DroMOOC

Control
Basic Level

PID control

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Quadrotor control

- Assumptions for control design
  - Small angles $\Rightarrow$ linearization of attitude dynamics
  - $+$ Diagonal inertia matrix $\Rightarrow$ decoupled control of attitude dynamics

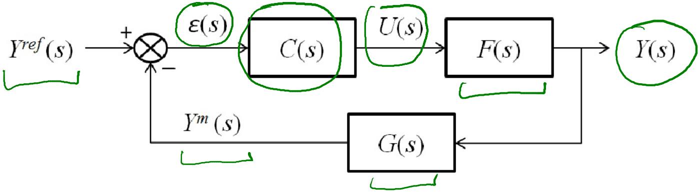
$$\begin{cases} \dot{p} = v \\ m\dot{v} = -\mathcal{T}Re_3 + mge_3 \quad (+F_{ext}) \\ \dot{R} = R\Omega_\times \\ J\dot{\Omega} = -\Omega \times J\Omega + \Gamma \quad (+\Gamma_{ext}) \end{cases} \Rightarrow \quad \begin{cases} \dot{p} = v \\ \dot{v} = u \quad (+\frac{F_{ext}}{m}) \end{cases} \quad \begin{cases} \dot{\eta} = \Omega \\ \dot{\Omega} = J^{-1}\Gamma \quad (+J^{-1}\Gamma_{ext}) \end{cases}$$

with $u = -\frac{\mathcal{T}}{m}Re_3 + ge_3$, $\quad \eta = [\phi \quad \theta \quad \psi]^T$, $\quad \Omega = [\omega_x \quad \omega_y \quad \omega_z]^T$, $\quad J = \mathrm{diag}(J_1, J_2, J_3)$

- Design of control laws
  - separately for translation and orientation dynamics
  - by considering double integrator systems

PID Control

- Most widely used control technique (output feedback)



- Three control actions
  - Proportional action: $C(s) = k_p$ ↑ speed, ↓ s.s. error, !
  - Integral action: $C(s) = k_i/s$ - accuracy
  - Derivative action: $C(s) = k_d s$ - stability
- PID control law: $U(s) = \left(k_p + k_d s + k_i \frac{1}{s}\right) \epsilon(s)$

PID for position control

Simplified model for control design of translation dynamics

$$\boxed{\begin{cases} \dot{p} = v \\ \dot{v} = u \end{cases}} \quad (+\frac{F_{ext}}{m})$$

Stabilization to a constant reference position: p^{ref} =cste, $v^{ref} = \dot{v}^{ref} = 0$

- Position error: $\epsilon_p = p - p^{ref}$
- Velocity error: $\epsilon_d = \dot{\epsilon}_p = v - v^{ref}$

Error dynamics:

$$\begin{cases} \dot{\epsilon}_p = \epsilon_d \\ \dot{\epsilon}_d = u \quad (+\frac{F_{ext}}{m}) \end{cases}$$

PID for position control

Without perturbation ($F_{ext} = 0$):

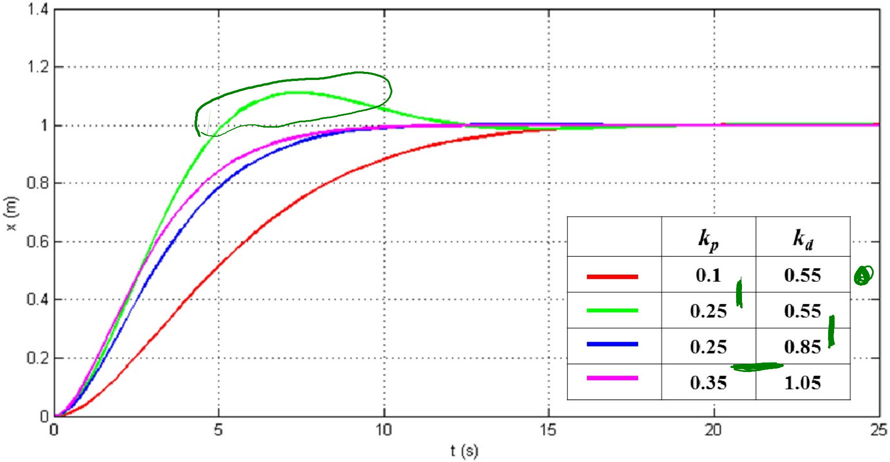
- PD control law: $u(t) = -k_p \epsilon_p(t) - k_d \epsilon_d(t)$ $\quad (k_p, k_d > 0)$
- closed loop error dynamics: $\ddot{\epsilon}_p + k_d \dot{\epsilon}_p + k_p \epsilon_p = 0$ (*)
- chose k_p and k_v to assign a desired behavior of a stable 2nd order system $\frac{1}{\omega_0^2}\ddot{\epsilon}_p + \frac{2\xi}{\omega_0}\dot{\epsilon}_p + \epsilon_p = 0$

With perturbation ($F_{ext} = cste$):

