Expressions for the Limber approximation

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1 Full-sky expressions

The angular power spectrum between two contributions is:

$$C_\ell^{ij} = 4\pi \int_0^\infty \frac{dk}{k}\,\mathcal{P}_\Phi(k)\Delta_\ell^i(k)\Delta_\ell^j(k). \tag{1}$$

The expressions for density, RSD, magnification, lensing convergence and CMB lensing are:

$$\Delta_\ell^D(k) = \int dz p_z(z) b(z) T_\delta(k,z) j_\ell(k\chi(z)) \tag{2}$$

$$\Delta_\ell^{RSD}(k) = \int dz \frac{(1+z)p_z(z)}{H(z)} T_\theta(k,z) j_\ell''(k\chi(z)) \tag{3}$$

$$\Delta_\ell^M(k) = -\ell(\ell+1)\int \frac{dz}{H(z)} W^M(z) T_{\phi+\psi}(k,z) j_\ell(k\chi(z)), \tag{4}$$

$$\Delta_\ell^L(k) = -\frac{\ell(\ell+1)}{2}\int \frac{dz}{H(z)} W^L(z) T_{\phi+\psi}(k,z) j_\ell(k\chi(z)), \tag{5}$$

$$\Delta_\ell^{IA}(k) = -\sqrt{\frac{(\ell+2)!}{(\ell-2)!}}\int dz\, p_z(z)\, b_{\rm IA}(z)\, T_\delta(k,z)\, \frac{j_\ell(k\chi(z))}{(k\chi(z))^2}, \tag{6}$$

$$\Delta_\ell^C(k) = \frac{\ell(\ell+1)}{2}\int_0^{\chi_*} d\chi \frac{\chi_*-\chi}{\chi\chi_*} T_{\phi+\psi}(k,z) j_\ell(k\chi), \tag{7}$$

where

$$W^M(z) \equiv \int_z^\infty dz' p_z(z') \frac{2-5s(z')}{2}\frac{\chi(z')-\chi(z)}{\chi(z')} \tag{8}$$

$$W^L(z) \equiv \int_z^\infty dz' p_z(z') \frac{\chi(z')-\chi(z)}{\chi(z')} \tag{9}$$

2 Limber approximation

The Limber approximation is

$$j_\ell(x) \simeq \sqrt{\frac{\pi}{2\ell+1}}\,\delta\left(\ell+\frac{1}{2}-x\right). \tag{10}$$

Thus for each k and ℓ we can define a radial distance $\chi_\ell \equiv (\ell+1/2)/k$

3 Expressions in the Limber approximation

The expressions above can be written as follows in the Limber approximation. First, the power spectrum can be rewritten as

$$C_\ell^{ij} = \frac{2}{2\ell+1}\int_0^\infty dk\, P_\delta\,(k, z=0)\, \tilde{\Delta}_\ell^i(k)\tilde{\Delta}_\ell^j(k). \tag{11}$$

where

$$\tilde{\Delta}_\ell^{D}(k) = p_z(\chi_\ell)\, b(\chi_\ell)\, D(\chi_\ell)\, H(\chi_\ell) \tag{12}$$

$$\tilde{\Delta}_\ell^{RSD}(k) = \frac{1+8\ell}{(2\ell+1)^2}\, p_z(\chi_\ell)\, f(\chi_\ell)\, D(\chi_\ell)\, H(\chi_\ell) - \frac{4}{2\ell+3}\sqrt{\frac{2\ell+1}{2\ell+3}} p_z(\chi_{\ell+1})\, f(\chi_{\ell+1})\, D(\chi_{\ell+1})\, H(\chi_{\ell+1}) \longrightarrow 0 \tag{13}$$

$$\tilde{\Delta}_\ell^{ISW}(k) = \frac{3\Omega_{M,0}H_0^2}{k^2} H(\chi_\ell) g(\chi_\ell)\, [1 - f(\chi_\ell)] \tag{14}$$

$$\tilde{\Delta}_\ell^{M}(k) = 3\Omega_{M,0}H_0^2 \frac{\ell(\ell+1)}{k^2}\, \frac{D(\chi_\ell)}{a(\chi_\ell)\chi_\ell} W^M(\chi_\ell) \longrightarrow 3\Omega_{M,0}H_0^2\, \frac{\chi_\ell D(\chi_\ell)}{a(\chi_\ell)} W^M(\chi_\ell) \tag{15}$$

$$\tilde{\Delta}_\ell^{L}(k) = \frac{3}{2}\Omega_{M,0}H_0^2 \sqrt{\frac{(\ell+2)!}{(\ell-2)}}\, \frac{1}{k^2}\, \frac{D(\chi_\ell)}{a(\chi_\ell)\chi_\ell} W^M(\chi_\ell) \longrightarrow \frac{3}{2}\Omega_{M,0}H_0^2\, \frac{\chi_\ell D(\chi_\ell)}{a(\chi_\ell)} W^M(\chi_\ell) \tag{16}$$

$$\tilde{\Delta}_\ell^{IA}(k) = p_z(\chi_\ell)\, b_I(\chi_\ell)\, D(\chi_\ell)\, H(\chi_\ell) \frac{\sqrt{(\ell+2)(\ell+1)\ell(\ell-1)}}{(\ell+1/2)^2} \tag{17}$$

$$\tilde{\Delta}_\ell^{C}(k) = \frac{3}{2}\Omega_{M,0}H_0^2 \frac{\ell(\ell+1)}{k^2}\, \frac{D(\chi_\ell)}{a(\chi_\ell)\chi_\ell}\, \frac{\chi_* - \chi_\ell}{\chi_*} \Theta(\chi_\ell - \chi_*) \longrightarrow \frac{3}{2}\Omega_{M,0}H_0^2\, \frac{\chi_\ell D(\chi_\ell)}{a(\chi_\ell)}\, \frac{\chi_* - \chi_\ell}{\chi_*} \Theta(\chi_\ell - \chi_*) \tag{18}$$