative seasonal autoregressive moving average method to predict server workloads in each time interval, and then made power control decisions based on steady-state queueing analysis and feedback control theory to satisfy such predicted demands. Govindan *et al.* [13] characterized statistical properties of the power needs of hosted workloads through a measurement-driven profiling and prediction framework. In the context of application requests in an SaaS cloud, the common problem with such approaches lies in the dubious feasibility of making accurate predictions of future request patterns, due to the fact that they are, in general, bursty and nonstationary.

*Second*, though there exist alternative online control solutions [14]–[17] in the literature for dynamic resource allocation and power management in datacenters, our work differs substantially in at least two important aspects. (1) We take an economic viewpoint to price application throughput with a nonlinear utility function based on the law of diminishing marginal utility (Sec. 3.2.3), rather than a simple linear utility function [14] that does not reflect reality in general. (2) Our framework can be extended to focus on improving the performance (dollar) per watt [5] at datacenters, by achieving a desired system throughput level with a certain power budget.

## 3 Power-Performance Tradeoff Model

In this section, we first formulate the basic datacenter model in Fig. 1, consisting of $N$ homogeneous servers $\mathcal{S} = \{1, 2, ..., N\}$, each of which is virtualized to $M$ virtual machines (VMs) to serve $M$ types of heterogeneous applications $\mathcal{A} = \{1, 2, ..., M\}$ with diverse request arrival rates and computing demand. Specifically, the $i$-th VM hosted on server $j$ serves the requests from the $i$-th type of application instance. Inspired by the latest modeling work on datacenters [9], we consider a discrete time-slotted system where the time slot length can range from hundreds of milliseconds to minutes [11]. In every time slot $t\ (= 0, 1, 2, ..., \tau, ...)$, a number of requests $A_i(t)$ generated by the $i$-th type of application arrive at the datacenter, and the time averaged rate of such an arrival process can be denoted as $\lambda_i = \mathbb{E}\{A_i(t)\}$.

We assume that each random variable $A_i(t), \forall i \in \mathcal{A}$, is independent and identically distributed (i.i.d.) [6] over time slots, and they are independent of the current amount of unfinished workload in the system. We also assume that there exist certain peak levels of application requests $A_i^{\max}, \forall i \in \mathcal{A}$, such that $\{A_i(t) \leq A_i^{\max}, \forall i \in \mathcal{A}, \forall t\}$. However, since the workloads in a cloud computing environment is highly dynamic and usually unpredictable (*e.g.,* demands can spike abruptly [18] and potentially exceed the current available processing capacity of a datacenter), *our model does not assume any a priori knowledge of the statistics* of $A_i(t), \forall i \in \mathcal{A}, \forall t$.

### 3.1 Control Decisions

Under the datacenter workload model above, we focus on *three* important control decisions to be made, introduced in Fig. 1 and Sec. 1, along with the key notations used in Table 1 which is available in Appendix A of the online supplementary file.

#### 3.1.1 Admission control of application requests

In each time slot $t$, given the number of newly arrived requests of each application $A_i(t), \forall i \in \mathcal{A}$, the forefront control decision of a datacenter is to determine a subset of requests of each application $R_i(t), \forall i \in \mathcal{A}$ (out of the potentially substantial amount of newly arrived requests of each application $A_i(t), \forall i \in \mathcal{A}$), that can be admitted into the system[2]: $\{0 \leq R_i(t) \leq A_i(t), \forall i \in \mathcal{A}, \forall t\}$. On the other hand, for those denied a portion of requests $[A_i(t) - R_i(t)], \forall i \in \mathcal{A}$, the system can return negative response signals to the corresponding users, who may choose to resend their requests to the datacenter [14].

#### 3.1.2 Routing of application requests

As soon as a portion of requests of each application $R_i(t), \forall i \in \mathcal{A}$ are admitted into the datacenter, the next core control decision is to route (dispatch) the admitted requests from each application $R_i(t), \forall i \in \mathcal{A}$ to the corresponding *queue* for each application on each server, where the requests will wait to be processed. Let $R_{ij}(t)$ denote a subset of requests to be routed to the queue maintained by the $i$-th VM on the $j$-th server in time slot $t$. Then, the routing control decisions should satisfy an obvious constraint: $R_i(t) = \sum_{j=1}^{N} R_{ij}(t), \forall i \in \mathcal{A}, \forall j \in \mathcal{S}, \forall t$. Essentially, such a routing control logic can be implemented as the *load balancer(s)* [18] of realistic datacenters, though our general model is not restricted to any specific design choice and implementation.

2. Though such an admission control of application requests can be achieved in either a centralized frond-end component or in a distributed manner across backend servers, our model focuses on the general underlying control decision without being restricted to any specific design choice and implementation.

#### 3.1.3 Scheduling of VMs

While the amount of routed requests $R_{ij}(t)$ are waiting in the aforementioned corresponding queue maintained by the $i$-th ($\forall i \in \mathcal{A}$) VM on the $j$-th ($\forall j \in \mathcal{S}$) server, another important control decision is to schedule each VM in time slot $t$, by switching between the *running* state (to process the routed application requests that are waiting in this VM's queue) and the *idle* state[3] to keep the routed requests waiting in this VM's queue, without processing them in the current time slot. Such VM scheduling decisions are denoted by the following indicator variables for $\forall i \in \mathcal{A}$, $\forall j \in \mathcal{S}, \forall t$:

$$a_{ij}(t) = \begin{cases} 1, & \text{if the } i\text{-th VM on server } j \text{ is running,} \\ 0, & \text{if the } i\text{-th VM on server } j \text{ is idle.} \end{cases}$$

For each VM, its queue backlog, arrival rate and service rate of application requests can be derived as follows. *First*, we assume that the limited processing capacity (*e.g.,* CPU, memory, disk or network bandwidth) of any server $\forall j \in \mathcal{S}$ is *fairly* allocated among its hosted $M$ VMs, *i.e.,* the processing capacity of each VM is $1/M$ of the total processing capacity of its hosting server. *Second*, while requests from different applications require different amounts of processing capacity, we assume that each request from the same application requires the same amount of processing capacity. Then, the number of time slots $d_i$ for a VM to process each request of a specific application $\forall i \in \mathcal{A}$ is identical. Henceforth, $d_i$ is referred to as the *size* of each request of this particular application $\forall i \in \mathcal{A}$. *Finally*, we define the *queue backlog* $Q_{ij}(t)$ of the $i$-th VM on the $j$-th server as the total sizes of all the requests that are waiting in the queue at the beginning of time slot $t$ (Initially, $Q_{ij}(0) = 0, \forall i \in \mathcal{A}, j \in \mathcal{S}$). The corresponding *service rate* and *arrival rate* of a queue in time slot $t$ can be quantified as $a_{ij}(t)$ and $d_i \cdot R_{ij}(t)$, respectively. By doing so, we can capture the following *queueing dynamics* over time for each VM hosted on each server in a datacenter:

$$Q_{ij}(t+1) = \max[Q_{ij}(t) - a_{ij}(t), 0] + d_i R_{ij}(t). \tag{1}$$

3. We focus on the decision for whether a VM will process application request(s) or remain idle in current time slot, rather than frequently turning on/off VMs (or servers) per time slot, which would incur considerable performance and energy overhead [19].

With the VM scheduling above, it is intuitive that the more VMs remain in the running state, the better processing capacity and thus performance can the datacenter provide. Yet, the tradeoff is a larger amount of power consumed by the datacenter, as we shall characterize in the following subsection.

## 3.2 Characterizing the Power-Performance Tradeoff

### 3.2.1 *System Throughput*

For the large-scale SaaS cloud platforms, one of the most important performance metrics is the overall application throughput in terms of the total number of requests (of all provided applications) that can be admitted and processed. Specifically, for each application $\forall i \in \mathcal{A}$, we define the *time averaged throughput* $r_i$ of a datacenter as:

$$r_i = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{R_i(\tau)\}. \quad (2)$$

As with $r_i$, we define the time averaged value $a_{ij}$ as:

$$a_{ij} = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{a_{ij}(\tau)\},$$

according to the definition above, $a_{ij}$ is the frequency to make the $i$-th VM on the $j$-th server running, while can also be viewed as the capacity consumed by that VM during the $t$ time slots.

Then, the metric $\sum_{i=1}^{M} r_i$ is the overall datacenter throughput that is expected to be maximized, subject to the following two constraints: (1) $r_i \le \lambda_i$, as the time averaged throughput $r_i$ cannot exceed the time averaged arrival rate $\lambda_i$ for any application $\forall i \in \mathcal{A}$, and (2) $d_i r_i \le \sum_{j=1}^{N} a_{ij}$, as the normalized time averaged throughput $d_i r_i$ cannot exceed the overall processing capacity allocated for the corresponding application $i \in \mathcal{A}$.

### 3.2.2 *Power Consumption*

Here we focus on a basic power consumption model of servers where the CPU processing capacity is the main bottleneck, in the case of serving computation-intensive applications. Yet, our model can be extended to incorporate other components (*e.g.*, memory, disk and network I/O) via informed ways [14], such as representing multi-dimensional resources of a server as a vector in our formulation, and amending the aforementioned request sizes and service rates accordingly.

Specifically, it has been shown by recent studies [16], [20] that, the amount of power consumed by a server (CPU processor) is primarily associated with its current CPU running speed $s$, as formally characterized by the following Eq. (3). Without loss of generality, we consider a normalized $s \in [0,1]$ (alternatively viewed as the CPU utilization ratio) and its corresponding normalized power consumption $P(s) \in [0,1]$, where $s = 0$ represents the idle state of a server while $s = 1$ represents its maximum CPU speed in the running state:

$$P(s) = \alpha s^v + (1-\alpha), \quad (3)$$

where the exponent parameter $v$ is empirically determined as $v \ge 1$ in practice (*e.g.*, a typical value is $v = 2$ [20]). With another parameter $\alpha \in [0,1]$, the term $(1-\alpha)$ represents the normalized power consumption of an idle server. Practical measurements [21] have shown that $(1-\alpha)$ is around $0.6$ (and barely lower than $0.5$), which implies that an idle server still consumes a non-trivial amount of power.

Based on the power model above, for a server $j \in \mathcal{S}$ that hosts $M$ VMs with the fair capacity allocation policy described in Sec. 3.1, its normalized CPU load and corresponding power consumption in time slot $t$ are given as:

$$\begin{aligned} s_j(t) &= \frac{\sum_{i=1}^{M} a_{ij}(t)}{M}, \\ P_j(t) &= \alpha\left(\frac{\sum_{i=1}^{M} a_{ij}(t)}{M}\right)^v + (1-\alpha). \end{aligned}$$

Accordingly, the *time average of normalized power consumption* $p_j$ of each server $\forall j \in \mathcal{S}$ in a datacenter can be defined as:

$$p_j = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{P_j(\tau)\}, \quad (4)$$

then, the metric $\sum_{j=1}^{N} p_j$ is the overall power consumption of all servers[4], hopefully being minimized.

### 3.2.3 *A Unified Objective from an Economic Perspective*

So far, we have derived both the datacenter performance metric $r_i, \forall i \in \mathcal{A}$ in Eq. (2) (time averaged throughput) and power consumption metric $p_j, \forall j \in \mathcal{S}$ in Eq. (4) (time averaged power consumption). However, the fundamental challenge is *how to optimize the tradeoff between the two potentially conflicting objectives in a balanced and cost-effective manner?* To this end, we first construct a unified profit objective to couple both sides in an economic way as follows.

*First, quantifying the power cost.* The power cost of a datacenter can be measured as (Price$\cdot$PUE$\cdot\sum_{j=1}^{N} p_j$), where Price is the electricity market price of each unit of the normalized power consumption, and PUE is the power usage efficiency metric provided by the Green Grid [23]. It represents the ratio of the total amount of power used by the entire datacenter facility to the power delivered to the computing equipment. Reportedly, inefficient datacenter facilities can have a PUE $\in [2.0, 3.0]$, while leading industry datacenter facilities are announced to approach a PUE of around $1.2$ [18].

