DroMOOC

Computer Vision

Basic Level

Introduction to Digital Images

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Introduction to Digital Images

Objectives of the Lecture

Understand the very basics of digital image processing: how to sample and sub-sample images, how to smooth an image or enhance its contrast, how to estimate the spatial derivatives, which are at the basis of image feature extraction and description.

Outline of the Lecture

- What is a digital image?
- Image Discretisation:
  - Sampling
  - Quantisation
- Image filtering:
  - Smoothing
  - Derivation / Enhancement
- Finite difference (convolution) based spatial derivatives

What is a Digital Image?

A 2d single-valued (e.g. gray level) image:

$$I : \underbrace{\{0, \dots, W-1\} \times \{0, \dots, H-1\}}_{\text{Finite Spatial Domain}} \longrightarrow \underbrace{\mathbb{T}}_{\text{Finite Tonal Domain}}$$

A 2d multi-valued (e.g. colour) image:

$$I : \{0, \dots, W-1\} \times \{0, \dots, H-1\} \longrightarrow \mathbb{T}_1 \times \mathbb{T}_2 \times \mathbb{T}_3$$

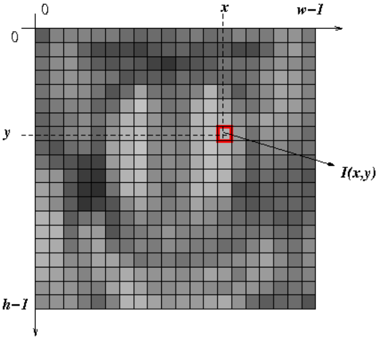


Image Sampling

The discretisation of the spatial domain is referred to as *Sampling*.
Sub-sampling means reducing the *resolution*, i.e. the number of spatial samples.

256x256 | 128x128 | 64x64 | 32x32 | 16x16

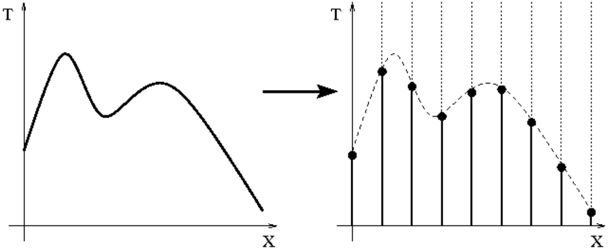
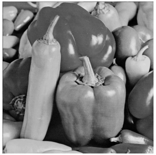
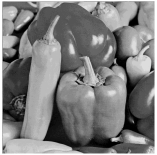


Image Quantisation

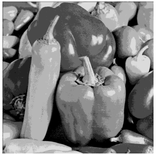
The discretisation of the tonal domain is referred to as *Quantisation*.



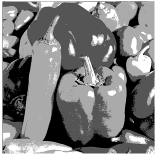
5 bits (32 levels)



4 bits (16 levels)

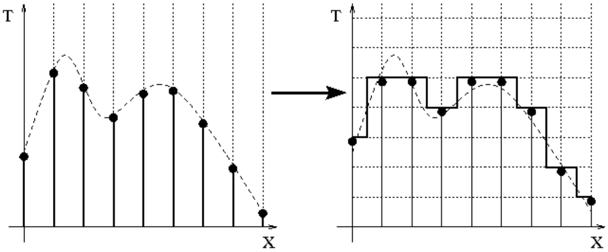


3 bits (8 levels)



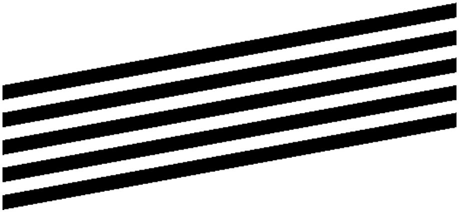
2 bits (4 levels)

1 bit (2 levels)

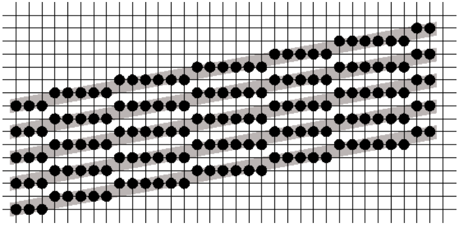


Rules for Image Sampling

According to the *Shannon-Nyquist* theorem, the sampling frequency of an image should be at least twice the highest frequency present in the image.



Original image (black stripes)

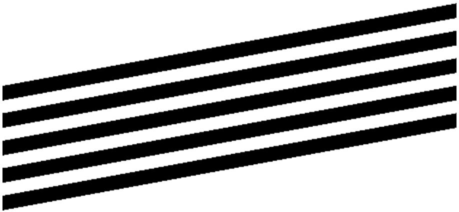


Sampled image (Nyquist frequency)

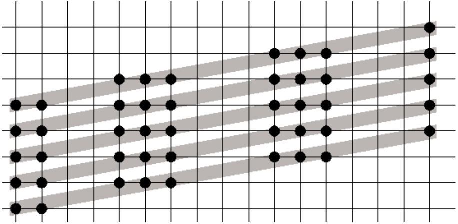
In the above example, the Nyquist-sampled digital image shows structures conform to the original (continuous) one.

