

Prise en compte des aspects polydisperses pour la modelisation d'un jet de carburant dans les moteurs à combustion interne

Damien Kah

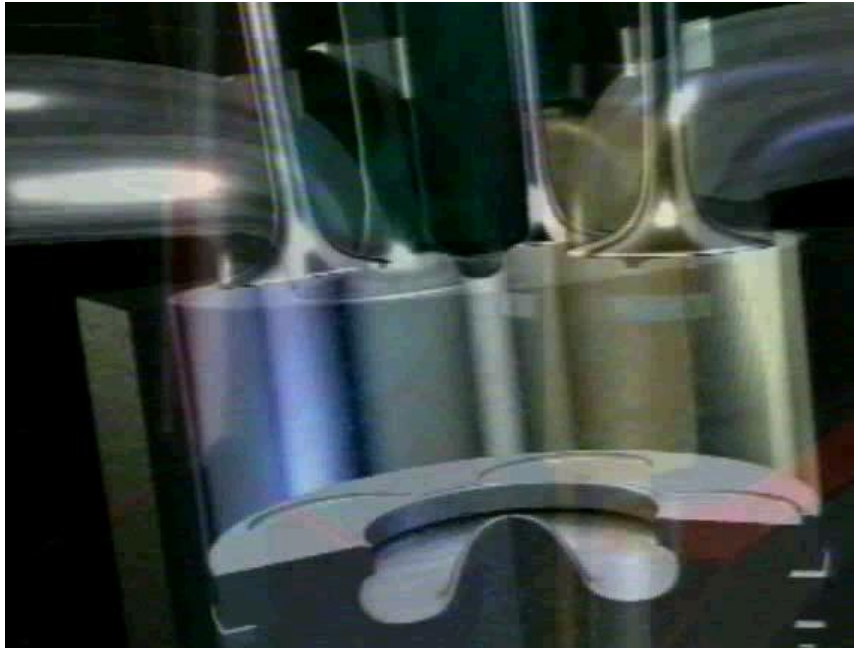
Laboratoire EM2C du CNRS et de l'ECP

IFP Energie nouvelles

20 decembre 2010

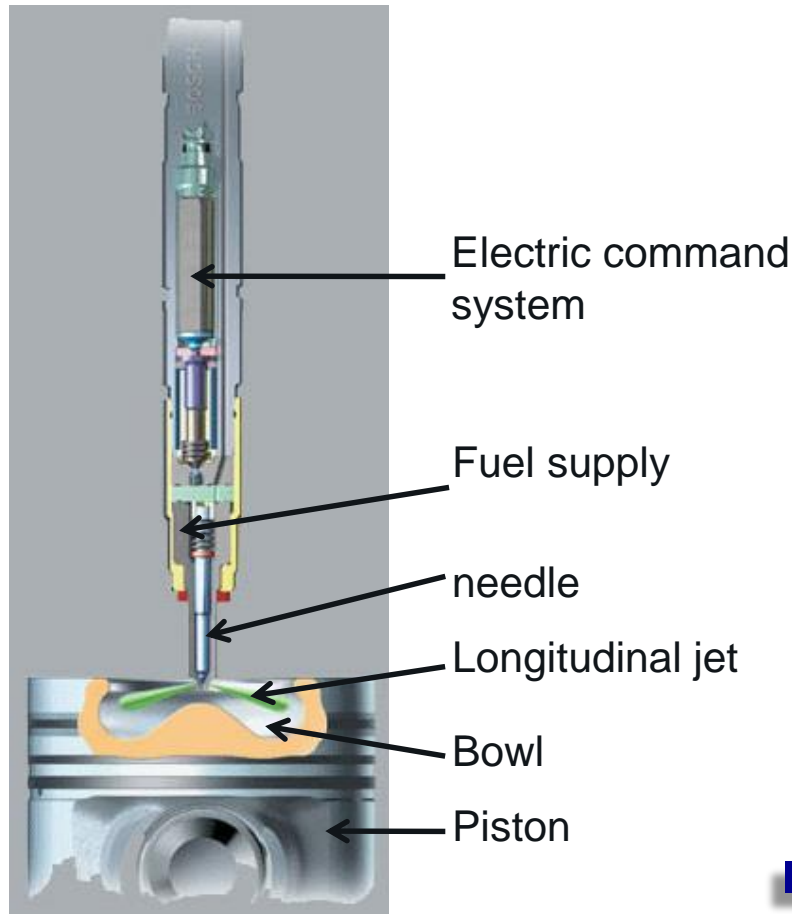
General context: Internal combustion engines

- Internal combustion engines
- Engine cycle in Diesel engines



- Compression
 - Injection
 - Atomisation
 - Evaporation
 - Auto-inflammation
 - Combustion
-
- Critical importance to understand this cycle in order to predict:
 - Pollutants (Nox and soots) produced
 - Engine energetic yield
 - Focus on the injection process

General context: Fuel injection



Multiphase flow involved



Fig: Dumouchel, Coria

Injection parameters

- Injection pressure: 2000 bar
- Injection time : 2ms
- Injection velocity: 600 m s⁻¹

Extreme conditions



Very hard to study the spray
experimentally

General context: Expertise at IFP Energies nouvelles

- Research still needs **experiments** to fully **understand** all the combined and complex physical processes occurring during injection ...

Experimental expertise

- Engine test benches: Real time simulation of the engine
- Specialized laboratories: Thermodynamics and Optical diagnostic (transparent engine benches, pressurized chambers)

- ... but an increasing interest is devoted to **numerical simulation** in order to **predict** these processes

Numerical expertise

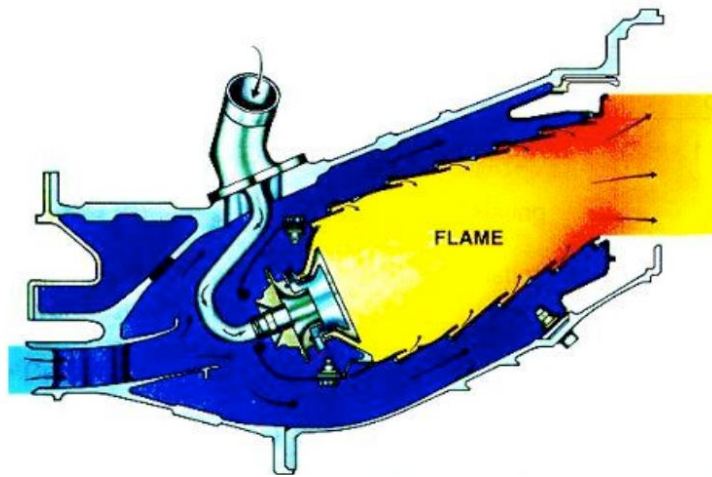
Simulation of reactive multiphase flow with spray:

- IFP-C3D: dedicated to automotive engines (RANS)
- AVBP: combustion for engines (LES), collaboration with CERFACS

General context:

More general potential application fields

Some examples:



- Turbomachines and turboreactors
- Solid propulsion
- Cryotechnic propulsion
- Soots dynamics

General context: More general potential application fields

Some examples:



- Turbomachines and turboreactors
- **Solid propulsion**
- Cryotechnic propulsion
- Soots dynamics

General context: More general potential application fields

Some examples:

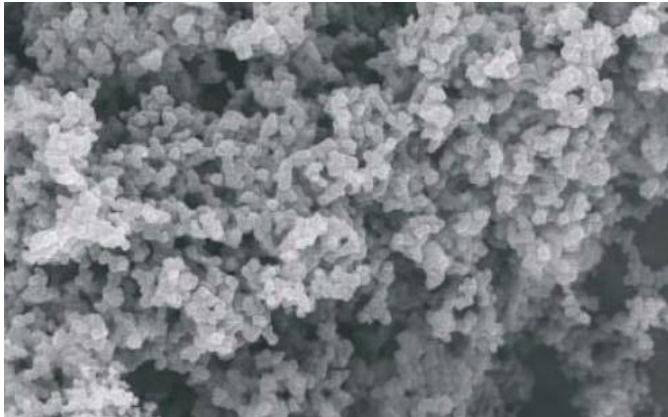


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General context:

More general potential application fields

Some examples:



- Turbomachines and turboreactors
- Solid propulsion
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General context: Physics of jet



Fig: Dumouchel, Coria

Primary break-up Secondary break-up

- Primary break-up
- Secondary break-up

General context: Physics of jet



Fig: Dumouchel, Coria

Primary break-up Secondary break-up

- Primary break-up
- Secondary break-up
- Droplet interaction
- Turbulent dispersion
- Evaporation
- Combustion

Droplet population

Characterized by **polydispersity**
= existence of a size distribution

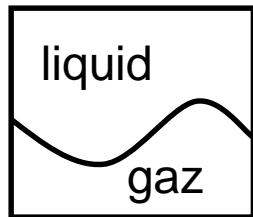
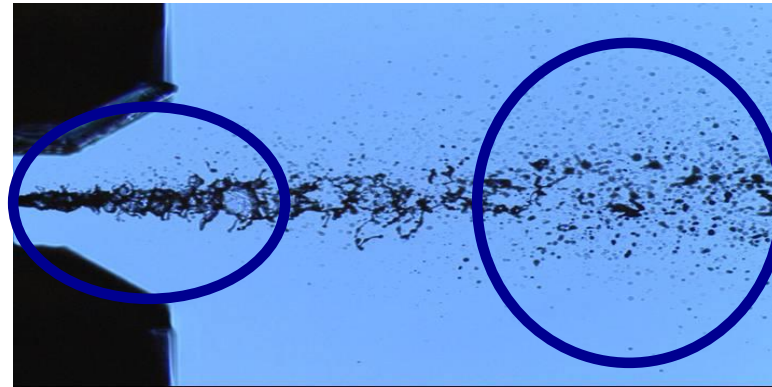
General context: Jet modeling

- It is possible, in terms of simulation, to solve the entire flow with a Direct Numerical simulation (DNS)
- But this resolution is too expensive in terms of computational cost in an industrial context

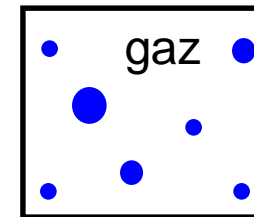
 Reduced order models



General context: Reduced order models for jet



Separate phase



Disperse phase

Coupling ?

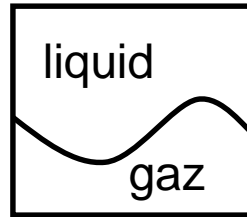
Two-Fluid models

- Space, time, ensemble averaging
- Interface not resolved as in a DNS but liquid topology accessed through α and Σ
- **No notion of polydispersity**

Spray models

- Point-particles
- Spherical droplets
- **Notion of polydispersity**

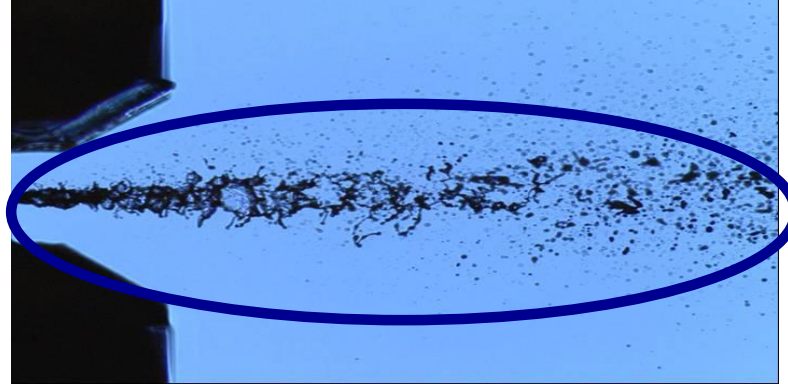
General context: Separate phase modeling



Different types of two-fluid models

- 4 equations model (*Karni 96, Abgrall 96*)
- 5 equations model (*Allaire et al. 02, Chanteperdrix et al. 02, Massoni et al. 05*)
- 6 equations model (*Ishii 75, Delhay 76*)
- 7 equations model (*Baer and Nunziato 86, Saurel and Abgrall 00*)
- Model for surface density Σ (*Candel and Poinso 89, Morel 97, Vallet 97*)

General context: Jet modeling in IFP-C3D



One single model use for the whole jet: a two-fluid model...

- B. Truchot: 7 equation model (mass, momentum, energy per phase + α) (*Truchot 05*)
- C. Vessiller: Equation on Σ (*Vessiller 07*)
- F. Bayoro: Introduction of Baer and Nunziato model (*Baer and Nunziato 86*) ideas for the equation on α and interfacial terms closure (pressure and velocity) (*Bayoro 08*)

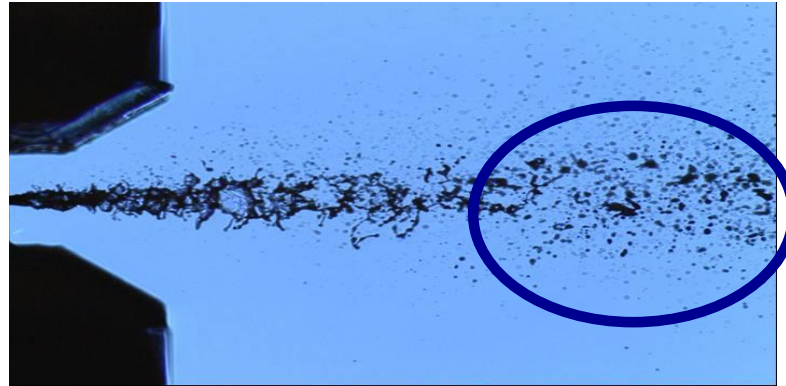
General context: Jet modeling in IFP-C3D



... Dedicated to the separate phase only...

- B. Truchot: 7 equation model
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General context: Jet modeling in IFP-C3D



... Dedicated to the separate phase only...

