DroMOOC

Sensor fusion and state estimation

Basic Level

Kalman filtering for attitude estimation

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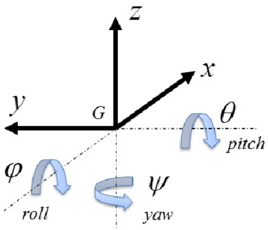
ONERA

Attitude estimation problem

φ θ ψ

Estimation of attitude angles (roll, pitch, yaw) is mandatory

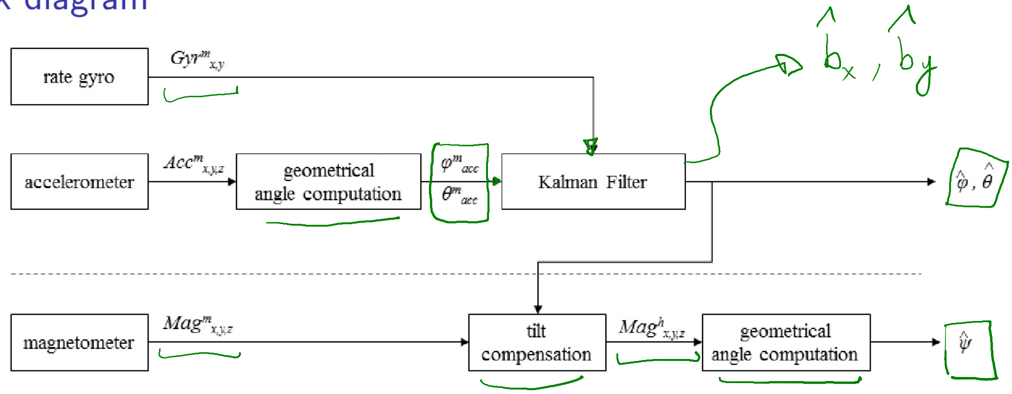
- for localization of the drone
- for control of the drone



In this lecture:

- Attitude estimation from measurements provided by an Inertial Measurement Unit (IMU) accelerometers, rate gyros + magnetometers
- Use of a Kalman Filter for estimation of roll and pitch from accelero and gyro measurements
- Estimation of yaw by geometrical computation from magnetometer measurements

Block diagram



Remarks

- Rate gyro bias estimated for roll and pitch
- Compensation by KF for possible initial error on pitch and roll estimates
- For yaw estimation: offset to be performed wrt to local reference frame (eg. zero yaw at take-off)

Roll and pitch angle estimation using KF

A Small angles

Model of rate gyro measurement: $Gyr_x^m = \dot{\phi} + b_x + n_x$ $\Leftrightarrow$ $\boxed{\dot{\phi} = Gyr_x^m - b_x - n_x}$

Assumption on the bias: $b_x \sim \text{cste} \Rightarrow \boxed{\dot{b}_x = n_x^b}$

Discrete-time model at sampling period T_s:

$$\begin{cases} \phi(k+1) = \phi(k) + T_s.Gyr_x^m(k) - T_s.b_x(k) - T_s.n_x(k) \\ b_x(k+1) = b_x(k) + T_s.n_x^b(k) \end{cases} \tag{1}$$

Roll and pitch angle estimation using KF

For roll and pitch:

$V(k)$

roll

$$\begin{cases} \phi(k+1) = \phi(k) - T_s.b_x(k) + T_s.Gyr_x^m(k) - T_s.n_x(k) \\ b_x(k+1) = b_x(k) + T_s.n_x^b(k) \end{cases} \quad (2)$$

pitch

$$\begin{cases} \theta(k+1) = \theta(k) - T_s.b_y(k) + T_s.Gyr_y^m(k) - T_s.n_y(k) \\ b_y(k+1) = b_y(k) + T_s.n_y^b(k) \end{cases} \quad (3)$$

$X(k+1)$ $\quad$ $X(k)$ $\quad$ $U(k)$

Model can be written as a LTI system: $X(k+1) = AX(k) + BU(k) + V(k)$
with

$$A = \left[\begin{array}{cc|cc} 1 & -T_s & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & -T_s \\ 0 & 0 & 0 & 1 \end{array}\right] \qquad B = \left[\begin{array}{c|c} T_s & 0 \\ 0 & 0 \\ \hline 0 & T_s \\ 0 & 0 \end{array}\right]$$

