The Simplest Math problem no one can solve !

The Collatz Conjecture

Also called the
3n + 1 problem

Named after Lothar Collatz

https://en.wikipedia.org/wiki/Lothar_Collatz

Proposed back in 1937

Unsolved for over 80 years.

Although mathematicians are closer than ever to solve it.

Maybe they will solve it in our lifetimes :)

The Collatz Conjecture also has a bunch of other names:

- Wondrous numbers
- 3n + 1 Conjecture
- Ulam Conjecture
- And many more ...

What is it ?

GIVEN:
Any randomly selected positive integer number.

WHEN:
Repeating two simple arithmetic operations - in sequence...

THEN:
Will eventually transform every positive integer into 1.

Steps

- Each term is based on previous term:

- IF previous term is EVEN -> next term is HALF PREVIOUS TERM

- IF previous term is ODD -> next term is 3 TIMES PREVIOUS TERM PLUS 1

Examples

If we start:

With 6 we get:
6, 3, 10, 5, 16, 8, 4, 2, 1
-> Counter/Steps = 9

With 9 we get:
9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1
-> Counter/Steps = 20

With 16 we get:
16, 8, 4, 2, 1
-> Counter/Steps = 5

THINK != PROOF

Although the conjecture has not been proven,
most mathematicians who have looked into the problem ...

They THINK the conjecture is True :)
Because experimental evidence and heuristic arguments support it.

Experimental/Computer Evidence

As of 2020, the conjecture has been checked by computers.
For all starting values up to 2^{68} ≈ 295 147 905 179 352 825 856.

All initial values tested so far eventually end in the repeating cycle (4; 2; 1) of period 3.

This Computer Evidence

Is not sufficient to prove that the conjecture is true for all starting values.

As in the case of some disproved conjectures, like the Pólya conjecture.

Counterexamples might be found when considering **very large/extremely large numbers**.

If the conjecture is false:

It can only be because there is a starting number which creates a sequence that doesn't end on 1.
It may be a sequence that either:

- Repeats in endless loop - *low chance, but still possible*
- Actually increases - *almost zero chance*

So far no such Starting Number/Sequence has been found !

Hard to solve

The mathematicians are still working on this problem - to this day.
Although many teachers/PhDs in the math circles say:

Don't lose your life/mind with this crazy/pesky math problem :)

Why I chose this math problem for my presentation

First of all - the problem may look simple on the surface.
But it seems very hard to actually prove it.

Even in our daily work.
Problems can look simple but in reality can be very complex.

Also:
Seemingly random sequences are often full of patterns if we look at them in different ways.

Even trying *(but not necessarily solving)* hard problems can lead to interesting/useful results.

The biggest problem we have in Computer Science

The Halting Problem
https://en.wikipedia.org/wiki/Halting_problem

In computability theory, the halting problem is the problem of determining:

GIVEN:
We have arbitrary computer program and an input.
Whether the program will finish running, or will continue to run forever.

Alan Turing proved in 1936 that a general algorithm to solve the halting problem for all possible program-input pairs **cannot exist !**

In other words

Rice's theorem
https://en.wikipedia.org/wiki/Rice%27s_theorem

In computability theory, Rice's theorem states
All non-trivial *semantic properties* of programs are undecidable.

The *Semantic Properties* may be - does the program terminate for all possible inputs.

In conclusion

There always will be bugs.
This is mathematically proven and 100% unavoidable.

What we can do is minimize the bugs.
And control the surface area of the bugs.

But we have to Assume *and* Live with the fact that:
- Bugs
- Randomness *and*
- Entropy

Are part of the **Universe *and* Life** as we know it.

