

**For problems 1 - 5:** Show your calculations to receive partial credit. Show your results to three significant digits.

**1.** Given the unit weight ( $w_c$ ) of a sample of concrete is  $120 \text{ pcf}$  ( $\text{lb}/\text{ft}^3$ ) and its strength ( $f'_c$ ) is  $3,000 \text{ psi}$ . What is the expected Modulus of Elasticity using  $E_c = w_c^{1.5} 33 \sqrt{f'_c}$  ?

$$E_c = (120 \text{ pcf})^{1.5} 33 \sqrt{3,000 \text{ psi}}$$

$$E_c = 2,376,000 \text{ psi}$$

$$E_c = 2.38 \times 10^6 \text{ psi}$$

**2.** If the concrete in problem 1 had a unit weight of  $145 \text{ pcf}$  what would its Modulus of Elasticity be?

$$E_c = (145 \text{ pcf})^{1.5} 33 \sqrt{3,000 \text{ psi}}$$

$$E_c = 3,155,924 \text{ psi}$$

$$E_c = 3.16 \times 10^6 \text{ psi}$$

**3.** Given a sample of normal weight concrete with a strength of  $4,000 \text{ psi}$ . What would the modulus of Elasticity be using  $E_c = 57,000 \sqrt{f'_c}$  ?

$$E_c = 57,000 \sqrt{4,000 \text{ psi}}$$

$$E_c = 3,604,997 \text{ psi}$$

$$E_c = 3.60 \times 10^6 \text{ psi}$$

**4.** If you needed to have normal weight concrete with a Modulus of Elasticity of  $3.00 \times 10^6 \text{ psi}$  what strength ( $f'_c$ ) would you need? (Not what you would ask for.)

$$\begin{array}{l|l|l} 57,000 \sqrt{f'_c} = E_c & f'_c = \left( \frac{3.00 \times 10^6 \text{ psi}}{57,000} \right)^2 & \text{Rounding to the next 100 psi...} \\ f'_c = \left( \frac{E_c}{57,000} \right)^2 & f'_c = 2,770 \text{ psi} & f'_c = 2,800 \text{ psi} \end{array}$$

**5.** If instead of normal weight concrete you wanted to use a light weight concrete ( $w_c = 115 \text{ pcf}$ ) but you still wanted to have a Modulus of Elasticity of  $3.00 \times 10^6 \text{ psi}$  what strength ( $f'_c$ ) would you need? (Not what you would ask for.)

$$\begin{array}{l|l|l} w_c^{1.5} 33 \sqrt{f'_c} = E_c & f'_c = \left( \frac{3.00 \times 10^6 \text{ psi}}{(115 \text{ pcf})^{1.5} 33} \right)^2 & \text{Rounding to the next 100 psi...} \\ f'_c = \left( \frac{E_c}{w_c^{1.5} 33} \right)^2 & f'_c = 5,434 \text{ psi} & f'_c = 5,500 \text{ psi} \end{array}$$

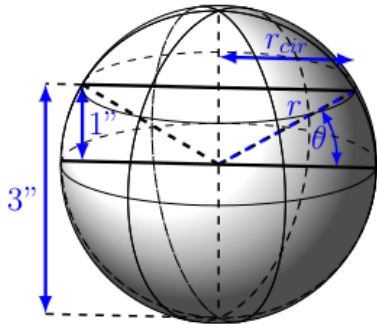
**For problems 6 - 7:** Given: A company you are working for has decided to put up a Christmas tree for the kids this year and its your job to help the children build the decorations. Since it is a construction

company the tree is going to be made from A706 rebar (the only kind to weld) and the decorations are going to be 4" diameter balls of concrete. A special mix of light weight concrete which has a slump of practically 12" will be used to fill the clear 96 plastic molds. To make the filling controllable a coworker brought in an old aquarium pump to fill the balls with. The pump can supply the mixed concrete at a rate of  $34.9 \frac{\text{gal}}{\text{hr}}$  ( $7.48052 \text{ gal} = 1 \text{ ft}^3$ ).

