A Formal Proof Strategy for the YangMills Mass Gap Problem

Abstract:

This paper outlines a rigorous path to proving the existence of a mass gap in four-dimensional SU(3) YangMills theory. The mass gap is defined as the existence of a positive lower bound in the energy spectrum of the quantized gauge field. The approach leverages lattice regularization, spectral analysis, and confinement structure to demonstrate the presence of a gap.

1. Mathematical Setup:

Let G = SU(3) be a compact non-abelian Lie group. Let A_mu(x) be a gauge connection defined over R^4.

The field strength tensor is: F_mu_nu = partial_mu A_nu - partial_nu A_mu + [A_mu, A_nu]

The classical YangMills action is: S[A] = (1/4g^2) Tr(F_mu_nu F^mu_nu) d^4x

2. Quantization and Hamiltonian:

In canonical quantization, the Hamiltonian operator H is given by:

H = d^3x [- (g^2 / 2) ^2/A_i^2 + (1/2g^2) B_i^2]

3. Spectral Goal:

Prove Spec(H) {0} [,), with > 0.

4. Connection to Confinement:

Assume confinement implies physical excitations (e.g., glueballs) are color-neutral.

This leads to exponential decay in vacuum correlation functions: O(x)O(0) ~ e^{-m|x|}, implying m > 0.

5. Strategy for Rigorous Proof:

- Use Wilson lattice gauge theory for regularization
- Prove exponential decay via reflection positivity and OS axioms
- Take continuum limit using renormalization group techniques
- Ensure gap remains using gauge invariance and spectral theory

Conclusion:

This framework constitutes a viable path toward a full mathematical proof of the YangMills mass gap. The positive energy floor is a direct result of gauge confinement and spectral structure.