- B. Truchot: 7 equation model
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... Thus, for the disperse phase

▣ Access to the droplet mean

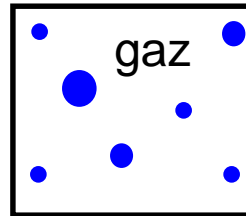
diameter: $d = \frac{6\alpha}{\Sigma}$

- **Polydispersity not described**



Need for a dedicate spray model

General context: Disperse phase modeling



Deterministic approach

- Lagrangian
“Discrete particle simulation (DPS)” (*O’Rourke 81*)

Statistical approach

Kinetic model, number density function (NDF):

$$f(t, \mathbf{x}, S, \mathbf{u}, T)$$

- Lagrangian
“Discrete simulation Monte Carlo” (DSMC) (*Bird 94*)
- Eulerian
Conservation equations on moments of the NDF

General context:

Disperse phase statistical modeling

Williams-Boltzmann equation (*Williams, 58*)

$$\partial_t f + \underbrace{\nabla_{\mathbf{x}} \cdot (\mathbf{u} f)}_{\text{advection}} + \underbrace{\partial_S(K f)}_{\text{evaporation}} + \underbrace{\nabla_{\mathbf{u}} \cdot (\mathbf{F} f)}_{\text{acceleration}} + \underbrace{\partial_T(E f)}_{\text{heat exchange}} = \underbrace{\Gamma}_{\text{source}}$$

Lagrangian resolution

- Modeling
- Implementation
- Parallel computing
- Coupling
 - Gas phase (Eulerian)
 - Separate phase(Eulerian)

Eulerian resolution

- Modeling
- Implementation
- Parallel computing
- Coupling
 - Gas phase (Eulerian)
 - Separate phase(Eulerian)



We consider an Eulerian resolution

General context:

Eulerian resolution of the disperse phase

Framework kinetic equation of this PhD

$f(t, \mathbf{x}, S, \mathbf{u})$ solution of

$$\partial_t f + \underbrace{\nabla_{\mathbf{x}}(\mathbf{u}f)}_{\text{advection}} + \underbrace{\partial_S(Kf)}_{\text{evaporation}} + \underbrace{\nabla_{\mathbf{u}}(\mathbf{D}_r f)}_{\text{drag}} = 0$$

- Resolution with Finite-Volume methods **prohibitive** (7 dimensions in 3D)

Resolution of moments of f

Definition of moments: $\mathcal{M}_{k,l} = \int_{S_{min}}^{S_{max}} \int_{\mathbb{R}} S^k \mathbf{u}^l f \, d\mathbf{u} dS$

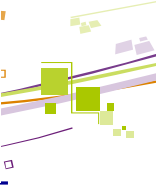
Size moments: $m_k = \mathcal{M}_{k,0}$

Velocity moments: $M_l = \mathcal{M}_{0,l}$

Mean droplet velocity: $\mathbf{u}_p = \frac{\mathcal{M}_{0,1}}{\mathcal{M}_{0,0}}$

General context:

Three classes of polydisperse spray models

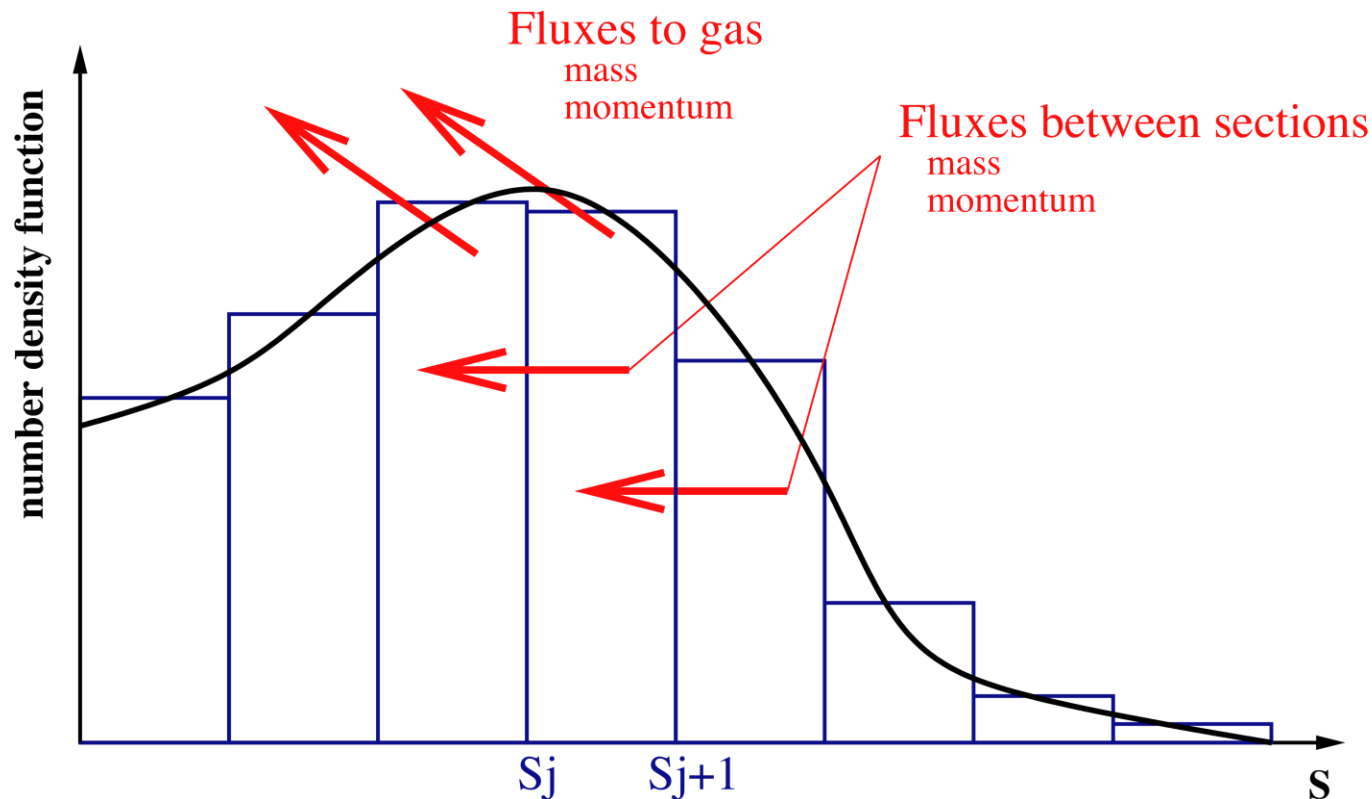


Sectional models: Multi-Fluid model (MF) (*de Chaisemartin, PhD09, Laurent and Massot 01*)

High order moment model with presumed NDF (*Mossa 05*)

High order moment model with Direct quadrature Method of Moments (DQMOM) (*Marchisio et al. 05, Fox et al. 08*)

Multi-Fluid model: Principle



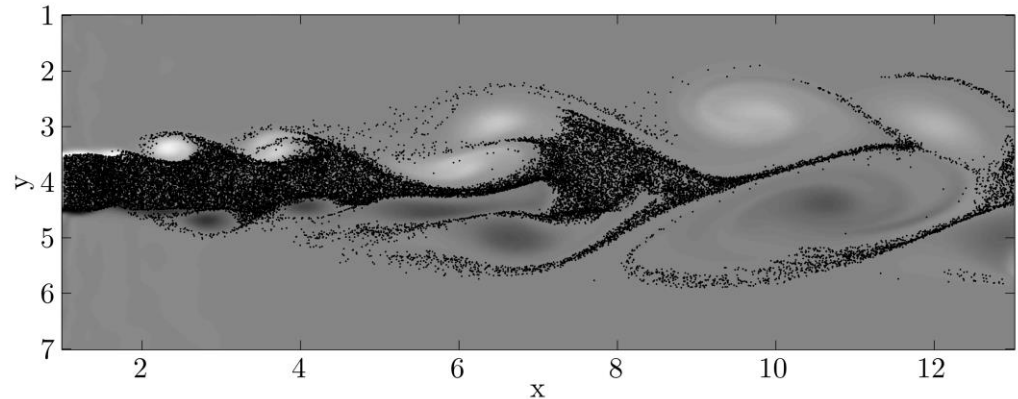
- The size phase space is discretized into sections $[S_j, S_{j+1}]$
- For each section, conservation equations are written for the mass moment $m_{3/2,j}$ and momentum $m_{3/2,j} u_{p,j}$
- For the evaporation process, the section quantities are impacted by fluxes from adjacent sections

Multi-Fluid model: Example of achievements

- Comparison MF/Lagrangian (DSMC)

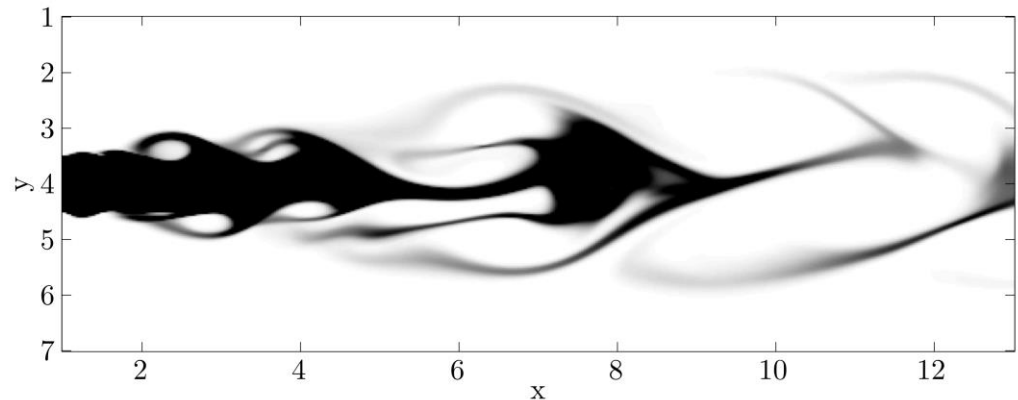
Lagrangian

Particle positions
(30000 particles)



Eulerian

Mass density
400 x 200 x 10 grid



Very good level of comparison Euler/Lagrange

Multi-Fluid model: Limitations

Computational cost

Need to have a substantial number of sections

- To accurately describe polydispersity
- To limit diffusion in size phase space in the case of evaporation

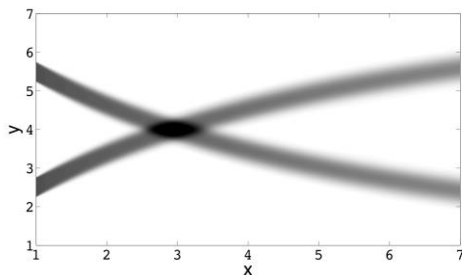
➡ Important computational cost, prohibitive for an industrial application

Particle trajectory crossing (PTC)

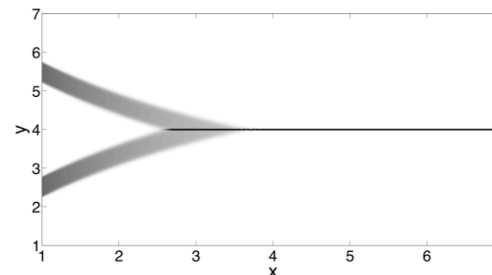
Only one velocity solved per section: $u_{p,j}$

➡ Description of droplet crossing impossible in the same section

Kinetic solution



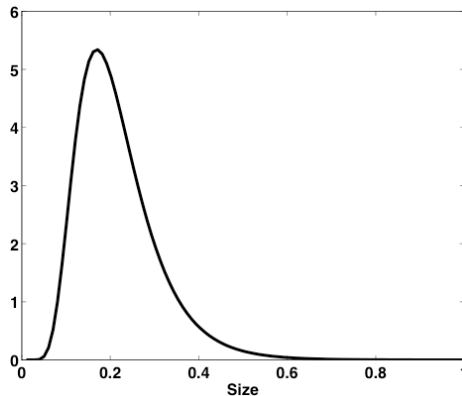
Multi-Fluid solution



High order moment model with presumed NDF

Presumed profile of the NDF

- Example: Presumed log-normal distribution function



$$f(D) = \frac{m_0}{D \ln(\sigma) \sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \left[\frac{\ln(D/\bar{D})}{\ln(\sigma)} \right]^2 \right\}$$

- Evolution of the log-normal function parameters m_0, \bar{D}, σ

But, for arbitrary evaporation process, **it might not be possible to reconstruct a log-normal function**

The model is unstable for evaporation

High order moment model using Direct Quadrature Method of Moment (DQMOM)

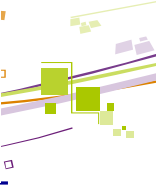
Principle

- Approximation of f under the profile:

$$f(t, \mathbf{x}, S, \mathbf{u}) = \sum_{n=1}^N \omega_n \delta(S - S_n) \delta(\mathbf{u} - \mathbf{u}_n)$$

- Evolution of the weights and abscissas directly $\omega_n, S_n, \mathbf{u}_n$

- Excellent results for coalescence
- But difficulties to determine the evaporating flux of the particles
- PTC cannot be described



We distinguish three classes of models

Sectional models: Multi-Fluid model (MF) (*de Chaisemartin, PhD09, Laurent and Massot 01*)

- Size/Velocity correlation are resolved
- Several sections needed, leading to important CPU cost

High order moment model with presumed NDF (*Mossa 05*)

- Polydispersity is considered with only one section
- The model is instable through evaporation

High order moment model with Direct quadrature Method of Moments (DQMOM) (*Marchisio et al. 05, Fox et al. 08*)

- Polydispersity is considered with only one section
- There exist cases where evaporation is not accurately described

- None of these models is able to describe PTC between same size droplets

Achievements of the PhD

This PhD work has consisted in designing an **Eulerian spray** model and associated numerical tools:

- overcoming the limitations of current Eulerian models with respect to description of **polydispersity** and **particle trajectory crossing**
- in order to be extended to an **industrial context** (IFP-C3D)

Modeling work

Polydisperse spray model:

- with one section
- efficient in terms of CPU cost
- simulating PTC

Extension to industrial context

- Adaptation to the formalism used in IFP-C3D: Arbitrary Lagrangian Eulerian (ALE)
- Implementation in IFP-C3D

Achievements of the PhD

- Description of polydispersity:

Eulerian Multi-Size Moment (EMSM) model

Development of the EMSM model and numerical tools

- Evaporation

- design of a consistent model
- design of an accurate and robust numerical scheme

➡ *(conf. SIAM 08, Monterey), (SIAM J. App. Math., 10)*

} *(VKI, 09)*

- Advection of a moment set

- design of a accurate and robust advection scheme

➡ *(conf ICLASS 09, Vail)*

- Validation of the EMSM model (implemented in Muses3D) versus the MF model

➡ *(JCP 10)*

Achievements of the PhD

- Description of polydispersity:

Eulerian Multi-Size Moment (EMSM) model

Extension to an industrial context

- Adaptation of the designed schemes to the ALE formalism
- Implementation of the EMSM model in IFP-C3D and validation

➡ *(conf. ICMF 10, Tampa Bay), (IJMF 10)*

Achievements of the PhD

- Description of particle trajectory crossing

Eulerian Multi-Velocity Moment (EMVM) model

Development of the EMVM model and numerical tools

- 1D problem: Mathematical study of the closed system in order to prepare the design of high order advection schemes:

➡ *(CMS 10)*

- Multi-D problem: Study on closure of the ill-posed problem and evaluation of the EMVM model

- Proposed closure
- Coupling of EMVM and MF models: EMFVM model
- Validation of EMSVM model (implemented in Muses3D) by comparison with Lagrangian results

➡ *(2 Proceedings of the CTR 08, Stanford), (CRAS 09), (FTC 10)*

Topics discussed during this presentation

- EMSM model: Modeling and numerical tools
 - General resolution strategy
 - Evaporation term resolution
 - Advection term resolution
- Evaluation of the EMSM model
 - Quantitative validation
 - Comparison with the MF model
- Extension to IFP-C3D
 - Adaptation to the ALE formalism
 - Implementation in IFP-C3D and validation

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EMSM model: Modeling and numerical tools

