DroMOOC

Control
Basic Level

Hierarchical control

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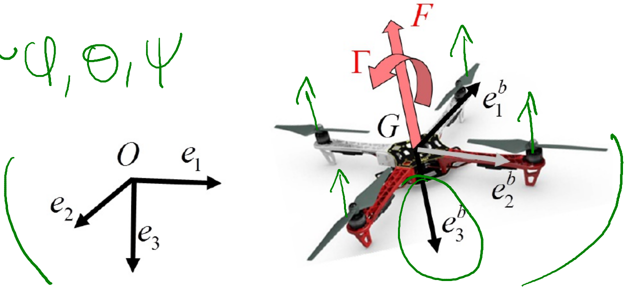
Quadrotor dynamics

$$
\begin{cases}
\dot{p} = v \\
m\dot{v} = F + mge_3 \quad (+\cancel{F_{ext}}) \\
\dot{R} = R\Omega_\times \\
J\dot{\Omega} = -\Omega \times J\Omega + \Gamma \quad (+\cancel{\Gamma_{ext}})
\end{cases}
$$

$$T = \|F\|$$

$$F = -T e_3^b$$

$$R_{\varphi, \theta, \psi}$$



Quadrotor dynamics (cont'd)

l

$$t \left\{ \begin{aligned} \dot{p} &= v \\ m\dot{v} &= -\mathcal{T} R e_3 + m g e_3 \end{aligned} \right.$$

$$\theta \left\{ \begin{aligned} \dot{R} &= R\Omega_\times \\ J\dot{\Omega} &= -\Omega \times J\Omega + \Gamma \end{aligned} \right.$$

$$u \begin{bmatrix} \mathcal{T} \\ \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -l.b & 0 & l.b \\ l.b & 0 & -l.b & 0 \\ d & -d & d & -d \end{bmatrix} \cdot \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} u$$

- Translational and orientation dynamics are coupled
- 6 DoF and only 4 control inputs $\Rightarrow$ quadrotor is <u>under-actuated</u>

  $p_x, p_y, p_z, \; \varphi, \theta, \psi$

- 4 controllable outputs + 2 "internal" variables

  $p_x, p_y, p_z, \; \psi \qquad \varphi, \theta$

Hierarchical control: main idea

- Control the translational and orientation dynamics in a hierarchical way

- Position control

  Compute the force $F = -\mathcal{T} R e_3$ such that $p \to p^d$ and $v \to v^d$

  $= cste$ $= 0$

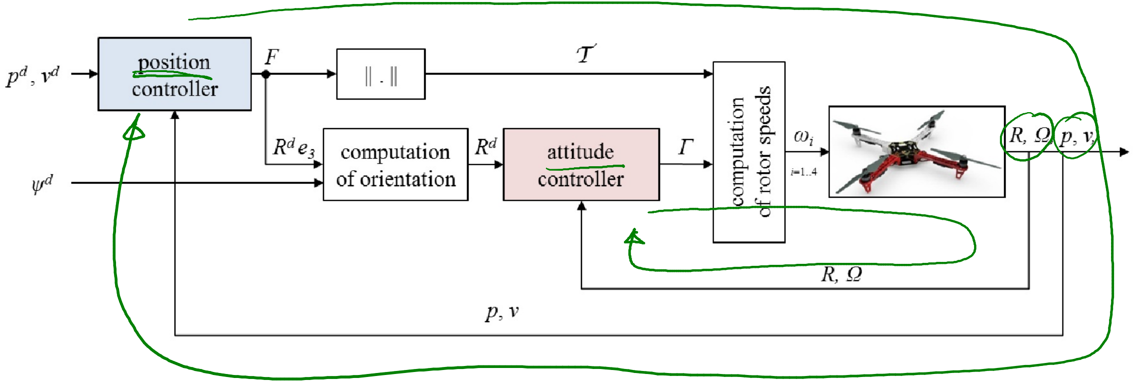
  Deduce the corresponding desired magnitude $\mathcal{T}^d$ and orientation R^d