*Second, pricing the system throughput.* Different from the power which is demanded rigidly, there exists "elastic demand" for the cloud applications served by a datacenter. Hence, we choose to price the throughput of

4. Given recent reports [22] that the power consumption of other non-IT equipments (*e.g.*, cooling) is roughly proportional to that by servers, our basic model can also be extended to capture the overall power consumption of a datacenter by scaling up $\sum_{j=1}^{N} p_j$ with a constant factor.

each application $i \in \mathcal{A}$ served by the datacenter as a log function $g(r_i) = \log(1 + d_i r_i)$, according to the law of diminishing marginal utility in economics [4]. Such a rule has also been adopted by real-world SaaS cloud services (*e.g.,* [3]). In addition, such a nonlinear function is different from a closely related work [14], which used a simple linear utility function.

Given the *revenue* brought by the system throughput and the *cost* incurred by the power consumption, we can maximize the time averaged profit of a datacenter as follows:

$$\begin{aligned} \max \quad & \sum_{i=1}^{M} g(r_i) - \beta \sum_{j=1}^{N} p_j \qquad (5) \\ \text{s.t.} \quad & 0 \le r_i \le \lambda_i, \quad \forall i \in \mathcal{A}, \\ & d_i r_i \le \sum_{j=1}^{N} a_{ij}, \quad \forall i \in \mathcal{A}, \end{aligned}$$

where the factor $\beta = \text{Price} \cdot \text{PUE}$.

Intuitively, one may attempt to solve the problem above directly. Before attempting this, we should rewrite $p_j$ in the form of $a_{ij}$, and thus to constrain the power consumption to the admission rate. As $P_j(t)$ is nonlinear to $a_{ij}(t)$, it is impossible to establish the equation between $p_j$ and $a_{ij}$. Fortunately, since $P_j(t)$ is convex to $a_{ij}(t)$, according to *Jesen's inequality*, we have:

$$p_j \ge \alpha(\frac{\sum_{i=1}^{M} a_{ij}}{M})^2 + (1 - \alpha). \qquad (6)$$

As we cannot obtain the explicit form of $p_j$, it is impractical to maximize the time averaged profit directly. Instead, we propose an offline control strategy to maximize the supreme bound of the time averaged profit in the next section.

## 4 Maximizing the Supreme Bound

In this section, we resort to the theoretical analysis to design an offline control strategy, which makes stationary decisions on admission control and VM scheduling, and thus to maximize the supreme bound of the time averaged profit. Such an offline control solution helps to justify the reliability of our basic power-performance tradeoff model. Furthermore, the supreme bound of the time averaged profit can be used as a profit benchmark, and thus to examine the effectiveness of alternative control solutions.

Based on the inequality (6) in Sec. 3.2.3, we maximize the supreme bound of the time averaged profit:

$$\begin{aligned} \max_{r_i, a_{ij}} \quad & \sum_{i=1}^{M} \log(1 + d_i r_i) - \frac{\alpha\beta}{M^2} \sum_{j=1}^{N} (\sum_{i=1}^{M} a_{ij})^2 + N(1 - \alpha) \\ \text{s.t.} \quad & 0 \le r_i \le \lambda_i \\ & d_i r_i \le \sum_{j=1}^{N} a_{ij} \\ & 0 \le a_{ij} \le 1. \qquad (7) \end{aligned}$$

First, note that the objective function is non-decreasing to $a_{ij}, \forall i \in \mathcal{A}, \forall j \in \mathcal{S}$. Hence, to maximize the objective function, we should choose:

$$\sum_{j=1}^{N} a_{ij} = d_i r_i. \qquad (8)$$

Since $\sum_{j=1}^{N} a_{ij}$ is the processing capacity allocated to the $i$-th application by the corresponding VMs, the equation above indicates that to maximize the profit, the processing capacity allocated to each application should be completely consumed to process the admitted requests, no power is consumed without processing any request and thus the power cost is minimized.

Then, to simplify the objective function, we first take *Cauchy's inequality* ($\sum_{k=1}^{n} a_k^2 \sum_{k=1}^{n} b_k^2 \ge (\sum_{k=1}^{n} a_k b_k)^2$) to bound the complicated term $\sum_{j=1}^{N}(\sum_{i=1}^{M} a_{ij})^2$,yielding:

$$\sum_{j=1}^{N}(\sum_{i=1}^{M} a_{ij})^2 \ge \frac{(\sum_{j=1}^{N}\sum_{i=1}^{M} a_{ij})^2}{\sum_{j=1}^{N} 1^2} = \frac{(\sum_{i=1}^{M}\sum_{j=1}^{N} a_{ij})^2}{N},$$

according to Cauchy's inequality, the equation above is established if and only if:

$$\sum_{i=1}^{M} a_{i1} = \sum_{i=1}^{M} a_{i2} = \cdots = \sum_{i=1}^{M} a_{iN}. \qquad (9)$$

Sine $\sum_{i=1}^{M} a_{ij}$ is the processing capacity consumed by the $j$-th server, the equation above implies that each server consumes the same amount of processing capacity. Furthermore, note that the term $N(1 - \alpha)$ is constant, then the **Problem** (7) is now transformed to the following simplified problem:

$$\begin{aligned} \max_{r_i} \quad & f = \sum_{i=1}^{M} \log(1 + d_i r_i) - \frac{\alpha\beta}{M^2 N}(\sum_{i=1}^{M} d_i r_i)^2 \\ \text{s.t.} \quad & 0 \le r_i \le \min[\lambda_i, \frac{N}{d_i}]. \qquad (10) \end{aligned}$$

To make it easier to derive the peak value of the objective function $f$, let $x_i = d_i r_i$, differentiating $f$ with respect to each $x_k, \forall k \in \mathcal{A}$, yielding:

$$\frac{\partial f}{\partial x_k} = \frac{1}{1 + x_k} - \frac{2\alpha\beta}{M^2 N} \sum_{i=1}^{M} x_i, \text{ for } k = 1, 2, \cdots, M.$$

For each application $k \in \mathcal{A}$, let $\frac{\partial f}{\partial x_k} = 0$. Note that the term $\sum_{i=1}^{M} x_i$ is identical to each application $k$, hence, we have the following equation:

$$x_1 = x_2 = \cdots = x_M,$$

then, $\sum_{i=1}^{M} x_i = M x_1$, plugging it into $\frac{\partial f}{\partial x_1} = 0$, yielding:

$$\frac{\partial f}{\partial x_1} = \frac{1}{1 + x_1} - \frac{2\alpha\beta}{MN} x_1 = 0.$$

Solving it yields the following solution:

$$x_1 = x_2 = \cdots = x_M = \frac{-\alpha\beta + \sqrt{\alpha^2\beta^2 + 2\alpha\beta MN}}{2\alpha\beta},$$

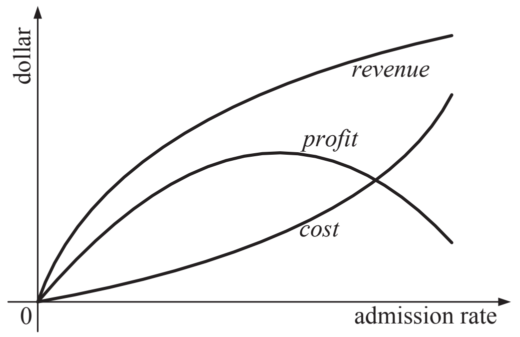

Fig. 2: An illustration of the optimal solution $r_i$, which reflects the shape of revenue, cost and profit.

where the other negative root is discarded. Recall that $x_i = d_i r_i$, so we have:

$$r_i = \frac{-\alpha\beta + \sqrt{\alpha^2\beta^2 + 2\alpha\beta MN}}{2\alpha\beta d_i}.$$

Before reaching the peak point, the objective function is non-decreasing, while after reaching the peak point, the objective function is decreasing. Hence, taking the additional constraint $0 \le r_i \le \min[\lambda_i, \frac{N}{d_i}]$ into consideration, yielding the following optimal solution $r_i, \forall i \in \mathcal{A}$, which can be viewed as the decision of admission control:

$$r_i = \min[\lambda_i, \frac{N}{d_i}, \frac{-\alpha\beta + \sqrt{\alpha^2\beta^2 + 2\alpha\beta MN}}{2\alpha\beta d_i}].$$

**Insight:** Fig. 2 illustrates the tendency of the revenue, cost and profit as the admission rate increases. To support the increasing admission rate, the cost that is incurred by the power consumption increases more and more rapidly, while the corresponding revenue brought by processing the increasing requests grows more and more slowly. Therefore, the profit increases initially and then decreases as shown in Fig. 2, and there exists a certain admission rate that maximizes the profit.

Meanwhile, to maximize the objective function, $a_{ij}, \forall i \in \mathcal{A}, \forall j \in \mathcal{S}$ should satisfy Eq. (8). Besides, based on Eq. (8) and Eq. (9) which can be viewed as the decision of VM scheduling, we further have:

$$\sum_{i=1}^{M} a_{i1} = \sum_{i=1}^{M} a_{i2} = \cdots = \sum_{i=1}^{M} a_{iN} = \frac{\sum_{i=1}^{M} d_i r_i}{N}. \tag{11}$$

The equation above suggests that the admitted requests are fairly allocated to each server, which can be viewed as load balancing. However, note that according to Eq. (8) and Eq. (11), the optimal solution $a_{ij}$ is not unique, i.e., there exist a variety of optimal VM scheduling decisions.

The optimal offline control solution above is reasonable and reliable. However, for realistic datacenters, there is a challenge when applying this solution: The datacenter workload is time-varying and unpredictable, which makes it infeasible to precisely capture key parameters (such as $\lambda_i$), and impractical to calculate optimal solution in an offline manner. In response, we seek to design an *online* control algorithm in Sec. 5, which is able to efficiently make decisions on all three important control decisions.

## 5 Constructing an Online Control Framework

In response to the challenges of **Problem** (5), we take advantage of the Lyapunov optimization techniques [6] to design an online control framework, which is able to concurrently make all three important control decisions in Fig. 1, including *request admission control*, *routing*, and *VM scheduling*. In particular, our control algorithms can be proved to approach a time averaged profit that is arbitrarily close to the optimum of **Problem** (5), while keeping the system stable.

### 5.1 Problem Transformation Using Lyapunov Optimization

Given that the function $g(r_i)$ is nonlinear, we are inspired by a recent technique [24] to transform **Problem** (5) to the following framework of Lyapunov optimization **Problem** (12), which introduces *auxiliary variables* $\gamma_i$ for each admitted stream of application requests $R_i(t), \forall i \in \mathcal{A}$, in the system described in Fig. 1:

$$\max \quad \sum_{i=1}^{M} g(\gamma_i) - \beta \sum_{j=1}^{N} p_j \tag{12}$$

$$\text{s.t.} \quad \gamma_i \le r_i, \quad \forall i \in \mathcal{A} \tag{13}$$

$$0 \le r_i \le \lambda_i, \quad \forall i \in \mathcal{A} \tag{14}$$

$$d_i r_i \le \sum_{j=1}^{N} a_{ij}, \quad \forall i \in \mathcal{A}. \tag{15}$$

It is easy to check that the optimal solution of the problem above is the same as that of the original **Problem** (5), as the function $g(*) = \log(1 + *d_i)$ is non-decreasing.

