DroMOOC

UAV dynamic modeling

BasicLevel

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ONERA

UAV dynamic modeling

- Trajectories of quadrotor UAV
- Representation in Frames
- Kinematics and dynamics

Quadrotor UAV trajectory

Representation of trajectories of UAV

Movement is classically decomposed as

- Movement of the centre of gravity
  - Evolution of its position wrt some a priori reference
  - Evolution of its speed and acceleration
- Rotation of the vehicle
  - Attitude: Angles between vehicle axes wrt a fixed triad
  - Angular velocity and acceleration

$\Rightarrow$ Kinematic representation

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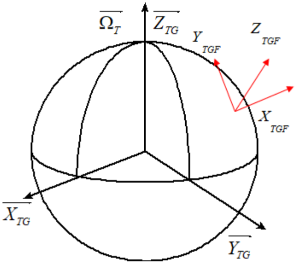
Evaluation of movement

Require definition of Reference Frames

Definition of Fixed Reference frames : Earth-linked Frames

Local East North Up: Considered inertial

- Origin: Fixed point located on Earth surface, e.g. Initial UAV location
- X axis: East direction
- Y axis: North direction
- Z axis: Local vertical up



Definition of Fixed Reference frames : Earth-linked Frames

Local North East Down: Considered inertial

- Origin: Fixed point located on Earth surface, e.g. Initial UAV location
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- Y axis: East direction
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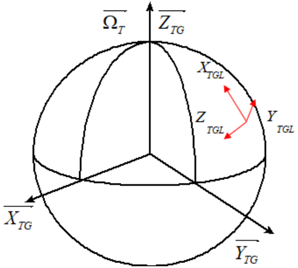
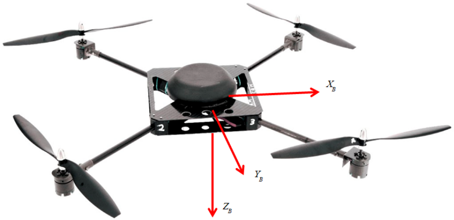


Definition of Vehicle-linked Frames

Body Frame: fixed on the vehicle

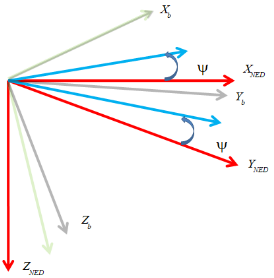
- Origin: Vehicle Center of Gravity (usual)
- X_b axis: Main body axis
- Z_b axis: Orthogonal to X in the symetry plane
- Y_b axis: Complete direct reference frame



Attitude: Inertial to Body Frame

Euler Angles ZYX

- Rotation of yaw angle ψ around Z
- Rotation of pitch angle θ around Y
- Rotation of roll angle ϕ around X



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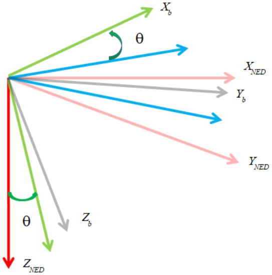

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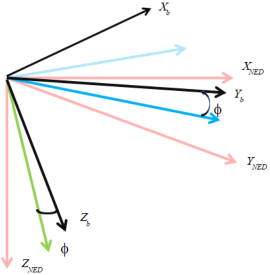

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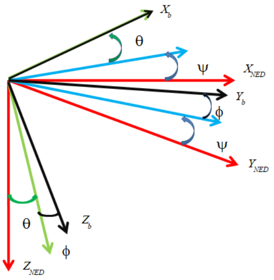

Kinematics

Attitude and Attitude variation of the vehicle

$\boldsymbol{\eta}_i = \begin{bmatrix} \psi & \theta & \phi \end{bmatrix}^t$

$\mathcal{R}$ Rotation matrix: Inertial to Body

$$\mathcal{R} = \begin{bmatrix} c\psi c\theta & c\psi s\theta s\phi - s\psi c\phi & c\psi s\theta c\phi + s\psi s\phi \\ s\psi c\theta & s\psi s\theta s\phi + c\psi c\phi & s\psi s\theta c\phi - c\psi s\phi) \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

Rotation speed $\boldsymbol{\omega}_i = \begin{bmatrix} p & q & r \end{bmatrix}^t$

Also be expressed as $\boldsymbol{\omega}_i = \dot{\psi}\,\overrightarrow{\boldsymbol{k}}_0 + \dot{\theta}\,\overrightarrow{\boldsymbol{i}}_1 + \dot{\phi}\,\overrightarrow{\boldsymbol{k}}_2$

$$\dot{\boldsymbol{\eta}}_i = N\left(\boldsymbol{\eta}_{\boldsymbol{i}}\right) \times \boldsymbol{\omega}_i$$

$$N\left(\boldsymbol{\eta}_{\boldsymbol{i}}\right) = \begin{bmatrix} 0 & \frac{s\phi}{s\theta} & \frac{c\phi}{c\theta} \\ 0 & c\phi & -s\phi \\ 1 & tan\theta.s\phi & tan\theta.c\phi \end{bmatrix}$$