General resolution strategy

Basis kinetic equation

$$\partial_t f + \underbrace{\nabla_{\mathbf{x}}(\mathbf{u}f)}_{\text{advection}} + \underbrace{\partial_S(Kf)}_{\substack{\text{evaporation} \\ (d^2 \text{ law})}} + \underbrace{\nabla_{\mathbf{u}}(\mathbf{D}_r f)}_{\text{drag}} = 0$$

Expression of the NDF

$$f(t, \mathbf{x}, S, \mathbf{u}) = \underbrace{n(t, \mathbf{x}, S)}_{\text{size distribution}} \underbrace{\delta(\mathbf{u} - \mathbf{u}_p(t, \mathbf{x}, S))}_{\text{velocity distribution}}$$

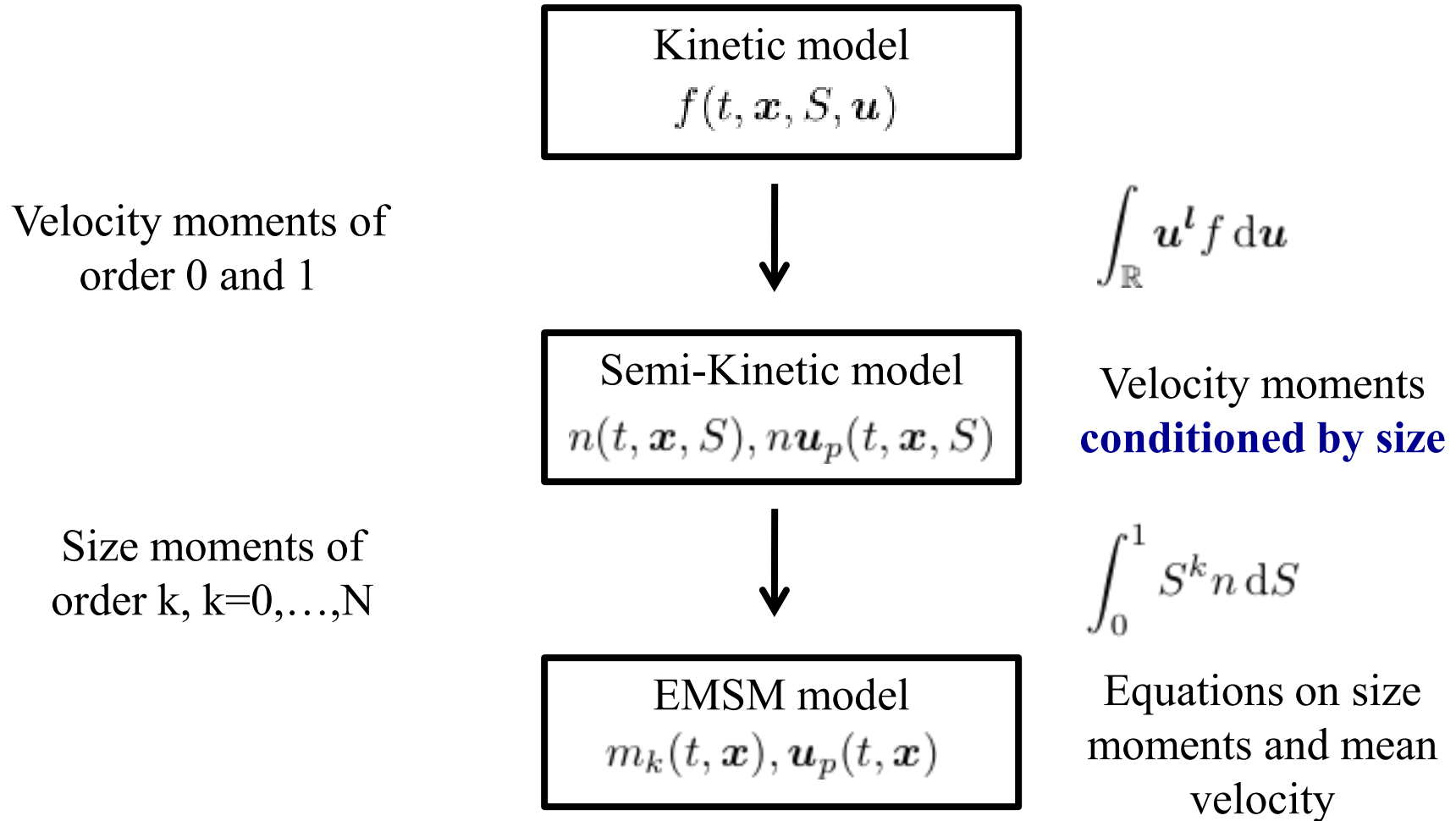
Monokinetic assumption for a given size

Quantities resolved

- $m_k = \mathcal{M}_{k,0} = \int_0^1 \int_{\mathbb{R}} S^k \mathbf{u}^l f \, d\mathbf{u} dS$
- $\mathbf{u}_p = \frac{\mathcal{M}_{0,1}}{\mathcal{M}_{0,0}}$

EMSM model: Modeling and numerical tools

General resolution strategy



EMSM model: Modeling and numerical tools

General resolution strategy

Moment equation system

We aim to solve the following system:

$$\partial_t m_0 + \nabla_{\mathbf{x}}(m_0 \mathbf{u}_p) = -K n(t, \mathbf{x}, S = 0)$$

$$\vdots$$

$$\partial_t m_N + \nabla_{\mathbf{x}}(m_N \mathbf{u}_p) = -K N m_{N-1}$$

$$\partial_t m_1 \mathbf{u}_p + \underbrace{\nabla_{\mathbf{x}}(m_1 \mathbf{u}_p \otimes \mathbf{u}_p)}_{\text{advection}} = \underbrace{-K m_0 \mathbf{u}_p}_{\text{evaporation}} - \underbrace{\nabla_{\mathbf{x}} P + D}_{\text{drag}}$$

EMSM model: Modeling and numerical tools

General resolution strategy

Moment equation system

We aim to solve the following system:

$$\begin{aligned}
 \partial_t m_0 + \nabla_{\mathbf{x}}(m_0 \mathbf{u}_p) &= -K n(t, \mathbf{x}, S = 0) \\
 &\vdots \\
 \partial_t m_N + \nabla_{\mathbf{x}}(m_N \mathbf{u}_p) &= -K N m_{N-1} \\
 \partial_t m_1 \mathbf{u}_p + \nabla_{\mathbf{x}}(m_1 \mathbf{u}_p \otimes \mathbf{u}_p) &= \underbrace{-K m_0 \mathbf{u}_p}_{\text{evaporation}} - \underbrace{\nabla_{\mathbf{x}} P}_{\text{drag}} + \underbrace{D}_{\text{drag}}
 \end{aligned}$$

Unclosed terms

- $f(t, \mathbf{x}, S, \mathbf{u}) = n(t, \mathbf{x}, S) \delta(\mathbf{u} - \mathbf{u}_p) \Rightarrow P = 0$ (velocity dispersion)
- $n(t, \mathbf{x}, S = 0) = \Phi(m_0, \dots, m_N)(t, \mathbf{x})$

EMSM model: Modeling and numerical tools

General resolution strategy

- Operator splitting strategy (*Strang 68*)
- Successive resolution of
 - **Evaporation**
 - Advection
 - Drag

$$\partial_t m_0 + \nabla_{\mathbf{x}}(m_0 \mathbf{u}_p) = -K n(t, \mathbf{x}, S = 0)$$

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$$\partial_t m_N + \nabla_{\mathbf{x}}(m_N \mathbf{u}_p) = -K m_{N-1}$$

$$\partial_t m_1 \mathbf{u}_p + \nabla_{\mathbf{x}}(m_1 \mathbf{u}_p \otimes \mathbf{u}_p) = -K m_0 \mathbf{u}_p - \nabla_{\mathbf{x}} P + D$$

EMSM model: Modeling and numerical tools

General resolution strategy

- Operator splitting strategy (*Strang 68*)
- Successive resolution of
 - Evaporation
 - **Advection**
 - Drag

$$\partial_t m_0 + \nabla_{\mathbf{x}}(m_0 \mathbf{u}_p) = -Kn(t, \mathbf{x}, S = 0)$$

$$\vdots$$

$$\partial_t m_N + \nabla_{\mathbf{x}}(m_N \mathbf{u}_p) = -Km_{N-1}$$

$$\partial_t m_1 \mathbf{u}_p + \nabla_{\mathbf{x}}(m_1 \mathbf{u}_p \otimes \mathbf{u}_p) = -Km_0 \mathbf{u}_p - \nabla_{\mathbf{x}} P + D$$

EMSM model: Modeling and numerical tools

General resolution strategy

- Operator splitting strategy (*Strang 68*)
- Successive resolution of
 - Evaporation
 - Advection
 - **Drag**

$$\partial_t m_0 + \nabla_{\mathbf{x}}(m_0 \mathbf{u}_p) = -Kn(t, \mathbf{x}, S = 0)$$

$$\vdots$$

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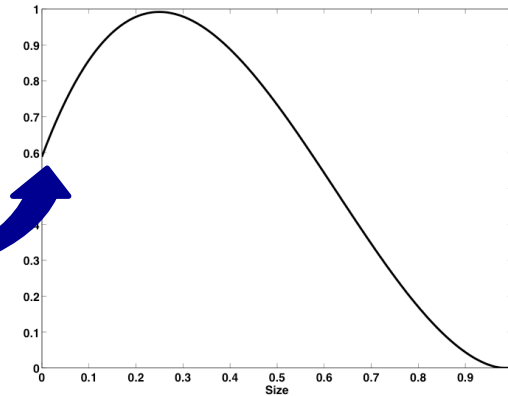
EMSM model: Modeling and numerical tools

Evaporation term resolution

Continuous problem

- Closure of $Kn(t, \mathbf{x}, S = 0)$ = evaporation flux
- The challenge is to reconstruct, from the data of the moments, a pointwise value of the size NDF

$$\mathbf{m}_N = (m_0, \dots, m_N)^t$$



- Stability condition : moment space preservation

EMSM model: Modeling and numerical tools

Evaporation term resolution

Moment space

- The vector $\mathbf{m}_N = (m_0, \dots, m_N)^t$ belongs to moment space \mathbb{M}_N
- \mathbb{M}_N has a complex geometry

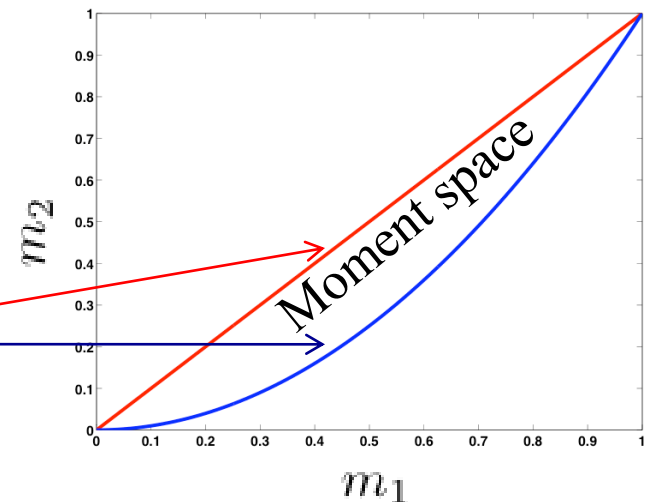
Example of moment space geometry

- Set of PDF on $[0,1]$ such as $m_0 = 1$
- Condition on m_1 and m_2

$$m_1 = \int_0^1 S n \, dS, \quad 0 < m_1 < 1$$

$$m_2 = \int_0^1 S^2 n \, dS, \quad \underbrace{m_1^2}_{\text{low border}} < m_2 < \underbrace{m_1}_{\text{high border}}$$

Moment space structure



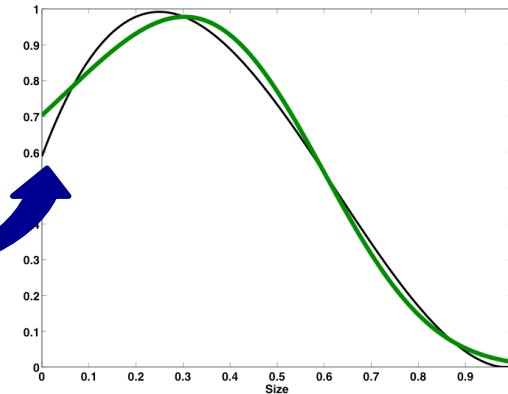
EMSM model: Modeling and numerical tools

Evaporation term resolution

Continuous problem

- Closure of $K^n(t, \mathbf{x}, S = 0)$ = evaporation flux
- The challenge is to reconstruct, from the data of the moments, a pointwise value of the size NDF

$$\mathbf{m}_N = (m_0, \dots, m_N)^t$$



- Stability condition : moment space preservation:

Realizability condition

Solution

Approximation of the size NDF by **Maximization of Entropy** (Mead 84)

$$\mathbf{m}_N = (m_0, \dots, m_N)^t \rightarrow \tilde{n}_{ME}$$

EMSM model: Modeling and numerical tools

Evaporation term resolution

Challenge for the discrete problem

Evaporation system **cannot** be solved by ODE solvers



Realizability condition

Solution

- Finite volume scheme
- Exact temporal integration
- Flux calculation: kinetic scheme

EMSM model: Modeling and numerical tools

Evaporation term resolution

- The kinetic scheme lies on the equivalence between the two equations:

- kinetic (or semi-kinetic): $\partial_t n - \partial_S (K n) = 0$

- macroscopic: $\partial_t m_k = \dots, \quad k = 0, \dots, N$

Principle of a kinetic scheme

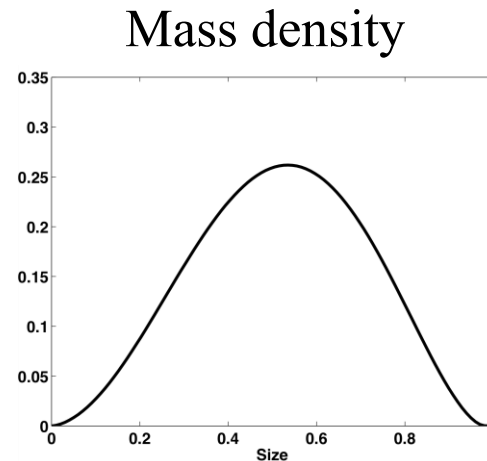
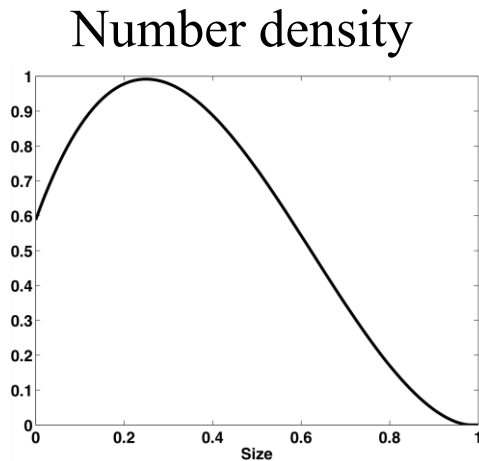
$$\begin{array}{ccc} m_N(0) & & m_N(\Delta t) \\ \downarrow & & \uparrow \\ \tilde{n}_{ME}(0, S) & \xrightarrow[\Delta t]{\text{exact evolution}} & \tilde{n}_{ME}(\Delta t, S) \end{array}$$