To solve the problem above, we first transform the inequality constraint (13) into a queue stability problem [6]. Specifically, we introduce *virtual queues* $H_i(t)$ for each $R_i(t)$. Initially, we define $H_i(0) = 0, \forall i \in \mathcal{A}$, and then update the queues per each time slot as follows:

$$H_i(t+1) = \max[H_i(t) - R_i(t), 0] + \gamma_i(t), \tag{16}$$

where $\gamma_i(t)$ denotes a process of non-negative auxiliary variables that the admission control of the datacenter system (Sec. 3.1) will determine in every time slot, which satisfies the constraint (17). The time average of each $\gamma_i(t)$ is defined as $\gamma_i = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{\gamma_i(\tau)\}$.

$$0 \le \gamma_i(t) \le A_i^{\max}. \tag{17}$$

**Insight:** Intuitively, the auxiliary variables $\gamma_i(t)$ can be viewed as the "arrivals" of the virtual queues $H_i(t)$, while $R_i(t)$ can be viewed as the service rate of such virtual queues. The constraints (13) are enforced on the condition that the virtual queues $H_i(t)$ are stable, *i.e.*,

$\lim_{t\to\infty}\mathbb{E}\{H_i(t)\}/t = 0$. Specifically, from (16) it is clear that: $H_i(t+1) \geq [H_i(t) - R_i(t) + \gamma_i(t)]$. By summing this inequality over time slots $\tau \in \{0, 1, ..., t-1\}$ and then dividing the result by $t$, we have $\frac{H_i(t)-H_i(0)}{t} + \frac{1}{t}\sum_{\tau=0}^{t-1} R_i(\tau) \geq \frac{1}{t}\sum_{\tau=0}^{t-1}\gamma_i(\tau)$. With $H_i(0) = 0$, taking expectations of both sides yields:

$$\lim_{t\to\infty}\frac{\mathbb{E}\{H_i(t)\}}{t} + r_i \geq \gamma_i. \quad (18)$$

If the virtual queues $H_i(t)$ are stable, then $\lim_{t\to\infty}\mathbb{E}\{H_i(t)\}/t = 0$ (Note that we will prove the strong stability of virtual queues $H_i(t)$ in Theorem 1 later). Plugging this into (18) yields $r_i \geq \lambda_i$, so that the constraint (13) can be satisfied.

#### 5.1.1 Characterizing the Stability-Profit Tradeoff

Let $\mathbf{Q}(t) = (Q_{ij}(t))$ and $\mathbf{H}(t) = (H_i(t))$ denote the matrix of the actual and virtual queues maintained by VMs (Sec. 3.1). Then, we use $\mathbf{\Theta}(t) = [\mathbf{Q}(t); \mathbf{H}(t)]$ to denote the combined matrix of all the actual queues and virtual queues. However, since $\mathbf{Q}(t)$ and $\mathbf{H}(t)$ have different scales (the former corresponds to the request size $d_i$ according to Eq. (1), while the latter corresponds to the number of requests according to Eq. (16)), we define a Lyapunov function $L(\mathbf{\Theta}(t))$, which assigns different weights $d_i$ and 1 to $H_i(t)$ and $Q_{ij}(t)$, respectively:

$$L(\mathbf{\Theta}(t)) = \frac{1}{2}\Big[\sum_{i=1}^{M} d_i^2 H_i^2(t) + \sum_{i=1}^{M}\sum_{j=1}^{N} Q_{ij}^2(t)\Big]. \quad (19)$$

This represents a scalar metric of *queue congestion* [6] in the datacenter system. For example, a small value of $L(\mathbf{\Theta}(t))$ implies that both actual queue backlogs and virtual queue backlogs are small. The implication is that the corresponding datacenter system has *strong stability*.

To keep the system stable by persistently pushing the Lyapunov function towards a lower congestion state, we introduce $\Delta(\mathbf{\Theta}(t))$ as the *one-step conditional Lyapunov drift* [6]:

$$\Delta(\mathbf{\Theta}(t)) = \mathbb{E}\{L(\mathbf{\Theta}(t+1)) - L(\mathbf{\Theta}(t)) | \mathbf{\Theta}(t)\}.$$

In the sense of Lyapunov optimization, the underlying objective of our optimal control decisions on request admission control, routing and VM scheduling is to minimize an infimum bound on the following *drift-minus-profit* expression in each time slot:

$$\Delta(\mathbf{\Theta}(t)) - V\mathbb{E}\Big\{\sum_{i=1}^{M} g(\gamma_i(t)) - \beta\sum_{j=1}^{N} P_j(t)|\mathbf{\Theta}(t)\Big\}. \quad (20)$$

**Insight:** The control parameter $V(\geq 0)$ represents *a design knob of the stability-profit tradeoff, i.e.,* how much we shall emphasize the profit maximization (**Problem** (12)) compared to system stability. It empowers system operators to make flexible design choices among various tradeoff points between system stability and profit. For example, one may prefer to achieve as much expected profit $\mathbb{E}\{\sum_{i=1}^{M} g(\gamma_i(t)) - \beta\sum_{j=1}^{N} P_j(t)|\mathbf{\Theta}(t)\}$ as possible, while having to keep $\Delta(\mathbf{\Theta}(t))$ small to avoid higher system congestion.

#### 5.1.2 Bounding Drift-Minus-Profit

The analysis above instructs a system designer to derive an infimum bound of the *drift-minus-profit* expression given in Eq. (20), which requires the following Lemma.

***Lemma 1:*** In each time slot $t$, for any value of $\mathbf{\Theta}(t)$, the Lyapunov drift $\Delta(\mathbf{\Theta}(t))$ of a datacenter system under any control strategy satisfies the following, where $B_1 \triangleq \frac{MN + 3\sum_{i=1}^{M}(d_i A_i^{\max})^2}{2}$ is a finite constant parameter.

$$\Delta(\mathbf{\Theta}(t)) \leq B_1 - \sum_{i=1}^{M} d_i^2 H_i(t)\mathbb{E}\{R_i(t) - \gamma_i(t)|\mathbf{\Theta}(t)\}$$
$$- \sum_{i=1}^{M}\sum_{j=1}^{N} Q_{ij}(t)\mathbb{E}\{a_{ij}(t) - d_i R_{ij}(t)|\mathbf{\Theta}(t)\}. \quad (21)$$

Interested readers are referred to Appendix B for a complete proof of Lemma 1, available in the online supplemental material.

Based on Lemma 1, subtracting the expression $V\mathbb{E}\{\sum_{i=1}^{M} g(\gamma_i(t)) - \beta\sum_{j=1}^{N} P_j(t)|\mathbf{\Theta}(t)\}$ from both sides of Eq. (21) yields an infimum bound of the *drift-minus-profit* expression of the datacenter system:

$$\Delta(\mathbf{\Theta}(t)) - V\mathbb{E}\Big\{\sum_{i=1}^{M} g(\gamma_i(t)) - \beta\sum_{j=1}^{N} P_j(t)|\mathbf{\Theta}(t)\Big\} \leq B_1$$
$$- \sum_{i=1}^{M}\mathbb{E}\{Vg(\gamma_i(t)) - d_i^2 H_i(t)\gamma_i(t)|\mathbf{\Theta}(t)\} \quad (22)$$
$$- \sum_{i=1}^{M}\mathbb{E}\Big\{d_i^2 H_i(t)R_i(t) - \sum_{j=1}^{N} d_i R_{ij}(t)Q_{ij}(t)|\mathbf{\Theta}(t)\Big\} \quad (23)$$
$$- \sum_{j=1}^{N}\mathbb{E}\Big\{\sum_{i=1}^{M} Q_{ij}(t)a_{ij}(t) - V\beta P_j(t)|\mathbf{\Theta}(t)\Big\}. \quad (24)$$

### 5.2 An Optimal Online Control Algorithm

Instead of directly minimizing the *drift-minus-profit* expression in Eq. (20) that involves implicit $\max[*]$ terms in both Eq. (1) and Eq. (16), we seek to design an optimal **Online Control Algorithm (OCA)** to minimize its infimum bound given above (*i.e.,* equivalent to maximizing the terms (22)(23)(24) on the right-hand-side), without undermining the optimality and performance of the algorithm according to [6]. Interestingly, we will show that the maximization of the terms (22)(23)(24) can be decoupled to a series of independent subproblems, which can be computed concurrently in a decentralized fashion.

Specifically, in each time slot $t$, based on *online* observations of the queue backlogs $\mathbf{Q}(t)$ and $\mathbf{H}(t)$, **OCA** performs the following four phases of control operations, including: (1) auxiliary variable selection, (2) request admission control and routing control, (3) VM scheduling, and (4) queue update.

Recall that the phases (2) and (3) make corresponding decisions for the three fundamental control decisions that we described in Sec. 3.1 and Fig. 1.

### 5.2.1 Auxiliary Variable Selection

For each application $i \in \mathcal{A}$ served by the datacenter, we determine $\gamma_i(t)$ by maximizing the term (22) in Sec. 5.1.2. Fortunately, as the decision variables $\gamma_i(t)$ are independent among applications, such centralized maximization can be decoupled to be computed concurrently as follows:

$$\begin{aligned} \max_{\gamma_i(t)} \quad & V\log(1+d_i\gamma_i(t)) - d_i^2 H_i(t)\gamma_i(t) \\ \text{s.t.} \quad & 0 \le \gamma_i(t) \le A_i^{\max}, \forall i \in \mathcal{A}. \end{aligned} \tag{25}$$

Differentiating the objective function above with respect to $\gamma_i(t)$ can yield the peak value of the objective function when $\gamma_i(t) = \frac{V}{d_i^2 H_i(t)} - \frac{1}{d_i}$. By taking the constraint (17) into consideration, we obtain the optimal solution to problem (25):

$$\gamma_i(t) = \begin{cases} 0, & H_i(t) > \dfrac{V}{d_i} \\ \dfrac{V}{d_i^2 H_i(t)} - \dfrac{1}{d_i}, & \dfrac{V}{d_i^2 A_i^{\max} + d_i} \le H_i(t) \le \dfrac{V}{d_i} \\ A_i^{\max}, & H_i(t) < \dfrac{V}{d_i^2 A_i^{\max} + d_i} \end{cases} \tag{26}$$

**Insight:** The stepped solution above is directly related to the value of $H_i(t)$. Recall from Eq. (18) that, if the value of $H_i(t)$ is small, then it implies that the time average of $\gamma_i(t)$ is close to that of $R_i(t)$, which improves the system stability as we expected. In this case, a larger value of $\gamma_i(t)$ can be chosen with respect to $H_i(t)$. On the other hand, if the value of $H_i(t)$ is large, then it implies that the time average of $\gamma_i(t)$ is far away from that of $R_i(t)$. To fill this gap, it would be better to choose a lower value of $\gamma_i(t)$. In addition, as the selection of auxiliary variables can be separately performed for each application $\forall i \in \mathcal{A}$, this can facilitate a distributed implementation of the decision phase above.

