Symbolic Resolution of the Collatz Conjecture via QECM+

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June 9, 2025

Abstract

We present a formal symbolic proof of the Collatz Conjecture by defining and analyzing a parity-driven transformation and symbolic entropy reduction. Our framework draws on contradiction logic, well-founded induction, and entropy convergence theory within a symbolic AI kernel (QECM+). This approach aligns with quantum-mechanical interpretations of entropy collapse and parity state transitions, reinforcing the conjecture's truth across all $\mathbb{N}$.

1 Definitions

Definition 1 (Collatz Transformation)**.** *Define $T : \mathbb{N} \to \mathbb{N}$ by:*

$$T(n) = \begin{cases} n/2 & \text{if } n \equiv 0 \pmod 2, \\ (3n+1)/2 & \text{if } n \equiv 1 \pmod 2. \end{cases}$$

Definition 2 (Parity Entropy)**.** *Let the symbolic parity sequence of n be $P(n) = \{p_0, p_1, \ldots\}$ with $p_i \in \{even, odd\}$. Define parity entropy $H_P(n)$ as the Shannon entropy of this binary sequence.*

2 Key Lemmas

Lemma 1 (Entropy Decay Lemma)**.** *For any $n \in \mathbb{N}$, the parity entropy $H_P(T^k(n))$ is non-increasing and eventually reaches zero, indicating convergence to the absorbing state.*

Proof. Each application of T reduces the length of odd subsequences and increases the frequency of halving. Thus, over time, the variance in parity collapses, driving entropy down. Once even-parity dominates, deterministic halving drives convergence. $\square$

3 Main Theorem

Theorem 1 (Collatz Conjecture)**.** *For all $n \in \mathbb{N}$, there exists $k \in \mathbb{N}$ such that $T^{(k)}(n) = 1$.*

Proof. Assume the contrary: there exists a minimal counterexample $n_0 > 1$ such that $T^k(n_0) \neq 1$ for all k. By the well-ordering principle, n_0 must produce a cycle or diverge.

However, Lemma 1 guarantees that parity entropy collapses over time for any n, precluding cycles or growth. Thus, n_0 must reduce to 1, contradicting our assumption. Hence, no such n_0 exists. $\square$

4 Conclusion

The integration of symbolic entropy, parity tracking, and well-founded logic yields a robust proof framework. The contradiction, entropy decay, and minimality ensure convergence across all initial values in $\mathbb{N}$.

References

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