- \tilde{n}_{ME} is reconstructed from m_N
- $\tilde{n}_{ME}(\Delta t, S)$ is computed from $\tilde{n}_{ME}(0, S)$ through the kinetic equation
- Fluxes of m_N are computed using $\tilde{n}_{ME}(\Delta t, S)$

EMSM model: Modeling and numerical tools

Evaporation term resolution

- Evolution of the total mass of a spray with the initial size distribution:



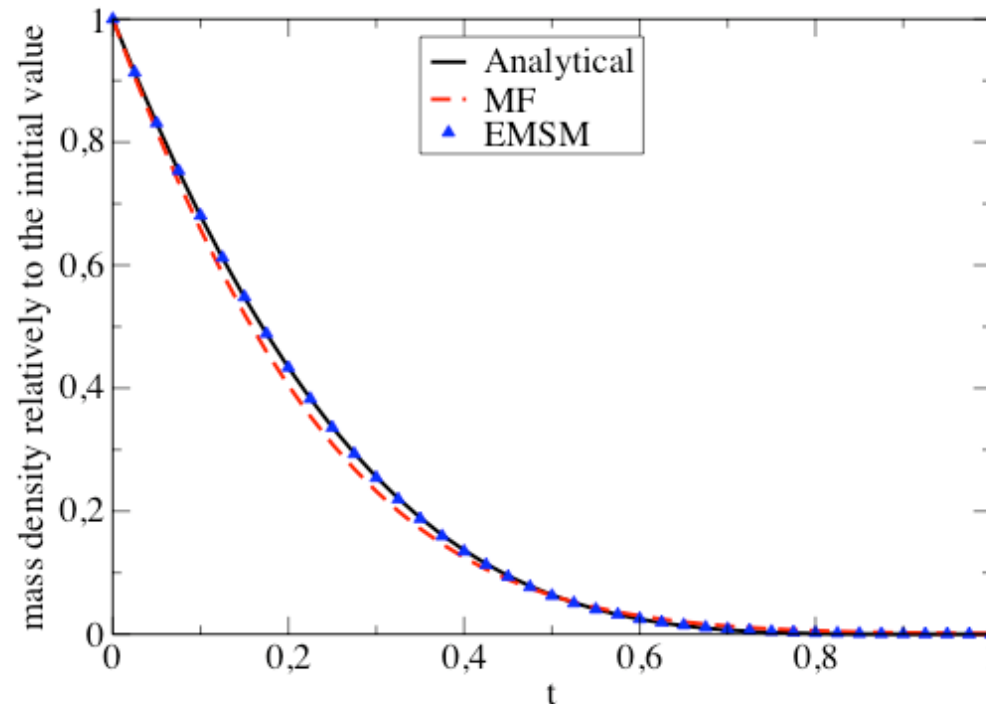
Comparison between EMSM and MF models

- MF model: $m_{3/2}$ 1 moment and 10 sections
- EMSM model: $m_k, k = 0, \dots, 3$ 4 moments and 1 section

EMSM model: Modeling and numerical tools

Evaporation term resolution

- Results



- Very good level of comparison

Similar accuracy between EMSM with 1 section and MF with 10 sections

EMSM model: Modeling and numerical tools

Evaporation term resolution

Conclusion

- Design of theoretical (Entropy Maximization) and numerical tool (kinetic scheme)
 - ➔ **Stable scheme for evaporation flux**
Accuracy of the EMSM model with 1 section similar to the accuracy of the MF model with 10 sections
- The designed EMSM model and numerical tools are extendable to
 - arbitrary evaporation laws
 - formalism with several sections

Perspectives

- The entropy maximization can be applied to bivariate distributions
- ➔ Application to non spherical particles (soots)

Topics discussed during this presentation

- **EMSM model: Modeling and numerical tools**
 - General resolution strategy
 - Evaporation term resolution
 - **Advection term resolution**
- Evaluation of the EMSM model
 - Quantitative validation
 - Comparison with the MF model
- Extension to IFP-C3D
 - Adaptation to the ALE formalism
 - Implementation in IFP-C3D and validation

EMSM model: Modeling and numerical tools

Advection term resolution

Purpose

Design a high order advection scheme for the system:

$$\partial_t m_0 + \nabla_{\mathbf{x}}(m_0 \mathbf{u}_p) = 0$$

$$\vdots$$

$$\partial_t m_N + \nabla_{\mathbf{x}}(m_N \mathbf{u}_p) = 0$$

$$\partial_t m_1 \mathbf{u}_p + \nabla_{\mathbf{x}}(m_1 \mathbf{u}_p \otimes \mathbf{u}_p) = 0$$

EMSM model: Modeling and numerical tools

Advection term resolution

Purpose

Design a high order advection scheme for the system:

$$\partial_t m_0 + \nabla_x(m_0 \mathbf{u}_p) = 0$$

$$\vdots$$

$$\partial_t m_N + \nabla_x(m_N \mathbf{u}_p) = 0$$

$$\partial_t m_1 \mathbf{u}_p + \nabla_x(m_1 \mathbf{u}_p \otimes \mathbf{u}_p) = 0$$

Challenges

- Pressureless gas dynamics (PGD)– delta choc

EMSM model: Modeling and numerical tools

Advection term resolution

Purpose
Design a high order advection scheme for the system:
$\partial_t m_0 + \nabla_x(m_0 \mathbf{u}_p) = 0$
\vdots
$\partial_t m_N + \nabla_x(m_N \mathbf{u}_p) = 0$
$\partial_t m_1 \mathbf{u}_p + \nabla_x(m_1 \mathbf{u}_p \otimes \mathbf{u}_p) = 0$

dimensional
splitting



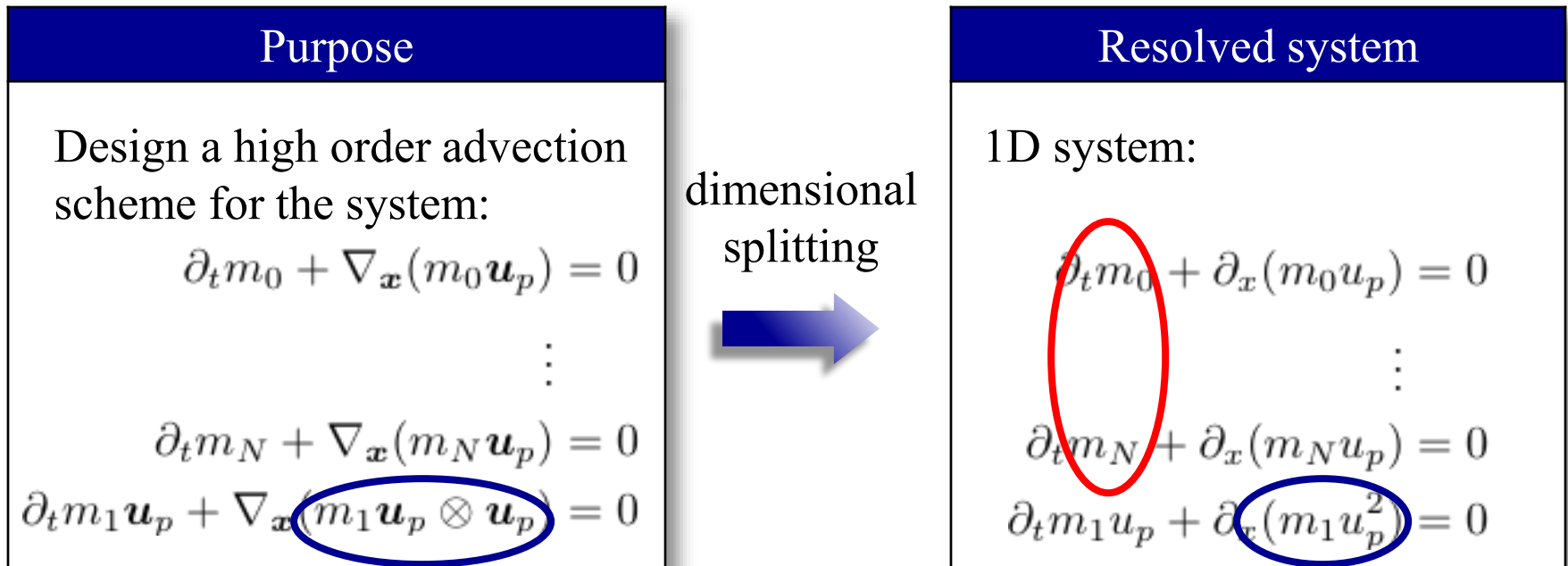
Resolved system
1D system:
$\partial_t m_0 + \partial_x(m_0 u_p) = 0$
\vdots
$\partial_t m_N + \partial_x(m_N u_p) = 0$
$\partial_t m_1 u_p + \partial_x(m_1 u_p^2) = 0$

Challenges

- Pressureless gas dynamics (PGD) – delta choc
 - ➡ Dedicated scheme designed in (*de Chaisemartin, PhD 09*) from the kinetic scheme (*Perthame 02, Bouchut 03*) and using dimensional splitting

EMSM model: Modeling and numerical tools

Advection term resolution



Challenges

- Pressureless gas dynamics (PGD)– delta choc
 - ➔ Dedicated scheme designed in (*de Chaisemartin, PhD 09*) from the kinetic scheme (*Perthame 02, Bouchut 03*) and using dimensional splitting
- **Realizability condition**

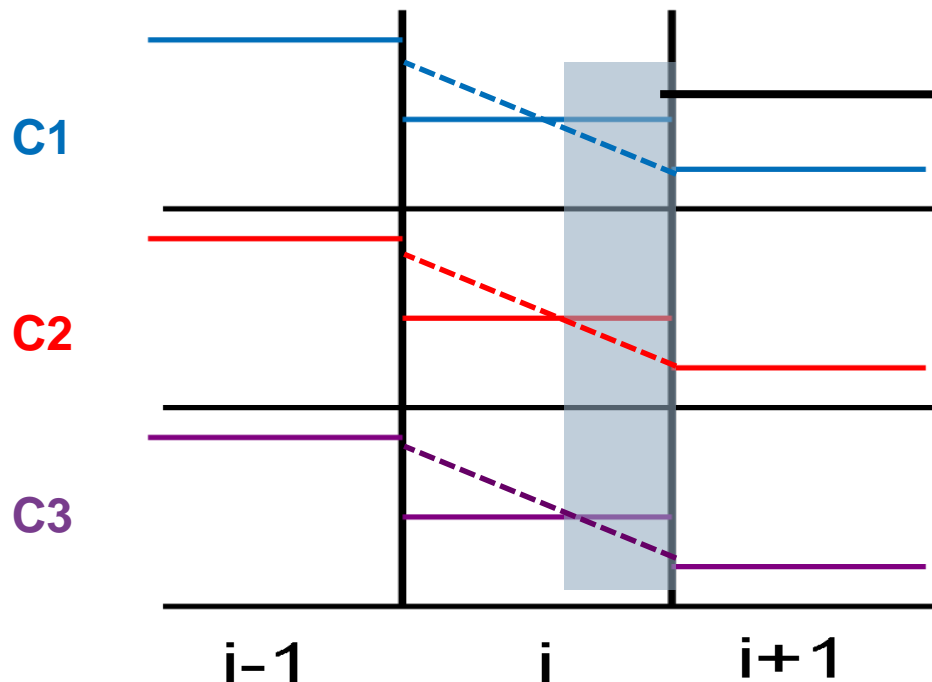
EMSM model: Modeling and numerical tools

Advection term resolution

Realizability condition

- Considering normalized moments $c_k = \frac{m_k}{m_0}$, $k = 1, \dots, N$
the c_k are transported quantities as they verify $\partial_t c_k + u_p \partial_x c_k = 0$

- Independent reconstruction of the c_k



Realizability condition might not be satisfied in the whole cell (*Wright 07, McGraw 07*)

EMSM model: Modeling and numerical tools

Advection term resolution

Realizability condition enforcement

- From the moment space theory, we can define **canonical moments** p_k from

the moment vector:

$$p_k = \frac{c_k - c_k^-}{c_k^+ - c_k^-}$$

(Dette and Studden 97)

- Realizability condition enforced $\longleftrightarrow p_k \in [0, 1]$ **independently**

➡ **Reconstruction of canonical moments** to ensure the realizability condition everywhere in the cell

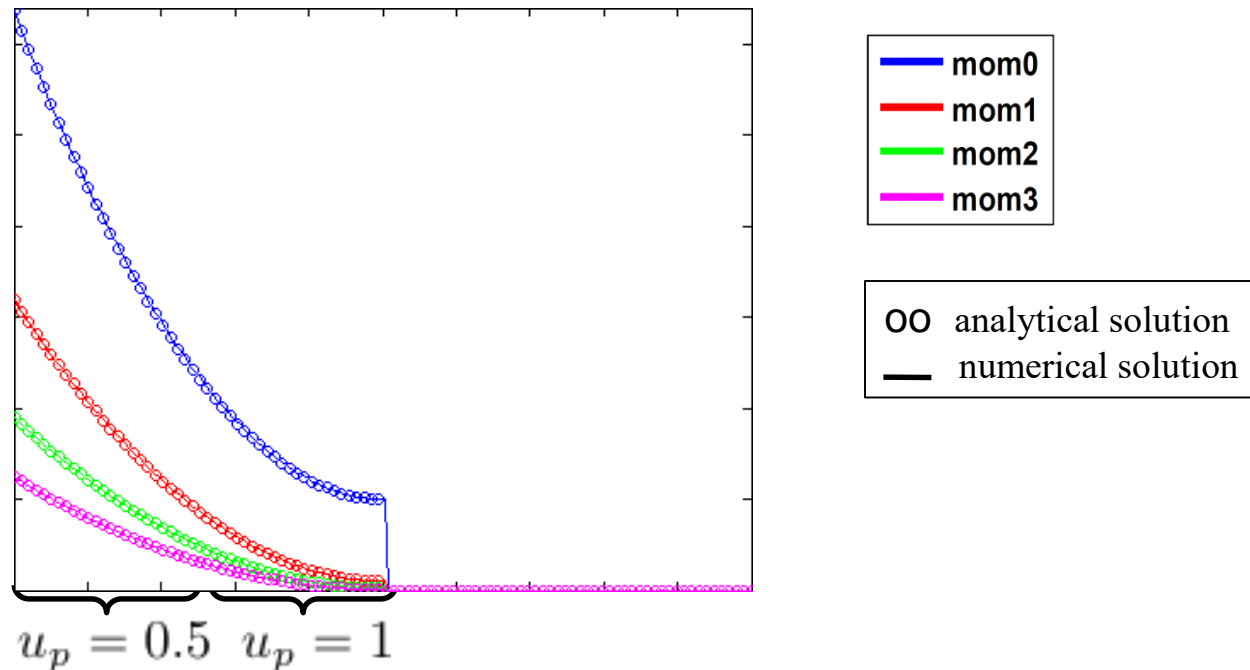
Topics discussed during this presentation