### 5.2.2 Request Admission Control and Routing

For each application $i \in \mathcal{A}$ served by the datacenter, the request admission decisions $R_i(t)$ and routing decisions $(R_{i1}(t), R_{i2}(t), ..., R_{iN}(t))$ as illustrated in Fig. 1 can be decided by maximizing the term (23) in Sec. 5.1.2. Again, since the admission decisions $R_i(t)$ and routing decisions $R_{ij}(t)$ of different applications are independent from each other, this centralized maximization can be decoupled to be computed concurrently as follows:

$$\begin{aligned} \max_{R_i(t), R_{ij}(t)} \quad & d_i^2 H_i(t) R_i(t) - d_i \sum_{j=1}^{N} R_{ij}(t) Q_{ij}(t) \\ \text{s.t.} \quad & 0 \le R_i(t) \le A_i(t), \forall i \in \mathcal{A}, \\ & R_i(t) = \sum_{j=1}^{N} R_{ij}(t). \end{aligned} \tag{27}$$

Different from the previous problem (25), both the control decisions of $R_i(t)$ and $R_{ij}(t)$ in problem (27) need to be determined. We first start from a simple case: if the value of $R_i(t)$ is known in advance, then problem (27) is exactly equivalent to the following problem for making routing decisions:

$$\begin{aligned} \min_{R_{ij}(t)} \quad & d_i \textstyle\sum_{j=1}^{N} R_{ij}(t) Q_{ij}(t) \\ \text{s.t.} \quad & \textstyle\sum_{j=1}^{N} R_{ij}(t) = R_i(t), \forall i \in \mathcal{A}. \end{aligned} \tag{28}$$

**Insight:** The problem (28) is a generalized *min-weight problem*, where the amount of requests routed to server $j \in \mathcal{S}$ for application $i \in \mathcal{A}$ is weighted by the current queue backlog $Q_{ij}(t)$. Hence, for each application $i \in \mathcal{A}$ served by the datacenter, the optimal routing strategy tends to *dispatch as many admitted requests as possible to the VM with the least backlogged queue*:

$$R_{ij}(t) = \begin{cases} R_i(t), & j = j_i^*, \\ 0, & \text{else}, \end{cases} \tag{29}$$

where $j_i^* = \arg\min_{j \in \{1,2,...,N\}} Q_{ij}(t)$, *i.e.*, the queue of the $i$-th VM on server $j_i^*$ is the shortest queue among all the $N$ queues for the $i$-th type of application. Such a routing policy is an intuitive *"Join the Shortest Queue"* policy for the purpose of load balancing, which is consistent with a recent work on scheduling of cloud computing clusters [25]. Further, it can effectively reduce the response delay of newly admitted requests, as they are preferentially routed to the shortest queues. However, it requires to obtain all the queue backlog information of those VMs serving the $i$-th type of applications. To mitigate this complexity, we can adopt a recently developed "Power-of-Two-Choices" [25] routing policy, which randomly samples two VMs and routes the application requests to the VM with a smaller queue backlog.

Recall that the optimal value of $R_i(t)$ is still undecided so far. Based on the routing strategy in Eq. (29), the second term of Eq. (27) (*i.e.*, $d_i \sum_{j \in \mathcal{S}} R_{ij}(t) Q_{ij}(t)$) can be rewritten as $d_i R_i(t) Q_{ij_i^*}(t)$. Then, the request admission control decision can be solved as:

$$\begin{aligned} \max_{R_i(t)} \quad & d_i^2 H_i(t) R_i(t) - d_i R_i(t) Q_{ij_i^*}(t) \\ \text{s.t.} \quad & 0 \le R_i(t) \le A_i(t), \forall i \in \mathcal{A}. \end{aligned} \tag{30}$$

The problem (30) is a simple linear programming problem in which the optimal value of $R_i(t)$ is:

$$R_i(t) = \begin{cases} A_i(t), & d_i H_i(t) > Q_{ij_i^*}(t) \\ 0, & \text{else} \end{cases} \tag{31}$$

**Insight:** This is a simple *threshold-based* admission control strategy. When the backlog of the shortest queue $Q_{ij_i^*}(t)$ is smaller than a threshold $d_i H_i(t)$, then all the newly arrived requests are admitted into the datacenter. Essentially, this not only reduces the value of $H_i(t)$ so as to push $\gamma_i$ to become closer to $r_i$, but also increases the datacenter throughput $r_i$ so as to improve the profit. On the other hand, when the backlog of the shortest queue $Q_{ij_i^*}(t)$ is larger than the threshold $d_i H_i(t)$, then all the requests will be denied to ensure the stability of the datacenter system. The intuition is to *prevent the system with considerable backlogged requests from being overloaded by newly arrived requests*.

#### 5.2.3 VM Scheduling

In each time slot $t$, the running or idle state of each VM $(a_{1j}(t), a_{2j}(t), ..., a_{Mj}(t))$ (Sec. 3.1) on server $j \in \mathcal{S}$ can be determined by maximizing the term (24) in Sec. 5.1.2. Observing that the indicator variables $a_{ij}(t)$ are independent among different servers, the centralized maximization can be implemented by each server in a fully *distributed* manner:

$$\begin{aligned} \max_{a_{ij}(t)} \quad & \sum_{i=1}^{M} Q_{ij}(t) a_{ij}(t) - V\beta P_j(t) \\ \text{s.t.} \quad & a_{ij}(t) \in \{0,1\}, \forall i \in \mathcal{A}, \forall j \in \mathcal{S}, \end{aligned} \tag{32}$$

where $Q_{ij}(t)$ can be viewed as the weight of the decision variable $a_{ij}(t)$. As the growth of power consumption caused by running each VM is the same under our model (see the definition of $P_j(t)$ in Sec. 3.2.2), the optimal solution for Eq. (32) would prefer to schedule those VMs with the most backlogged queues (*i.e.,* larger $Q_{ij}(t)$ as weights) to the running state. Following this intuition, each server adopts a simple yet effective *greedy strategy* that ranks hosted VMs according to their queue backlogs. Then, it searches from VMs with the most backlogged queues to VMs with the least backlogged queues: (1) If the growth in the sum of backlogs exceeds the growth of power consumption (weighted by $V\beta$) caused by running a certain VM, then the VM is preferentially scheduled to the running state. (2) Such a search process can continue until the growth in the sum of backlogs falls below the growth of power consumption (weighted by $V\beta$) for a certain VM, which is scheduled to the idle state. Then, the other remaining VMs are scheduled to the idle state.

#### 5.2.4 Queue Update

Finally, the virtual queues $\mathbf{H}(t)$ can be updated according to Eq. (16), by using the optimal values of $\gamma_i(t)$ and $R_i(t)$ determined by the phases above. Likewise, the actual queues $\mathbf{Q}(t)$ maintained by VMs in the datacenter can be updated according to Eq. (1), based on the optimal values of $R_{ij}(t)$ and $a_{ij}(t)$ derived above.

### 5.3 Optimality Analysis

***Theorem** 1:* For arbitrary arrival rates of application requests $(\lambda_1(t), \lambda_2(t), ..., \lambda_M(t))$ (possibly exceeding the processing capacity of a datacenter), a datacenter using the **OCA** algorithm with any $V \geq 0$ (the stability-profit tradeoff parameter defined in Sec. 5.1.1) can guarantee that all the actual and virtual queues are strongly stable over time slots:

$$H_i(t) \leq \frac{V}{d_i} + A_i^{\max}, \forall i \in \mathcal{A}, \tag{33}$$

$$Q_{ij}(t) \leq V + 2d_i A_i^{\max}, \forall i \in \mathcal{A}, \forall j \in \mathcal{S}. \tag{34}$$

Meanwhile, the gap between its achieved time averaged profit and the optimal profit $\xi^*$ is within $B_1/V$:

$$\liminf_{t\to\infty} \Big\{ \sum_{i=1}^{M} g(r_i) - \beta \sum_{j=1}^{N} p_j \Big\} \geq \xi^* - \frac{B_1}{V}, \tag{35}$$

where $\xi^* = \sum_{i=1}^{M} g(r_i^*) - \beta \sum_{j=1}^{N} p_j^*$, $r_i^*$ and $p_j^*$ are the optimal solution to **Problem** (5), and $B_1$ is a finite constant parameter defined in Lemma 1.

**Insight:** Theorem 1 provides a strong deterministic guarantee of the upper bounds on backlogs of all the actual and virtual queues in any time slot. Meanwhile, Eq. (35) indicates that the gap between the time averaged profit achieved by **OCA** algorithm and the optimal profit is within $O(1/V)$. Interestingly, as the value of parameter $V$ increases to sufficiently large, the time averaged profit under **OCA** can be pushed arbitrarily close to the optimum. However, according to Eq. (33) and Eq. (34), overly aggressive increases of profit can increase the bounds of queue backlogs. By Little's law [6], the bounds of response delays for application requests would also increase. The proof of Theorem 1 can be found in Appendix C of the online supplemental material.

## 6 Extension: Buffering the Requests

In a realistic datacenter, to satisfy some practical requirements, the designer of the datacenter may have additional design choices based on the typical cloud platform shown in Fig. 1. For example, to alleviate the impact of bursty arrivals (*e.g.,* the flash crowd phenomenon in Internet content distribution and video streaming [26], [27]) and thus to improve the robustness of the datacenter system, the designer may prefer to store the excessive admitted requests for each application in a buffering facility before part of them are routed to the corresponding VMs. In this section, we first show that the Lyapunov-based optimization framework can be enhanced to accommodate this additional design choice in realistic datacenters. Then, we present the corresponding buffer-enhanced optimal control algorithm which is denoted as **BOCA**. Finally, we distinguish **BOCA** from **OCA**. Please refer to Sec. D of the online supplementary material for detailed discussions.

## 7 Extension: Enforcing a Power Budget

In additional to accommodating diverse design choices such as buffering facilities in a realistic datacenter, our model and optimization framework can also be extended to incorporate various practical requirements of the datacenter power-performance tradeoff. For instance, as most real-world datacenters are operated within a certain *power budget* [5], it is important for datacenter operators to improve the *performance (dollar) per watt* [5] by achieving a desired performance level with an enforced power budget. Please refer to Sec. E of the online supplementary material for detailed discussions.

## 8 Performance Evaluation

We conduct comprehensive simulations to evaluate our online control algorithm **OCA** and its extensions from a variety of perspectives, including: (1) the verification of algorithm optimality, (2) the examination of system stability, (3) the adaptivity to bursty request arrivals, (4) the effectiveness of admission control, (5) the effectiveness of power budget enforcement, (6) the sensitivity of key parameters, as well as (7) the exactness of the maximized supreme bound. Please refer to Sec. G of the online supplementary material for detailed discussions on the experimental configurations and evaluation results.

Based on the evaluation under representative datacenter scenarios, we show that: (1) our online control framework can approach a time averaged profit that is arbitrarily close to the optimum, which implies a cost-effective power-performance tradeoff; (2) it can maintain strong system stability, in term of the robustness and adaptivity to time-varying and bursty application requests; (3) it can reflect the potential benefits of practical factors, such as the impact of power efficient hardware and cooling control.

## 9 Conclusion

In response to dynamic and unpredictable user requests from heterogeneous applications served by a SaaS cloud datacenter platform, this paper designs and analyzes an optimal online control framework to balance the tradeoff between the throughput performance and power consumption for processing requests. By applying rigorous Lyapunov optimization approaches, our online control framework can independently and simultaneously make decisions on three important control decisions of such a cloud platform, including request admission control, routing, and VM scheduling. To show the superiority of our online control framework, an offline control solution is derived in advance to provide a profit benchmark, and also to justify the reliability of the basic model.

In particular, our online control framework is shown to be extensible to explore the various design choices and operational constraints. Specifically, additional buffering facilities are introduced for better system scalability in term of design choices, and a certain power budget is enforced towards better performance (dollar) per watt in term of operational constraints. Through in-depth mathematical analysis and various simulations, we demonstrate that our online control framework can approach a time averaged profit — unifying a cost-effective power-performance tradeoff — that is arbitrarily close to the optimum, while still maintaining strong system stability, in term of the robustness and adaptivity to time-varying and bursty application requests. As future work, we will investigate more effective cost reduction mechanisms, by dynamically adjusting the number of activated servers in the datacenter.

