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SPATIAL DENSITY APPROACH FOR MODELLING OF THE SPACE DEBRIS POPULATION

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This article proposes a continuum density approach for space debris modelling. The debris population in Low Earth Orbit (LEO) is represented through its density in semi-major axis, eccentricity and inclination. The time evolution of the density in orbital elements is modelled through the continuity equation. The perturbing effect of aerodynamic drag is included in the divergence term, while the effect of fragmentation can be seen as source term in the equation. The spatial density is then calculated from the orbital element density at each time. The proposed continuum method is used to analyse the evolution of the debris population in LEO; as initial condition the debris 2013 population is used. Then, the effect of a breakup event is superimposed onto the global population of space debris and its effect analysed; the fragment distribution caused by the breakup up of satellite DMSP-F13 is considered as test case scenario.

INTRODUCTION

The space surrounding our planet is densely populated by an increasing number of man-made space debris, most of which have been generated from the break-up of operational satellites, abandoned spacecraft or upper stages of launchers. Space debris is internationally recognised as a hazard to current and future space activities and space agencies are currently cooperating to identify appropriate and sustainable space debris mitigation measures.

The debris evolution in Low Earth Orbit (LEO) is dominated by the effects of the Earth's oblateness and the atmospheric drag, which is the only natural way debris objects are removed from their orbits, to re-enter and burn in the atmosphere. Long-term studies of the debris environment perform simulation of the space debris populations over 100 to 200 years to observe the effects of the growing space activities (e.g. launches), the uncertainty of the physical environment (e.g. atmosphere model, changes in the Earth's atmosphere due to the solar activities) and the spacecraft parameters (such as attitude, solar and drag coefficient, material deterioration), the consequences of fragmentation and explosion of inoperative objects and active satellites[1]. From the other side, these long-term studies aim at evaluating the efficacy of mitigation rules, such as passive disposal, collision avoidance manoeuvres, end-of-life guidelines, active debris removal, to reduce the risk to operating satellites and ensure the long term sustainability of space.

Surveys of the existing evolution models are available[2,3]. Most of these evolutionary debris models[4,5,6,7] use semi-analytical methods to propagate the dynamics under orbit perturbations and

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make some assumptions on natural phenomena, the future evolution space activities, compliance with mitigation guidelines, and debris interaction (e.g. criteria for collision, number of fragmentation events per year). To ensure the analyses are robust to these uncertainties and to overcome the absence of a complete set of experimental data through observations several Monte Carlo runs are used to consider an large number of evolution scenarios[8,9,10,4]. This dramatically increases the computational time and limits the variety of the possible analyses. To overcome this limitation, simplified were proposed based on a grid discretization of the debris population in altitude bins and a variational approach to allow for a quick evaluation of the debris evolution[11,12]. In some cases the computation of the collision risk for a target spacecraft is done starting from the number of objects in each bin through a Poisson distribution. Some other models are instead based on a fitting process of the deterministic high-fidelity models[13,14]. At the other end, a fully analytical model for LEO was proposed by McInnes[15], borrowing the use of the continuity equation from fluid dynamics and planetary science. In his work the evolution of debris is described through their spatial density. Letizia et al.[16] extended the continuity equation method to more than a single variable and applied it to the fragment clouds generated by a single collision or fragmentation event in space. The knowledge of the spatial density and the distribution of relative velocities (between the cloud and a target spacecraft) within the cloud was used to compute the collision probability, via the kinetic gas theory[17]. In this ways, maps of collision risk can be produced in a very short computational time; these maps can be used for evaluating the risk on operative spacecraft.