- EMSM model: Modeling and numerical tools
 - General resolution strategy
 - Evaporation term resolution
 - Advection term resolution
- Evaluation of the EMSM model
 - Quantitative validation
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Evaluation of the EMVM model

Quantitative validation

Validation of the evaporation and advection operator



Interest of the test case

- Velocity discontinuity → Creation of Vacuum zone
- Inhomogeneous size NDF profile

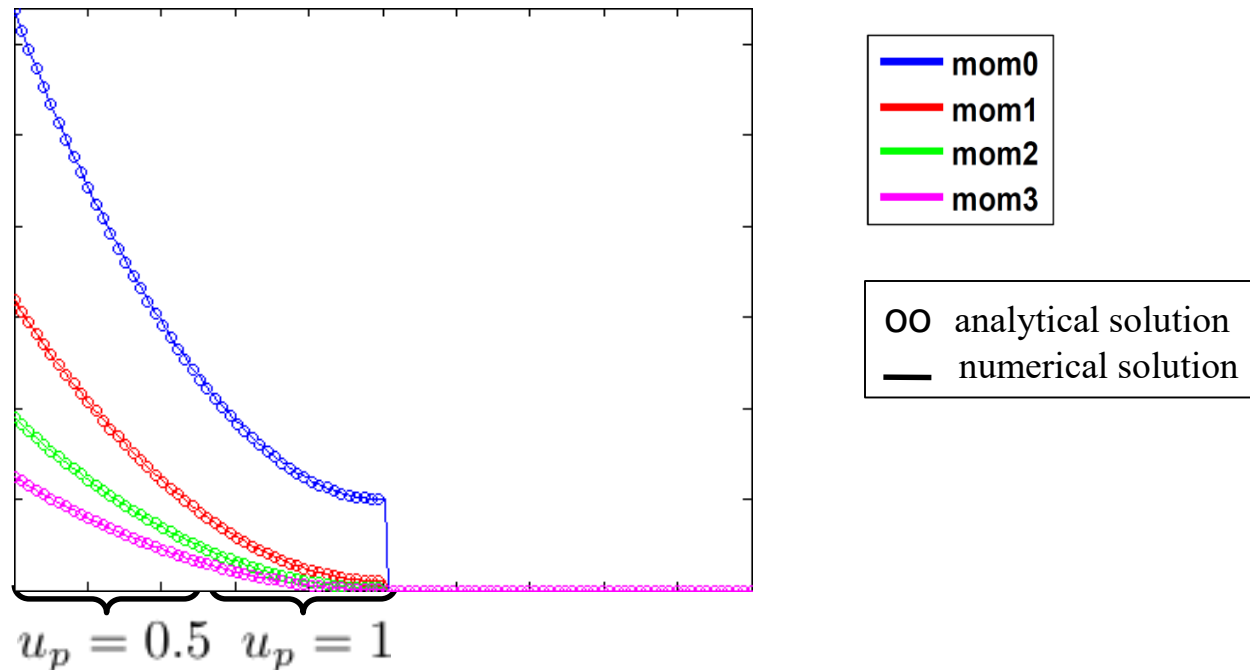
$$n^0(x, S) = \begin{cases} \lambda(x) \sin(\pi S) + (1 - \lambda(x)) \exp(-10 S) & \text{if } x \leq 0.5 \\ 0 & \text{otherwise} \end{cases}$$

→ Actual reconstruction of canonical moment with non null slope

Evaluation of the EMVM model

Quantitative validation

Validation of the evaporation and advection operator



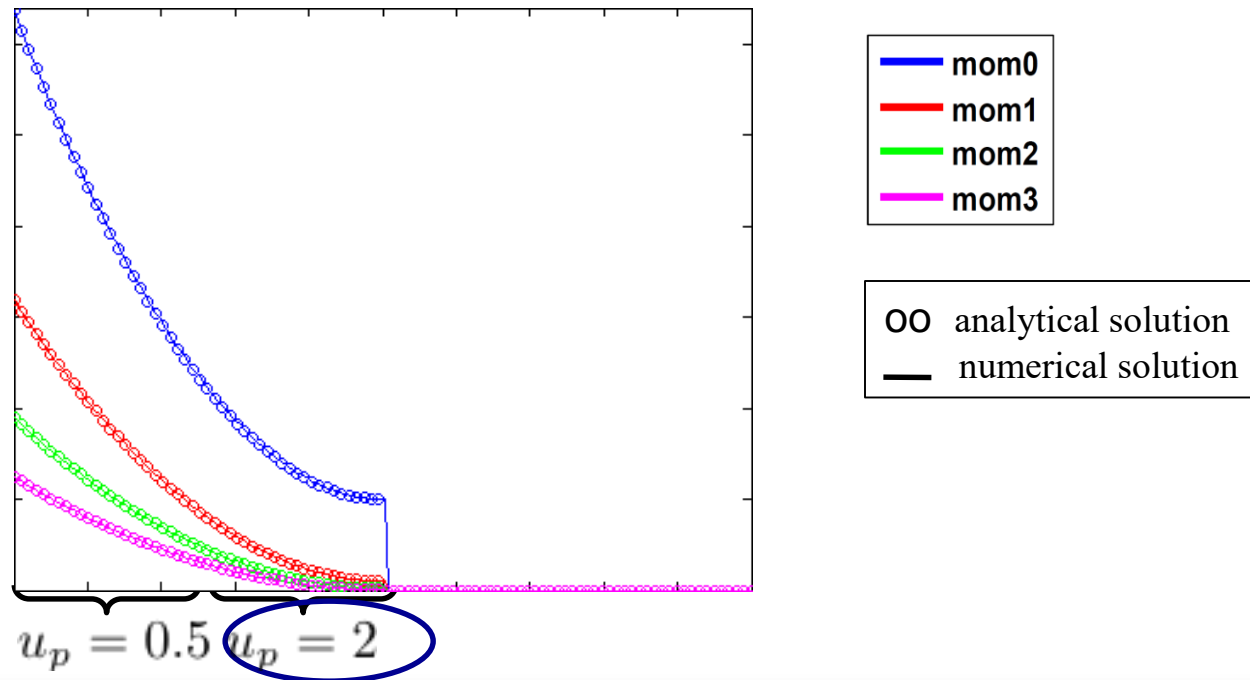
Conclusion

- Vacuum zone well described
- Good level of comparison with the analytical solution

Evaluation of the EMVM model

Quantitative validation

Validation of the evaporation and advection operator



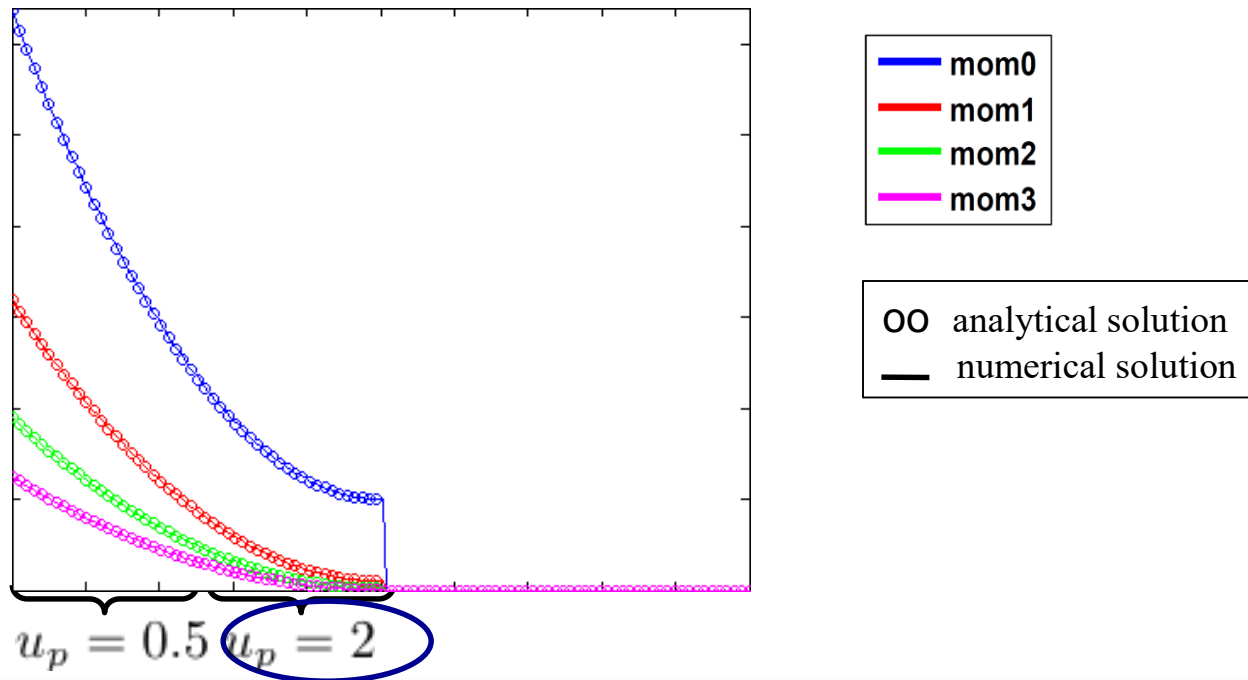
Interest of the test case

- Periodic boundary conditions
 - ➡ Cloud at $u_p = 2$ catches up the cloud at $u_p = 0.5$
 - ➡ Singularity formation

Evaluation of the EMVM model

Quantitative validation

Validation of the evaporation and advection operator



Conclusion

- **Singularity (delta-shock) well captured**
- **Illustration of the monokinetic assumption limitation**

Topics discussed during this presentation

- EMSM model: Modeling and numerical tools
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Comparison with the MF model

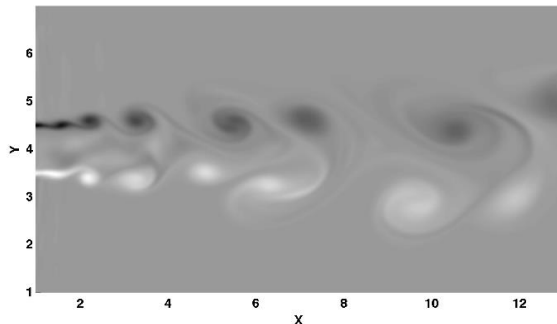
Free jet test case

Case run with Muses3D coupled with asphodele (*Reveillon 07*)

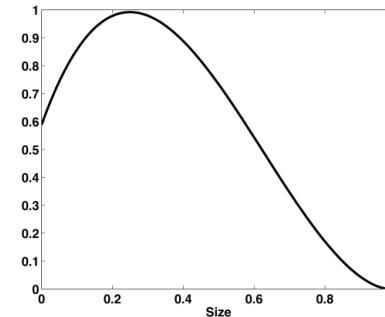
- Complex case: injected turbulence in the gas phase
- Comparison with the MF model, validated by comparison with a lagrangian model
- Time resolved dynamics
- One way coupling

Presentation of the configuration

- Gas vortices field at time $t=20$



- Size NDF for the droplet



- $St_{max} = 0.75$
- Comparison of $m_{3/2}, Y_F$ (evaporated fuel mass fraction)

Comparison with the MF model

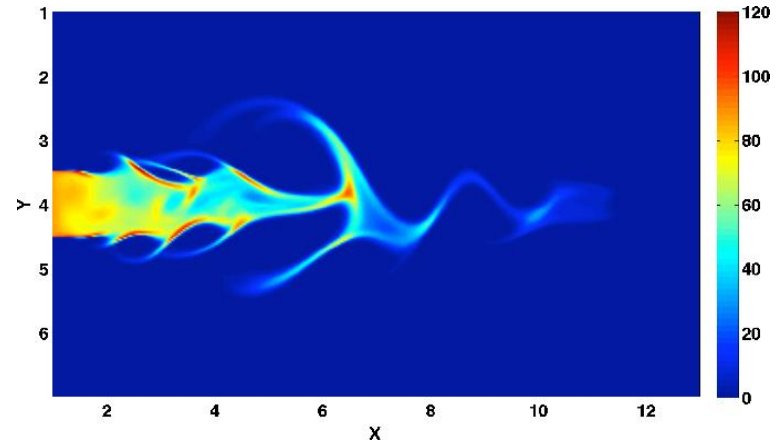
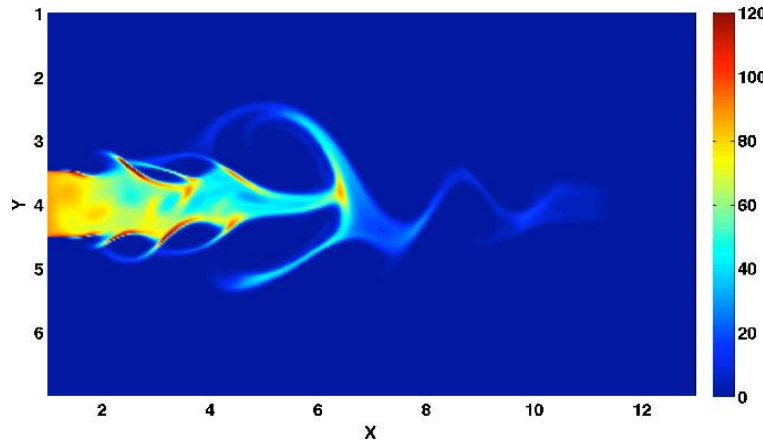
Free jet test case

Results at $t = 10$

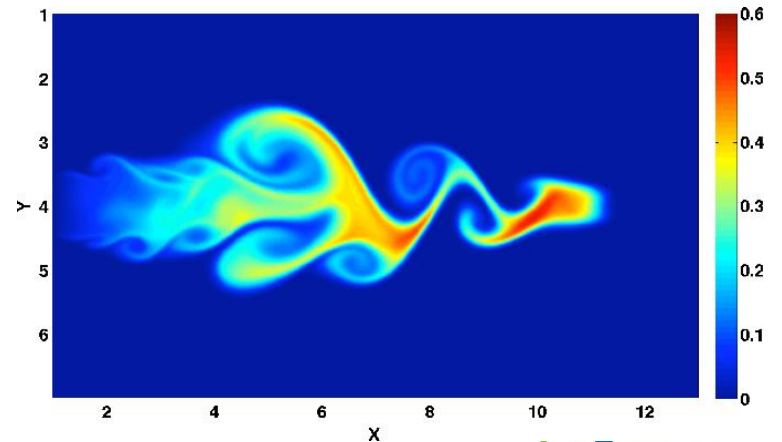
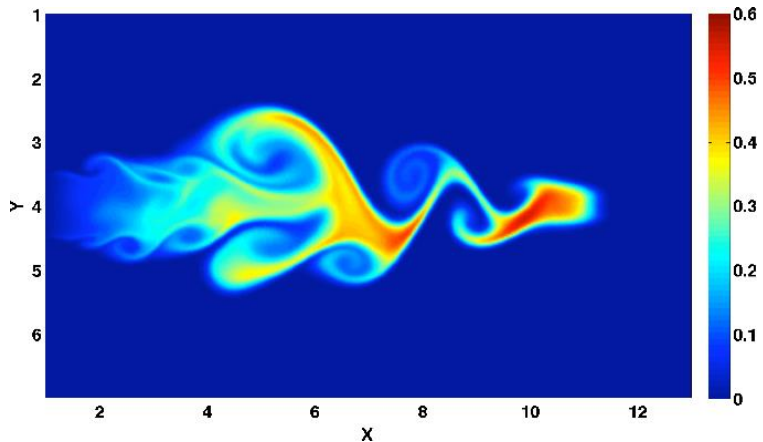
● EMSM model

