DroMOOC

Control
Advanced Level

Nonlinear attitude control & Trajectory tracking

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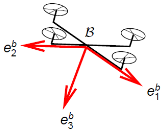
ONERA

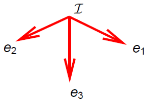
Why nonlinear control?

Reminder on UAV modeling

$$\dot{p} = v$$
$$m\dot{v} = -\mathcal{T} R e_3 + m g e_3$$
$$\dot{R} = R\Omega_\times$$
$$J\dot{\Omega} = -\Omega \times J\Omega + \Gamma$$

$p = [p_x\, p_y,\, p_z]^T$	UAV position in $\mathcal{F}_\mathcal{I}$
$v = [v_x\, v_y,\, v_z]^T$	UAV velocity in $\mathcal{F}_\mathcal{I}$
$\Omega = [\Omega_p\, \Omega_q\, \Omega_r]^T$	UAV angular velocity in $\mathcal{F}_\mathcal{B}$
R	orientation matrix (from $\mathcal{F}_\mathcal{B}$ to $\mathcal{F}_\mathcal{I}$)
m	UAV mass
J	UAV inertia matrix
$\mathcal{T}$	thrust magnitude
$\Gamma = [\Gamma_1\, \Gamma_2\, \Gamma_3]^T$	torques in $\mathcal{F}_\mathcal{B}$
g=9.81 m.s^{-2}	gravity constant
$e_3 = [001]^T$	downwards vertical unit vector in $\mathcal{F}_\mathcal{I}$





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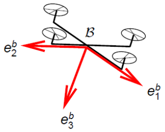

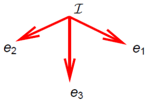

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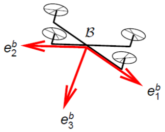

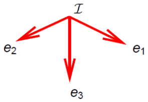

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Ideal model

The only external forces present are:

- i) gravity
- ii) control torques/forces

Why nonlinear control?

Nonlinear UAV model

$$\begin{aligned}\dot{p} &= v\\ m\dot{v} &= -\mathcal{T}Re_3 + mge_3 + F_a\\ \dot{R} &= R\Omega_\times\\ J\dot{\Omega} &= -\Omega\times J\Omega + \Gamma + \Gamma_a\end{aligned}$$

- F_a: perturbing forces in $\mathcal{F}_\mathcal{I}$
- Γ_a: perturbing torques in $\mathcal{F}_\mathcal{B}$

Nonlinearities

- Rotational kinematics
- Coriolis forces
- Aerodynamic forces/torques

Additional issues

i) Underactuation (4 inputs, 6 DOF); ii) Parameter uncertainty; iii) Unmeasured signals; iv) Idealized control inputs

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Control objective

The control law has to ensure that the closed-loop system ensures the desired performance and stability characteristics while being robust wrt uncertainties and disturbances.

Controllability analysis

Since 4 inputs while 6 DOF (η, p) it can be shown that

- (p, ψ) are controllable inputs
- (θ, ϕ) are internal variables

Approach: Hierarchical control

Exploit the cascaded structure of the UAV model to:

1. Calculate Γ to ensure $R \to R_d$
2. Calculate $\mathcal{T}$, R_d s.t. $p \to p_d$
3. Gain selection for faster convergence of rotational dynamics compared to translational dynamics

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Trajectory tracking

Nominal Model

$$\dot{p} = v$$
$$m\dot{v} = -\mathcal{T} R_d e_3 + mge_3 =: u$$

PD control

$$u = -k_p(p - p_d) - k_d(v - v_d) + m\dot{v}_d \quad k_p > 0, k_d > 0 \implies$$
$$\mathcal{T} R_d e_3 = mge_3 + k_p(p - p_d) + k_d(v - v_d) + m\dot{v}_d \implies$$
$$\mathcal{T} = |\mathcal{T} R_d e_3|, \qquad R_d e_3 = \frac{\mathcal{T} R_d e_3}{|\mathcal{T} R_d e_3|}$$

Since ψ_d is given we can deduce (θ_d, ϕ_d) hence R_d!!!

Closed-loop

$$\dot{p} - \dot{p}_d = v - v_d$$
$$m(\dot{v} - \dot{v}_d) = -k_p(p - p_d) - k_d(v - v_d)$$

Trajectory tracking (cont'd)

Model with additive disturbances

$$\dot{p} = v$$
$$m\dot{v} = -\mathcal{T}Re_3 + mge_3 + F_a =: u + F_a$$

- F_a: perturbations in $\mathcal{F}_\mathcal{I}$
- Assumption: $|F_a| \leq c_d$, with known bound $c_d > 0$

Modified PD control

$$u = -k_p(p - p_d) - k_d(v - v_d) + m\dot{v}_d - (k_s + c_d)\mathrm{sgn}(m(v - v_d) + k_d(p - p_d))$$
$$k_s > 0, \mathrm{sgn}(x) := (\mathrm{sgn}(x_1), \ldots, \mathrm{sgn}(x_n))^T, x = (x_1, \ldots, x_n)$$

R_d derivation

Same as in the nominal case

Trajectory tracking (cont'd)

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Trajectory tracking (cont'd)

Stability proof

- Apply coordinate change: $\tilde{v} = m(v - v_d) + k_d(p - p_d)$
- Closed-loop dynamics

$$\begin{aligned} \dot{p} - \dot{p}_d &= -k_d(p - p_d) + \tilde{v} \\ \dot{\tilde{v}} &= -k_p(p - p_d) - k_s \operatorname{sgn}(\tilde{v}) - c_d \operatorname{sgn}(\tilde{v}) + F_a \end{aligned}$$

