

A decorative graphic on the left side of the slide consisting of two overlapping parallelograms. The front one is blue and the back one is light green. They are set against a dark blue background with faint, lighter blue diagonal stripes.

The Simplest Math problem no one can solve !

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The Collatz Conjecture



Also called the
 $3n + 1$ problem



Named after
Lothar Collatz

https://en.wikipedia.org/wiki/Lothar_Collatz



Proposed back in 1937




Unsolved for over 80 years.





Although mathematicians are closer than ever to solve it.

Maybe they will solve it in our lifetimes :)



The Collatz Conjecture also has a bunch of other names:

- Wondrous numbers
- $3n + 1$ Conjecture
- Ulam Conjecture
- And many more ...



What is it ?

GIVEN:

Any randomly selected positive integer number.

WHEN:

Repeating two simple arithmetic operations - in sequence...

THEN:

Will eventually transform every positive integer into 1.



Steps

- Each term is based on previous term:
- IF previous term is EVEN \rightarrow next term is HALF PREVIOUS TERM
- IF previous term is ODD \rightarrow next term is 3 TIMES PREVIOUS TERM PLUS 1



Examples

If we start:

With **6** we get:

6, 3, 10, 5, 16, 8, 4, 2, 1

-> Counter/Steps = 9

With **9** we get:

9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

-> Counter/Steps = 20

With **16** we get:

16, 8, 4, 2, 1

-> Counter/Steps = 5



THINK != PROOF

Although the conjecture has not been proven,
most mathematicians who have looked into the problem ...

They THINK the conjecture is True :)
Because experimental evidence and heuristic arguments support it.



Experimental/Computer Evidence

As of 2020, the conjecture has been checked by computers.

For all starting values up to $2^{68} \approx 295\,147\,905\,179\,352\,825\,856$.

All initial values tested so far eventually end in the repeating cycle (4; 2; 1) of period 3.



This Computer Evidence

Is not sufficient to prove that the conjecture is true for all starting values.

As in the case of some disproved conjectures, like the Pólya conjecture.

Counterexamples might be found when considering **very large/extremely large numbers**.



If the conjecture is false:

It can only be because there is a starting number which creates a sequence that doesn't end on 1.
It may be a sequence that either:

- Repeats in endless loop - *low chance, but still possible*
- Actually increases - *almost zero chance*


So far no such Starting Number/Sequence has been found !



Hard to solve

The mathematicians are still working on this problem - to this day.
Although many teachers/PhDs in the math circles say:

Don't lose your life/mind with this crazy/pesky math problem :)



Why I chose this math problem for my presentation

First of all - the problem may look simple on the surface.
But it seems very hard to actually prove it.


Even in our daily work.

Problems can look simple but in reality can be very complex.

Also:

Seemingly random sequences are often full of patterns if we look at them in different ways.

Even trying (*but not necessarily solving*) hard problems can lead to interesting/useful results.



The biggest problem we have in Computer Science

The Halting Problem

https://en.wikipedia.org/wiki/Halting_problem

In computability theory, the halting problem is the problem of determining:

GIVEN:

We have arbitrary computer program and an input.

Whether the program will finish running, or will continue to run forever.

Alan Turing proved in 1936 that a general algorithm to solve the halting problem for all possible program-input pairs **cannot exist !**



In other words

Rice's theorem

https://en.wikipedia.org/wiki/Rice%27s_theorem

In computability theory, Rice's theorem states

All non-trivial semantic properties of programs are undecidable.

The Semantic Properties may be - does the program terminate for all possible inputs.



In conclusion

There always will be bugs.

This is mathematically proven and 100% unavoidable.

What we can do is minimize the bugs.

And control the surface area and the blast radius of the bugs.

But we have to Assume *and* Live with the fact that:

- Bugs
- Randomness *and*
- Entropy

Are part of the **Universe** *and* **Life** as we know it.



Trivia 1 - Hailstone

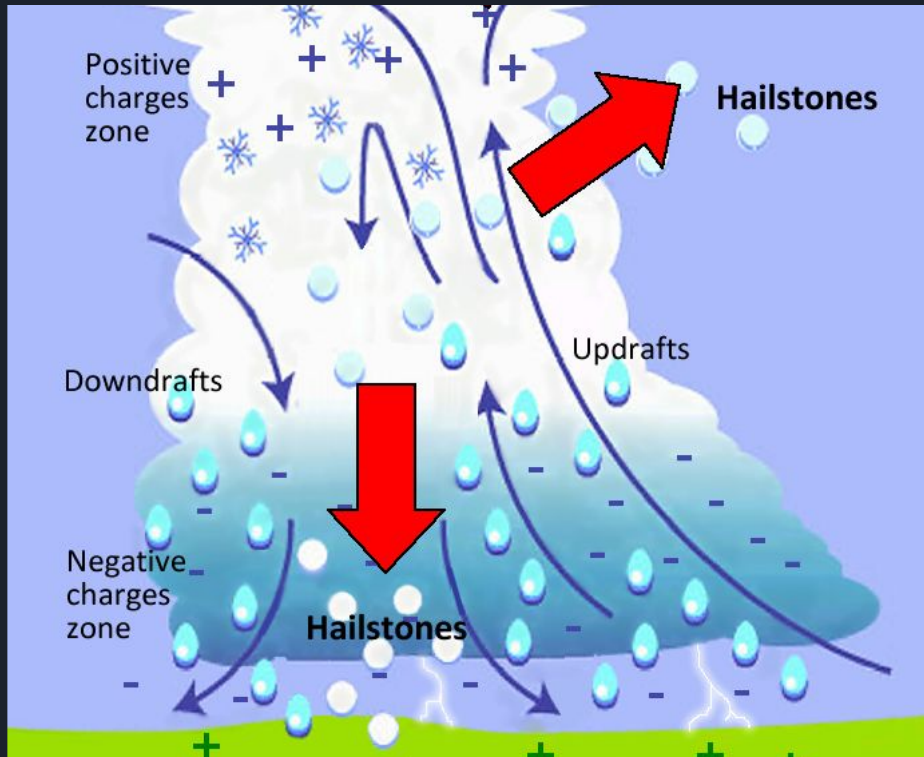
The Collatz sequence of numbers involved is sometimes referred to as the **hailstone sequence**.
Or the **Hailstone Numbers**.