Roll and pitch angle estimation using KF

(A) Small accelerations

Geometrical computation of roll and pitch from accelerometer measurements

$$\tan(\phi^m_{Acc}) = \frac{Acc^m_y}{Acc^m_z} \qquad \tan(\theta^m_{Acc}) = \frac{-Acc^m_x}{\sqrt{(Acc^m_y)^2 + (Acc^m_z)^2}} \tag{4}$$

Taking

$$Y = \begin{bmatrix} \phi^m_{Acc} \\ \theta^m_{Acc} \end{bmatrix} = \begin{bmatrix} \operatorname{atan2}\left(Acc^m_y, Acc^m_z\right) \\ \operatorname{atan2}\left(-Acc^m_x, \sqrt{(Acc^m_y)^2 + (Acc^m_z)^2}\right) \end{bmatrix} = \begin{bmatrix} \phi \\ \theta \end{bmatrix} + \overset{W}{\begin{bmatrix} w_\phi \\ w_\theta \end{bmatrix}}$$

one has the measurement equation

$$Y(k) = CX(k) + W(k) \qquad \text{with} \qquad C = \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \end{array}\right]$$

$$X = [\phi \;\; b_x \;\; \theta \;\; b_y]^T$$

Reminder on Kalman filtering

"Required information" to implement and run the filter

- model of the system (discrete-time LTI): A, B, C ✓
- state and measurement noise characteristics: Q, R ✓

$$V(k) \sim \mathcal{N}(0, Q) \qquad W(k) \sim \mathcal{N}(0, R)$$

- initial state estimate and estimation error covariance matrix: $\hat{X}(0)$, $\boxed{P(0)}$

OFFLINE

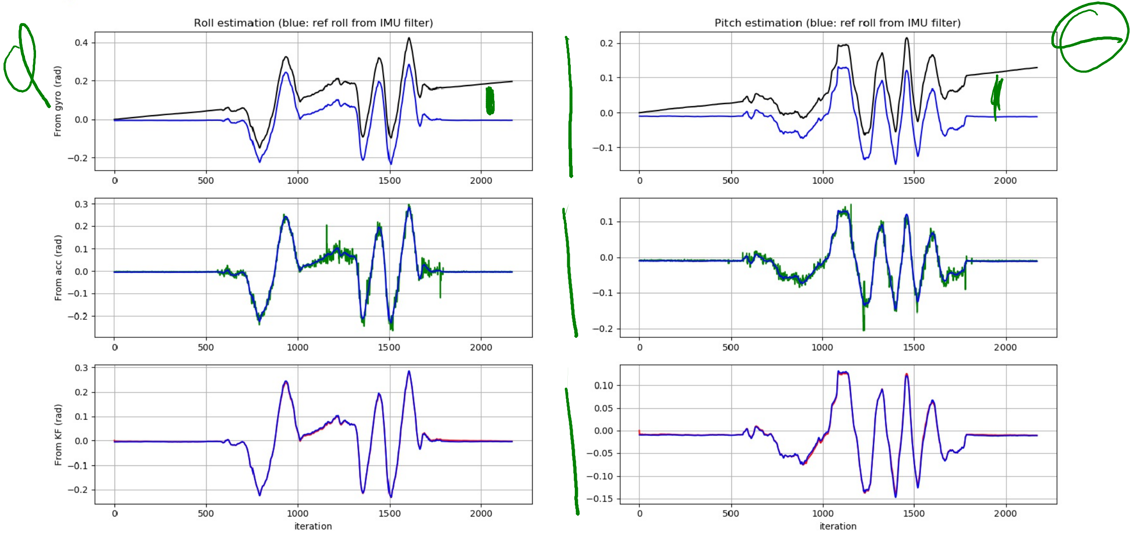
- online information: $U(k)$ (for prediction), $Y(k)$ (for update)

ONLINE

Output provided by the filter:

- state estimate $\hat{X}(k)$ $= [\hat{\phi}\ \hat{b}_x\ \hat{\theta}\ \hat{b}_y]^T$
- estimation error covariance $P(k)$