In this paper, we extend our previous research on the modelling of clouds through a continuum approach, which demonstrated to be an efficient way to propagate the density of particles in the space of orbital elements. We use a semi-analytical continuum density approach for debris modelling; the debris population in LEO is represented through its spatial density in orbital elements of semi-major axis, eccentricity and inclination. The time evolution of the density in orbital elements is modelled through the continuity equation that describes the debris flow evolution through a local representation via the Jacobian of the dynamics equations. With respect to existing particle-in-a-box approaches, where some representative objects are propagated to then rebuild the spatial density a posteriori, here an additional equation is added to the system ${dn}/{dt}$ that describes the time history of the density of space debris in the phase space, similarly to was was done by Nazarenko[18] and Smirnov et al.[19]. The proposed continuum method is validated though comparison with the actual debris evolution fully propagated element-wise by a semi-analytical propagator. As initial condition the debris population in January 2013 is used. Then, a source term is added to the continuum equation, which represent a fragmentation. New fragments are thus added onto the population and their effect is superimposed onto the whole debris population; the case of the breakup of DMSP-F13 is considered. This paper will briefly describes, in the first Section, the approach developed for the propagation of debris fragments. The second Section will detail the application of the density based method to the description of the debris density evolution. The method will be applied in the third Section to study the evolution of the debris population in 2013 provided by the European Space Agency.

DENSITY-BASED PROPAGATION FOR A CLOUD OF DEBRIS FRAGMENTS

The propagation method CiELO (debris Cloud Evolution in Low Orbits)[20] was developed to the aim of describing the evolution of space debris fragments resulting from breakup and collision in space and to assess the risk that they pose to operative spacecraft. Indeed, even in case of low intensity fragmentations, thousand of objects of dimension smaller than $5\,\mathrm{cm}$ are created. The

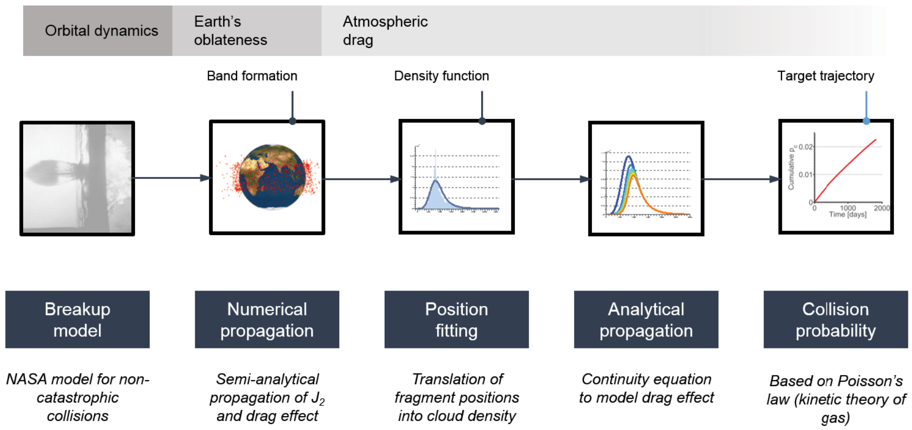

Figure 1: Schematics of the CiELO method.

inclusion of all these objects in long-term evolutionary studies would be prohibitive in terms of computational time. Within our approach the fragmentation cloud is described in terms of its spatial density, whose evolution in time under the effect of drag is obtained by applying the continuity equation, following the approach proposed by McInnes[15].

The breakup is modelled through the standard NASA breakup model[21,22] that gives the distribution of objects in terms of their relative velocity with respect to the nominal orbital velocity where the fragmentation took place (which is function of the kinetic energy contribution from the event) and the distribution of area-to-mass. The following evolution of the fragments in LEO is dominated by Earth's oblateness and atmospheric drag. In particular the effect of the Earth's oblateness causes the distribution of the anomaly of the ascending node and the anomaly of the perigee of the fragments orbits. This phenomenon takes place over a period of time in the order of months, until the objects form a band around the Earth with minimum and maximum latitude approximately equal to the inclination where the initial fragmentation took place. For the following phase of the evolution, the atmospheric drag can be considered as the main perturbation and it works as a natural sink mechanics which removes fragments from their original orbits. In this regime, the continuous method can be applied to find an analytical expression which describes the time evolution of the spatial density. Compared to formulation by[15], where the debris density is function of the radial distance from the Earth (r) only, the continuum method was extended to express the cloud density as function of semi-major axis (a) and eccentricity (e)[23]. Apart of giving an insight into the evolution of fragments as a whole, the proposed approach drastically reduces the computational time, allowing the study of many fragmentation scenarios. In Fig. 1 a schematic the CiELO method is shown.

