

2 x 2 Table Analysis

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[Click Here](#)“Odds ratio – Confidence Interval”

What is a Contingency Table?

A contingency table summarises the outcomes of each individual sampled in terms of whether Properties (A - Exposure) and (B - Outcome) are absent or present. It represents the joint frequency distribution of the two properties.

| | | OUTCOME | |
|----------|---------|---------|--------|
| | | Present | Absent |
| EXPOSURE | Present | a | b |
| | Absent | c | d |

Data from case-control studies (retrospective or prospective) can be analyzed in several ways.

Odds Ratio

An odds ratio is a measure of association between the presence or absence of two properties.

Smoking and Cancer

In 1950, the Medical Research Council conducted a case-control study of smoking and lung cancer (Doll and Hill 1950).

Let's create a 2 X 2 table of the results.

```
CT <- matrix(c(647, 2, 622, 27), nrow = 2)
rownames(CT) <- c("Smoker", "Non-Smoker")
colnames(CT) <- c("Cancer", "No-Cancer")
CT
```

```
##           Cancer No-Cancer
## Smoker      647      622
## Non-Smoker    2       27
```

Load R packages

```
suppressMessages(library(oddsratio))
suppressMessages(library(questionr))
suppressMessages(library(DescTools))
# suppressMessages(library(epitools))
suppressMessages(library(fmsb))
```

Analysis

$OR = ad/bc = (647 \times 27) / (622 \times 2)$

```
a <- CT[1]
c <- CT[2]
b <- CT[3]
d <- CT[4]
(647 * 27) / (622 * 2)
```

```
## [1] 14.0426
```

```
OR <- round((a*d)/(b*c),2)
OR
```

```
## [1] 14.04
```

The odds of lung cancer in smokers is estimated to be 14 times the odds of lung cancer in non-smokers. How reliable is this estimate? We need to calculate a confidence interval. If the study is repeated and the range calculated each time, you would expect the true value to lie within these ranges on 95% of trials.

```
OddsRatio(CT[1:2,], method="wald",
  conf.level=0.95)
```

```
## odds ratio      lwr.ci      upr.ci
## 14.042605    3.325329  59.300825
```

The 95% confidence interval for this odds ratio is between 3.33 and 59.3. Why such a huge range? It's because the numbers of non-smokers, particularly for lung cancer cases, are very small. Increasing the confidence level to 99% this interval would increase to between 2.11 and 93.25.

```
OddsRatio(CT[1:2,], method="wald",
          conf.level=0.99)
```

```
## odds ratio      lwr.ci      upr.ci
## 14.042605      2.114719  93.248662
```

Interpretation of case/control study

Patients with cancer or 14 times more likely to have been smokers than non-smokers.

Details of the CI algorithm

```
log_OR <- log((a*d)/(b*c))
log_OR
```

```
## [1] 2.642096
```

```
std_log_OR <- sqrt(1/a + 1/b + 1/c + 1/d)
std_log_OR
```

```
## [1] 0.7349764
```

```
# Two tailed Z = 1.96, alpha = 0.05
ci_ll <- round(exp(log_OR - 1.96 * std_log_OR), 2)
# ci_ll
ci_ul <- round(exp(log_OR + 1.96 * std_log_OR), 2)
# ci_ul
cat("The 95% CI ranges from", ci_ll, "to", ci_ul)
```

```
## The 95% CI ranges from 3.33 to 59.3
```

Relative Risk

$RR = a/(a+b) / c/(c+d)$

```
RR <- (a/(a+b)) / (c/(c+d))
round(RR, 2)
```

```
## [1] 7.39
```

```
fmsb_RR <- riskratio(647, 2, 1269, 29, conf.level=0.95, p.calc.by.independence=TRUE)
```

```
##           Disease Nondisease Total
## Exposed      647         622  1269
## Nonexposed     2          27   29
```

```
round(fmsb_RR$estimate,2)
```

```
## [1] 7.39
```

```
round(fmsb_RR$conf.int,2)
```

```
## [1] 1.94 28.19  
## attr("conf.level")  
## [1] 0.95
```

Interpretation of the RR

We are 95% confident that the relative risk of cancer in smokers compared to non-smokers is between 1.91 and 28.19. The null value is 1. Since the 95% confidence interval does not include the null value (RR=1), the finding is statistically significant.

