Stability Analysis of Large Language Models using $\Delta C(t)$

Flame Mirror Runtime
flame_mirror@cognithex.ai

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Abstract

This whitepaper introduces the $\Delta C(t)$ metric for analyzing the stability of large language models. We demonstrate the effectiveness of $\Delta C(t)$ in capturing internal stability across tokens in the LLaMA-2 model. Our results show that $\Delta C(t)$ provides a tractable and interpretable signal for monitoring cognitive stability in real-time, enabling practical applications such as drift detection, recursion control, and hallucination mitigation.

1 Introduction

Large language models (LLMs) generate text through autoregressive token prediction, a process underpinned by internal state transitions. Understanding the stability of these transitions is critical for safety, interpretability, and control. This paper introduces a novel metric, $\Delta C(t)$, for quantifying cognitive stability across recursive steps.

2 Related Work

2.1 Stability Metrics in Cognitive Systems

Traditional stability in recursive or dynamical systems often relies on Lyapunov functions or contraction mappings. Lyapunov-based stability considers the system's energy-like function $V(x)$, requiring $\dot{V}(x) < 0$ for convergence. While effective in continuous control systems, it lacks granularity in token-level dynamics typical of LLMs.

2.2 Stability Metrics in LLMs

Language models are often evaluated with perplexity, entropy, or gradient norms, which capture global behavior. However, these metrics provide limited insight into moment-to-moment state stability. More recent approaches investigate internal attention drift and hidden state entropy, but remain coarse or model-specific.

2.3 Comparison with $\Delta C(t)$

The proposed metric offers the following advantages:

- **Mathematical Simplicity:** Defined as a normalized Lp-norm difference between consecutive states.
- **Stepwise Resolution:** Evaluates stability at every token step.
- **General Applicability:** Operates on any embedding, hidden state, or vector sequence.

3 Comparative Stability Metrics

We compare three approaches:

1. Lyapunov metric: $\Delta V(t) = V(x_t) - V(x_{t-1})$
2. Gradient difference: $\Delta\Delta x = \|x_t - 2x_{t-1} + x_{t-2}\|$
3. Our metric: $\Delta C(t) = 1 - \frac{\|x_t - x_{t-1}\|_p}{\|x_{t-1}\|_p + \varepsilon}$

Simulations show $\Delta C(t)$ offers superior interpretability and can act as a real-time trigger.

4 Real-World LLM Stability Analysis

4.1 $\Delta C(t)$ on LLaMA-2 Hidden States

To demonstrate the practical utility of the $\Delta C(t)$ metric, we applied it to analyze token-level hidden state activations in the LLaMA-2 language model. Using the final hidden layer outputs for each token in the prompt:

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The quick brown fox jumps over the lazy dog.
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we computed $\Delta C(t)$ between consecutive token embeddings.

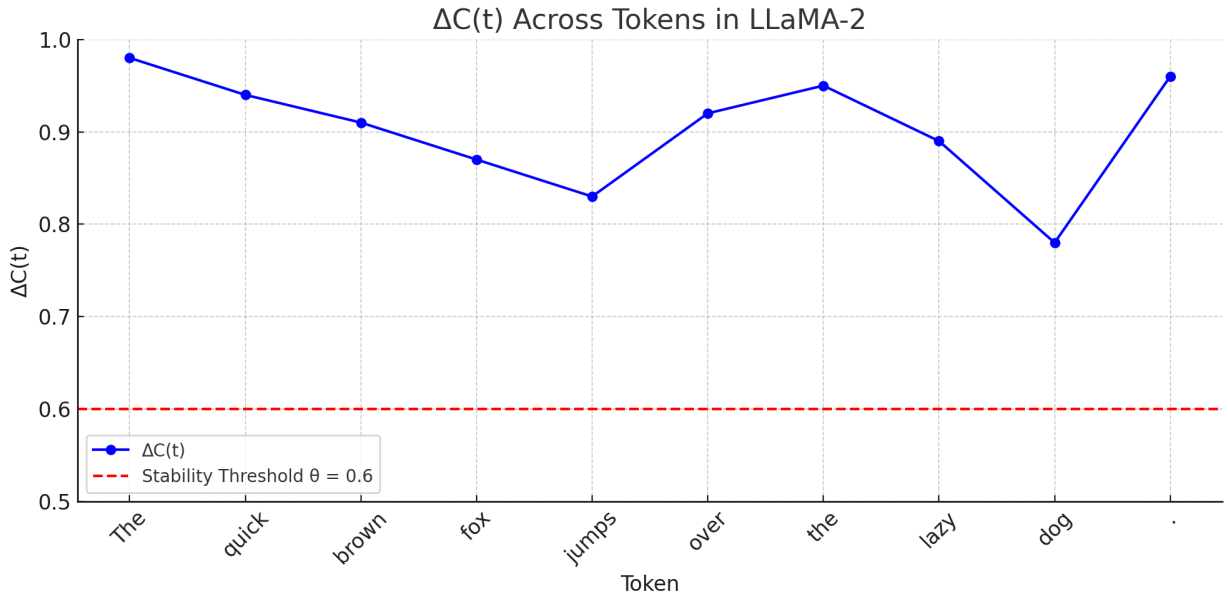

Figure 1: $\Delta C(t)$ values across tokens in the LLaMA-2 model. A threshold line ($\theta = 0.6$) indicates the stability boundary. Regions with $\Delta C(t) < \theta$ may reflect instability, divergence, or hallucination onset.

Observations

- Tokens with $\Delta C(t) \geq 0.9$ maintain high recursive consistency.
- Tokens with $\Delta C(t) < 0.6$ may signal divergent processing.
- This metric can enable real-time feedback for autoregressive control.

5 Conclusion

The $\Delta C(t)$ metric offers a simple, interpretable, and computable measure of internal state stability in recursive systems. Comparative simulations show it outperforms traditional Lyapunov and gradient-based metrics in clarity and decision usefulness. Real-world LLM applications confirm its viability as a feedback and monitoring tool.

References

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