Continuum approach

The evolution of the density is obtained through the continuity equation that describes the change in the density of a dispersed set starting from the knowledge of the velocities of the particles. In particular, if n represents the fragments density, the continuity equation can be written as

$$\frac{\partial n}{\partial t} + \nabla \bullet \mathbf{f} = \dot{n}^{+} - \dot{n}^{-} \tag{1}$$

where $\nabla \bullet \mathbf{f}$ models the forces acting on the system and accounts for *slow*/ continuous phenomena (such as orbit perturbations) and $\dot{n}^+ - \dot{n}^-$ represents the sources and the sinks of the system, so it can models *fast*/discontinuous events (e.g., the injection of new fragments due to launches). Once the initial condition for n is known, the continuity equation is used to obtain its evolution with time, with very low computational effort. The method was previously applied to describe the evolution of interplanetary dust[24,25], nano-satellites constellations[26] and high area-to-mass spacecraft[27]. The multi-dimension extension of the continuity equation Eq. 1 was fully derived in[16], following the approach by Gor'kavyi[28]. The idea is to work in the phase space of the orbital elements by simply writing the divergence in rectangular coordinates; this simplifies by far the mathematical formulation. In the last phase of the cloud evolution, when drag is the dominant factor, the semi-major axis and the eccentricity can be chosen as phase space variables. The vector **f** in Eq. 1 can be written as a vector field with two components, respectively, the rate of variation of the semi-major axis a and eccentricity e caused by drag:

$$\mathbf{f} = n(a, e; t) \begin{pmatrix} v_a(a, e; t) \\ v_e(a, e; t) \end{pmatrix} \tag{2}$$

The expressions of the velocities were further simplified by Letizia et al.[16], to obtain an explicit analytical solution:

$$\begin{cases} v_a = -\sqrt{\mu R_H} \frac{c_D A}{M} \rho_0 \exp\left(- \frac{a - R_H}{H} \right) f(R_H, \tilde{e}(a), H) \\ v_e = 0 \end{cases} \tag{3}$$

where $\tilde{e}(a)$ expresses a fixed reference value of the eccentricity for each value of the semi-major axis. The value of $\tilde{e}(a)$ was set starting from the initial distribution $n_0(a, e)$. Given the expression in Eq. 3, the continuity equation Eq. 1 can be solved adopting the method of characteristics obtaining the following expression for the density:

$$n(a, e; t) = n_0(a_i, e_i) \frac{v_a(a_i)}{v_a(a)} \tag{4}$$

with n_0 is the initial density at the band formation and a_i, e_i are two functions obtained by inverting the characteristics of the system at the initial time[16]:

$$a_i(a, t) = -H \log \left[\exp\left(\frac{a - R_H}{H} \right) + \varepsilon \left(R_H + \tilde{e}(a), H \right) \frac{\sqrt{R_H}}{H} t \right] \tag{5}$$

$$e_i(e, t) = e. \tag{6}$$

With Equation 4 the value of the density in the phase space at any time is known once the initial condition is given. As an example, Fig. 2 shows the value of the density in the phase space at the band formation and after 1000 days for a fragmentation at $700\,\mathrm{km}$.

Once the phase space density is known at any time t in any point of the domain, the spatial density can be retrieved from the phase space density by the transformations developed by Sykes[29] and Kessler[30].

DENSITY PROPAGATION FOR THE WHOLE DEBRIS POPULATION

This fully-analytical density propagation method described in the previous Section can be applied between 700 and $1000\,\mathrm{km}$[16], therefore it can also be employed to describe the whole LEO region.

