# A Eulerian Model for the Dynamics of Polydisperse Evaporating sprays: Combining the Multi-Fluid Model with the Quadrature Method of Moments

**D. Kah**[1] F. Laurent[1] M. Massot[1] R.Fox[2], S. Jay[3]

[1]Laboratoire EM2C, CNRS, Ecole Centrale Paris

[2]Iowa State University

[3]Institut Français du Pétrole

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IOWA STATE UNIVERSITY

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# Outline

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# Context

- Presence of fuel droplets in many industrial problems
  - aeronautics or automotive combustion chambers
  - industrial furnaces.

Modelling of droplets: key issue
Highly instationnary problems

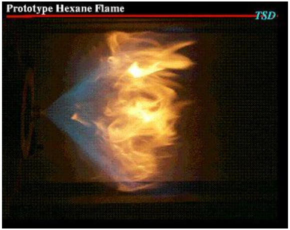



Figure: Source Prof. Edwards (Stanford)

Figure: Source C. Dumouchel (CORIA Rouen)

# Context

Influence of the liquid phase on the physical phenomenon

- flame structure and dynamics
- combustion efficiency
- pollutants produced

modelling problems

- droplets-gas interactions
  - drag force, evaporation, heat transfer
- droplets-droplets interactions
  - coalescence, breakup

key issue: polydispersion

# Statistical Description

- $\rightarrow$ statistical description (spherical droplets)
  Introduction of the number density function $f(t, \boldsymbol{x}, T, \boldsymbol{u}, \phi)$
  $\rightarrow \phi$ droplets radius, surface or volume

$$f^R dR = f^S dS = f^V dV$$

following a Williams-Boltzmann equation : Kinetic Model
Williams,1958

$$\underbrace{\partial_t f + \partial_x \cdot (fu)}_{\text{free transport}} + \underbrace{\partial_S (R_S f)}_{\text{vaporisation}} + \underbrace{\partial_u (Ff)}_{\text{drag}} + \underbrace{\partial_T (Ef)}_{\text{heat exchange}} = \underbrace{\Gamma}_{\text{collision operator}}$$

- $\boxed{\text{Lagrangian Model}}$ $\boxed{\text{Eulerian Multi-Fluid Model}}$

## Eulerian Multi-Fluid Model

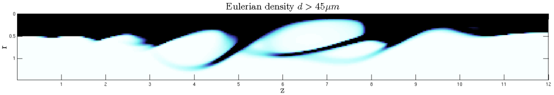
- Equilibrium hypothesis on velocity at fixed $(t, x, S)$
- Discretization in S by finite volume
  $\rightarrow$ equations on moments

Advantages:

- accurate description of polydispersion at moderate cost
- easy coupling between gas phase and liquid phase

Realisations:

- Free jet DNS calulation



- Limitations:
  - $1^{st}/2^{nd}$ order accurate method
    $\rightarrow$ Need to have a lot of sections in $\phi$ to correctly model polydispersion
  - Existence of a single velocity (conditioned by size)
    $\rightarrow$ Finite Stokes number droplet of same size have the same velocity (singularities)
    De Chaisemartin et al, *Submitted to Journal of Computational Physics*
- Objectives:
  - Increase the accuracy of the multi-fluid method
    - Massot et al *Submitted to Journal of Aerosol Science*
  - Allow droplet crossings in collaboration with R . Fox
    - Desjardin et al, *Journal of Computational Physics*

# Context of the study

Simplified Williams equation: $f(t, x, S, u)$

$$\partial_t f + \underbrace{\partial_x u f}_{\text{transport}} + \underbrace{\partial_u F f}_{\text{drag}} + \underbrace{\partial_S R_S f}_{\text{evaporation}} = 0$$

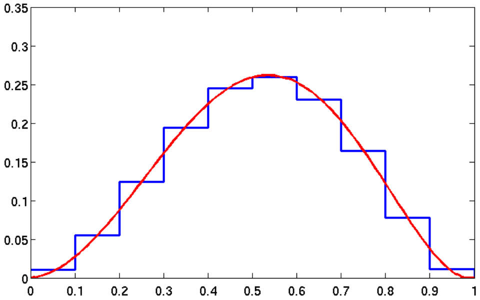
- Mass Moments: $m_k = \int_S S^k f(t, x, S, u)\, dSdu$
- Velocity Moments: $\mathcal{M}_k = \int_u u^k f(t, x, S, u)\, dSdu$
- $m_0 = \mathcal{M}_0$
- Coupling Mass and Velocity Moments:

$$f(t, x, u, S) \rightarrow \int_{S,u} S^k u^k f\, dSdu$$

# Outline

# Multi-Fluid Method

- Several sections $\rightarrow$ mass ($m_{3/2}$) and momentum ($m_{3/2}\frac{\mathcal{M}_1}{\mathcal{M}_0}$)
- Section = independant fluid, interacting through fluxes
- Flux $\rightarrow$ piecewise constant reconstruction