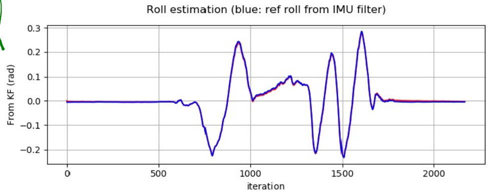
Example from real IMU measurements

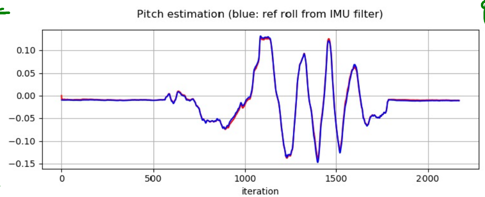


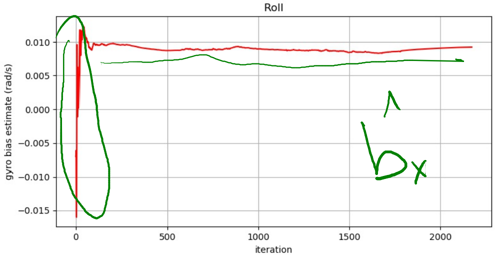
Example from real IMU measurements

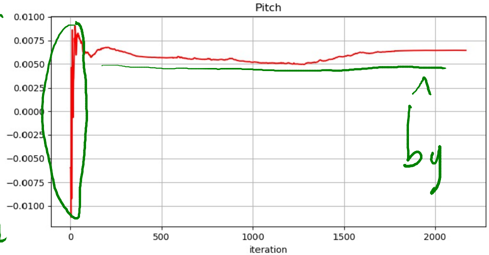
ϕ

θ









Yaw angle estimation

Geometrical computation of yaw angle from magnetometer measurements

- If no roll and pitch:

$$\tan(\psi) = \frac{-Mag_y^m}{Mag_x^m}$$

- Tilt compensation due to non zero roll and pitch:

$$Mag^h = R_{\hat{\theta},\hat{\phi}} Mag^m \qquad \hat{\psi} = \text{atan2}\left(-Mag_y^h, Mag_x^h\right) \tag{5}$$

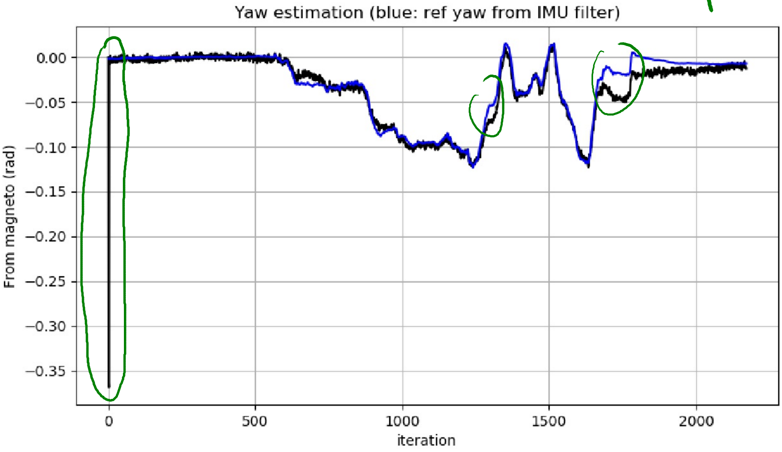
with

$$R_{\hat{\theta},\hat{\phi}} = \begin{bmatrix} c_{\hat{\theta}} & s_{\hat{\phi}}.s_{\hat{\theta}} & c_{\hat{\phi}}.s_{\hat{\theta}} \\ 0 & c_{\hat{\phi}} & -s_{\hat{\phi}} \\ -s_{\hat{\theta}} & s_{\hat{\phi}}.c_{\hat{\theta}} & c_{\hat{\phi}}.c_{\hat{\theta}} \end{bmatrix}$$

$c_\alpha = \cos\alpha$

$s_\alpha = \sin\alpha$

Example from real IMU measurements



Attitude estimation: conclusions

- Simple approach for attitude estimation using IMU measurements
  - Assumption: small angles and small accelerations
  - Not robust to magnetic perturbations
- Improvements and more adavanced methods
  - Use additional sensors (eg. GPS, vision, etc.)
  - Advanced Kalman Filters (EKF, UKF)
  - Complementary filters, nonlinear observers
  - Other attitude representations (eg. quaternions)