- PID control law: $u(t) = -k_p \epsilon_p(t) - k_d \epsilon_d(t) - k_i \int_0^t \epsilon_p(\tau) d\tau$ $(k_p, k_d, k_i > 0)$

PID for position control

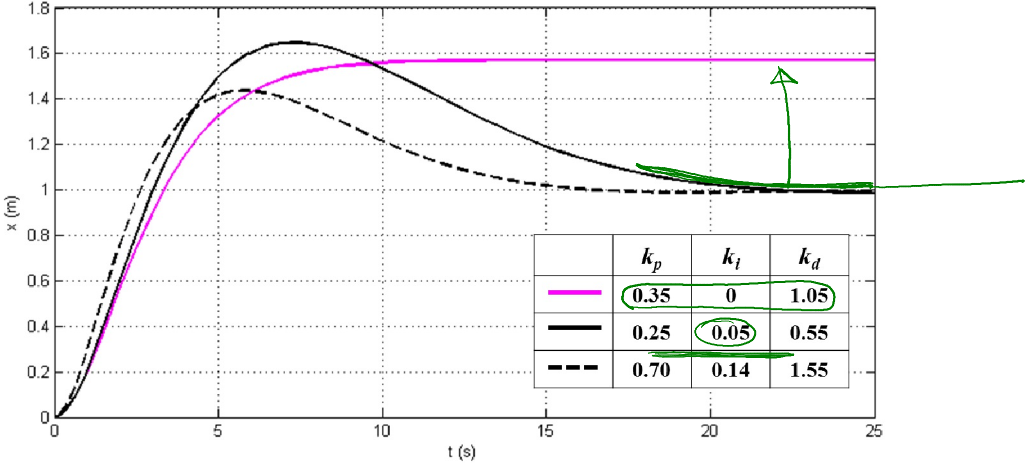
$F_{ext} = 0$, PD controller. Double integrator system Motion along x-axis, $x^{ref} = 1$m



	k_p	k_d
	0.1	0.55
	0.25	0.55
	0.25	0.85
	0.35	1.05

PID for position control

$F_{ext} = 0.2$ N, PID controller. Double integrator system Motion along x-axis, $x^{ref} = 1$m



PID for attitude control

Simplified model for control design of attitude dynamics (with $J = \mathrm{diag}(J_1, J_2, J_3)$)

$$\begin{cases} \dot{\eta} = \Omega \\ \dot{\Omega} = J^{-1}\Gamma \quad (+J^{-1}\Gamma_{ext}) \end{cases} \Leftrightarrow \underbrace{\begin{cases} \dot{\phi} = \omega_x \\ \dot{\omega}_x = J_1^{-1}\Gamma_1 \end{cases}}, \underbrace{\begin{cases} \dot{\theta} = \omega_y \\ \dot{\omega}_y = J_2^{-1}\Gamma_2 \end{cases}}, \underbrace{\begin{cases} \dot{\psi} = \omega_z \\ \dot{\omega}_z = J_3^{-1}\Gamma_3 \end{cases}}$$

Stabilization to a given angular reference

- $(\phi^{ref}, \theta^{ref})$: from desired direction for the thrust (position control input)
- ψ^{ref}: from user

assumed to be constant $\Rightarrow \dot{\phi}^{ref}, \dot{\theta}^{ref}, \dot{\psi}^{ref} = 0$

PID for attitude control

Without perturbation ($\Gamma_{ext} = 0$):

- PD control laws stabilize the attitude dynamics
- for roll (or pitch): small angle errors

$\omega_x^{ref} = 0$

$$\begin{aligned}
\Gamma_1 &= -k_p^1(\phi - \phi^{ref}) - k_d^1\omega_x \quad (k_p^1, k_d^1 > 0) \\
\Gamma_2 &= -k_p^2(\theta - \theta^{ref}) - k_d^2\omega_y \quad (k_p^2, k_d^2 > 0)
\end{aligned}$$

- for yaw:

$$\Gamma_3 = -k_p^3(\psi - \psi^{ref}) - k_d^3\omega_z \quad (k_p^3, k_d^3 > 0)$$

  - angle errors may be large! Be careful !

Yaw control

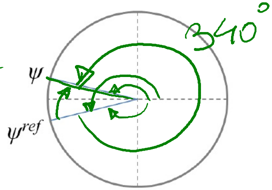
$$\Gamma_3 = -k_p^3 \underbrace{(\psi - \psi^{ref})}_{\varepsilon_\psi} - k_d^3 \omega_z \quad (k_p^3, k_d^3 > 0)$$

A simple example: $\psi = 170\text{deg}$

- $\psi^{ref} = -170\text{deg} \Rightarrow \epsilon_\psi = \psi - \psi^{ref} = 340 \text{ deg}$
- $\psi^{ref} = 190\text{deg} \Rightarrow \epsilon_\psi = \psi - \psi^{ref} = -20 \text{ deg}$