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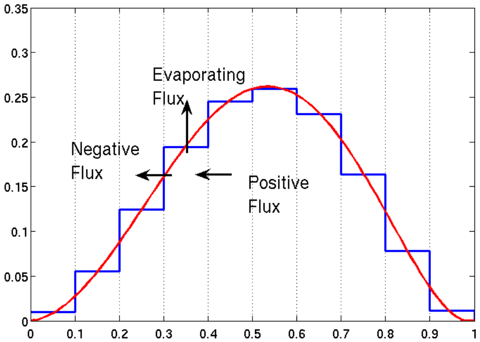
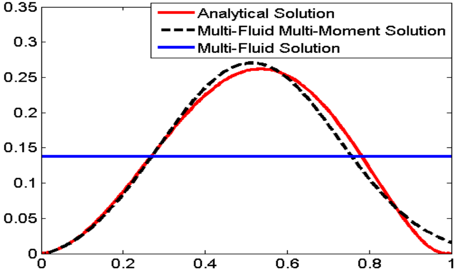


# Principles

Objective:

$\boxed{\text{Increase the amount of information in a section}}$

- Multi-Fluid method : 1 moment
- Multi-Fluid Moment method : 4 moments




- M-F: 1 section
- M-F Moment: 1 section

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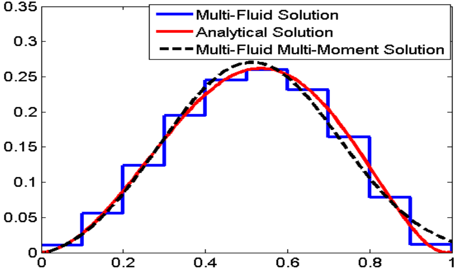



- M-F: 10 sections
- M-F Moment: 1 section

# M-F Moment equation system

$$\boxed{\partial_t f + \partial_x uf + \partial_S R_S = 0}$$ Phase space (x,S,u)

Equilibrium hypothesis $\Big\downarrow \int_u$

$$\boxed{\partial_t n + \partial_S R_S n + \partial_x n\bar{u} = 0}$$ Phase space (x,S)

Discretization in S, finite Volume $\Big\downarrow \int_S S^k$

$$\boxed{\begin{aligned}
\partial_t(m_0) + \partial_x \cdot (m_0 \tfrac{\mathcal{M}_1}{\mathcal{M}_0}) &= R_S n(t, S=0)\\
\partial_t(m_1) + \partial_x \cdot (m_1 \tfrac{\mathcal{M}_1}{\mathcal{M}_0}) &= m_0\\
\partial_t(m_2) + \partial_x \cdot (m_2 \tfrac{\mathcal{M}_1}{\mathcal{M}_0}) &= 2m_1\\
\partial_t(m_3) + \partial_x \cdot (m_3 \tfrac{\mathcal{M}_1}{\mathcal{M}_0}) &= 3m_2\\
\partial_t(\mathcal{M}_1) + \partial_x (\tfrac{\mathcal{M}_1}{\mathcal{M}_0}^2) &= R_S n(t, S=0) \tfrac{\mathcal{M}_1}{\mathcal{M}_0}
\end{aligned}}$$

Phase space (x)

$R_S n(t, S=0)$
$=$ Evaporating Flux

# Difficulties

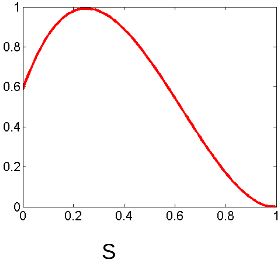
- From $(m_0, m_1, m_2, m_3)^t$ in a section, how to calculate the evaporating flux ($R_S n(t, S = 0)$)?

- $(m_0, m_1, m_2, m_3)^t$ must stay in the Surface **Moment Space**
  A number density, $f$ must exist, such as: $\int_S S^k f \, dS = m_k$
  Dette,Studden,1997

# Evaporative flux

## Entropy Maximisation.

For a given moment vector $(m_0, m_1, m_2, m_3)^t$, $f$ is reconstructed under the profil $f = exp(\sum_{i=0}^{3} \lambda_i S^i)$,
with $\lambda_i = \lambda_i(m_0, m_1, m_2, m_3)$
Mead, 1984



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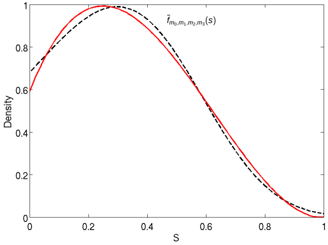


