Probability - The Addition Rule

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Probability of compound events. The Addition Rule. Created by Sal Khan.

English video

Probability-Addition Rule "Venn Diagrams"

Let's put some cubes and spheres into a bag.

Green: 8 cubes and 9 spheres

Yellow: 5 cubes and 7 spheres

```
bag <- data.frame(objects = c(rep("gc", 8),rep("gs",9),rep("yc",5),rep("ys",7)))
bag
```

```
##    objects
## 1       gc
## 2       gc
## 3       gc
## 4       gc
## 5       gc
## 6       gc
## 7       gc
## 8       gc
## 9       gs
## 10      gs
## 11      gs
## 12      gs
## 13      gs
## 14      gs
## 15      gs
## 16      gs
## 17      gs
## 18      yc
## 19      yc
## 20      yc
## 21      yc
## 22      yc
## 23      ys
## 24      ys
## 25      ys
## 26      ys
## 27      ys
## 28      ys
## 29      ys
```

Shake the bag and pour out one item. What are the probabilies of getting different types of objects?

What is the probability of getting a cube of any color?

All equally probable possibilities equal the sum of the objects in the bag.

```
P_total <- nrow(bag)
paste("Total possibilities =", P_total,"objects")
```

```
## [1] "Total possibilities = 29 objects"
```

P(cube of any color)=Events that meet the criteria/Total possible events = (8+5)/29 = 0.448

```
options (digits=3)
trial_1 <- subset(bag, objects == "gc" | objects == "yc")
P_trial_1 = nrow(trial_1) / P_total
P_trial_1
```

```
## [1] 0.448
```

Simulate the experiment 10,000 times

```
n <- 10000
trial_1_s <- data.frame(objects = sample(bag$objects, n, replace=TRUE))
head(trial_1_s) # Look at first 6 results
```

```
##   objects
## 1      ys
## 2      ys
## 3      ys
## 4      gs
## 5      gs
## 6      yc
```

```
p_any_cube <- ifelse(trial_1_s$objects == "gc" | trial_1_s$objects == "yc",1,0)
head(p_any_cube)
```

```
## [1] 0 0 0 0 0 1
```

```
p_any_cube <- sum(p_any_cube) / n
p_any_cube
```

```
## [1] 0.446
```

```
p_any_cube = round(p_any_cube * 100, digits = 1)
paste("Probability of drawing a cube of any color =", p_any_cube,"%")
```

```
## [1] "Probability of drawing a cube of any color = 44.6 %"
```

What is the probability of getting any yellow object?

P(any yellow object) = (5+7)/29 = 12/29 = 0.414

```
options (digits=3)
trial_2 <- subset(bag, objects == "yc" | objects == "ys")
P_trial_2 = nrow(trial_2) / P_total
P_trial_2
```

```
## [1] 0.414
```

Simulate the experiment 10,000 times

```
n <- 10000
trial_2_s <- data.frame(objects = sample(bag$objects, n, replace=TRUE))
head(trial_2_s) # Look at first 6 results
```

```
##   objects
## 1      gs
## 2      ys
## 3      yc
## 4      ys
## 5      yc
## 6      gc
```

```
p_yellow <- ifelse(trial_2_s$objects == "yc" | trial_2_s$objects == "ys",1,0)
p_yellow <- sum(p_yellow) / n
p_yellow
```

```
## [1] 0.42
```

```
p_yellow = round(p_yellow * 100, digits = 1)
paste("Probability of drawing a yellow object =", p_yellow,"%")
```

```
## [1] "Probability of drawing a yellow object = 42 %"
```

What is the probability of getting a yellow object or a cube of any color?