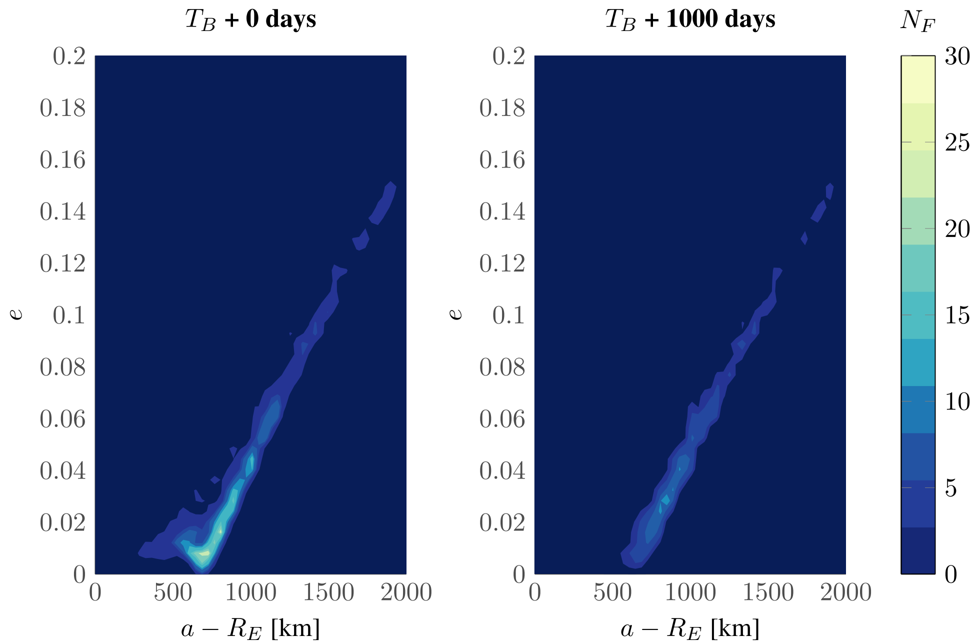

Figure 2: Visualisation of cloud density (in number of fragments) following a fragmentation at 700 km at the band formation (TB = 92 days) and after 1000 days [16].

The initial debris population at time t_0 is known. For each object in the population we know its type (i.e. mission related object, payload, debris, rocket body), the area-to-mass A/M and the orbit condition and orbital elements.

In time, the Earth's oblateness causes the debris' orbits to rotate with a precession rate that depends on the object's orbital parameters and that is, therefore, different among the objects in the population. We can expect that, after a certain time, the right ascension of the ascending node and the anomaly of the perigee will be equally distributed among objects of the same kind, due to differences in launching time and conditions.

As a first attempt in applying the proposed continuous technique to the global population of space debris, some simplifying assumptions will be made and they are justified here. The mean anomaly of the objects in long-term propagation studies is usually randomised, while many Monte Carlo runs are used to take into account differences in initial conditions, together with the uncertainties in the models [8,9,10,4]. In the continuum approach this is equivalent to assume that the mean anomaly of the object can be considered to be uniformly distributed across each orbit, therefore it can be removed as a variable from the continuity equation. The argument of perigee and the longitude of the ascending node are also randomised. Therefore, M, ω and Ω can be excluded from the dependence of $\mathbf{f}$ in Eq. 3. With the hypothesis of a non-rotating atmosphere, the dependence on the inclination i can also be removed. Under these assumption, $\mathbf{f}$ can be written as a vector field with two components in a and e as in Eq. 3.

Eq. 4 can be now used on each point of the initial grid of the a and e domain to compute how the phase-space density evolve over time. Note that, in this work, we are assuming that no further launches are recorded after time t_0, no collision among satellites are considered and no objects are removed from the population due to active debris removal. Each one of these terms will be added in a future extension of this work as the continuity equation can be also handle this cases though the term $\dot{n}^+ - \dot{n}^-$ in Eq. 1.

As said, the only perturbation on the debris population is due to the effect of drag. The effect will be different depending on the object's A/M. This is tackled by dividing the considered domain in A/M bins. Note that in Ref.[23] the multi-dimensional extension of the continuity equation was also applied to record the evolution of object with different area-to-mass; A/M, indeed can be added as a further phase-space variable, with a zero variation in time (i.e. the area-to-mass does not change over time). However, it was demonstrated that this did not result in an improvement in the computational time; for this reason, the binning approach is used here for the A/M.

Fragmentation as superimposition of the effects

Now, let's suppose that a fragmentation takes place at a given time t_f. The NASA breakup model[22,21] can be used to obtain the fragment distribution given the mass of the projectile an its impact velocity v_c. The method previously described can be now used to compute the evolution of the density of the fragment cloud for any time $t > t_f$. Therefore, at time t_f^+ the fragments resulting from the fragmentation event add up to the whole debris population. From a mathematical point of view, this means that the new objects are added to the $a - e$ grid and used to compute the new phase-space density at time t_f^+. The propagation is then continued to evaluate the effect of the fragmentation cloud on the whole debris population.