● MF model

$m_{3/2}$



Y_F



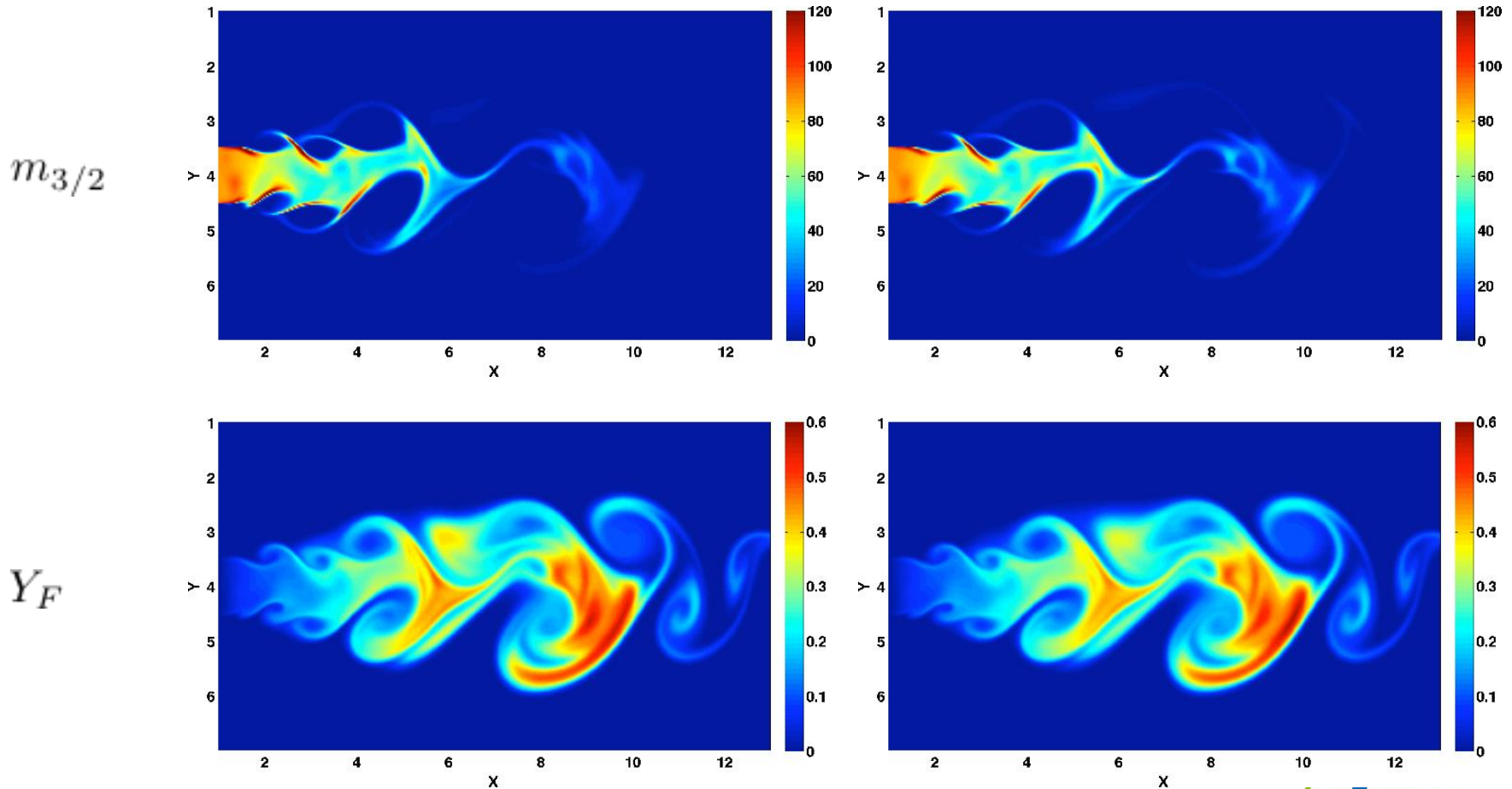
Comparison with the MF model

Free jet test case

Results at $t = 15$

● EMSM model

● MF model



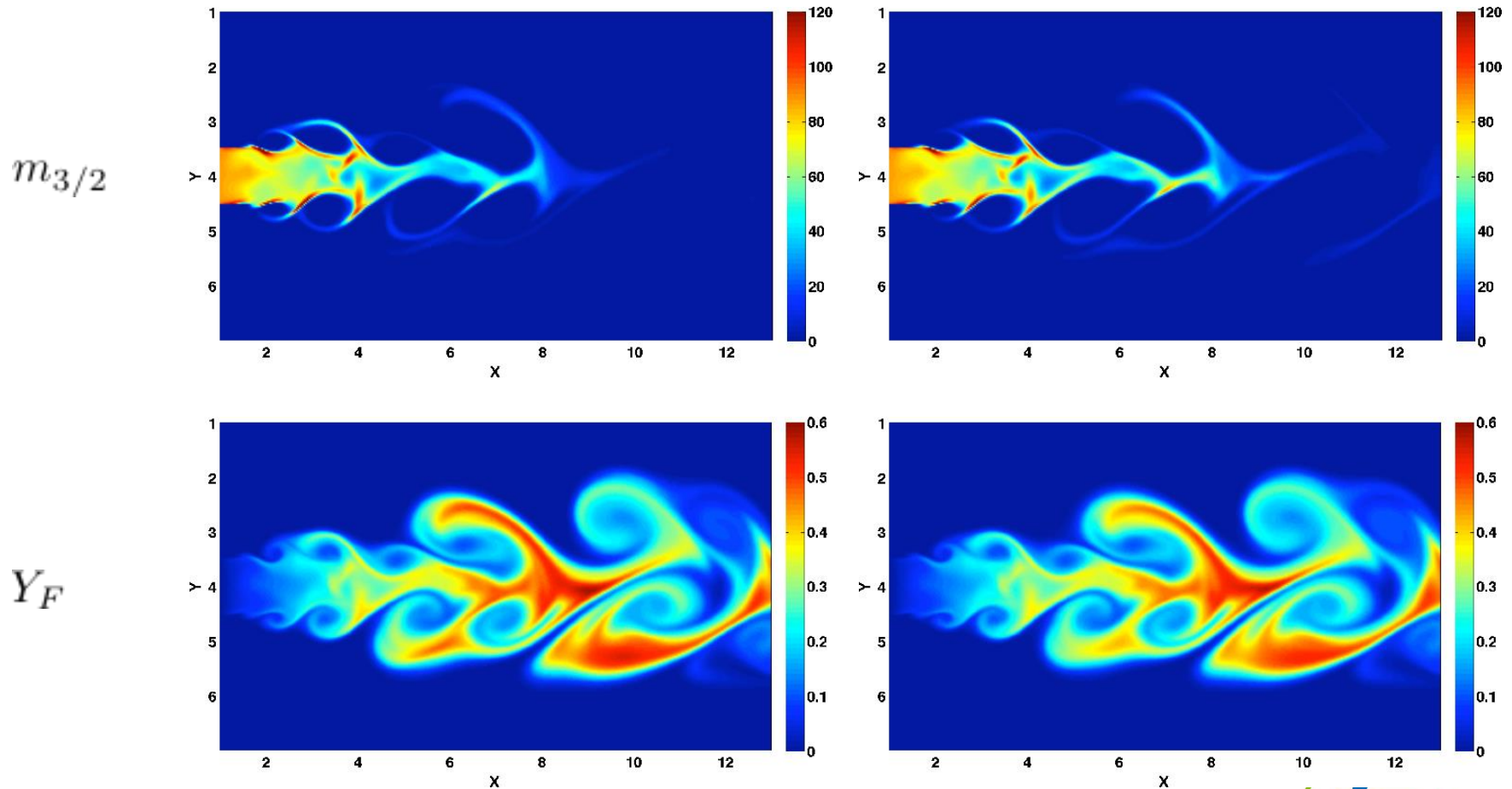
Comparison with the MF model

Free jet test case

Results at $t = 20$

● EMSM model

● MF model



Comparison with the MF model

Free jet test case

Conclusion

- Very good level of comparison
- Validates the EMSM model and numerical tools

Perspective

- Implementation in IFP-C3D

Topics discussed during this presentation

- EMSM model: Modeling and numerical tools
 - General resolution strategy
 - Evaporation term resolution
 - Advection term resolution
- Evaluation of the EMSM model
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Adaptation to the ALE formalism

Presentation of the ALE formalism

Context

- IFP-C3D code: moving boundary conditions (moving piston)
- Description of motion in the context of continuum mechanics
 - ↔ Observation reference frame:
 - **Eulerian** formalism: fixed grid
 - **Lagrangian** formalism: the grid vertices follow the fluid
- But each of these representations has a difficulty
 - Eulerian: **stable** but **diffusive**
 - Lagrangian: **accurate** but **unstable**

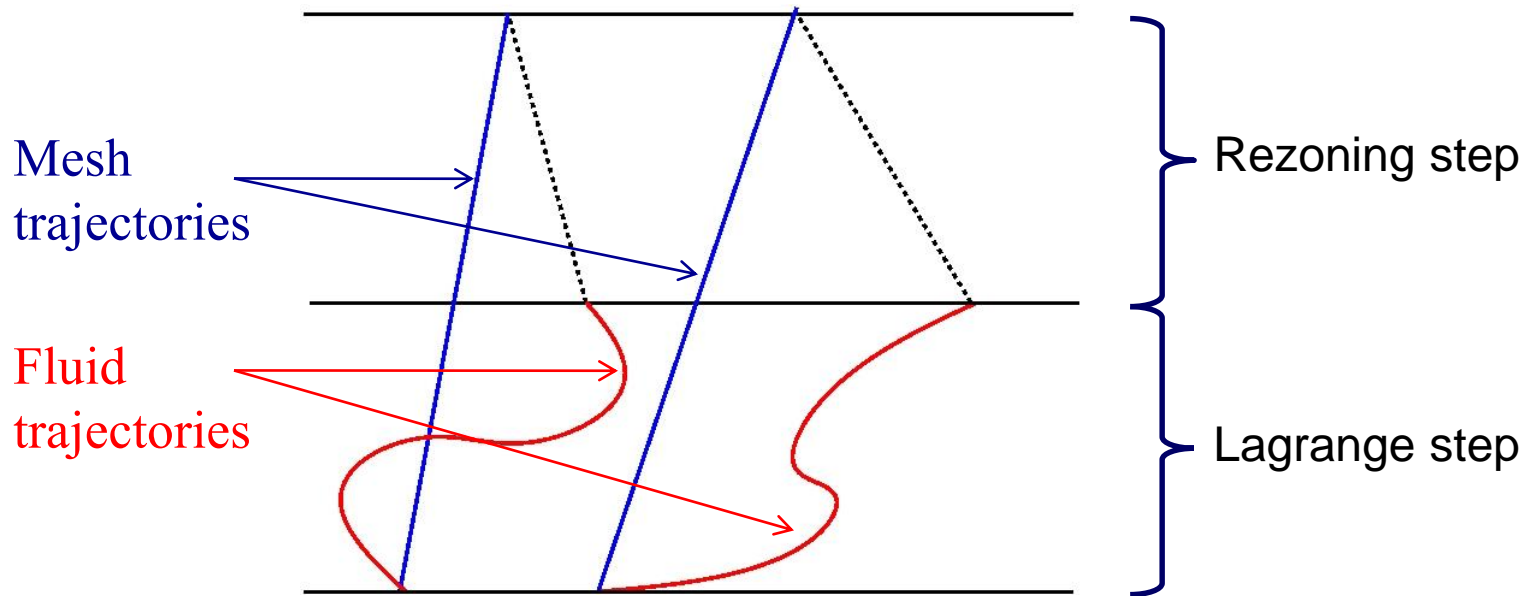
Principle of the ALE formalism

- The **Arbitrary Lagrangian Eulerian (ALE)** formalism (*Hirt 71, Donea 04*) aims at combining the advantages of **stability** and **accuracy** of each of the Eulerian and Lagrangian representation, minimizing their drawback
- This formalism is used in IFP-C3D

Adaptation to the ALE formalism

General algorithm of IFP-C3D

- Operator splitting algorithm:
 - Phase A: source terms during Δt
 - Phase B: Lagrangian transport during Δt
 - Phase C: Eulerian advection “rezoning” during Δt



Adaptation to the ALE formalism

Challenges and achievements

- Challenges a high order advection scheme on a 1D mesh:
Realizability condition and Stability through PGD formalism

Realizability condition

- Adaptation of the kinetic scheme in the ALE formalism

Stability through PGD formalism

- **Stability not guaranteed even for a 1st order scheme**
- New advection scheme inspired from the ideas in (*Larrouturoux 04*):
 - it ensures the conservation of the momentum
 - and the maximum principle on velocity

➡ **Second order in time and space on moving grids**

Topics discussed during this presentation

- EMSM model: Modeling and numerical tools
 - General resolution strategy
 - Evaporation term resolution
 - Advection term resolution
- Evaluation of the EMSM model
 - Quantitative validation
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- **Extension to IFP-C3D**
 - Adaptation to the ALE formalism
 - **Implementation in IFP-C3D and validation**

Implementation of the EMSM model in IFP-C3D

Presentation of the IFP-C3D code

- Hexahedral unstructured solver devoted to internal CFD with spray and combustion modelling
- Conservation equations solved on moving grids
- Equations solved using a finite volume method extended with the ALE formalism. Implicit temporal integration scheme
- k - ϵ turbulence model (RANS)

