Year 13 Pure D Homework Sheet 6

Section A: Mathematical Health Check

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The questions are in the test itself for this one

OR Whatever your teacher asks you to do for section A

Section B: Past Paper Style Questions

1.

The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C, find

(a) the coordinates of C

(3)

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$.

(b) Find the size of the angle ACB. Give your answer in degrees to 2 decimal places.

(4)

(c) Hence, or otherwise, find the area of the triangle ABC.

(5)
(Total 12 marks)

2.

On separate diagrams, sketch the curves with equations

(a) $y = \arcsin x, \quad -1 \le x \le 1,$

(b) $y = \sec x, \quad -\frac{\pi}{3} \le x \le \frac{\pi}{3}$, stating the coordinates of the end points of your curves in each case.

(4)

Use the trapezium rule with five equally spaced ordinates to estimate the area of the region bounded by the curve with equation $y = \sec x$, the x-axis and the lines $x = \frac{\pi}{3}$ and $x = -\frac{\pi}{3}$, giving your answer to two decimal places.

(4)
(Total 8 marks)

3.

(i) Use de Moivre's theorem to prove that $\tan 3\theta \equiv \dfrac{\tan\theta(3-\tan^2\theta)}{1-3\tan^2\theta}$. [4]

(ii) (a) By putting $\theta = \frac{1}{12}\pi$ in the identity in part (i), show that $\tan \frac{1}{12}\pi$ is a solution of the equation $t^3 - 3t^2 - 3t + 1 = 0.$ [1]

(b) Hence show that $\tan\frac{1}{12}\pi = 2 - \sqrt{3}$. [4]

(iii) Use the substitution $t = \tan\theta$ to show that $\displaystyle\int_0^{2-\sqrt{3}} \frac{t(3-t^2)}{(1-3t^2)(1+t^2)}\,dt = a\ln b,$

where a and b are positive constants to be determined. [5]

Section C: More Interesting Questions

4.

The straight line l has vector equation

$$\mathbf{r} = 3\mathbf{i} + 3\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}),$$

where λ is a scalar parameter.

The point A has coordinates $(3,3,-3)$, relative to a fixed origin O.

The points P and Q lie on the l so that $|AP| = |AQ|$.

Given further that $\measuredangle PAQ = 90°$, find the coordinates of P and the coordinates of Q.

5. (*starts OK but gets harder*)

Points A, B, C in three dimensions have coordinate vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, respectively. Show that the lines joining the vertices of the triangle ABC to the mid-points of the opposite sides meet at a point R.

P is a point which is **not** in the plane ABC. Lines are drawn through the mid-points of BC, CA and AB parallel to PA, PB and PC respectively. Write down the vector equations of the lines and show by inspection that these lines meet at a common point Q.

Prove further that the line PQ meets the plane ABC at R.