RESULTS

The debris population for January 2013 is used here as initial condition; this is limited to objects larger than $10\,\mathrm{cm}$. Only objects in LEO are considered with semi-major axis $a \leq 2000\,\mathrm{km}$. Figure 3 shows the initial distribution in semi-major axis and inclination. In Figure 4 instead, the initial debris distribution is shown in semi-major axis and eccentricity, distinguishing among the object types (MRO = Mission related Object, PL = payload, DEB = Debris, RB = Rocket Body). It can be noted that the objects in LEO larger than $10\,\mathrm{cm}$ have much lower eccentricity values than for the case of single breakups where the fragments have higher area-to-mass ratio. The grid considered for the computation is discretized with bin sizes of $20\,\mathrm{km}$ for semi-major axis and 0.0002 for eccentricity. The objects were divided in 15 bins of area to mass ratio in the rage of 0.001-13.35 m^2/kg such that each bin contains the same number of objects (the difference in number of objects in each bin is less than 5%).

Debris long-term evolution

The evolution of the debris population can be computed with the method proposed, the density in phase space is propagated with the continuity equation, then the spatial density is calculated. Figure 5 show the debris population evolution over 25 years. The results obtained with the analytical method (continuous line) are compared with the results obtained by a numerical method (dashed line) which integrates each single objects and reconstruct the spatial density though a binning approach. The continuous method is able to accurately follow the debris evolution. The aerodynamic drag acts differently depending on the value of the area-to-mass ratio of the objects as visible from Figure 6. This was already noted by McInnes[15], however here the real debris population is used

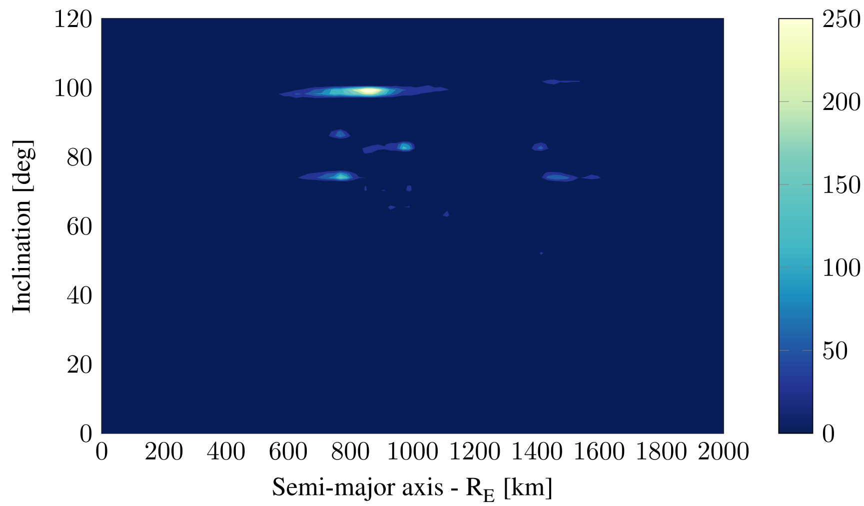

Figure 3: Initial debris distribution in LEO in semi-major axis and inclination.