P(yellow cube) = 12/29 + 13/29 - 5/29 = 20/29 = 0.690

```
options (digits=3)
all_yellow <- ifelse(grep("y", bag$objects), 1)
all_cubes <- ifelse(grep("c", bag$objects), 1)
yellow_cubes <- ifelse(grep("yc", bag$objects), 1)

p_yellow_or_any_cube <- (sum(all_yellow) + sum(all_cubes) -  sum(yellow_cubes)) / P_total
p_yellow_or_any_cube
```

```
## [1] 0.69
```

Simulate the experiment 10,000 times

```
n <- 10000
trial_3_s <- data.frame(objects = sample(bag$objects, n, replace=TRUE))
head(trial_3_s) # Look at first 6 results
```

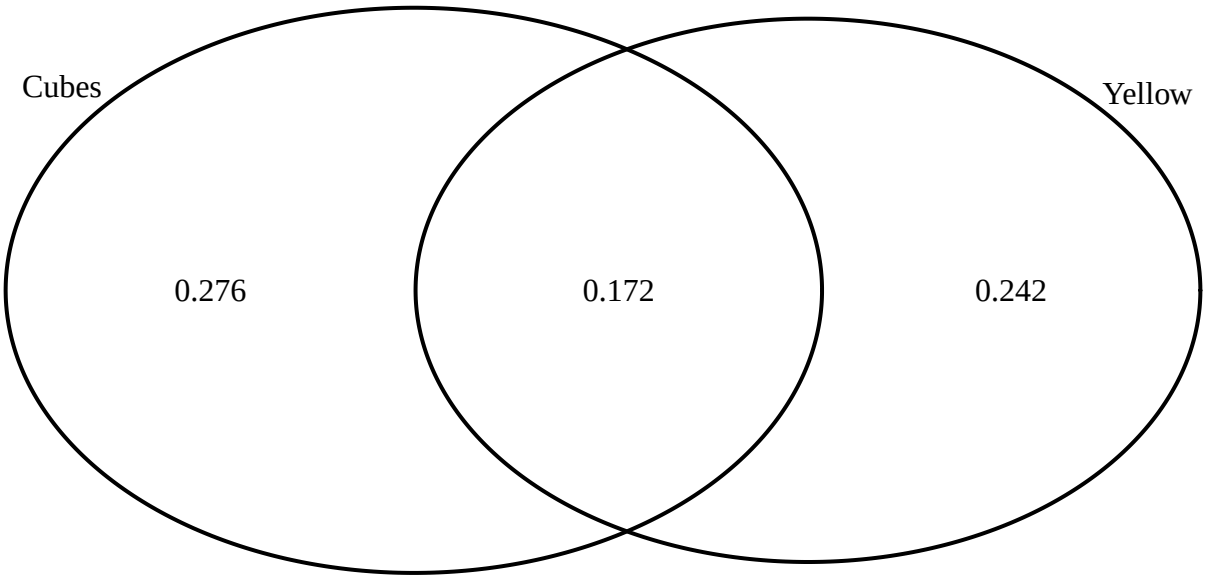
```
##   objects
## 1      gc
## 2      ys
## 3      gc
## 4      ys
## 5      yc
## 6      gs
```

```
all_yellow <- ifelse(grep("y", trial_3_s$objects), 1)
all_cubes <- ifelse(grep("c", trial_3_s$objects), 1)
yellow_cubes <- ifelse(grep("yc", trial_3_s$objects), 1)
p_trial_3 <- sum(all_yellow) + sum(all_cubes) -  sum(yellow_cubes)
p_trial_3 <- sum(p_trial_3) / n
p_trial_3 = round(p_trial_3 * 100, digits = 1)
paste("Probability of drawing a yellow object or any cube =", p_trial_3,"%")
```

```
## [1] "Probability of drawing a yellow object or any cube = 69.1 %"
```

Venn Diagram

```
## Warning: package 'VennDiagram' was built under R version 3.2.5
```




```
## (polygon[GRID.polygon.1], polygon[GRID.polygon.2], polygon[GRID.polygon.3], polygon[GRID.polygon.4],
```

```
## [1] "Probability of yellow object or any cube = 0.448 + 0.414 - 0.172 = 69 %"
```

Summing up the Addition Rule

P(Yellow + Cubes) = P(Yellow) + P(Cubes) - P(Yellow and Cubes)

If the criteria are mutually exclusive (no overlap)

Then P(Yellow + Cubes) = P(Yellow) + P(Cubes)

General Rule: The probability of being a member of Set a or Set b

P(A or B) = P(A) + P(B) - P(A and B)

You must subtract one version of the overlap to avoid double counting

If there is mutual exclusivitiy (no overlap), P(A and B) = 0

Both events can't happen at the same time