Rules for Image Sampling

According to the *Shannon-Nyquist* theorem, the sampling frequency of an image should be at least twice the highest frequency present in the image.



Original image (black stripes)



Sampled image (Low frequency)

In the above example, the badly sampled digital image shows different structures from the original image. These artefact structures are called *aliasing*.

Image Sub-sampling and Aliasing

Original (512x512)

Obviously those rules also apply to sub-sampling a digital image:

$(1/2)^2$ sampled
(256x256)

$(1/4)^2$ sampled
(128x128)

Rules for Image Sampling

It is then necessary to know:

1. how to analyse the frequency content of an image
2. how to reduce the highest frequencies (i.e. filter) an image

Frequency Analysis and the Fourier Transform

The Fourier Transform allows to analyse a signal in terms of frequency content, by decomposing it as a sum of basic periodic functions (sinusoidal signals).

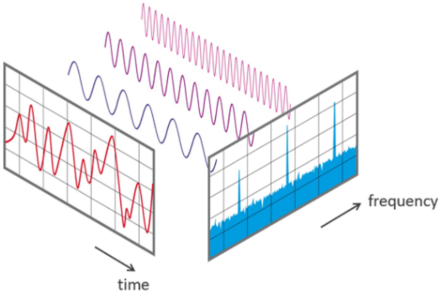


Illustration ©Wikipedia

Frequency Analysis and the Fourier Transform

Accordingly, the 2d Discrete Fourier Transform decomposes an image into a sum of basic 2d periodic functions (complex sine maps).

Inverse Fourier Transform

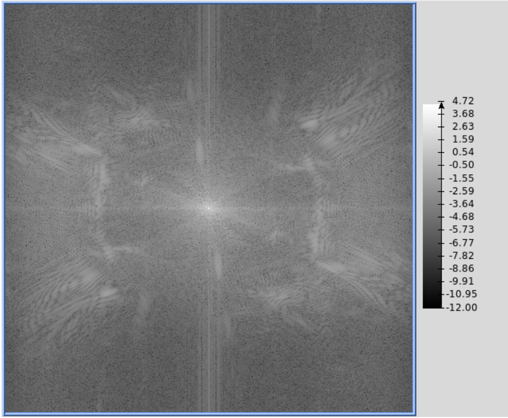
$$I(x, y) = \frac{1}{wh} \sum_{u=0}^{w-1} \sum_{v=0}^{h-1} F(u, v) e^{2j\pi(ux/w + vy/h)}$$

Direct Fourier Transform

$$F(u, v) = \sum_{x=0}^{w-1} \sum_{y=0}^{h-1} I(x, y) e^{-2j\pi(ux/w + vy/h)}$$

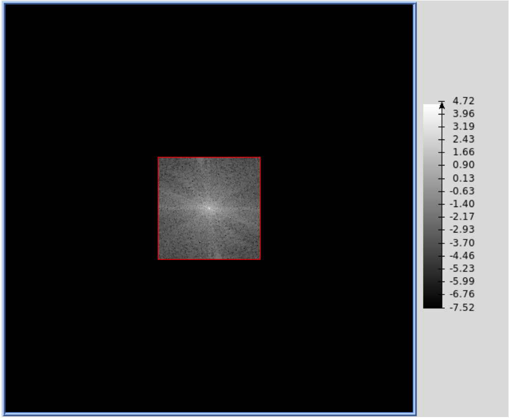
Frequency Analysis and the Fourier Transform

$I(x, y)$



$\log(||F(u, v)||)$

Inverse Fourier Transform and Image Filtering



The Fourier framework allows to filter the images by direct selection of the frequencies.

Above, a low pass filter with $u_{max} = v_{max} = 64$, that allows to sub-sample the resulting image (on the right) to 128×128.

Image Filtering and Sub-sampling

Original Low-Pass Filtered (512x512)

$(1/4)^2$ Sampled
(128x128)

Image Filtering and Convolution

Thanks to the equivalence multiplication / convolution of the Fourier Transform, Image Filtering can also be done directly in the spatial domain by applying convolution:

Convolution

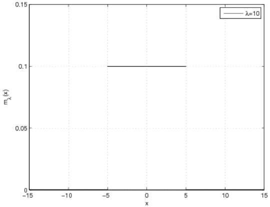
$$(I \star g)(x,y) = \sum_{i=x_1}^{x_2} \sum_{j=y_1}^{y_2} I(x-i, y-j).g(i,j)$$

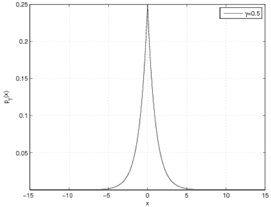
Equivalence Convolution / Multiplication

