1. Norm, Dot Product and Distance in R

- Norm (Length of a vector): ||v|| = sqrt(v1² + v2² + ... + vn²)

- Dot Product: u.v = u1v1 + u2v2 + ... + unvn

- Distance: d(u, v) = ||u - v||

2. Orthogonality

- Two vectors are orthogonal if their dot product is zero: u.v = 0

3. Cross Product (only in R³)

- u x v = (u2v3 - u3v2, u3v1 - u1v3, u1v2 - u2v1)

4. Eigenvalues and Eigenvectors

- Av = λv, where A is a matrix, v is an eigenvector, and λ is an eigenvalue

- Eigenvalues are solutions to the characteristic equation: det(A - λI) = 0

5. Diagonalization

- A matrix A is diagonalizable if there exists an invertible matrix P such that P⁻¹AP is a diagonal matrix D. The columns of P are the eigenvectors of A.

6. Complex Vector Spaces

- These are vector spaces where the scalars are complex numbers. All the rules of real vector spaces apply, but with complex arithmetic.

7. Inner Products

- An inner product on a vector space V is a function that associates a number, denoted as ⟨u, v⟩, with each pair of vectors u and v of V. It satisfies certain properties like commutativity, linearity in the first argument, and positive-definiteness.

8. Angle and Orthogonality in Inner Product Spaces

- Angle: cos θ = ⟨u, v⟩ / (||u|| ||v||)

- Orthogonality: Two vectors are orthogonal if their inner product is zero: ⟨u, v⟩ = 0