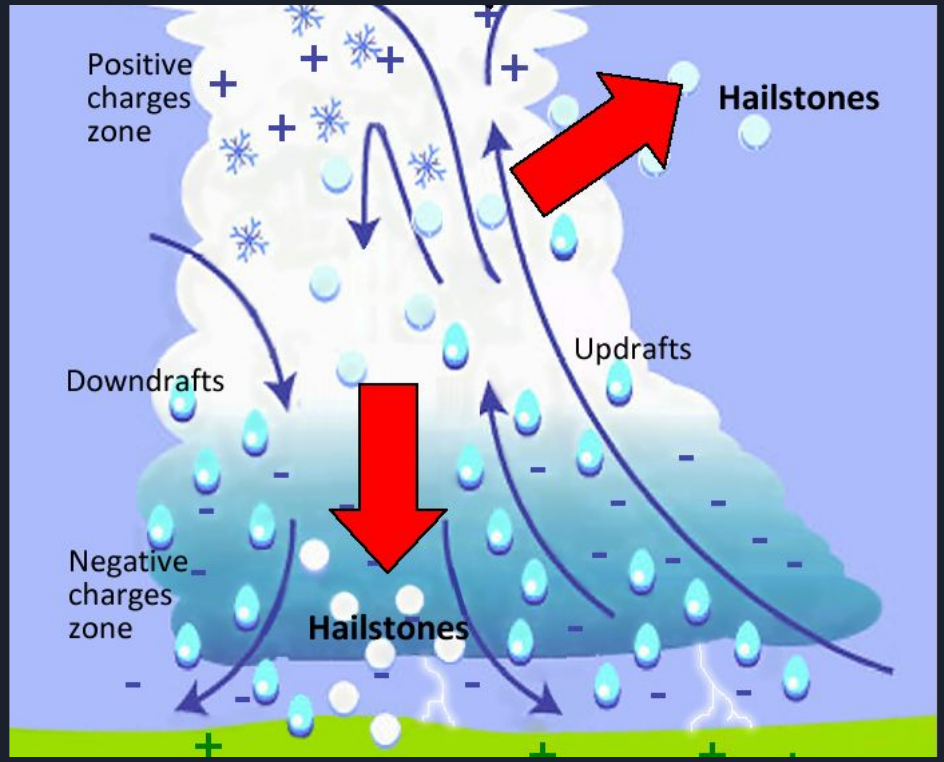
Trivia 1 - Hailstone

The Collatz sequence of numbers involved is sometimes referred to as the **hailstone sequence**. **Or the Hailstone Numbers.**

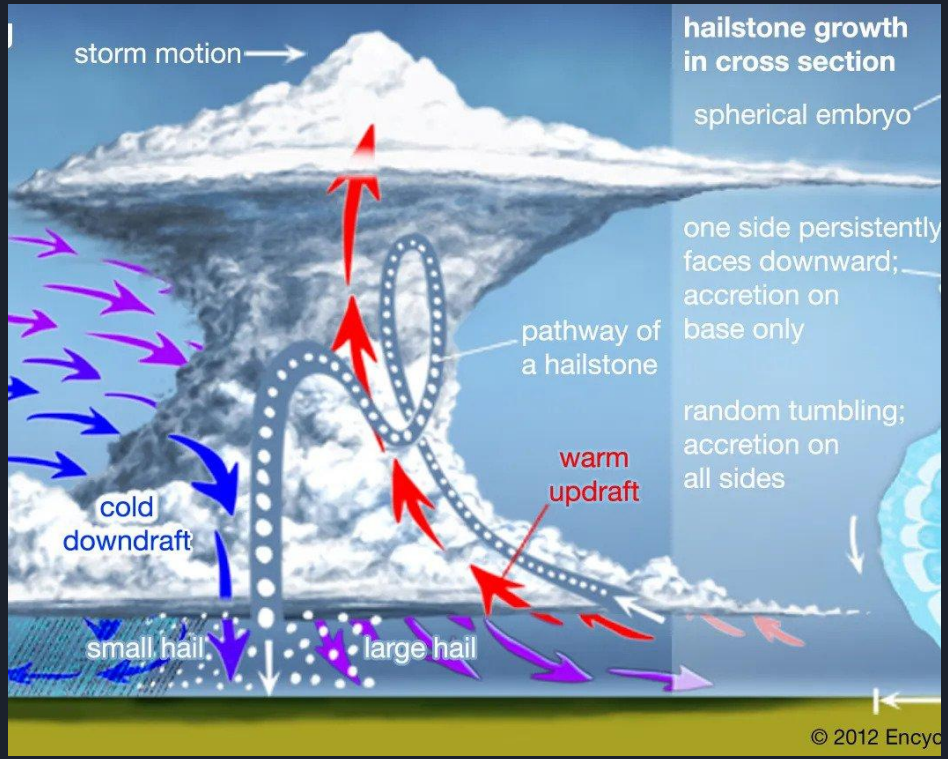
Because the values are usually subject to **multiple descents *and* ascents** like hailstones in a cloud. Here are some images to illustrate this.

Trivia 1 - Hailstone - Image 1

Trivia 1 - Hailstone - Image 2



Trivia 1 - Hailstone - Image 3



Trivia 2 - List of unsolved problems in mathematics

The Holy Grail of the Mathematical Sherlock Holmes problems.
Or Mathematical Doctor House problems. :)

Whoever of the 2 geniuses you actually like more :)
https://en.wikipedia.org/wiki/List_of_unsolved_problems_in_mathematics

Links 1

The Original Wikipedia page:
https://en.wikipedia.org/wiki/Collatz_conjecture

The best YouTube video on the subject:
https://www.youtube.com/watch?v=094y1Z2wpJg

The Simplest Math problem no one can solve.
Or The Most Dangerous problem in Math.

Links 2

Some Python Implementations:

https://hackernoon.com/implementing-3x1-in-python
https://medium.com/@aseemkohli_36958/collatz-conjecture-with-python-834b97171e2a

Python Chart/Graph Libraries:

https://www.askpython.com/python/python-plotting-and-graph-libraries
https://realpython.com/python-data-visualization-bokeh/
https://wiki.python.org/moin/PythonGraphLibraries

Links 3

Other PP Presentations
https://www.slideserve.com/peri/the-collatz-problem

Other Interesting Math Problems / Theorems:
https://en.wikipedia.org/wiki/Halting_problem
https://en.wikipedia.org/wiki/Rice%27s_theorem
https://en.wikipedia.org/wiki/List_of_unsolved_problems_in_mathematics