
Dynamic Programming (Value Iteration, Policy Iteration, Q-Learning)

0.1 Value Iteration

- For each state s , we need to figure out the expected reward of strating in s an acting optimally.
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Algorithm 1: Policy Improvement

Input: MDP, value function V
Output: policy π'
for $s \in \mathcal{S}$ **do**
 for $a \in \mathcal{A}(s)$ **do**
 $Q(s, a) \leftarrow \sum_{s' \in \mathcal{S}, r \in \mathcal{R}} p(s', r | s, a)(r + \gamma V(s'))$
 end
 $\pi'(s) \leftarrow \arg \max_{a \in \mathcal{A}(s)} Q(s, a)$
end
return π'

Algorithm 2: Value Iteration

Input: MDP, small positive number θ
Output: policy $\pi \approx \pi_*$
Initialize V arbitrarily (e.g., $V(s) = 0$ for all $s \in \mathcal{S}^+$)
repeat
 $\Delta \leftarrow 0$
 for $s \in \mathcal{S}$ **do**
 $v \leftarrow V(s)$
 $V(s) \leftarrow \max_{a \in \mathcal{A}(s)} \sum_{s' \in \mathcal{S}, r \in \mathcal{R}} p(s', r | s, a)(r + \gamma V(s'))$
 $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
 end
until $\Delta < \theta$;
 $\pi \leftarrow \text{Policy_Improvement}(\text{MDP}, V)$
return π

- $p(s', r | s, a)$: probability of next state s' and reward r , given current state s and current action a ($\mathbb{P}(S_{t+1} = s', R_{t+1} = r | S_t = s, A_t = a)$)

0.2 Policy Iteration