Lyapunov stability

$$\begin{aligned} E &:= \frac{k_p}{2}|p - p_d|^2 + \frac{1}{2}|\tilde{v}|^2 \\ \dot{E} &= -k_p k_d |p - p_d|^2 - k_s \tilde{v}^T \operatorname{sgn}(\tilde{v}) \\ &\quad - (c_d \tilde{v}^T \operatorname{sgn}(\tilde{v}) - \tilde{v}^T F_a) \\ &\leq -k_p k_d |p - p_d|^2 - k_s |\tilde{v}|_1 - c_d(|\tilde{v}|_1 - |\tilde{v}|) \leq 0 \end{aligned}$$

$\rightarrow$ Similarly for rotational motion

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Attitude control

Euler angle parametrization

$$\dot{\eta} = T(\eta)\Omega$$
$$J\dot{\Omega} = -\Omega \times J\Omega + \Gamma$$

$$T(\eta) := \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix},$$

$$\det(T(\eta)) \neq 0 \text{ for } |\theta| < \pi/2$$

Approach:Backstepping

- Consider Ω as virtual input $\Omega = \Omega_d$
- $\Omega_d := T(\eta)\Big(-K_\eta(\eta - \eta_d) + \dot{\eta}_d\Big) \implies$
  $\dot{\eta} - \dot{\eta}_d = -K_\eta(\eta - \eta_d)$
- Γ designed s.t. $\Omega \to \Omega_d$
- $\Gamma := \Omega \times J\Omega + J\dot{\Omega}_d - JK_\Omega(\Omega - \Omega_d) \implies$
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Attitude control (cont'd)

Euler angle parametrization - Complete closed-loop

$$\dot{\eta} - \dot{\eta}_d = -K_\eta(\eta - \eta_d) + T^{-1}(\eta)(\Omega - \Omega_d)$$
$$\dot{\Omega} - \dot{\Omega}_d = -K_\Omega(\Omega - \Omega_d)$$

Lyapunov stability

$$E := \frac{1}{2}|\eta - \eta_d|^2 + \frac{\rho}{2}|\Omega - \Omega_d|^2$$
$$\dot{E} = -\lambda_{min}(K_\eta)|\eta - \eta_d|^2 + (\eta - \eta_d)^T T^{-1}(\eta)(\Omega - \Omega_d)$$
$$- \lambda_{min}(K_\Omega)|\Omega - \Omega_d|^2$$
$$\leq -(\lambda_{min}(K_\eta) - \frac{\epsilon}{2})|\eta - \eta_d|^2 - \rho(\lambda_{min}(K_\Omega)$$
$$- \frac{c_T}{2\epsilon\rho})|\Omega - \Omega_d|^2, \quad ||T^{-1}(\eta)|| \leq c_T$$

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Attitude control (cont'd)

Rotation matrix parametrization

$$\dot{R} = R\Omega_\times$$
$$J\dot{\Omega} = -\Omega \times J\Omega + \Gamma$$

Definition of "vee" operation:

$$M := \begin{bmatrix} 0 & -m_{12} & m_{13} \\ m_{12} & 0 & -m_{23} \\ -m_{13} & m_{23} & 0 \end{bmatrix} \mapsto M^\vee = \begin{bmatrix} m_{23} \\ m_{13} \\ m_{12} \end{bmatrix}$$

- Error definition:

$$e_R := \frac{1}{2}\left[R_d^T R - R^T R_d\right]^\vee$$
$$e_\Omega := \Omega - R^T R_d \Omega_d$$

- Control law:

$$\Gamma := J(-K_R e_R - K_\Omega e_\Omega) + \Omega \times J\Omega$$
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Attitude control (cont'd)

Closed-loop dynamics

$$\dot{e}_R = \frac{1}{2}\left(\mathrm{trace}(R^T R_d) I_3 - R^T R_d\right) e_\Omega$$
$$\dot{e}_\Omega = -K_R e_\Omega - K_\Omega e_\Omega$$

Lyapunov stability

$$E := \frac{1}{2} e_\Omega^T J e_\Omega + K_R \frac{1}{2}\mathrm{trace}(I_3 - R_d^T R) + \epsilon e_R^T e_\Omega$$
$$\dot{E} \leq -[|e_R||e_\Omega|] M [|e_R||e_\Omega|]^T \leq 0$$
$$M := \begin{bmatrix} \frac{\epsilon K_R}{\lambda_{max}(J)} & -\frac{\epsilon K_\Omega}{2\lambda_{min}(J)} \\ -\frac{\epsilon K_\Omega}{2\lambda_{min}(J)} & K_\Omega - \epsilon \end{bmatrix}$$
$$K_\Omega > \epsilon$$
$$K_R > \frac{\epsilon K_\Omega^2 \lambda_{max}(J)}{4\lambda_{min}(J)(K_\Omega - \epsilon)}$$

Attitude control (cont'd)

Closed-loop dynamics

$$\dot{e}_R = \frac{1}{2}\left(\mathrm{trace}(R^T R_d) I_3 - R^T R_d\right) e_\Omega$$

$$\dot{e}_\Omega = -K_R e_\Omega - K_\Omega e_\Omega$$

Lyapunov stability

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$$K_\Omega > \epsilon$$

$$K_R > \frac{\epsilon K_\Omega^2 \lambda_{max}(J)}{4\lambda_{min}(J)(K_\Omega - \epsilon)}$$

Conclusions

- Design of nonlinear laws for attitude control and trajectory tracking
- Hierarchical control approach
- Two popular parametrizations for attitude control
- Guaranteed stability through a Lyapunov analysis
- Illustration of how one can modify a nominal law to account for bounded, additive perturbations