# Preservation of the moment space

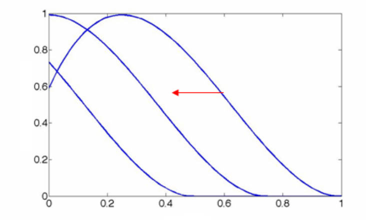
- Scheme caracteristics:
  - Finite volume
  - Kinetic scheme
  - Flux determined by computer algebra (Maple)

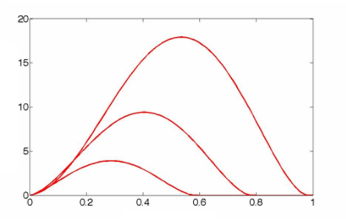
- Proprieties of the scheme:
  - positivity on density
  - maximum principle on velocity
  - preservation of moment space

# Comparison of moments

Solved kinetic equation: $\partial_t n + \partial_S R_S n = 0$, with $R_S = -1$

- Analytical solution:



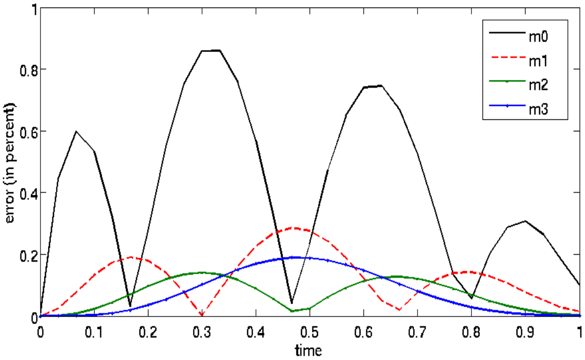




**number density** **mass density**

- Evolution in terms of moments:
  - Multi-Fluid: 12 sections, 1 moment
  - Multi-Fluid Moment: 1 section, 4 moments

# Comparison of mass density

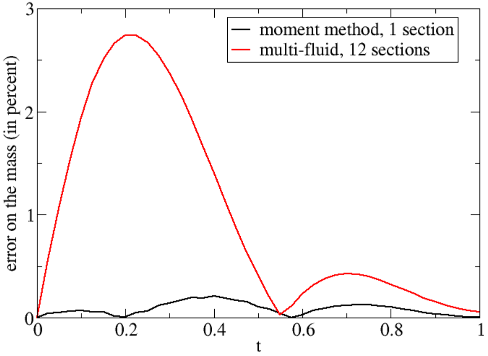
Error in size moments (Mutli-Fluid Moment method)




Precision less than 1 percent for the density.
Precison less than 0.4 percent for higher order moments

# Comparison of mass density

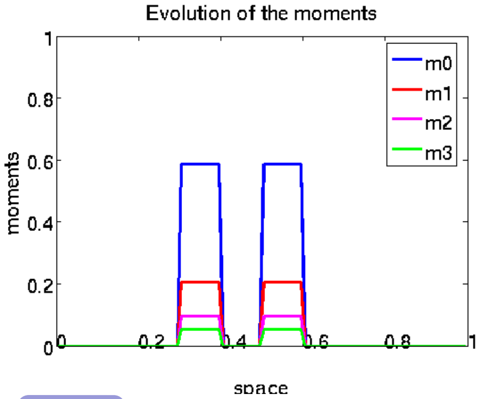
Comparison of error on mass.




MF Moment method much more precise than the simple MF method.

# Multi-Fluid Moment: evaporation and transport

Evaporation and transport of a droplet flow



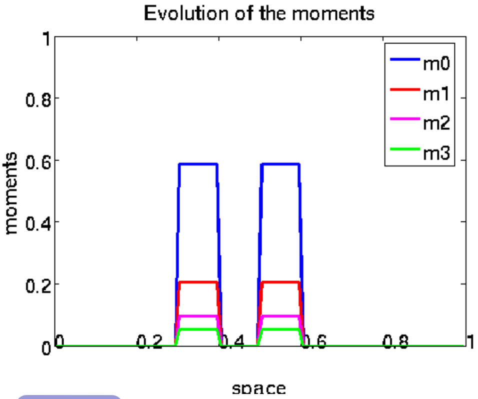

MF Moment

Dilute spray without collisions
$\rightarrow$ The two clouds should cross each other

- Free flow:Evaporation and Transport well simulated
- Impinging droplets: $\delta$-shock creation (non physical)

# Multi-Fluid Moment: evaporation and transport

Evaporation and transport of a droplet flow




MF Moment

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→ The two clouds should cross each other

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# Different quadrature methods of moments