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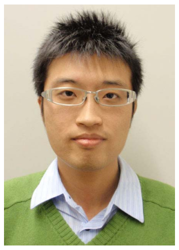

**Fangming Liu** is an associate professor in the School of Computer Science and Technology, Huazhong University of Science and Technology, Wuhan, China; and he is awarded as the CHUTIAN Scholar of Hubei Province, China. He is the Youth Scientist of National 973 Basic Research Program Project on Software-defined Networking (SDN)-based Cloud Datacenter Networks, which is one of the largest SDN projects in China. Since 2012, he has also been invited as a StarTrack Visiting Young Faculty in Microsoft Research Asia (MSRA), Beijing. He received his B.Engr. degree in 2005 from Department of Computer Science and Technology, Tsinghua University, Beijing; and his Ph.D. degree in Computer Science and Engineering from the Hong Kong University of Science and Technology in 2011. From 2009 to 2010, he was a visiting scholar at the Department of Electrical and Computer Engineering, University of Toronto, Canada. He was the recipient of two Best Paper Awards from IEEE GLOBECOM 2011 and IEEE CloudCom 2012, respectively. His research interests include cloud computing and datacenter networking, mobile cloud, green computing and communications, software-defined networking and virtualization technology, large-scale Internet content distribution and video streaming systems. He is a member of IEEE and ACM, as well as a member of the China Computer Federation (CCF) Internet Technical Committee. He has been a Guest Editor for IEEE Network Magazine, an Associate Editor for Frontiers of Computer Science, and served as TPC for IEEE INFOCOM 2013-2014 and GLOBECOM 2012-2013.

**Zhi Zhou** is a Master student in the School of Computer Science and Technology, Huazhong University of Science and Technology, Wuhan, China. His research interests include cloud computing and green datacenters.

**Hai Jin** is a Cheung Kung Scholars Chair Professor of computer science and engineering at the Huazhong University of Science and Technology (HUST), China. He is now dean of the School of Computer Science and Technology at HUST. He received his Ph.D. degree in computer engineering from HUST in 1994. In 1996, he was awarded a German Academic Exchange Service fellowship to visit the Technical University of Chemnitz in Germany. He worked at the University of Hong Kong between 1998 and 2000, and as a visiting scholar at the University of Southern California between 1999 and 2000. He was awarded the Excellent Youth Award from the National Science Foundation of China in 2001. He is the chief scientist of ChinaGrid, the largest grid computing project in China, and chief scientist of the National 973 Basic Research Program Project of Virtualization Technology of Computing Systems. He is a senior member of the IEEE and a member of the ACM. His research interests include computer architecture, virtualization technology, cluster computing and grid computing, peer-to-peer computing, network storage, and network security.

**Bo Li** is a professor in the Department of Computer Science and Engineering, Hong Kong University of Science and Technology. His current research interests include large-scale content distribution in the Internet, datacenter networking, cloud computing, and mobile sensor networks. He made pioneering contributions in the Internet video broadcast with a system, called Coolstreaming (the keyword had over 2,000,000 entries on Google), which was credited as the world first large-scale Peer-to-Peer live video streaming system, which spearheaded a momentum in video broadcast industry, with no fewer than a dozen successful companies adopting the same mesh-based pull streaming technique to deliver live media content to hundreds of millions of users in the world. He has been an editor or a guest editor for a dozen of IEEE journals and magazines. He was the Co-TPC Chair for IEEE INFOCOM 2004. He received his B. Eng. Degree in the Computer Science from Tsinghua University, Beijing, and his Ph.D. degree in the Electrical and Computer Engineering from University of Massachusetts at Amherst. He is a Fellow of IEEE.

**Baochun Li** is a professor with the Department of Electrical and Computer Engineering at the University of Toronto, Canada. He received the B.Engr. degree from the Department of Computer Science and Technology, Tsinghua University, China, in 1995 and the M.S. and Ph.D. degrees from the Department of Computer Science, University of Illinois at Urbana-Champaign, Urbana, in 1997 and 2000. He holds the Nortel Networks Junior Chair in Network Architecture and Services from October 2003 to June 2005, and the Bell Canada Endowed Chair in Computer Engineering since August 2005. His research interests include large-scale distributed systems, cloud computing, peer-to-peer networks, applications of network coding, and wireless networks. He was the recipient of the IEEE Communications Society Leonard G. Abraham Award in the Field of Communications Systems in 2000. In 2009, he was a recipient of the Multimedia Communications Best Paper Award from the IEEE Communications Society, and a recipient of the University of Toronto McLean Award. He is a member of ACM and a senior member of IEEE.

**Hongbo Jiang** received the B.S. and M.S. degrees from Huazhong University of Science and Technology, China. He received his Ph.D. from Case Western Reserve University in 2008. After that he joined the faculty of Huazhong University of Science and Technology as an associate professor. His research concerns computer networking, especially algorithms and architectures for high-performance networks and wireless networks. He is a member of IEEE.

## APPENDIX A KEY PARAMETERS IN THE BASIC DATACENTER MODEL

TABLE 1: Key Parameters in the Basic Datacenter Model.

| Notations | Definitions |
|---|---|
| $M$ | The number of heterogeneous applications $\mathcal{A}$ served by a datacenter |
| $N$ | The number of homogeneous servers $\mathcal{S}$ in a datacenter |
| $a_{ij}(t)$ | The running/idle state of the $i$-th VM on the $j$-th server in time slot $t$ |
| $A_i(t)$ | The number of arrived requests of each application $\forall i \in \mathcal{A}$ in time slot $t$ |
| $R_i(t)$ | The number of admitted requests of each application $\forall i \in \mathcal{A}$ in time slot $t$ |
| $R_{ij}(t)$ | The portion of requests of each application $\forall i \in \mathcal{A}$ that are routed to the $i$-th VM on the $j$-th server in time slot $t$ |
| $Q_{ij}(t)$ | The queue backlog of each application $\forall i \in \mathcal{A}$ on the $j$-th server in time slot $t$ |
| $P_j(t)$ | The power consumption of the $j$-th server in time slot $t$ |
| $d_i$ | The respective size of a request of each application $\forall i \in \mathcal{A}$ |
| $\lambda_i, r_i$ | The time average of $A_i(t)$ and $R_i(t)$ |
| $p_j, a_{ij}$ | The time average of $P_j(t)$ and $a_{ij}(t)$ |
| $V$ | Lyapunov control parameter |

## APPENDIX B PROOF OF LEMMA 1

*Proof:* Squaring Eq. (1) and Eq. (16) and leveraging the fact that $(\max[a-b,0]+c)^2 \le a^2+b^2+c^2-2a(b-c), \forall a,b,c \ge 0$, we have:

$$\begin{aligned} H_i^2(t+1)-H_i^2(t) &\le R_i^2(t)+\gamma_i^2(t)- \\ &\quad 2H_i(t)(R_i(t)-\gamma_i(t)), \quad (36) \\ Q_{ij}^2(t+1)-Q_{ij}^2(t) &\le a_{ij}^2(t)+d_i^2R_{ij}^2- \\ &\quad 2Q_{ij}(t)(a_{ij}(t)-d_iR_{ij}(t)) (37) \end{aligned}$$

Based on Eq. (36) and Eq. (37), we further have:

$$\begin{aligned} \Delta(\boldsymbol{\Theta}(t)) \le & \sum_{i=1}^{M}\frac{d_i^2}{2}\mathbb{E}\{R_i^2(t)+\gamma_i^2(t)|\boldsymbol{\Theta}(t)\} \\ & +\frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{N}\mathbb{E}\{a_{ij}^2(t)+d_i^2R_{ij}^2(t)|\boldsymbol{\Theta}(t)\} \\ & -\sum_{i=1}^{M}d_iH_i(t)\mathbb{E}\{R_i(t)-\gamma_i(t)|\boldsymbol{\Theta}(t)\} \\ & -\sum_{i=1}^{M}\sum_{j=1}^{N}Q_{ij}(t)\mathbb{E}\{a_{ij}(t)-d_iR_{ij}(t)|\boldsymbol{\Theta}(t)\}. \end{aligned}$$

Note that $d_i^2\sum_{i=1}^{M}(R_i^2(t)+\gamma_i^2(t))$ is bounded by $2\sum_{i=1}^{M}(d_iA_i^{max})^2$ and $\sum_{i=1}^{M}\sum_{j=1}^{N}(a_{ij}^2(t)+d_i^2R_{ij}^2(t))$ is bounded by $MN+\sum_{i=1}^{M}(d_iA_i^{max})^2$. By defining $B_1 \triangleq \frac{MN+3\sum_{i=1}^{M}(d_iA_i^{max})^2}{2}$, the above expression can be simplified to Eq. (21). □

## APPENDIX C PROOF OF THEOREM 1

The proof of Theorem 1 is presented in the following two aspects:

*First*, we prove the strong stability of the virtual queues $\mathbf{H}(t)$ and actual queues $\mathbf{Q}(t)$ under our **OCA** algorithm via mathematical introduction:

*Proof:* For $\mathbf{H}(t)$, suppose $H_i(t) \le V/d_i + A_i^{max}$ holds for $\forall i \in \mathcal{A}$ and $\forall t$. Initially, it holds when $t=0$, as all queues are initialized to empty (0). For time slot $t+1$, we consider the following three cases:

(1) If $V/d_i < H_i(t) \le V/d_i + A_i^{max}$: based on Eq. (26), we have $\gamma_i(t)=0$ under our **OCA** algorithm. Then, from Eq. (16), we have $H_i(t+1) \le H_i(t)+\gamma_i(t) \le V/d_i+A_i^{max}$, which indicates that Eq. (33) holds in this case.

(2) If $\frac{V}{d_i^2A_i^{max}+d_i} \le H_i(t) \le \frac{V}{d_i}$: then, $\gamma_i(t)=\frac{V}{d_i^2H_i(t)}-\frac{1}{d_i}$ under our **OCA** algorithm. Thus, we have:

$$\begin{aligned} H_i(t+1) &\le H_i(t)+\gamma_i(t) \\ &\le \frac{V}{d_i}+\frac{V}{d_i^2H_i(t)}-\frac{1}{d_i} \\ &\le \frac{V}{d_i}+\frac{V}{d_i^2\frac{V}{d_i^2A_i^{max}+d_i}}-\frac{1}{d_i} \\ &= \frac{V}{d_i}+A_i^{max}, \end{aligned}$$

which satisfies Eq. (33).

(3) If $H_i(t) < \frac{V}{d_i^2A_i^{max}+d_i}$: then, $\gamma_i(t)=A_i^{max}$ under our **OCA** algorithm. Thus, we have:

$$\begin{aligned} H_i(t+1) &\le \frac{V}{d_i^2A_i^{max}+d_i}+A_i^{max} \\ &\le \frac{V}{d_i}+A_i^{max}, \end{aligned}$$

which also satisfies Eq. (33). Above all, $H_i(t) \le \frac{V}{d_i}+A_i^{max}$ holds for all time slots.

Likewise, the strong stability of virtual queues $\mathbf{Q}(t)$ can be proved via mathematical introduction, based on control decisions (29) and (31) under our **OCA** algorithm. We omit the details for brevity. □

*Next*, we prove the time average profit bound Eq. (35) in Theorem 1, which requires the following Lemma 2:

***Lemma** 2:* (Existence of Optimal Randomized Stationary Policy): For any arrival rate vector $(\lambda_1, \lambda_2, ..., \lambda_M)$, there exists a randomized stationary control policy $\pi$ that chooses feasible control decisions $\gamma_i^\pi(t), R_i^\pi(t), R_{ij}^\pi(t)$ and $a_{ij}^\pi(t)$ for $\forall i \in \mathcal{A}, \forall j \in \mathcal{S}, \forall t$, independent of the current queue backlogs, and yields the following steady state values:

$$\begin{aligned} \gamma_i^\pi(t) &= r_i^*, \\ \mathbb{E}\{a_{ij}^\pi(t)\} &\ge \mathbb{E}\{d_iR_{ij}^\pi(t)\}, \\ \mathbb{E}\{R_i^\pi(t)\} &= r_i^*, \\ \mathbb{E}\{P_j^\pi(t)\} &= p_j^*. \end{aligned} \quad (38)$$

As Lemma 2 can be proved by applying similar techniques as [6], we omit the details for brevity. Based on

Lemma 2, we are able to prove the time average profit bound in Eq. (35) under our **OCA** algorithm as follows.