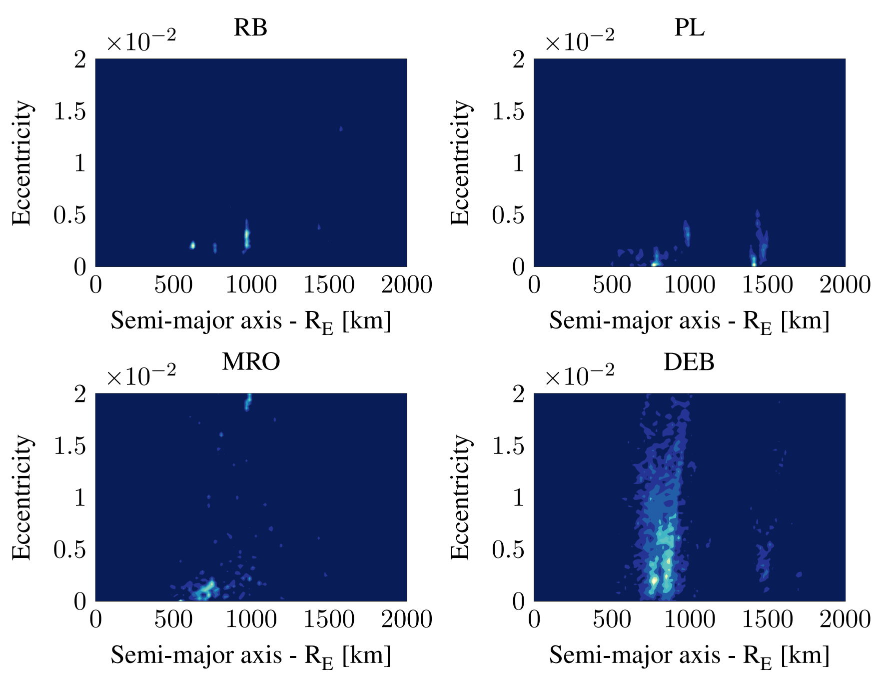

Figure 4: Initial debris distribution in LEO in semi-major axis and eccentricity, distinguishing among the object types.

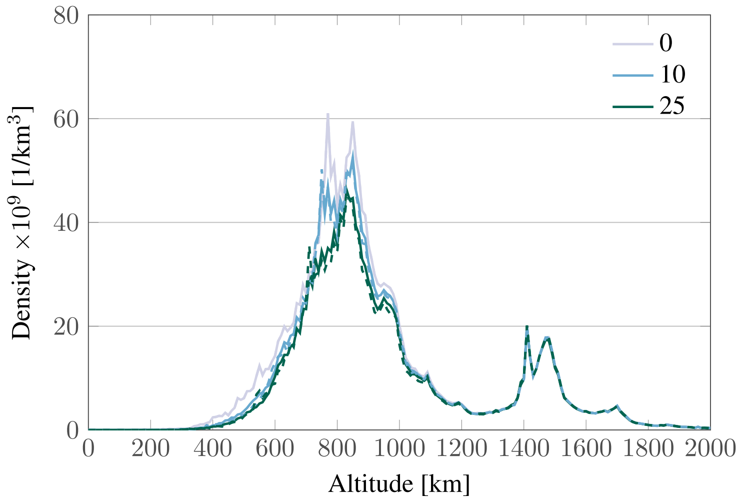

Figure 5: Debris population evolution over 25 years. Continuous line: analytical, dashed line: numerical

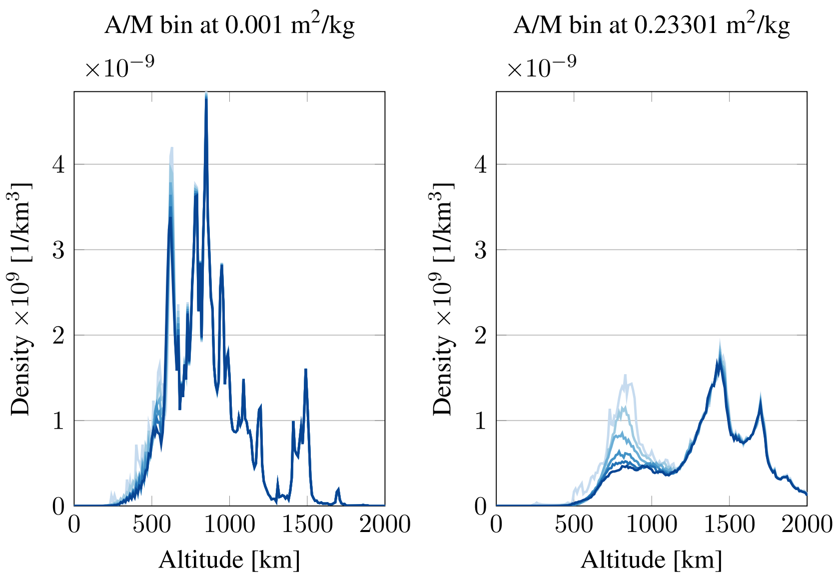

Figure 6: Debris population evolution over 25 years (analytical propagation). The evolution of two different area-to-mass bins is shown.

and the propagation is performed in a and e, not only in r. The method can also be applied to longer term studied of 100 years as shown in Figures 7 and 8.