Equilibrium hypothesis: $\rightarrow \begin{pmatrix} \mathcal{M}_0 \\ \mathcal{M}_1 \end{pmatrix} : f(u) = \mathcal{M}_0 \delta(u - \frac{\mathcal{M}_1}{\mathcal{M}_0}(S))$

Monodisperse flow: equations on $(\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3)^t$

$$\begin{aligned} \partial_t(\mathcal{M}_0) + \partial_x(\mathcal{M}_1) &= 0 \\ \partial_t(\mathcal{M}_1) + \partial_x(\mathcal{M}_2) &= 0 \\ \partial_t(\mathcal{M}_2) + \partial_x(\mathcal{M}_3) &= 0 \\ \partial_t(\mathcal{M}_3) + \partial_x(\mathcal{M}_4) &= 0 \end{aligned}$$

Closure:
$\Rightarrow f(u) = w_1 \delta(u - \bar{u}_1) + w_2 \delta(u - \bar{u}_2) \rightarrow$ droplet crossings
R. Fox, 2008

# Outline

# Multi-Fluid Moment method with droplet crossings

- M-F Moment: polydispersion but one velocity
- Multi-Velocity(R . Fox): droplet crossings but monodisperse

Coupling of the two models: one section for the example

$$f(t,x,S,u) = \underbrace{n(t,x,S)}_{\text{size distribution}} \quad \underbrace{\phi(t,x,u)}_{\text{velocity distribution}}$$

$$\begin{aligned}
\partial_t m_0 + \partial_x \mathcal{M}_1 &= R_S n(t, S=0) \\
\partial_t m_1 + \partial_x m_1 \mathcal{M}_1/\mathcal{M}_0 &= m_0 \\
\partial_t m_2 + \partial_x m_2 \mathcal{M}_1/\mathcal{M}_0 &= 2m_1 \\
\partial_t m_3 + \partial_x m_3 \mathcal{M}_1/\mathcal{M}_0 &= 3m_2 \\
\partial_t \mathcal{M}_1 + \partial_x \mathcal{M}_2 &= R_S n(t, S=0)\mathcal{M}_1/\mathcal{M}_0 \\
\partial_t \mathcal{M}_2 + \partial_x \mathcal{M}_3 &= R_S n(t, S=0)\mathcal{M}_2/\mathcal{M}_0 \\
\partial_t \mathcal{M}_3 + \partial_x \mathcal{M}_4 &= R_S n(t, S=0)\mathcal{M}_3/\mathcal{M}_0
\end{aligned}$$

# Preservation of the moment space conserved

- Scheme caracteristics:
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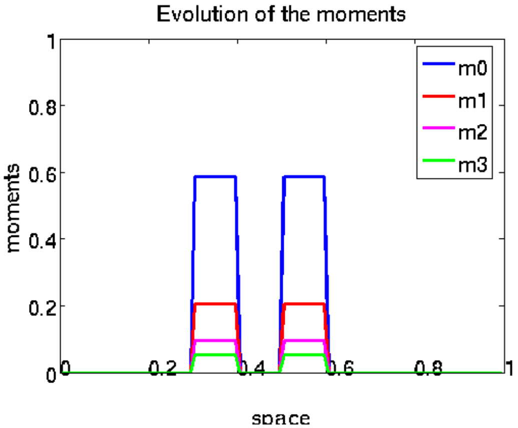
# Multi-Fluid Moment method with droplet crossings: evaporation and transport

Evaporation and transport of a droplet flow




MF Moment

Droplet crossing well resolved

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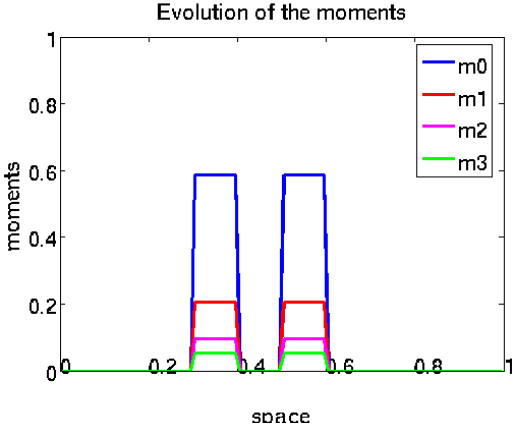


MF Moment

Droplet crossing well resolved

# Conclusions and Perspectives

Conclusions:

- Solutions to Multi-Fluid Model limitations

Perspectives:

- Simulation of $2D/3D$ configuration with the Multi-Fluid Multi-Moment Multi-Velocity model
- Extending this model to the collision operator
- Including these models in a global code (Summer Program 2008, CTR, Stanford)

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