- Attitude control

  Compute the torque Γ such that $R \to R^d$ and $\Omega \to \Omega^d$

- Rotors' speed control (assumed to be done)

Hierarchical control: block diagram



- Two control loops:
  - inner loop for attitude control
  - outer loop for position control
- Constraint on the speeds of convergence of the two control loops

Position control: main idea

$$\begin{cases} \dot{p} = v \\ \dot{v} = -\frac{\mathcal{T}}{m} R e_3 + g e_3 = u \end{cases}$$

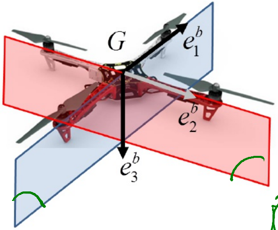
$$\begin{cases} \dot{p} = v \\ \dot{v} = u \end{cases}$$

- Choose $u = -\frac{\mathcal{T}}{m} R e_3 + g e_3$ as control vector
- Design a control law $u = f(p, p^d, v, v^d)$ (e.g. for position stabilization)
- Deduce the desired force $F^d = (-\mathcal{T} R e_3)^d = m(u - g e_3)$
  - its magnitude $\mathcal{T}^d = \|F^d\|$
  - its direction $(R e_3)^d = -F^d / \mathcal{T}^d$

$\mathcal{T}^d \neq 0$
$(\mathcal{T}^d > 0)$

- Assume that ψ^d is given
  - one can deduce a desired orientation R^d (i.e. ϕ^d, θ^d, ψ^d)

Attitude control: main idea



$$\begin{cases} \dot{R} = R\Omega_\times \\ J\dot{\Omega} = -\Omega \times J\Omega + \Gamma \end{cases}$$

- Design a control law $\Gamma = g(R, R^d, \Omega, \Omega^d)$ (e.g. for attitude stabilization) $J = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix}$

For a quadrotor, (e_1^b, e_3^b) and (e_2^b, e_3^b) are symmetry planes $J = diag(J_1, J_2, J_3)$ and $J_1 = J_2$

$$\Omega \times J\Omega = \begin{bmatrix} (J_3 - J_2)\omega_y\omega_z \\ (J_1 - J_3)\omega_x\omega_z \\ \underbrace{(J_2 - J_1)}_{=0}\omega_x\omega_y \end{bmatrix} \qquad \Rightarrow \boxed{J_3\dot{\omega}_z = \Gamma_3}$$

$\Rightarrow$ Control of the yaw can be decoupled

Back to hierarchical control

$$\text{slow}\left\{\begin{array}{l}\dot p = v\\ m\dot v = -\mathcal{T}Re_3 + mge_3 \pm (\mathcal{T}Re_3)^d = \overbrace{\boxed{-\mathcal{T}R^d e_3 + mge_3}}^{u}\,\underbrace{\overbrace{-\mathcal{T}(R-R^d)e_3}^{\text{error}}}_{=0}\end{array}\right.$$

$$\text{fast}\left\{\begin{array}{l}\dot R = R\Omega_\times\\ J\dot\Omega = -\Omega\times J\Omega + \Gamma\end{array}\right.$$

- Closed loop orientation dynamics must converge (much) faster than the closed loop translational dynamics ("inner loop faster than outer loop")

- Simplifications for the design of the control laws:
  - Design of position control law assuming $R = R^d$
  - Design of attitude control law assuming $R^d \sim$ cste
    $\Rightarrow \dot R^d = 0,\ \Omega^d = 0$ $\qquad \dot R^d = R^d\Omega^d_\times$

"Time-scale" separation between translational and orientation dynamics

Hierarchical control: conclusions

What we have learned so far

- A quadrotor is an underactuated system $\Rightarrow$ only 4 controllable outputs
- Translational and orientation dynamics are coupled
- Hierarchical approach: position control, attitude control
- Control laws can be designed separately
- But: specific tuning of the control laws to ensure closed loop stability of the whole system

Upcoming Lectures

- Design of position and attitude control laws
  - Linear methods PID
  - Advanced methods