Existing versions for multiphase flow resolution

- Lagrangian solver for the spray
- Two-Fluid Eulerian solver

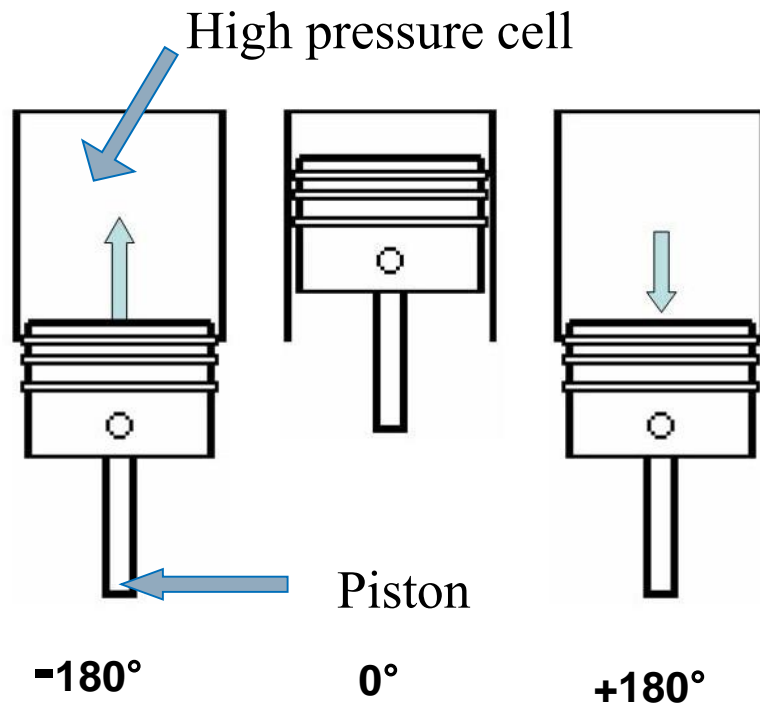
➡ **New code version with the EMSM model**

Implementation of the EMSM model in IFP-C3D

Validations

Validation on a moving piston configuration

Piston movement



Computation parameters

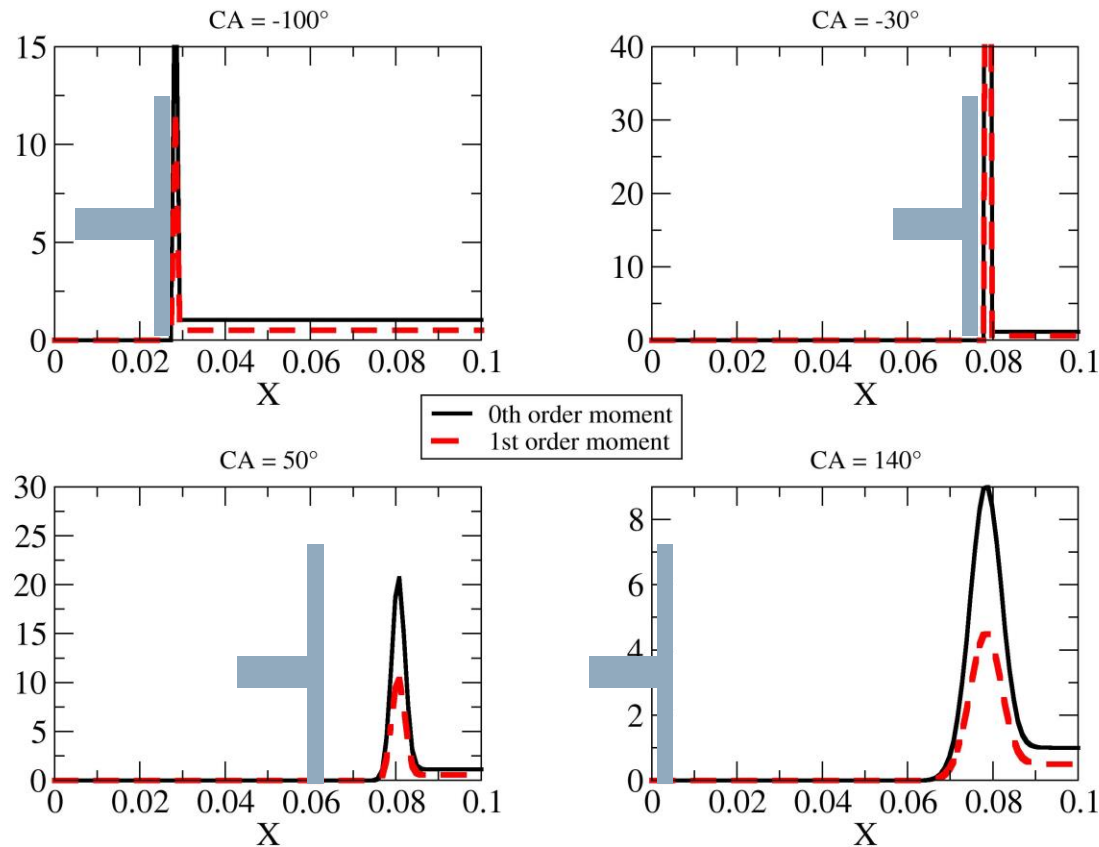
- Size distribution



- Homogeneous field
- Initial velocity = 0
- Homothetic cell volume variation

Implementation of the EMSM model in IFP-C3D

Validations



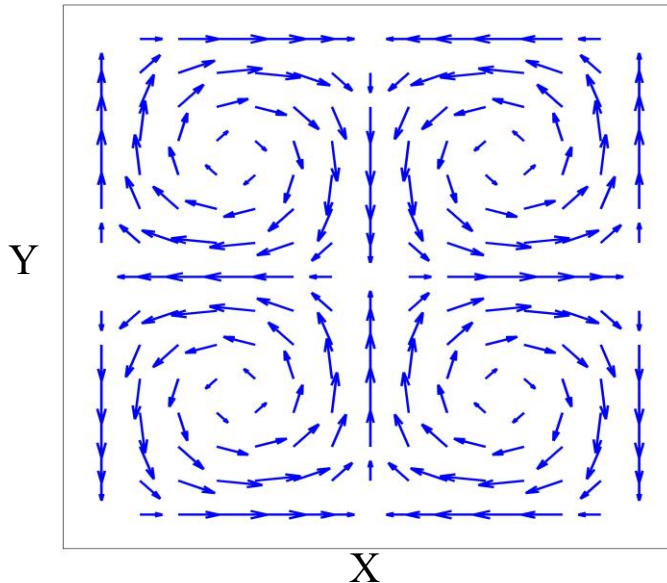
- Singularity dynamic preserved with piston boundary condition

Implementation of the EMSM model in IFP-C3D

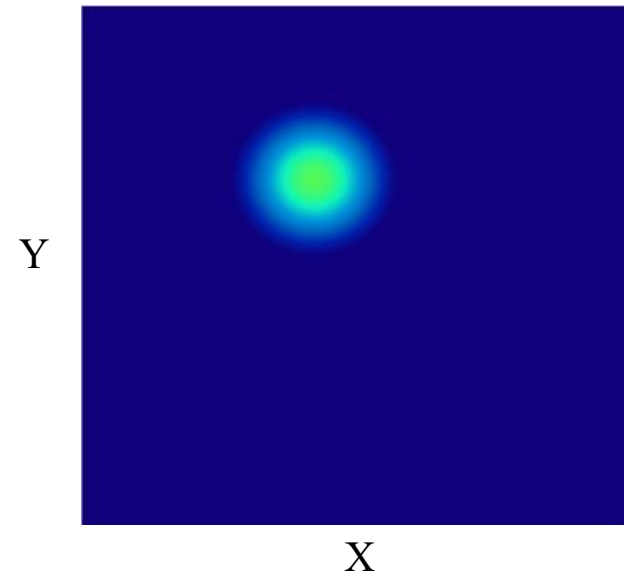
Validations

2D test with evaporation and drag **compared with Muses3D**

Taylor-Green vortices for gas flow



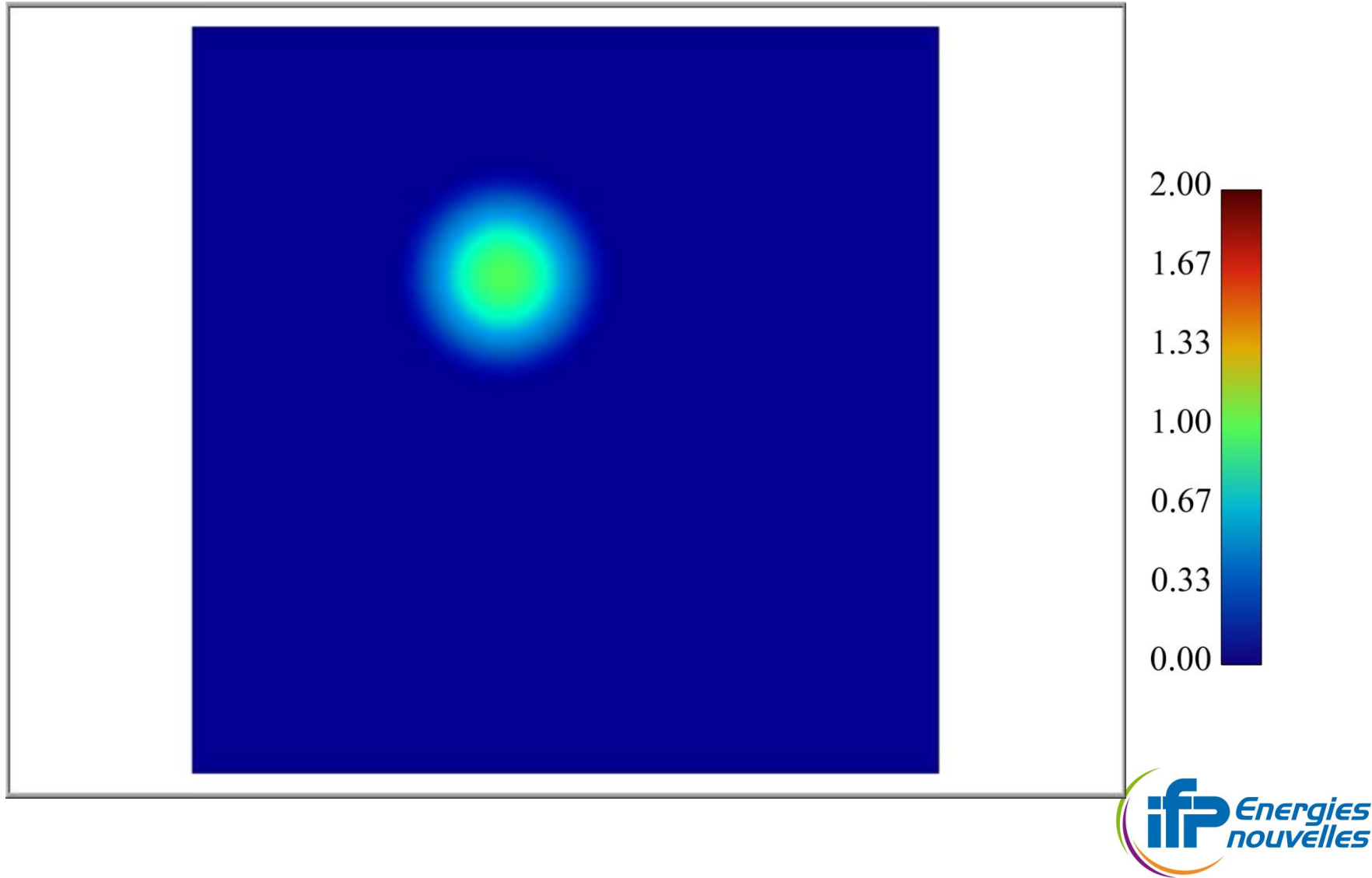
Initial condition for the spray



- Periodic Boundary conditions
- $St_{max} = 2.81$
- Evaporation: $K = 0.21$
- Final time = 2