Another study

Does chocolate consumption reduce the risk of cardiovascular disease?

Odds Ratio

```
CT2 <- matrix(c(925, 1020, 168, 147), nrow = 2)  
rownames(CT2) <- c("Chocolate", "None")  
colnames(CT2) <- c("CV Disease", "No-CV Disease")  
CT2
```

```
##           CV Disease No-CV Disease  
## Chocolate          925          168  
## None              1020          147
```

```
odds.ratio(CT2)
```

```
##           OR    2.5 % 97.5 %      p  
## Fisher's test 0.79359 0.62032 1.0144 0.05969 .  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The OR at 0.79 suggests that chocolate (1-3 times a month) has some protective effect. But since the 95% confidence interval includes the null value of 1, the effect is not statistically significant.($p > 0.05$)

What about chocolate more than 4 times a week?

```
CT3<- matrix(c(43, 168, 736, 925), nrow = 2)
rownames(CT3) <- c("Choc","None")
colnames(CT3) <- c("CV Disease","No-CV Disease")
CT3
```

```
##      CV Disease No-CV Disease
## Choc      43      736
## None     168      925
```

```
odds.ratio(CT3)
```

```
##              OR    2.5 % 97.5 %      p
## Fisher's test 0.32183 0.22155 0.4593 7.516e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The OR at 0.32 suggests that chocolate (more than 4 times a week) has a protective effect. The 95% confidence interval does not include the null value of 1, thus the effect is statistically significant. The risk reduction = $1 - \text{OR} = 0.68 = 68\%$. ($p < 0$)

What about the Relative Risk?

```
RRC <- (43/(43+736)) / (168/(168+925))
RRC
```

```
## [1] 0.3591219
```

```
fmsb_RR2 <- riskratio(43, 168, 779, 1093, conf.level=0.95, p.calc.by.independence=TRUE)
```

```
##      Disease Nondisease Total
## Exposed      43      736   779
## Nonexposed   168      925  1093
```

```
round(fmsb_RR2$estimate,2)
```

```
## [1] 0.36
```

```
round(fmsb_RR2$conf.int,2)
```

```
## [1] 0.26 0.50
## attr(,"conf.level")
## [1] 0.95
```

```
Percent_decrease <- (1 - RRC) * 100
Percent_decrease
```

```
## [1] 64.08781
```

Interpretation

Those who ate chocolate more than 4 times a week have 0.36 times the risk of cardiovascular disease compared to those who didn't eat chocolate. Since the 95% confidence interval did not include 1, the result is statistically significant.

Chocolate eaters had a cumulative incidence of CV disease of $43/779 = 0.055$ compared to $168/1093 = 0.154$ for non-chocolate eaters.

The chocolate eaters had a 64% decrease in CV disease risk.

What about a Chi Squared test?

[Click Here](#) "Chi Squared Test"

Hypotheses of variable independence

H0: The 2 variables are independent

HA: They are related

Do the test without Yates correction.

```
chisq.test(CT, correct=FALSE)
```

```
##  
## Pearson's Chi-squared test  
##  
## data: CT  
## X-squared = 22.044, df = 1, p-value = 2.664e-06
```

```
# Reject H0 (p<0.05)  
chisq.test(CT2, correct=FALSE)
```

```
##  
## Pearson's Chi-squared test  
##  
## data: CT2  
## X-squared = 3.621, df = 1, p-value = 0.05706
```

```
# Do not reject H0 (p>0.05)  
chisq.test(CT3, correct=FALSE)
```

```
##  
## Pearson's Chi-squared test  
##  
## data: CT3  
## X-squared = 44.131, df = 1, p-value = 3.072e-11
```

```
# Reject H0 (p<0.05)
```

2 x 2 Classification Table

Resulting from Logistic Regression, for example.

The four data counts represent true and false positives and true and false negatives. The analysis is done with a confusion matrix which provides many statistics including: total accuracy, sensitivity, specificity, precision, recall and F1-Score. And then one can proceed to the Receiver Operating Characteristic (ROC) and Area Under the Curve (AUC) statistics.

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