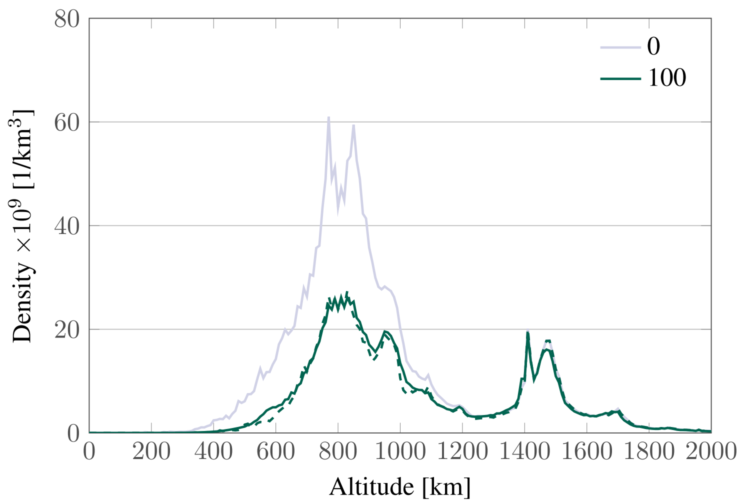

Figure 7: Debris population evolution over 100 years. Continuous line: analytical, dashed line: numerical.

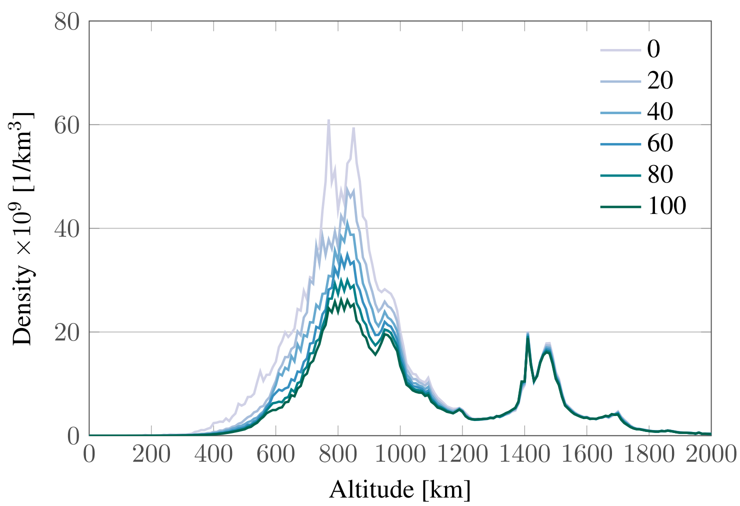

Figure 8: Debris population evolution over 100 years continuous line with analytical propagation.

Effect of a fragmentation

A breakup on an 800 km Sun-synchronous orbit is now considered and the feedback of the event on the whole debris population assessed. For the fragments distribution, the case of the fragmentation of DMSP-F13 on 25 February 2015 is used a test case scenario. The object had an orbit of perigee and apogee altitude of respectively 842 and $856\,\mathrm{km}$ and a mass of $830\,\mathrm{kg}$. The proposed method can be used for a quick estimation of the future development of space debris population also under the effects of a fragmentation event. Figure 9 shows the spatial density of the space debris at the moment the fragmentation takes place and a zoom on the altitude where the fragmentation happens, where a distinct jump in spatial density can be recognised. The light blue line shows the initial population (January 2013), the darker blue line the debris evolution without fragmentation (in February 2015) and the red line adds up the fragmentation (in February 2015). The effect of the fragmentation after 20 year since January 2013 can be seen in Figure 10, a density difference of 6.5% at the altitude of the fragmentation at time t_f^+ is diluted over time and in 2038 is decreased to 3.8% (visible in the peak at altitude of around 750 km). So, over time the effect of drag level out the cloud peak to the envelope of the whole debris population.