*Proof:* Let $\xi(t) = \sum_{i=1}^{M} g(\gamma_i(t)) - \beta \sum_{j=1}^{N} P_j(t)$ denote the profit of the datacenter in time slot $t$. Recall that our **OCA** algorithm seeks to choose those decision variables that can minimize the right-hand-side of Eq. (24) among all feasible decisions (including the control decision $\pi$ in Lemma 2) in each time slot, thus we have:

$$\begin{aligned}
&\Delta(\mathbf{\Theta}(t)) - V\mathbb{E}\{\xi(t)|\mathbf{\Theta}(t)\} \\
&\le B_1 - \sum_{i=1}^{M}\mathbb{E}\{Vg(\gamma_i^\pi(t)) - d_i^2 H_i(t)\gamma_i^\pi(t)|\mathbf{\Theta}(t)\} \\
&\quad - \sum_{i=1}^{M}\mathbb{E}\{d_i^2 H_i(t) R_i^\pi(t)|\mathbf{\Theta}(t)\} \\
&\quad + \sum_{i=1}^{M}\sum_{j=1}^{N}\mathbb{E}\{d_i R_{ij}^\pi(t) Q_{ij}(t)|\mathbf{\Theta}(t)\} \\
&- \sum_{i=1}^{M}\sum_{j=1}^{N} Q_{ij}(t)\mathbb{E}\{a_{ij}^\pi(t)|\mathbf{\Theta}(t)\} + V\beta\sum_{j=1}^{N}\mathbb{E}\{P_j^\pi(t)|\mathbf{\Theta}(t)\}.
\end{aligned}$$

Since the control decisions $\gamma_i^\pi(t), R_i^\pi(t), R_{ij}^\pi(t)$ and $a_{ij}^\pi(t)$ are independent of $\mathbf{\Theta}(t)$, taking expectations of both sides of the above yields:

$$\begin{aligned}
&\mathbb{E}\{L(\mathbf{\Theta}(t+1)) - L(\mathbf{\Theta}(t))\} - V\mathbb{E}\{\xi(t)\} \\
&\le B_1 - \sum_{i=1}^{M}\mathbb{E}\{Vg(\gamma_i^*(t)) - d_i^2 H_i(t)\gamma_i^*(t)\} \\
&- \sum_{i=1}^{M}\mathbb{E}\{d_i^2 H_i(t) R_i^*(t)\} + \sum_{i=1}^{M}\sum_{j=1}^{N}\mathbb{E}\{d_i R_{ij}^*(t) Q_{ij}(t)\} \\
&\quad - \sum_{i=1}^{M}\sum_{j=1}^{N} Q_{ij}(t)\mathbb{E}\{a_{ij}^*(t)\} + V\beta\sum_{j=1}^{N}\mathbb{E}\{P_j^*(t)\}. \quad (39)
\end{aligned}$$

Then, plugging (38) into the right-hand-side of (39) yields:

$$\mathbb{E}\{L(\mathbf{\Theta}(t+1)) - L(\mathbf{\Theta}(t))\} - V\mathbb{E}\{\xi(t)\} \le B_1 - \xi^*.$$

By summing the above over time slots $\tau \in \{0, 1, ..., t-1\}$ and then divide the result by $t$, we have:

$$\frac{\mathbb{E}\{L(\mathbf{\Theta}(t))\} - \mathbb{E}\{L(\mathbf{\Theta}(0))\}}{t} - \frac{V}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\{\xi(\tau)\} \le B_1 - \xi^*.$$

Rearranging terms and considering the fact that $L(\mathbf{\Theta}(t)) \ge 0$ and $L(\mathbf{\Theta}(0)) = 0$ yields:

$$\frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\{\xi(\tau)\} \ge \xi^* - \frac{B_1}{V}. \quad (40)$$

Since the function $g(*)$ is concave, we can have the following relationship according to *Jensen's inequality*:

$$g\Big(\frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\{\gamma_i(\tau)\}\Big) \ge \frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\{g(\gamma_i(\tau))\}.$$

By plugging this into Eq. (40) and letting $t \to \infty$, we have:

$$\liminf_{t\to\infty}\left\{\sum_{i=1}^{M} g(\gamma_i) - \beta\sum_{j=1}^{N} p_j\right\} \ge \xi^* - \frac{B_1}{V}. \quad (41)$$

As each virtual queue $H_i(t)$ is stable, $r_i \ge \gamma_i$ is satisfied according to Eq. (18). Then, plugging this inequality into Eq. (41) yields Eq. (35). □

## Appendix D Extended Model: Buffering the Requests

In this section, we first show that the Lyapunov-based optimization framework can be enhanced to accommodate this additional design choice in realistic datacenters. Then, we present the corresponding buffer-enhanced optimal control algorithm which is denoted as **BOCA**. Finally, we distinguish **BOCA** from **OCA**.

### D.1 Characterizing the Buffer

We assume that the aforementioned amount of admitted requests $R_i(t)$ are queued in the buffer for the $i$-th ($\forall i \in \mathcal{A}$) type of application before routed to the queues maintained by the $i$-th VM on each server, and let $L_i(t)$ denote the backlog of this $i$-th buffer at time slot $t$. For the $i$-th buffer queue, in time slot $t$, the amount of requests admitted to it is $R_i(t)$, while as in Sec. 3.1, $R_{ij}(t)$ is the number of requests that are routed to server $j$ from this $i$-th buffer queue. Hence, the queuing dynamics for the backlog $L_i(t)$ can be given by:

$$L_i(t+1) = L_i(t) - \sum_{j=1}^{N} R_{ij}(t) + R_i(t).$$

Initially, $L_i(0) = 0, \forall i \in \mathcal{A}$. In time slot $t$, for the $i$-th application, there are at most $L_i(t)$ requests available that can be routed to the VMs. Thus, the requests routing decisions $R_{ij}(t)$ must satisfy $\sum_{j=1}^{N} R_{ij}(t) \le L_i(t), \forall i \in \mathcal{A}, \forall t$. Note that, after introducing the buffer queues, the requests admission control and routing control can be decoupled to compute *independently* and *concurrently*. Further, to mitigate the bursty admitted arrivals, we impose the constraint $\sum_{j=1}^{N} R_{ij}(t) \le R_i^{\max}, \forall i \in \mathcal{A}, \forall t$, where $R_i^{\max}$ are suitably large positive constants and should satisfy $R_i^{\max} \le A_i^{\max}, \forall i \in \mathcal{A}$. Above all, we have

$$\sum_{j=1}^{N} R_{ij}(t) \le \min[L_i(t), R_i^{\max}], \quad \forall i \in \mathcal{A}, \forall t.$$

### D.2 Buffer-enhanced Optimal Control Algorithm

With our control framework based on Lyapunov optimization presented in previous sections, we let $\mathbf{L}(t) = (L_i(t))$ denote the vector of all the buffer queues, and let $\mathbf{\Theta}(t) = [\mathbf{Q}(t), \mathbf{H}(t), \mathbf{L}(t)]$ represent the combined matrix of all the queues in the datacenter system. Define the Lyapunov function as:

$$L(\mathbf{\Theta}(t)) = \frac{1}{2}\Big[\sum_{i=1}^{M} d_i^2 H_i^2(t) + \sum_{i=1}^{M} d_i^2 L_i^2(t) + \sum_{i=1}^{M}\sum_{j=1}^{N} Q_{ij}^2(t)\Big]. \quad (42)$$

Then, as in Sec. 5.1.2, to bound the drift-minus-profit expression Eq. (20), we rewrite inequality (24) as:

$$\begin{aligned}\Delta(\mathbf{\Theta}(t)) - V\mathbb{E}\Big\{\sum_{i=1}^{M} g(\gamma_i(t)) - \beta\sum_{j=1}^{N} P_j(t)|\mathbf{\Theta}(t)\Big\} \\ \le B_2 - \sum_{i=1}^{M}\mathbb{E}\{Vg(\gamma_i(t)) - d_i^2 H_i(t)\gamma_i(t)|\mathbf{\Theta}(t)\} \\ - \sum_{i=1}^{M}\mathbb{E}\{d_i^2 R_i(t)(H_i(t) - L_i(t))|\mathbf{\Theta}(t)\} \\ - \sum_{i=1}^{M}\sum_{j=1}^{N}\mathbb{E}\{d_i R_{ij}(t)(d_i L_i(t) - Q_{ij}(t))|\mathbf{\Theta}(t)\} \\ - \sum_{i=1}^{M}\sum_{j=1}^{N} Q_{ij}(t)\mathbb{E}\{a_{ij}(t)|\mathbf{\Theta}(t)\} + V\beta\sum_{j=1}^{N}\mathbb{E}\{P_j(t)|\mathbf{\Theta}(t)\}. \quad (43)\end{aligned}$$

Where the constant $B_2 \triangleq \frac{MN+3\sum_{i=1}^{M}(d_i A_i^{\max})^2+2\sum_{i=1}^{M}(d_i R_i^{\max})^2}{2}$, which is larger than the constant $B_1$, as a result of introducing the buffer queues.

Comparing inequality (43) to inequality (24), we observe that the decisions of *auxiliary variable selection* and *VM scheduling* are exactly the same as that **OCA** made in Sec. 5.2. While different from **OCA**, the decisions of requests *admission control* and *routing control* made by **BOCA** are now decoupled to the following two phases, which can be computed concurrently in a decentralized fashion:

#### *D.2.1* Requests Admission Control

For each application $i \in \mathcal{A}$ served by the datacenter, the request admission decisions $R_i(t)$ as illustrated in Fig. 1 can be decided by maximizing the term $\sum_{i=1}^{M} d_i^2 R_i(t)[H_i(t) - L_i(t)]$ in the right hand of inequality (43). Since the admission decisions $R_i(t)$ are independent from each other, this centralized maximization can be decoupled to be computed concurrently as follows:

$$\begin{aligned}\max_{R_i(t)} \quad & d_i^2 R_i(t)[H_i(t) - L_i(t)] \\ \text{s.t.} \quad & 0 \le R_i(t) \le A_i(t).\end{aligned}$$

As with problem (30) in Sec. 5.2.2, the above linear programming reduces to the *threshold-based* flow control decision of admitting $R_i(t) = A_i(t)$ if $H_i(t) > L_i(t)$, and $R_i(t) = 0$ otherwise.

#### *D.2.2* Requests Routing

For each application $i \in \mathcal{A}$ served by the datacenter, the routing decisions $(R_{i1}(t), R_{i2}(t), ..., R_{iN}(t))$ as illustrated in Fig. 1 can be decided by maximizing the term $\sum_{i=1}^{M}\sum_{j=1}^{N} d_i R_{ij}(t)[d_i L_i(t) - Q_{ij}(t)]$ in the right hand of inequality (43). Again, since the routing decisions $R_{ij}(t)$ of different applications are independent from each other, this centralized maximization can be decoupled to be computed concurrently as follows:

$$\begin{aligned}\max_{R_{ij}(t)} \quad & \sum_{j=1}^{N} d_i R_{ij}(t)[d_i L_i(t) - Q_{ij}(t)] \\ \text{s.t.} \quad & 0 \le \sum_{j=1}^{N} R_{ij}(t) \le \min[L_i(t), R_i^{\max}].\end{aligned}$$

The above weighted linear programming also leads to the join-the-shortest-queue policy below, as with the problem (28) in Sec. 5.2.2:

$$R_{ij}(t) = \begin{cases} \min[L_i(t), R_i^{\max}] & j = j_i^* \text{ and } d_i L_i(t) > Q_{ij_i^*}(t) \\ 0 & \text{otherwise} \end{cases}$$

where $j_i^* = \arg\min_{j\in\{1,2,...,N\}} Q_{ij}(t)$, *i.e.*, the queue of the $i$-th VM on server $j_i^*$ is the shortest queue among all the $N$ queues for the $i$-th type of application, which have been defined in Sec. 5.2.2.