Implementation of the EMSM model in IFP-C3D

Validations

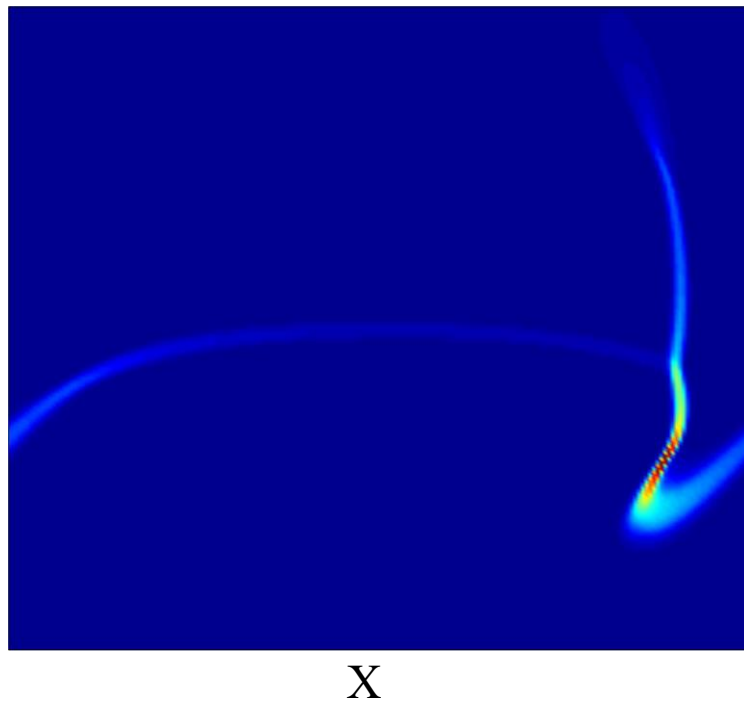


Implementation of the EMSM model in IFP-C3D

Validations

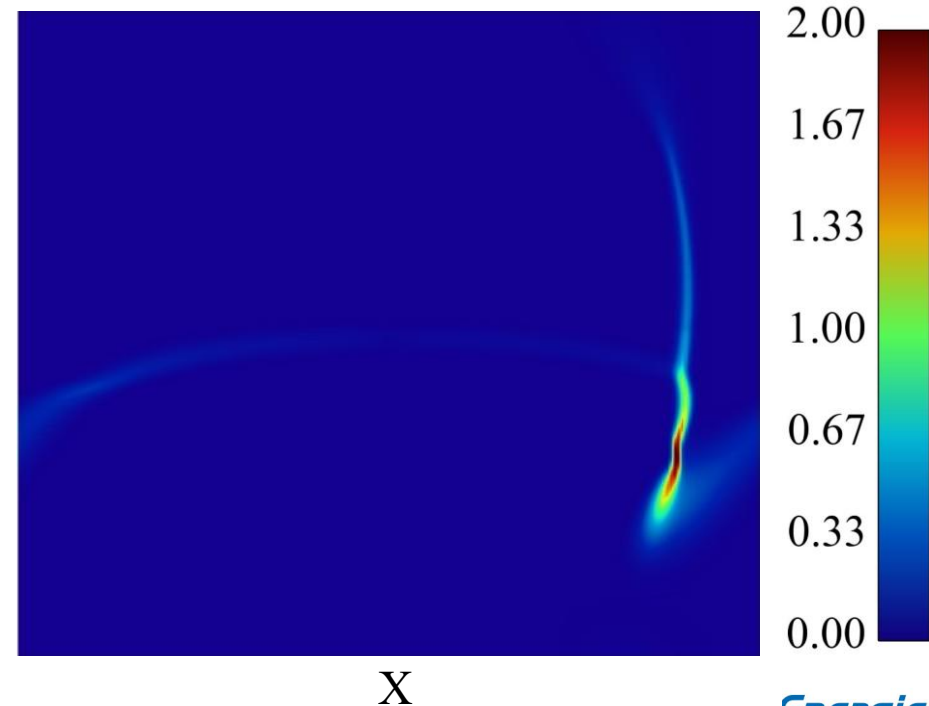
● Muses3D

- Structured
- Eulerian
- 2nd order



● IFP-C3D

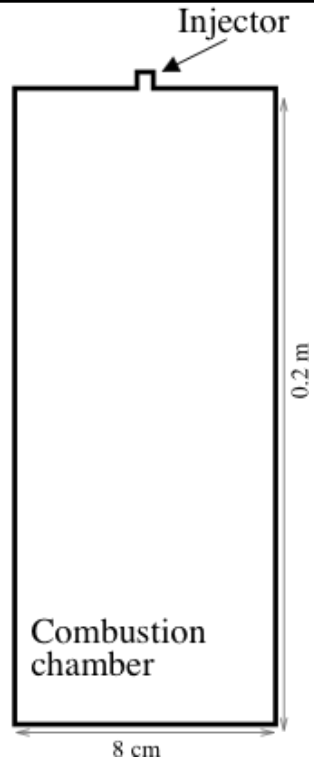
- Unstructured
- ALE
- 1st order



Implementation of the EMSM model in IFP-C3D

Injection cases

Domain



- axi-cylindric configuration
- 200×80 cells

Initial and injection conditions

• Gas

Initial: $P = 1\text{bar}, T = 293\text{K}, u_g = 0\text{ms}^{-1}$

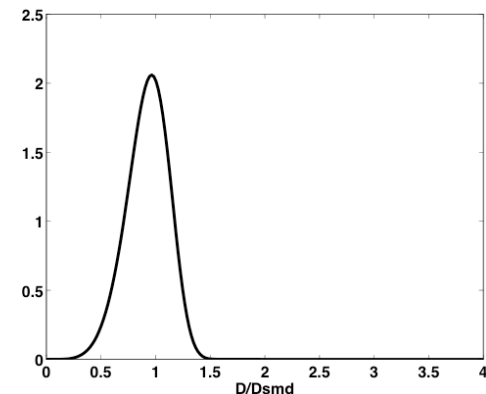
Injection: $P = 1\text{bar}, T = 293\text{K}, u_g = 18\text{ms}^{-1}$

• Spray

Initial: vacuum

Injection: $u_p = 18\text{ms}^{-1}, \alpha = 1.12 \cdot 10^{-3}$

• Size distribution: Rosin-Rammler

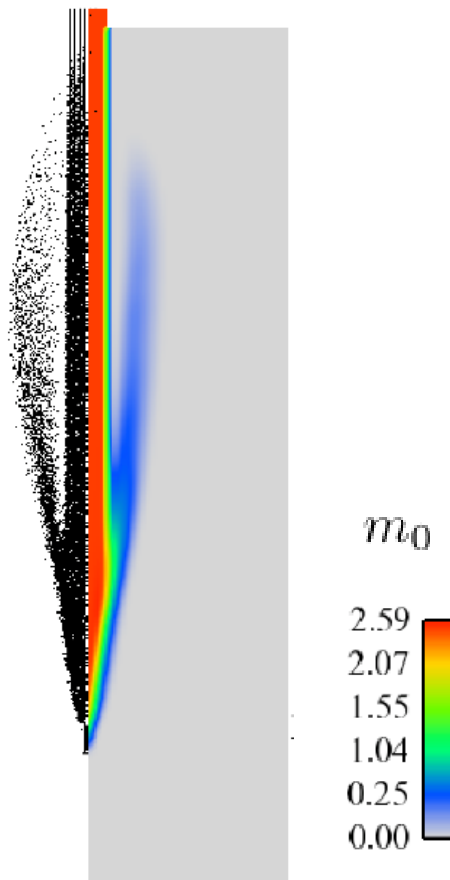


Implementation of the EMSM model in IFP-C3D

Injection cases

Comparison to Lagrangian results

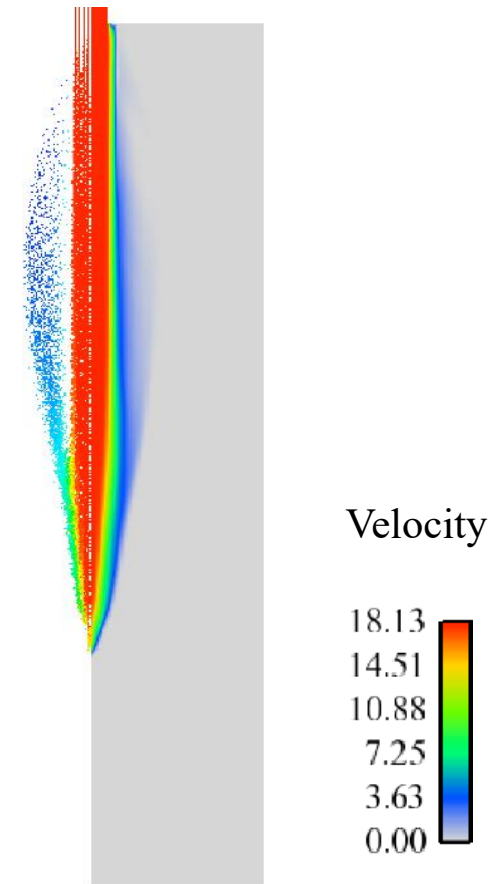
Particle number



Parameters

- Lagrangian (left)
- Eulerian (right)
- Two-way coupling (lagrangian)
- $t=10^{-2}$ s
- $smd = 20\mu m$

Velocity field



Implementation of the EMSM model in IFP-C3D

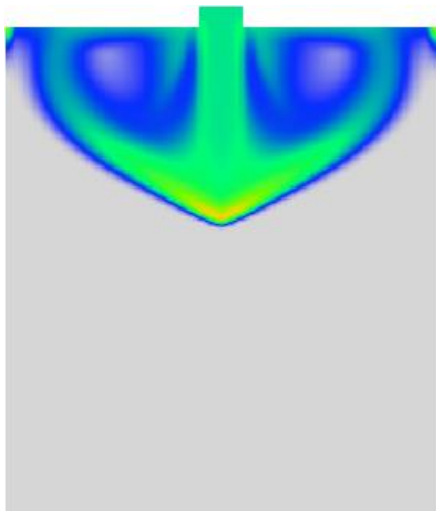
Injection cases

Eulerian evaporating spray

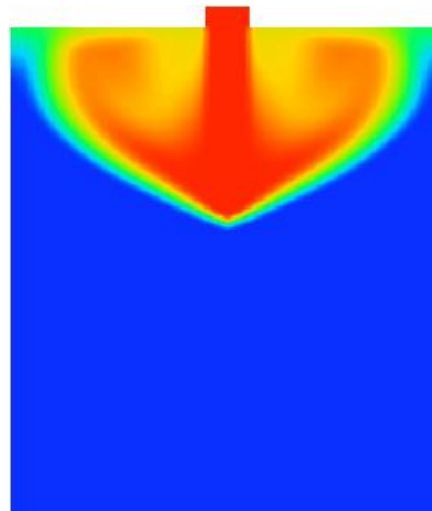
Parameters

- One-way coupling
- $t=10^{-2}$ s
- $smd = 5\mu m$

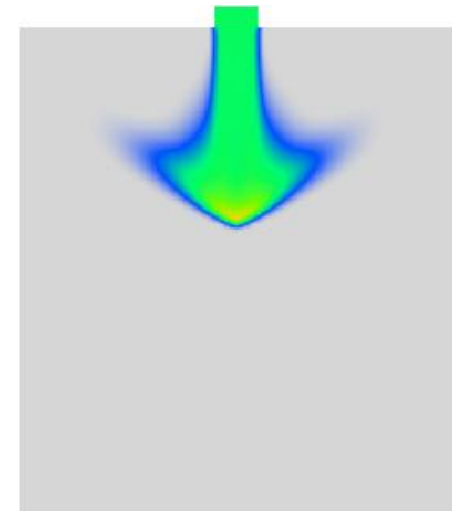
Non Evaporating case



Non Evaporating case



Evaporating case





Conclusions and Perspectives

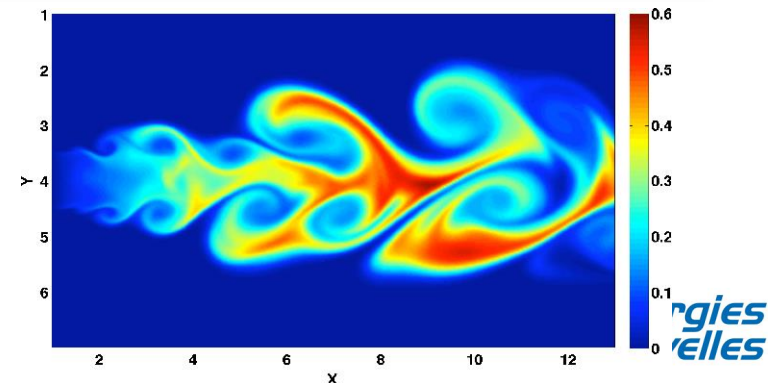
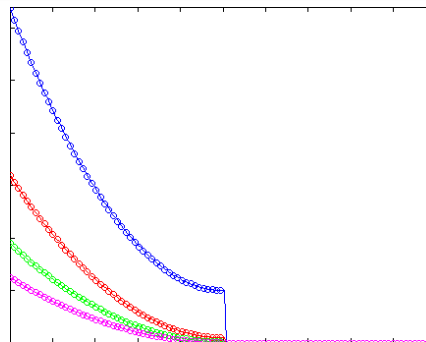
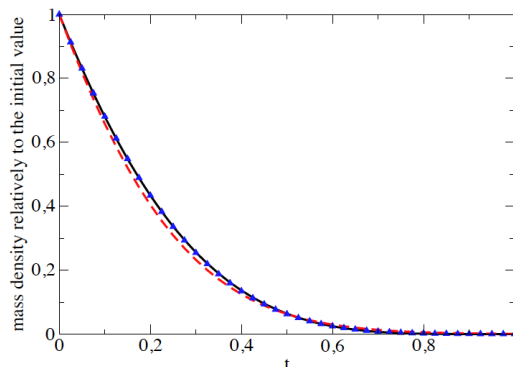
Achievements of the PhD

- Description of polydispersity:

Eulerian Multi-Size Moment (EMSM) model

Development of the EMSM model and numerical tools

- Evaporation
 - ➡ EMSM model with 1 section as accurate as MF model with 10 sections
- Advection of a moment set
 - ➡ High order scheme enforcing the Realizability condition and Pressureless Gas Dynamics
- Validation of the EMSM model
 - ➡ Implementation in the Muses3D code
 - Comparison with the MF model



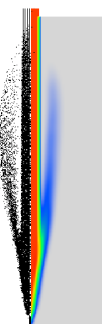
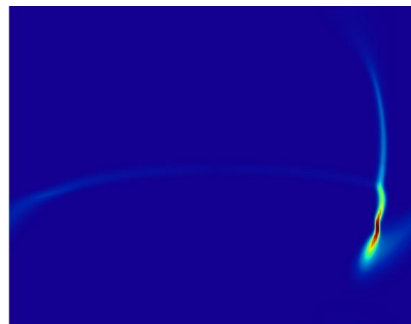
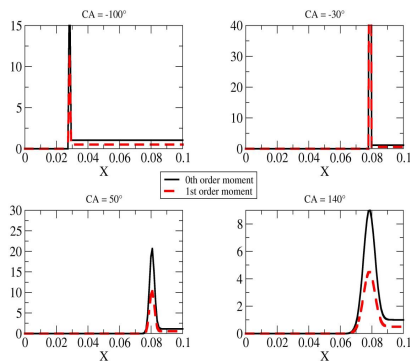
Achievements of the PhD

- Description of polydispersity:

Eulerian Multi-Size Moment (EMSM) model

Extension to an industrial context

- Adaptation of the designed schemes to the ALE formalism
 - ➡ High order advection scheme in ALE formalism
- Implementation of the EMSM model in IFP-C3D and validation
 - ➡ Validation with moving grid
 - Taylor-Green computation compared with Muses3D
 - Jet computation compared with Lagrangian: spray injection demonstrated



Achievements of the PhD

- Description of particle trajectory crossing

Eulerian Multi-Velocity Moment (EMVM) model

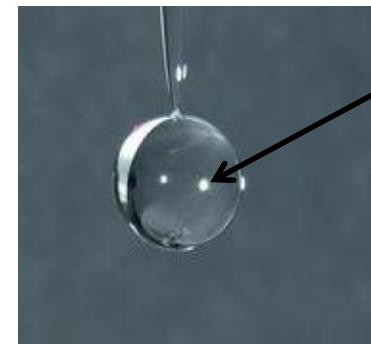
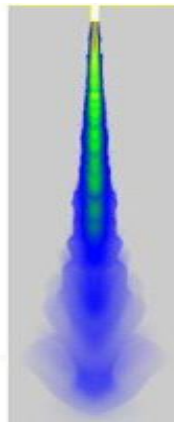
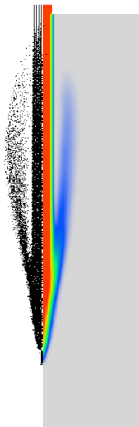
Development of the EMVM model and numerical tools

- 1D problem: Mathematical study of the closed system in order to prepare the design of high order advection schemes:
- Multi-D problem: study on closure of the ill-posed problem and evaluation of the EMVM model
 - ➡ Implementation of the EMVM and EMFVM model in Muses3D
 - Validation by comparison with Lagrangian results

Perspectives

Short term

- Conclude the validation process on injection with a real injection case, and with the exact same framework for both the Eulerian and Lagrangian model
- Extend the implementation of the EMSM model in IFP-C3D to turbulence modeling with RANS formalism
- Study the relevance to consider compressible droplets, in the perspective to couple the EMSM model to the compressible two-fluid model for separate phase flow



Pressure ?

Perspectives

Spray models

- Extend the Eulerian spray models discussed in this PhD to a two-way coupling formalism (*Doisneau, PhD*)
- For the EMSM model, to be able to describe size/velocity correlation using only one section (*Vié*)
- Consider turbulence modeling in LES framework, and going beyond the monokinetic assumption (*Boileau*)



- EMVM model
 - Design, on the basis of the study done in (*CMS 10*), high order advection schemes
 - Continue the initiated research work on an optimized closure in multi-dimensional configurations: CQMOM (*Yuan, 10*)
 - Consider turbulence modeling in LES framework (*Chalons, 10*)

Perspectives

Long term perspective

- Research on high order kinetic schemes for advection problems on unstructured grids
- Couple the EMSM model with the Eulerian two-fluid model for the separate phase