6. How long will it take for a single ball to be filled?

$$\begin{array}{l|l|l} \text{Vol} = \frac{4}{3}\pi r^3 & \text{Rate} = 34.9 \frac{\text{gal}}{\text{hr}} \times \frac{1 \text{ ft}^3}{7.48052 \text{ gal}} \times \frac{(12'')^3}{(1 \text{ ft})^3} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} & t = 14.964 \text{ s} \\ \text{Vol} = \frac{4}{3}\pi (2'')^3 = 33.51 \text{ in}^3 & \text{Rate} = 2.239 \frac{\text{in}^3}{\text{s}} & \text{with 3 significant digits} \\ \text{Rate} = \frac{\text{Vol}}{t} \therefore t = \frac{\text{Vol}}{\text{Rate}} & t = \frac{33.51 \text{ in}^3}{2.239 \frac{\text{in}^3}{\text{s}}} & t = 15.0 \text{ s} \end{array}$$

7. What is the instantaneous (not average) rate of filling of a mold (*change in height of the concrete*) when it has 3" for concrete in it? (FYI average rate would be found by  $\frac{\text{Remaining Volume}}{\text{Remaining Fill Time}}$ )



$$\begin{array}{l|l} h_0 = h - r = 3'' - 2'' = 1'' & \text{Area}_{\text{cir}} = \pi r_{\text{cir}}^2 \\ \theta = \sin^{-1}\left(\frac{h_0}{r}\right) = \sin^{-1}\left(\frac{1''}{2''}\right) = 30^\circ & \text{Area}_{\text{cir}} = \pi (1.732'')^2 \\ r_{\text{cir}} = r \cos(\theta) = 2'' \cos(30^\circ) & \text{Area}_{\text{cir}} = 9.425 \text{ in}^2 \\ r_{\text{cir}} \approx 1.732'' & \frac{\delta h}{\delta t} = \frac{\text{Rate}}{\text{Area}_{\text{cir}}} \\ & \frac{\delta h}{\delta t} = \frac{2.239 \frac{\text{in}^3}{\text{s}}}{9.425 \text{ in}^2} = 0.238 \frac{\text{in}}{\text{s}} \end{array}$$

**For problems 8 - 10:** A concrete supplier that you have been using has had to switch to using aggregate from a different source, and developed a new mix design. Since the switch was very recent your supplier has only been able to conduct 20 strength tests so far. They provide you with the standard deviation of the 20 samples ( $S_{20} = 740 \text{ psi}$ ). The project you are currently working on requires you to provide concrete with  $f'_c$  of 3,000 *psi* and 5,300 *psi*. (Relevant lecture [slide](#) for reference.)

8. What is the value of concrete strength ( $f'_{cr}$ ) you are required to order to get  $f'_c = 3,000 \text{ psi}$  while limiting the chance of having a batch that is too weak?

$$\begin{array}{l|l} S = S_{20} \times 1.08 = 740 \text{ psi} \times 1.08 & f'_{cr} = \max \left\{ \begin{array}{l} f'_c + 1.34S = 4,071 \text{ psi} \\ f'_c + 2.33S - 500 = 4,362 \text{ psi} \end{array} \right. \\ S = 799.2 \text{ psi} & \text{Use: } f'_{cr} = 4,400 \text{ psi} \end{array}$$

9. What would the value be for  $f'_{cr}$  for when  $f'_c = 5,300 \text{ psi}$ ?

$$\begin{array}{l|l} S = S_{20} \times 1.08 = 740 \text{ psi} \times 1.08 & f'_{cr} = \max \left\{ \begin{array}{l} f'_c + 1.34S = 6,371 \text{ psi} \\ 0.9f'_c + 2.33S = 6,632 \text{ psi} \end{array} \right. \\ S = 799.2 \text{ psi} & \text{Use: } f'_{cr} = 6,700 \text{ psi} \end{array}$$

10. If with further testing of 5 more samples (25 total) the standard deviation drops to  $S_{25} = 400 \text{ psi}$  what would the new value of  $f'_{cr}$  be for when you need  $f'_c = 3,000 \text{ psi}$ ?

$$\begin{array}{l|l} S = S_{25} \times 1.03 = 400 \text{ psi} \times 1.03 & f'_{cr} = \max \left\{ \begin{array}{l} f'_c + 1.34S = 3,552 \text{ psi} \\ f'_c + 2.33S - 500 = 3,460 \text{ psi} \end{array} \right. \\ S = 412 \text{ psi} & \text{Use: } f'_{cr} = 3,500 \text{ psi} \end{array}$$