1 Question 1

$$n + d = 2011 <=> d = 2011 - n$$
$$\frac{n}{d} < \frac{1}{3}$$
$$\frac{n}{2011 - n} < \frac{1}{3}$$
$$3n < 2011 - n$$
$$4n < 2011$$
$$n < 502.75$$

Due to the constraint that n and d are positive integers, the maximum value for n is 502. Therefore $d = 2011 - n = 2011 - 502 = 1509$.

2 Question 2

Let X be the random variable modeling the outcome of the bet, where x_1 is the amount by which one's money, m, is increased in the event that the bet is favorable, and x_2 is the amount by which m is increased if the bet is not favorable. We know that x_1 and x_2 are complements. Therefore, the expectation of such a variable X is:

$$E[X; m] = \sum_i x_i P(x_i) = 3mP(x_1) - mP(x_2) = 3mP(x_1) - m(1 - P(x_1)) = 4mP(x_1) - m$$
$$= m(4P(x_1) - 1)$$

In order for the bet to be favorable, we would need the expectation of X to be positive, i.e.,:

$$E[X; m] = m(4P(x_1) - 1) > 0$$
$$P(x_1) > 0.25$$

Let n be the number of face cards in the first 5 cards, and let c be the event where the sixth card is a face card, i.e., one's money is increased. Then:

$$P(c|n = i) = \frac{12 - i}{47}$$

Therefore:

$$P(c|n=0) = 0.255$$

$$P(c|n=1) = 0.234$$

Thus we can see that the only way the bet is favorable (i.e., $P(x_1) = P(c|n=i) > 0.25$) is when $i = 0$. We now need to figure out the probability of such an occurrence:

$$P(n=0) = \frac{\binom{12}{0}\binom{40}{5}}{\binom{52}{5}} = 0.25318$$

3 Question 3

We have the following parameters for the Poisson distribution:

$$\lambda = 100/5 = 20$$

$$\alpha = 0.05$$

$$n = 60$$

$$k = n\lambda = (60)(20) = 1200$$

$$CI = [\frac{1}{2}\chi^2(\frac{\alpha}{2}; 2k), \frac{1}{2}\chi^2(1-\frac{\alpha}{2}; 2k+2)] = [\frac{1}{2}\chi^2(0.025; 2400), \frac{1}{2}\chi^2(0.975; 2402)] = [1133.1, 1269.9]$$

4 Question 4

This is a one-sided Z-test.

$$Z^* = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{(\frac{26}{200} - 0.1)}{\sqrt{\frac{0.1(0.9)}{200}}} = 1.414$$

$$p = P(Z \geq Z^*) = 0.0787 > 0.05$$

Therefore we fail to reject the null hypothesis.