Kinematics

Variation of Euler angles and rotation speed

$$\dot{\theta} = q\cos\phi - r\sin\phi$$
$$\dot{\phi} = p + \tan\theta(q\sin\phi + r\cos\phi)$$
$$\dot{\psi} = \frac{1}{\cos\theta}(q\sin\phi + r\cos\phi)$$

and

$$p = \dot{\psi}\sin\theta + \dot{\phi}$$
$$q = \dot{\psi}\cos\theta\sin\phi + \dot{\theta}\cos\phi$$
$$r = \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi$$

Kinematics

Position and Speed

$\overrightarrow{\boldsymbol{p}}_i$, Position of UAV COG in Inertial Frame (NED)
$\overrightarrow{\boldsymbol{V}}_i$, Speed of UAV in Inertial Frame (NED) $\overrightarrow{\boldsymbol{V}}_i = \frac{d\overrightarrow{\boldsymbol{p}}_i}{dt}$
$\overrightarrow{\boldsymbol{v}}_B$, Speed of UAV in Body Frame: $\overrightarrow{\boldsymbol{V}}_i = \mathcal{R}\overrightarrow{\boldsymbol{v}}_B$

$\frac{d\overrightarrow{\boldsymbol{V}}_i}{dt} = \frac{d\mathcal{R}\overrightarrow{\boldsymbol{v}}_B}{dt}$

$\left[\frac{d\overrightarrow{\boldsymbol{V}}_i}{dt}\right]_i = \left[\frac{d\overrightarrow{\boldsymbol{v}}_B}{dt}\right]_B + \boldsymbol{\omega}_i \times \overrightarrow{\boldsymbol{v}}_B$

Dynamical Model

Newton laws at the vehicle COG

$$m\dot{\vec{\boldsymbol{V}}}_i = \sum Forces_i$$

$m =$ mass of the vehicle

$$J^{\mathcal{B}} \times \dot{\boldsymbol{\omega}}_i + \boldsymbol{\omega}_i \times J^{\mathcal{B}}\boldsymbol{\omega}_i = \sum Moments_B$$

$J^{\mathcal{B}}$ Inertia in Body, assumed $\mathrm{diag}\,(I_x,\ I_y,\ I_z)$

Forces

Newton laws at the vehicle COG

- Weigth mg
- Aerodynamical forces: $\overrightarrow{\boldsymbol{F}}_i^A = \begin{bmatrix} F_x^a & F_y^a & F_z^a \end{bmatrix}^t$ (often neglected)
- Thrust of all the four rotors $\overrightarrow{\boldsymbol{F}}_i^T = -\mathcal{T} z_{\mathcal{B}}$

Thrust of a rotor: function of the rotation speed of rotor
$\mathcal{T}_i = b\,\omega_{ri}^2 z_{\mathcal{B}}$
ω_{ri} proportional to power of $motor_i$

$$Force_{Xi} = -\left(\cos\left(\psi_i\right)\cos\left(\theta_i\right)\sin\left(\phi_i\right) + \sin\left(\psi_i\right)\sin\left(\phi_i\right)\right)\mathcal{T}$$

$$Force_{Yi} = -\left(\sin\left(\psi_i\right)\cos\left(\theta_i\right)\sin\left(\phi_i\right) + \cos\left(\psi_i\right)\sin\left(\phi_i\right)\right)\mathcal{T}$$

$$Force_{Zi} = mg - \left(\cos\left(\theta_i\right)\cos\left(\phi_i\right)\right)\mathcal{T}$$

Moments

Newton laws at the vehicle COG

- Γ_1: Thrust Left rotors minus thrust Right rotors times d_{mot} around $x_{\mathcal{B}_i}$
- Γ_2: Thrust Front rotors minus thrust Back rotors times d_{mot} around $y_{\mathcal{B}_i}$
- Γ_3: Sum of clockwise and counter-clockwise torques around $z_{\mathcal{B}_i}$
- Aerodynamics moments: $\begin{bmatrix}\Gamma_p^A & \Gamma_q^A & \Gamma_r^A\end{bmatrix}^t$ (neglected)

Torque of rotor: function of the rotation speed of rotor
$\tau_i = (-1)^{i+1}\ d\,\omega_{ri}^2$, sign: clockwise $+$ vs anticlockwise $-$
Distance d_{mot} between rotors location and COG

$$Moment_{I_B} = \Gamma_1$$
$$Moment_{J_B} = \Gamma_2$$
$$Moment_{K_B} = \Gamma_3$$

Equations of movement

Kinematics and dynamics equations

- $\dot{\overrightarrow{\boldsymbol{p}}}_i = \overrightarrow{\boldsymbol{V}}_i$
- $\overrightarrow{\boldsymbol{V}}_i = \mathcal{R}\overrightarrow{\boldsymbol{v}}_B$
- $m\dot{\overrightarrow{\boldsymbol{V}}}_i = mg - \mathcal{T}\mathcal{R}\overrightarrow{\boldsymbol{K}}_B$
- $\dot{\boldsymbol{\eta}}_i = N\left(\boldsymbol{\eta_i}\right) \times \boldsymbol{\omega}_i$
- $J^{\mathcal{B}} \times \dot{\boldsymbol{\omega}}_i + \boldsymbol{\omega}_i \times J^{\mathcal{B}}\boldsymbol{\omega}_i = \overrightarrow{\boldsymbol{\Gamma}}_1 + \overrightarrow{\boldsymbol{\Gamma}}_2 + \overrightarrow{\boldsymbol{\Gamma}}_3$