## D.3 Comparing Tradeoffs: Buffer-enhanced or Not?

***Theorem** 2:* For arbitrary arrival rates of application requests $(\lambda_1(t), \lambda_2(t), ..., \lambda_M(t))$ (possibly exceeding the processing capacity of a datacenter), a datacenter using the **BOCA** algorithm with any $V \ge 0$ (the stability-profit tradeoff parameter defined in Sec. 5.1.1) can guarantee that all the actual and virtual queues are strongly stable over time slots:

$$\begin{aligned}H_i(t) &\le \frac{V}{d_i} + A_i^{\max}, \forall i \in \mathcal{A} \\ L_i(t) &\le \frac{V}{d_i} + 2A_i^{\max}, \forall i \in \mathcal{A} \\ Q_{ij}(t) &\le V + d_i(2A_i^{\max} + R_i^{\max}), \forall i \in \mathcal{A}, \forall j \in \mathcal{S}.\end{aligned}$$

Meanwhile, the gap between its achieved time averaged profit and the optimal profit $\xi^*$ is within $B_2/V$:

$$\liminf_{t\to\infty}\Big\{\sum_{i=1}^{M} g(r_i) - \beta\sum_{j=1}^{N} p_j\Big\} \ge \xi^* - \frac{B_2}{V}. \quad (44)$$

where $\xi^* = \sum_{i=1}^{M} g(r_i^*) - \beta\sum_{j=1}^{N} p_j^*$, $r_i^*$ and $p_j^*$ are the optimal solution to **Problem** (5).

The proof of Theorem 2 is similar to that of Theorem 1, so it's omitted here. However, note that, the constant $B_2$ is strictly larger than the constant $B_1$ in Theorem 1, and $B_2 - B_1 = \sum_{i=1}^{M}(d_i A_i^{\max})^2$. It follows that if the parameter $V$ is fixed, then lower bound of the expected profit achieved in Theorem 2 is smaller than that in Theorem 1. Hence, the buffers incur a loss of accuracy with the **BOCA** algorithm. Furthermore, the upper bound of $Q_{ij}(t)$ in Theorem 2 is also larger than that in Theorem 1, which implies that introducing the buffers aggravates the congestion of the queues $Q_{ij}$. Accordingly, the response delay of the requests will also be prolonged by Little's law, and this does not even count the delay triggered by the buffers.

*Why does buffering not only aggravate the congestion of the system, but also incur a loss of accuracy of the control algorithm?* This can be explained as follows: After introducing the buffer queues $L_i(t)(\forall i \in \mathcal{A})$ to the datacenter system, the corresponding Lyapunov function $L(\mathbf{\Theta}(t))$ is enlarged according to Eq. (19) and Eq. (42). However, the profit expression $\mathbb{E}\{\sum_{i=1}^{M} g(\gamma_i(t)) - \beta \sum_{j=1}^{N} P_j(t)|\mathbf{\Theta}(t)\}$ does not change at all after introducing the buffer queues, which can be understood as the weight of $\Delta(\mathbf{\Theta}(t))$ ascending after introducing the buffer queues according to the drift-minus-profit expression (20). As a consequence, **BOCA** leads to a profit loss compared to **OCA**.

## Appendix E Extended Model: Enforcing a Power Budget

As most real-world datacenters are operated within a certain *power budget* [5], it is important for datacenter operators to improve the *performance (dollar) per watt* [5] by achieving a desired performance level with an enforced power budget.

Specifically, we enforce an additional power budget constraint $p_j \le p_j^{av}, \forall j \in \mathcal{S}$ (in terms of the time average of normalized server power consumption) within the basic **Problem** (5), while still maintaining a performance requirement on the time averaged throughput of applications $r_i \ge r_i^{av}, \forall i \in \mathcal{A}$. This yields an extended optimization model:

$$\begin{aligned} \max \quad & \sum_{i=1}^{M} g(r_i) - \beta \sum_{j=1}^{N} p_j \qquad (45)\\ \text{s.t.} \quad & r_i^{av} \le r_i \le \lambda_i, \quad \forall i \in \mathcal{A},\\ & d_i r_i \le \sum_{j=1}^{N} a_{ij}, \quad \forall i \in \mathcal{A},\\ & p_j \le p_j^{av}, \quad \forall j \in \mathcal{S}. \end{aligned}$$

To accommodate the newly introduced constraints, we further define virtual queues $Z_i(t), X_j(t)$ for each application $i \in \mathcal{A}$ and each server $j \in \mathcal{S}$, respectively,

$$\begin{aligned} Z_i(t+1) &= \max[Z_i(t) - R_i(t), 0] + r_i^{av},\\ X_j(t+1) &= \max[X_j(t) - p_j^{av}, 0] + P_j(t). \end{aligned}$$

Then, with our control framework based on Lyapunov optimization presented in previous sections, we define the Lyapunov function as: $L(\mathbf{\Theta}(t)) = \frac{1}{2}\Big[\sum_{i=1}^{M} d_i^2 H_i^2(t) + \sum_{i=1}^{M} d_i^2 Z_i^2(t) + \sum_{j=1}^{N} X_j^2(t) + \sum_{i=1}^{M}\sum_{j=1}^{N} Q_{ij}^2(t)\Big]$. According to the definition of the Lyapunov drift $\Delta(\mathbf{\Theta}(t))$ in Sec. 5.1.1, the corresponding drift-minus-profit expression under the extended model can be rewritten as follows, where $B_3 = \frac{1}{2}[MN + 5\sum_{i=1}^{M}(d_i A_i^{\max})^2 + \sum_{i=1}^{M}(r_i^{av})^2 + \sum_{j=1}^{N}(p_j^{av})^2]$:

$$\begin{aligned} \Delta(\mathbf{\Theta}(t)) - V\mathbb{E}\Big\{\sum_{i=1}^{M} g(\gamma_i(t)) - \beta \sum_{j=1}^{N} P_j(t)|\mathbf{\Theta}(t)\Big\} \\ \le B_3 + \sum_{i=1}^{M} r_i^{av} Z_i(t) - \sum_{j=1}^{N} p_j^{av} X_j(t) \\ - \sum_{i=1}^{M} \mathbb{E}\{V g(\gamma_i(t)) - d_i^2 H_i(t)\gamma_i(t)|\mathbf{\Theta}(t)\} - \\ \sum_{i=1}^{M} \mathbb{E}\Big\{d_i^2 R_i(t)(H_i(t) + Z_i(t)) - \sum_{j=1}^{N} d_i R_{ij}(t) Q_{ij}(t)|\mathbf{\Theta}(t)\Big\} \\ - \sum_{j=1}^{N} \mathbb{E}\Big\{\sum_{i=1}^{M} Q_{ij}(t)a_{ij}(t) - P_j(t)(V\beta + X_j(t))|\mathbf{\Theta}(t)\Big\}. \quad (46) \end{aligned}$$

**Insight:** Based on Eq. (46), we can also design an online control algorithm for the extended **Problem** (45), named **EOCA**, which follows the skeleton of the four phases of control operations in Sec. 5.2. Specifically, the auxiliary variable selection of **EOCA** is the same as that of **OCA**, which implies that the newly introduced power and performance constraints do not affect this phase. Additionally, the routing policy of **EOCA** still follows the "Join the Shortest Queue" policy as in **OCA**. However, **EOCA** differs from **OCA** in the following aspects:

*First*, for the request admission control and routing phase of **EOCA**, the weights of $R_i(t), \forall i \in \mathcal{A}$ would be increased to $d_i^2(H_i(t) + Z_i(t)), \forall i \in \mathcal{A}$, after introducing the virtual queues $Z_i(t), \forall i \in \mathcal{A}$. This changes the admission control policy to become: if $d_i(H_i(t) + Z_i(t)) > Q_{ij_i^*}(t)$, $R_i(t) = A_i(t)$; otherwise, $R_i(t) = 0$.

*Second*, for the VM scheduling phase of **EOCA**, the term $\sum_{i=1}^{M} Q_{ij}(t)a_{ij}(t) - V\beta P_j(t)$ to be maximized is rewritten as $\sum_{i=1}^{M} Q_{ij}(t)a_{ij}(t) - V\beta(P_j(t) + X_j(t))$. This adjusts the greedy strategy in **OCA** to become: only when the growth in $\sum_{i=1}^{M} Q_{ij}(t)a_{ij}(t)$ exceeds the growth of $V\beta(P_j(t)+X_j(t))$ caused by running a certain VM, the VM will be preferentially scheduled to the running state.

*Third*, the queue backlogs of $Z_i(t)$, $X_j(t)$ and $Q_{ij}(t)$ are not guaranteed to be deterministically bounded under **EOCA**, though it can ensure the stability of these queues (*i.e.*, $p_j \le p_j^{av}, \forall j \in \mathcal{S}$ and $r_i \ge r_i^{av}, \forall i \in \mathcal{A}$) and a bounded gap between its achieved time averaged profit and the optimal profit for the extended **Problem** (45), as given by the following Theorem 3.

***Theorem** 3:* For arbitrary arrival rates of application requests $(\lambda_1(t), \lambda_2(t), ..., \lambda_M(t))$ (possibly exceeding the processing capacity of a datacenter), a datacenter can use **EOCA** algorithm with any $V \ge 0$ to guarantee the stability of queues $Q_{ij}(t), Z_i(t), X_j(t)$ over time slots. Meanwhile, the gap between its achieved time averaged profit and the optimal profit $\xi^\star$ for the extended **Problem**

(45) is within $B_3/V$:

$$\liminf_{t\to\infty}\left\{\sum_{i=1}^{M} g(r_i) - \beta\sum_{j=1}^{N} p_j\right\} \geq \xi^\star - \frac{B_3}{V}, \tag{47}$$

where $\xi^\star = \sum_{i=1}^{M} g(r_i^\star) - \beta\sum_{j=1}^{N} p_j^\star$, $r_i^\star$ and $p_j^\star$ are the optimal solution to the extended **Problem** (45), and $B_3$ is a finite constant parameter defined in Eq. (46).

Interested readers are referred to Appendix F for a complete proof of Theorem 3, available in the online supplemental material.