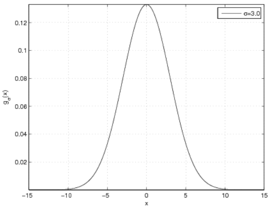
$$I_1 \star I_2 \xrightarrow{\text{Fourier Transform}} F_1.F_2$$

Smoothing Convolution Kernels

Box (λ)	Exponential (γ)	Gaussian (σ)







$$g(x,y) = \begin{cases} \frac{1}{\lambda^2} \text{ if } \max(|x|,|y|) < \frac{\lambda}{2} \\ 0 \text{ otherwise} \end{cases}$$

$$g(x,y) = \frac{\gamma^2}{4} e^{-\gamma(|x|+|y|)}$$

$$g(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

- All these kernels are 2d-separable.
- Exponential kernel is implementable recursively.
- Gaussian kernel is derivable everywhere.

Smoothing filters for images

Smoothing filters are low-pass, or symmetric, unit-sum and positive convolution kernels. They can be used to:

- reduce higher frequencies, e.g. to sub-sample the image.
- decrease the level of details / increase the scale.
- reduce the impact of additive noise: $(I + n) \star g = I \star g + n \star g \simeq I \star g$.

$$\star \frac{1}{25}\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & \mathbf{1} & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} =$$

Smoothing by a 5×5 Box Filter.

Convolution filter for contrast

Convolution Filters can also be used for estimating derivatives and enhancing contrast.

- $\begin{pmatrix} -1 & \mathbf{1} \end{pmatrix}$, or $\begin{pmatrix} -1 & \mathbf{0} & 1 \end{pmatrix}$, or $\begin{pmatrix} -1 & 0 & 1 \\ -1 & \mathbf{0} & 1 \\ -1 & 0 & 1 \end{pmatrix}$ can be used to approximate $I_x = \frac{\partial I}{\partial x}$.
- $\begin{pmatrix} -1 \\ \mathbf{1} \end{pmatrix}$, or $\begin{pmatrix} -1 \\ \mathbf{0} \\ 1 \end{pmatrix}$, or $\begin{pmatrix} -1 & -1 & -1 \\ 0 & \mathbf{0} & 0 \\ 1 & 1 & 1 \end{pmatrix}$ can be used to approximate $I_y = \frac{\partial I}{\partial y}$.

$$\star \begin{pmatrix} -1 & 0 & 1 \\ -1 & \mathbf{0} & 1 \\ -1 & 0 & 1 \end{pmatrix} =$$

Approximation of $I_x = \frac{\partial I}{\partial x}$.

Convolution filter for contrast

Convolution Filters can also be used for estimating derivatives and enhancing contrast.

- The *norm of the gradient* $||\nabla I|| = \sqrt{I_x^2 + I_y^2}$ is a measure of the local contrast.
- The *argument of the gradient* $\arg \nabla I = \arctan\left(\frac{I_y}{I_x}\right)$ provides the local orientation.

Approximation of the gradient norm.

Convolution filter for contrast

Second order spatial derivatives can also be approximated by convolution filters:

- $\begin{pmatrix} 1 & -\mathbf{2} & 1 \end{pmatrix}$ can be used to approximate $I_{xx} = \frac{\partial^2 I}{\partial x^2}$.
- $\begin{pmatrix} 1 \\ -\mathbf{2} \\ 1 \end{pmatrix}$ can be used to approximate $I_{yy} = \frac{\partial^2 I}{\partial y^2}$.
- and then, the Laplacian $\Delta I = I_{xx} + I_{yy}$ can be approximated by $\begin{pmatrix} 0 & 1 & 0 \\ 1 & -\mathbf{4} & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$$\star \begin{pmatrix} 0 & 1 & 0 \\ 1 & -\mathbf{4} & 1 \\ 0 & 1 & 0 \end{pmatrix} =$$

Approximation of the Laplacian.

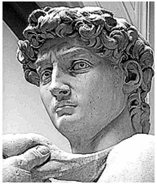
Convolution filter for contrast

Contrast sharpening can be done by combining the image with its Laplacian:

$$I_{sharpened} = I - \gamma \Delta I,$$

it can then also be approximated by a convolution filter:

$$\star \begin{pmatrix} 0 & -1 & 0 \\ -1 & \mathbf{5} & -1 \\ 0 & -1 & 0 \end{pmatrix} =$$



Contrast sharpening convolution filter.

Introduction to Digital Images - CONCLUSION

What have we learned so far

- Formal definition of a digital image
- How to properly sample and sub-sample images
- How to smooth an image or enhance its contrast by convolution
- How to estimate the spatial derivatives by convolution based finite differences

Upcoming Lecture

- Scale space derivatives
- Image Feature Detection
  - Contours
  - Interest Points
- Feature Description