Foreground cleaning in power-spectrum space

John Locke

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1 Foreground model

Let $\mathbf{a}^i_{\ell m}$ be the SHT of the i-th channel map with frequency ν_i. We represent it as a vector with components T, E and B, and we model it as:

$$\mathbf{a}^i_{\ell m} = \sum_\alpha \mathbf{b}^\alpha_{\ell m}\, f^i_\alpha, \tag{1}$$

where α runs over three different components: CMB ($\alpha = c$), synchrotron ($\alpha = s$) and dust ($\alpha = d$). f^i_α is the frequency evolution of the α-th component in the i-th frequency channel, which we model as:

$$f_c(\nu) = \frac{x^2\, e^x}{(e^x-1)^2}, \quad f_s(\nu) = \left(\frac{\nu}{\nu^s_*}\right)^{\beta_s}, \quad f_d(\nu) = \left(\frac{\nu}{\nu^d_*}\right)^{\beta_d+1} \frac{\exp[h\nu^d_*/(k_B T_d)]-1}{\exp[h\nu/(k_B T_d)]-1}, \tag{2}$$

where $x \equiv h\nu/(k_B\, T_{\rm CMB})$.

The cross-power spectrum of all channels is then:

$$\langle \mathbf{a}^i_{\ell m}(\mathbf{a}^j_{\ell m})^\dagger\rangle \equiv \mathsf{C}^{ij}_\ell = \sum_{\alpha_1,\alpha_2} \mathsf{C}^{\alpha_1,\alpha_2}_\ell\, f^i_{\alpha_1} f^j_{\alpha_2}, \tag{3}$$

where we model the cross-power spectrum of the different components as:

$$\mathsf{C}^{c,c}_\ell = \text{bandpowers or parametrized model (CAMB)}, \tag{4}$$

$$\mathsf{C}^{c,\alpha_2\neq c}_\ell = \mathsf{C}^{\alpha_1\neq c,c}_\ell = 0, \tag{5}$$

$$\mathsf{C}^{\alpha_1=(s,d),\alpha_2=(s,d)}_\ell = \text{bandpowers or parametrized model (power law?)}. \tag{6}$$

1.1 Degrees of freedom

Let there be N_ν frequency channels and N_c components, N_p polarization channels, and assume that the power spectra have been measured in N_ℓ bandpowers. The total number of degrees of freedom would be

$$\#\text{dof} = N_\ell\, [N_\nu N_p (N_\nu N_p + 1)/2 - (\#\text{spec}) - (\#\text{param CMB}) - N_c \times 2] \tag{7}$$

if one uses parametrized models for the component power spectra, and

$$\#\text{dof} = N_\ell\, [N_\nu N_p (N_\nu N_p + 1)/2 - N_c N_p (N_c N_p + 1)/2 - (\#\text{spec})] \tag{8}$$

if bandpowers are used instead. In the latter case, the free parameters of the model are:

$$\theta \equiv \beta_s,\, \beta_d,\, T_d,\, \mathsf{C}^{cc}_{\ell_k},\, \mathsf{C}^{ss}_{\ell_k},\, \mathsf{C}^{sd}_{\ell_k},\, \mathsf{C}^{dd}_{\ell_k}. \tag{9}$$

2 Likelihood

We will assume (wrongly) a Gaussian likelihood for the parameters:

$$\chi^2 \equiv -2\log p(\theta|\hat{\mathsf{C}}^{ij}_{\ell_k}) = \sum_{k,k'}\sum_{a,b} [\mathsf{C}^a_{\ell_k}(\theta) - \hat{\mathsf{C}}^a_{\ell_k}]\, (M^{-1})^{a,b}_{k,k'}\, [\mathsf{C}^b_{\ell_{k'}}(\theta) - \hat{\mathsf{C}}^b_{\ell_{k'}}] \tag{10}$$

where we have labelled each bandpower by an index k and all quantities labelled by ℓ_k are averaged over the corresponding bandpower. The indices a and b label here all possible combinations of frequency and polarization channels. Here M is the covariance matrix of the power spectrum, given below, and $\mathsf{C}_\ell(\theta)$ is the model power spectra as described above.

2.1 Covariance

Here we will assume a theoretical Gaussian covariance matrix, given by:

$$M_{k,k'}^{(iP_i,jP_j),(pQ_p,qQ_q)} \equiv \left\langle \Delta\hat{\mathsf{C}}_{\ell_k}^{iP_i,jP_j}\, \Delta\hat{\mathsf{C}}_{\ell_{k'}}^{pQ_p,qQ_q} \right\rangle = \frac{\left[\mathsf{C}_{\ell_k}^{iP_i,pQ_p}\mathsf{C}_{\ell_k}^{jP_j,qQ_q} + \mathsf{C}_{\ell_k}^{iP_i,qQ_q}\mathsf{C}_{\ell_k}^{jP_j,pQ_p}\right]}{(2\ell_k+1)\Delta\ell_k f_{\rm sky}}\delta_{kk'}, \qquad (11)$$

where $\Delta\ell_k$ is the bandpower width, and P_i, Q_i label the polarization channels. We have assumed that the bandpowers are wide enough so that correlations between different bandpowers caused by the incomplete sky coverage are negligible, and that its effects can be encapsulated in the factor $f_{\rm sky}$. We will estimate the covariance matrix using the power spectra measured from the data.