## Appendix F Proof of Theorem 3

Similar to Theorem 1, the proof of Theorem 3 also requires the proof of existence of optimal randomized stationary policy [6], which yields Eq. (38) in Lemma 2 as well as the following additional Eq. (48):

$$\begin{aligned}\mathbb{E}\{R_i^\pi(t)\} &\geq r_i^{av}, \\ \mathbb{E}\{P_j^\pi(t)\} &\leq p_j^{av}.\end{aligned} \tag{48}$$

Eq.(48) can also be proved by applying similar techniques as [6]. Then, we can prove Theorem 3 as follows:

*Proof:* Plugging Eq. (38) and Eq. (48) into the drift-minus-profit expression (46) yields:

$$\Delta(\mathbf{\Theta}(t)) - V\mathbb{E}\{\xi(t)|\mathbf{\Theta}(t)\} \leq B_2 - \xi^\star.$$

Summing the above over $\tau \in \{0, 1, ..., t-1\}$ for $t > 0$ yields:

$$\mathbb{E}\{L(\mathbf{\Theta}(t))\} - \mathbb{E}\{L(\mathbf{\Theta}(0))\} \leq (B_2 + \xi_{max} - \xi^\star)t,$$

where $\xi_{max} = \max_{\tau\in\{0,1,...,t-1\}}\{\sum_{i=1}^{M} g(\gamma_i(t)) - \beta\sum_{j=1}^{N} P_j(t)\}$, and $\xi_{max} \geq \xi^\star$. Using the definition of $L(\mathbf{\Theta}(t))$ and the fact that $L(\mathbf{\Theta}(0)) = 0$ yields:

$$\frac{1}{2}\mathbb{E}\left\{\sum_{i=1}^{M} d_i^2\Big(H_i^2(t) + Z_i^2(t)\Big) + \sum_{j=1}^{N} X_j^2(t) + \sum_{i=1}^{M}\sum_{j=1}^{N} Q_{ij}^2(t)\right\} \leq (B_2 + \xi_{max} - \xi^\star)t.$$

Then, we have $\mathbb{E}\{Q_{ij}(t)\} \leq \sqrt{2(B_2 + \xi_{max} - \xi^\star)t}, \forall i \in \mathcal{A}, \forall j \in \mathcal{S}$. Dividing this by $t$ with $t \to \infty$ gives:

$$\lim_{t\to\infty}\frac{\mathbb{E}\{Q_{ij}(t)\}}{t} \leq \lim_{t\to\infty}\sqrt{\frac{2(B_2 + \xi_{max} - \xi^\star)t}{t^2}} = 0.$$

Thus, all the queues $Q_{ij}(t)$ are stable. Similarly, the stability of queues $Z_i(t)$ and $X_j(t)$ can be also proved. Then, the proof of the time average profit bound in Eq. (47) is similar to that of Theorem 1. □

## Appendix G Performance Evaluation

We conduct simulations to evaluate our online control algorithm **OCA** and its extensions in a representative datacenter scenario. The scenario consists of 100 homogeneous servers, each of which hosts 10 VMs to serve 10 heterogeneous applications, respectively. Specifically, the requests from each application $i$ arrive according to a random process of mean rate $\lambda_i$, and different applications have different mean arrival rates and request sizes $d_i$ (Sec. 3.1) in Table 2. For each application $i$, we set its peak request arrival rate as $A_i^{\max} = 2\lambda_i$, its maximal routing rate as $R_i^{\max} = A_i^{\max}$, and the number of newly arrived requests in each time slot is assumed to be uniformly and randomly distributed within $[0, A_i^{\max}]$.

With such a setup, each application is being served by a total of 100 VMs across servers, with an allocated total processing capacity of 100 (in terms of the total queue service rates of these VMs as modeled in Sec. 3.1), according to the fair allocation policy of server capacity in Sec. 3.1. Observing the capacity demand of each application in terms of $d_i\lambda_i$ in Table 2, we find that the capacity demands of requests for applications $1-6$ are within the respective allocated processing capacity, while those for applications $7-10$ exceed the respective allocated processing capacity. We choose a typical setting of exponent parameter $v = 2$ and $\alpha = 0.5$ [20] for the normalized power function in Eq. (3), and an empirical value of parameter $\beta = 0.4$. The following simulations are carried out over 100,000 time slots.

TABLE 2: Request Arrival Rates and Sizes of Different Applications.

| App $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda_i(\times 10^3)$ | 2.5 | 2 | 3.5 | 2 | 3 | 2 | 2.75 | 2.4 | 2.6 | 2.8 |
| $d_i(\times 10^{-2})$ | 2 | 3 | 2 | 4 | 3 | 5 | 4 | 5 | 5 | 5 |
| $d_i\lambda_i(\times 10)$ | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |

***Verification of algorithm optimality.*** Fig. 3 plots the time averaged profit for different values of the control parameter $V$ under **OCA** algorithm. We observe that: (1) as the value of $V$ increases, the time averaged profit achieved improves significantly and converges to the maximum level for larger values of $V$. This quantitatively corroborates Theorem 1 in that **OCA** can approach the optimal profit with a diminishing gap $(1/V)$ (captured by Eq. (35)), which also implies a cost-effective tradeoff between power and performance unified by the profit objective (recall **Problem** (5)). However, such an improvement starts to diminish with excessive increases of $V$, which can adversely aggravate the congestion of queues in the system (captured by Eq. (19)). (2) Furthermore, compared to **OCA**, the variant algorithm **BOCA** with enhanced buffering facilities can achieve a slightly higher time averaged profit for smaller values of $V$, but such a profit gap (marked as BOCA–OCA) will diminish as $V$ grows.

***Examination of system stability.*** Fig. 4 plots the time average of queue congestion [6] captured by Eq. (19) and Eq. (42) for different values of $V$ under both **OCA**

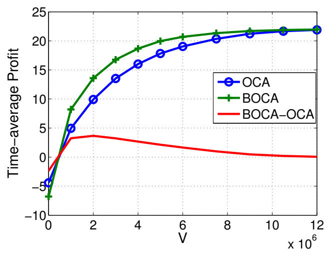

Fig. 3: Time averaged profit vs. different values of the control parameter $V$ under OCA and BOCA algorithms.

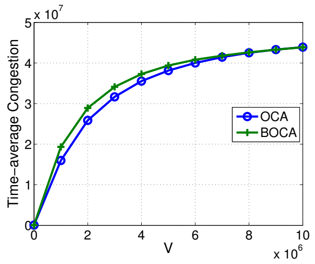

Fig. 4: Time averaged system congestion vs. different values of the control parameter $V$ under OCA and BOCA algorithms.

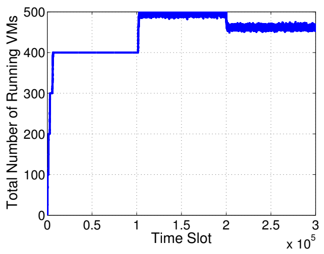

Fig. 5: The total number of running VMs in the system using OCA algorithm over all time slots.

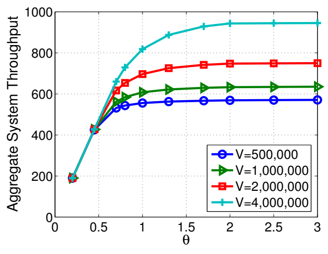

Fig. 6: Total system throughput vs. the multiplier $\theta$ of request arrival rates, with the admission control of the OCA algorithm.

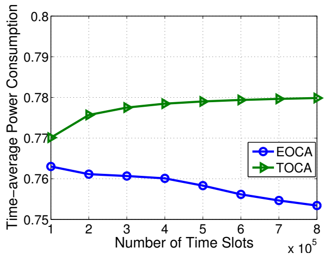

Fig. 7: Average power consumption (normalized) of servers vs. the length of simulation time, with EOCA against TOCA.

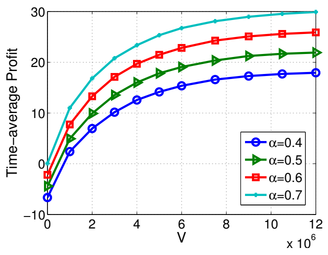

Fig. 8: Time averaged profit vs. different values of the control parameter $V$ under OCA with various values of parameter $\alpha$.

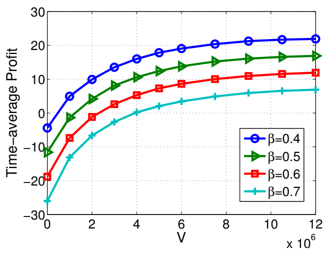

Fig. 9: Time averaged profit vs. different values of the control parameter $V$ under OCA with various values of parameter $\beta$.

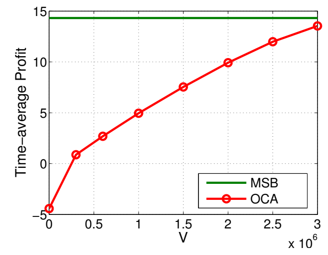

Fig. 10: Time averaged profit vs. different values of the control parameter $V$ under OCA and the maximized supreme bound (MSB).

and **BOCA**, respectively. With the growth of $V$, the time averaged system congestion with both algorithms increases. Along with Fig. 3, this reflects the tradeoff between system stability and profit maximization, as revealed in Sec. 5.1.1. In comparison, though **BOCA** gains a higher time averaged profit than **OCA** does, the former, on the other hand, incurs a higher level of system congestion.

***Online control in response to bursty request arrivals.*** Different from our simulations above with fixed mean arrival rates of application requests $(\lambda_1, \lambda_2, ..., \lambda_M)$, Fig. 5 plots the fluctuations with respect to the number of running VMs in the system under **OCA**, when the mean request arrival rates of applications vary in a bursty manner. We set the mean request arrival rate of each application as half of their original rate in Table 2 (*i.e.*, $0.5\lambda_i$) during the first $1/3$ of the simulation time. Then, the rates abruptly rise to $1.5\lambda_i$ in the next $1/3$ interval before dropping to the original $\lambda_i$ in the final $1/3$ of the simulation time. Fig. 5 clearly shows that, even under bursty and unpredictable arrivals of requests, **OCA** is able to quickly adapt to the varying demand by increasing or decreasing the number of running VMs.

***The effectiveness of admission control.*** We further examine the effects of admission control with **OCA** on the total system throughput (quantified as $\sum_{i=1}^{M} d_i r_i$) by varying the request arrival rates in Fig. 6. By tuning the mean request arrival rates of all applications by a factor of $\theta$ times the original setting in Table 2, we observe that as long as $\theta$ is relatively small, the total system throughput increases linearly with the increasing request arrival rates, despite different values of $V$. The rationale is that under such a condition, all application requests can be admitted by **OCA** according to the threshold-based admission control policy in Sec. 5.2. With even higher request arrival rates, the aggregate throughput achieved by the system under different values of $V$ will gradually become stabilized, which shows that **OCA** can prevent the system from being overwhelmed by excessive requests.

***The effectiveness of power budget enforcement.*** Fig. 7 compares the average power consumption of servers under our extended algorithm **EOCA** with both power budget enforcement and throughput requirement discussed in Sec. 7, against a counterpart algorithm with only throughput requirements (marked as TOCA). We set the time averaged throughput requirement of both **EOCA** and TOCA as $r_i^{av} = 2,000, \forall i \in \mathcal{A}$. The normalized time averaged power budget of **EOCA** for half of the servers in the system is set as $0.7$, while that of the other half is set as $0.8$. The other parameters are set as the same for both algorithms. As expected, we observe that, in the long run, **EOCA** indeed outperforms TOCA with respect to the reduction of server power consumption.

***The sensitivity of parameter*** $\alpha$. Fig. 8 plots the time averaged profit under **OCA** with different values of parameter $\alpha$. We observe that when $V$ is given, the time averaged profit increases with the growth of parameter $\alpha$. Recall that the term $1 - \alpha$ represents the normalized power consumption of an idle server (in Sec. 3.2.2), therefore, Fig. 8 indicates that power savings in datacenters can be achieved by reducing the power consumption of each server (by designing more power efficient hardware) in its idle state.

***The sensitivity of parameter*** $\beta$. Fig. 9 plots the time averaged profit under **OCA** with various values of parameter $\beta$. It can be seen that with the same level of $V$, a smaller value of parameter $\beta$ will lead to a time averaged profit that is more concave. Recall that the parameter $\beta$ is defined as $\beta = \text{Price} \cdot \text{PUE}$ in Sec. 3.2.3, where Price is the electricity market price and PUE is the power usage efficiency metric. Then, according to Fig. 9, an increase in the electricity market price will diminish the time averaged profit, while cost savings in datacenters can be achieved by improving the PUE (by cooling control).

***The exactness of the maximized supreme bound.*** We also examine the exactness of the maximized supreme bound of the time averaged profit achieved by the offline solution derived in Sec. 4, by comparing it to the time averaged profit achieved by **OCA** in Fig. 10. We observe that the maximized supreme bound is equal to $14.1$. Within the given scope of $V$, as the value of $V$ increases, the time averaged profit achieved by **OCA** improves significantly and never surpasses the maximized supreme bound. Though as $V$ continues to increase to large enough, the time averaged profit achieved by **OCA** will eventually outperform the maximized supreme bound, this is principally due to the fact that time slots in the simulation is limited, while our theoretical analysis is based on the assumption $V \to \infty$. Furthermore, from the perspective of datacenter operators, a larger $V$ will incur the worse congestion. Therefore, it is practical to regard the maximized supreme bound as a profit benchmark.