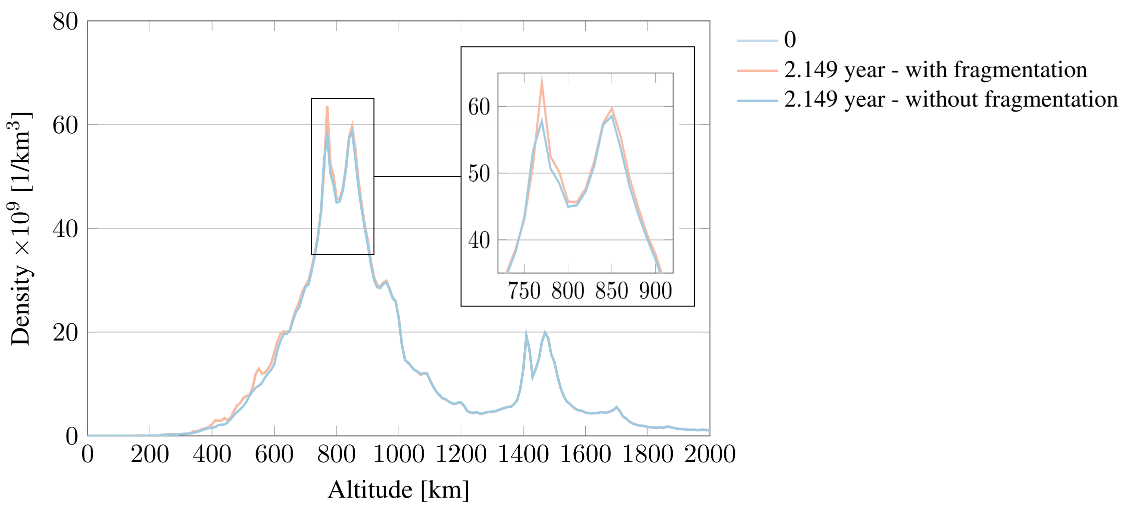

Figure 9: Debris population + DMSP-F13 fragmentation after 2.149 years.

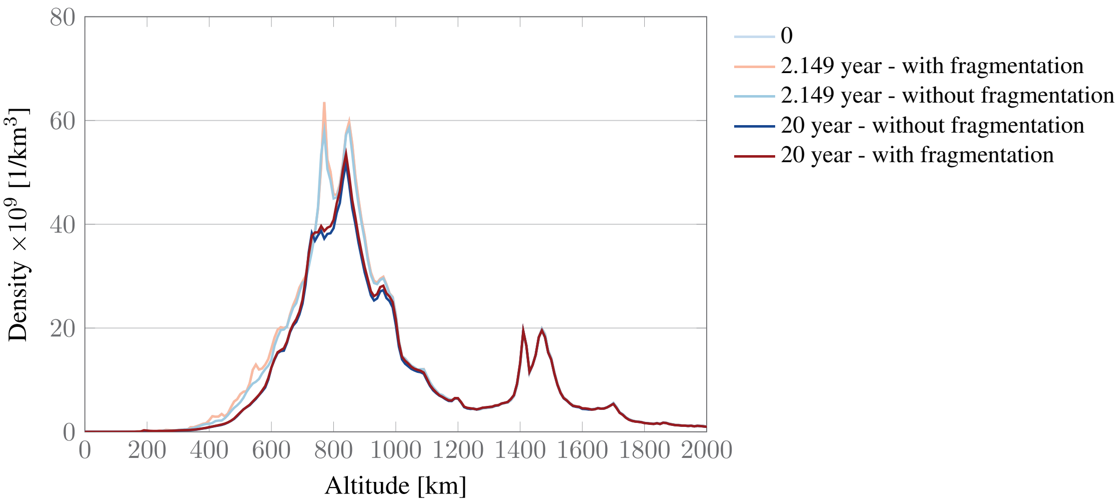

Figure 10: Debris population + DMSP-F13 fragmentation over 20 years from t_0.

CONCLUSIONS

The modelling of the contribution of small debris fragments to the collision risk requires methods that do not rely on the propagation of single objects; in this case, density-based models offer an interesting alternative. A method based on the continuity equation, previously developed to describe the evolution of the density of debris clouds produced by single fragmentations, is here extended to the study of the global debris population. The results presented show the feasibility of this approach for such applications with long term propagation. The accuracy of the method is demonstrated over long time span of 100 years so it is suitable for environment evolution studies. The effect of a fragmentation on the background population can be easily modelled though the superimposition of the effects. Future work will be devoted to complete model the sources and sinks of the debris population system and to measure the collision risk for spacecraft considering also background population. Such an approach has a potential application to perform collision risk analysis for small satellites.

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