$170° =$

$190° =$



Avoid "U-turns" by keeping $|\epsilon_\psi|$ below 180deg

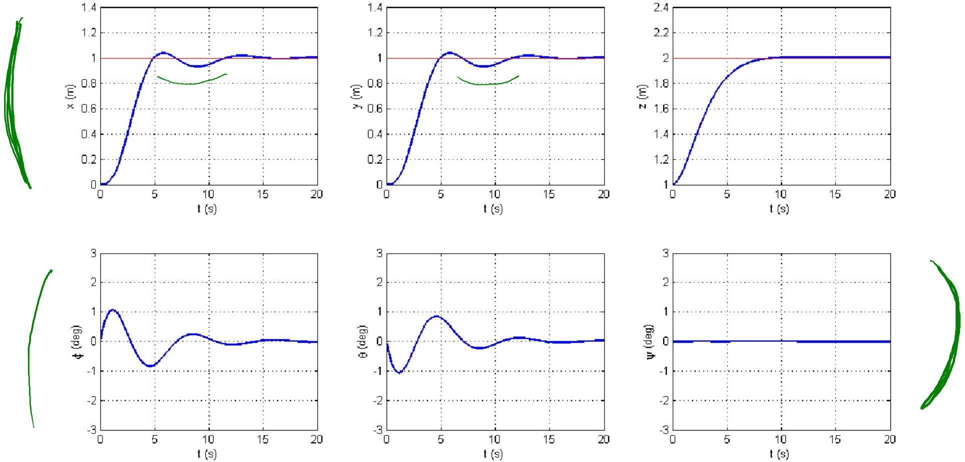
$-170° + (+1) \times 360° = 190°$

$$\begin{cases} \text{if } \ |\epsilon_\psi| \geq \pi \text{ then } \psi'^{ref} = \psi^{ref} + sign(\epsilon_\psi).2\pi \\ \text{else } \psi'^{ref} = \psi^{ref} \end{cases}$$

$$\Rightarrow \boxed{\Gamma_3 = -k_p^3(\psi - \psi'^{ref}) - k_d^3 \omega_z}$$

Hierarchical control

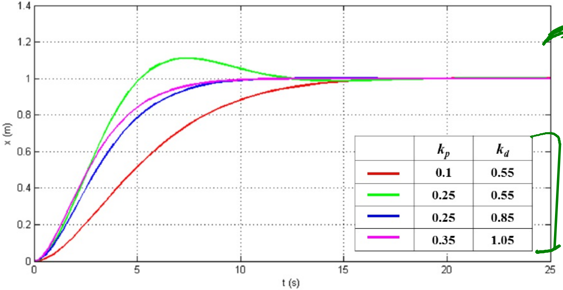
- Position and attitude control laws designed separately (simplified sub-systems)
- Validation on the full dynamical model: $(\underbrace{x^{ref} = y^{ref} = 1\text{m}}, \underbrace{z^{ref} = 2\text{m}}, \underbrace{\psi^{ref} = 0})$



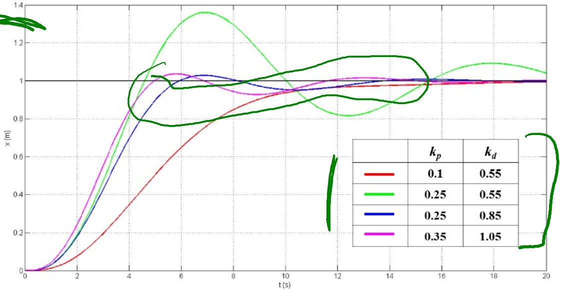
Hierarchical control

- Influence of the attitude dynamics

PD



Double integrator model



Full UAV model

Implementation concerns

- PD control law: $u(t) = -k_p(p(t) - p^{ref}) - k_d v(t)$
  - requires measurements/estimates on position and velocity of the drone
  - (possibly) different gains for x,y and z motions
- Saturations of the control input
  - if error between position and reference is too large (distant WPs!)
  - if integral action: use of an anti windup system
- Discrete-time control
  - Control design for the discrete-time model of the translation dynamics

$$\begin{cases} p(k+1) = p(k) + T_s v(k) + \frac{T_s^2}{2} u(k) \\ v(k+1) = v(k) + T_s u(k) \end{cases}$$

$\begin{cases} \dot{p} = v \\ \dot{v} = u \end{cases}$ T_s : sampling period.

  - Discrete-time version of the PID controller

$$e.g. \quad u(k) = -k_p \epsilon_p(k) - k_i T_s \sum_{j=0}^{k} \epsilon_p(j) - \frac{k_d}{T_s}(\epsilon_p(k) - \epsilon_p(k-1))$$

Conclusions

- PID control for regulation problems (fixed references)
- Extension to trajectory tracking (LQR, MPC)
- Extension to obstacle avoidance (potential fields)
- Coupling with state estimators
- Examples of more advanced control techniques
  - Constraint handling: MPC
  - Control of the full nonlinear cascaded system: backstepping
  - Parameter uncertainties - perturbations: H_∞, adaptive control, etc.
  - Model free controllers (machine learning)