Because the values are usually subject to multiple descents *and* ascents like hailstones in a cloud. Here are some images to illustrate this.

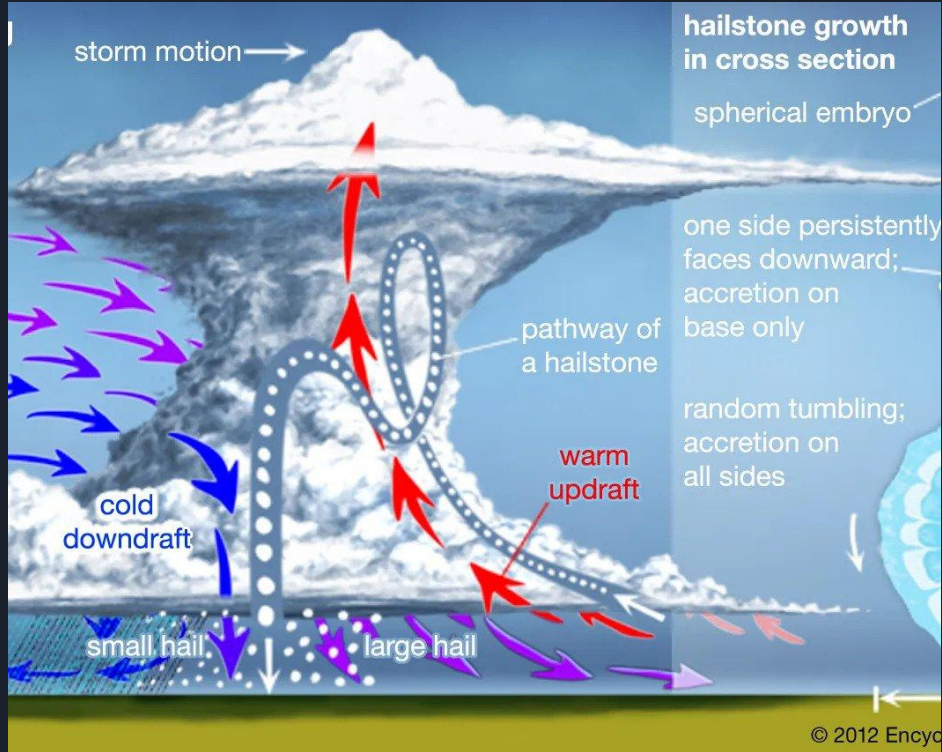
Trivia 1 - Hailstone - Image 1



Trivia 1 - Hailstone - Image 2



Trivia 1 - Hailstone - Image 3





Trivia 2 - List of unsolved problems in mathematics

The Holy Grail of the Mathematical Sherlock Holmes problems.
Or Mathematical Doctor House problems. :)

Whoever of the 2 geniuses you actually like more :)

https://en.wikipedia.org/wiki/List_of_unsolved_problems_in_mathematics



Links 1

The Original Wikipedia page:

https://en.wikipedia.org/wiki/Collatz_conjecture

The best YouTube video on the subject:

<https://www.youtube.com/watch?v=094y1Z2wpJg>

The Simplest Math problem no one can solve.

Or The Most Dangerous problem in Math.



Links 2

Some Python Implementations:

<https://hackernoon.com/implementing-3x1-in-python>

https://medium.com/@aseemkohli_36958/collatz-conjecture-with-python-834b97171e2a

Python Chart/Graph Libraries:

<https://www.askpython.com/python/python-plotting-and-graph-libraries>

<https://realpython.com/python-data-visualization-bokeh/>

<https://wiki.python.org/moin/PythonGraphLibraries>



Links 3

Other PP Presentations

<https://www.slideserve.com/peri/the-collatz-problem>

Other Interesting Math Problems / Theorems:

https://en.wikipedia.org/wiki/Halting_problem

[https://en.wikipedia.org/wiki/Rice%27s theorem](https://en.wikipedia.org/wiki/Rice%27s_theorem)

https://en.wikipedia.org/wiki/List_of_unsolved_problems_in_mathematics