

# Josephson Junctions and Neuromorphic Computing

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## Background

Josephson Junctions are superconducting circuit elements whose behavior is described by a second-order, non-linear differential equation:

$$I = I_c \sin \phi + \frac{\Phi_0}{2\pi R} \dot{\phi} + \frac{\Phi_0 C}{2\pi} \ddot{\phi},$$

This makes them an ideal candidate for modeling complicated non-linear systems - such as neurons.

A Josephson Junction is formed by two superconducting niobium plates separated by a thin aluminum oxide insulator. At a certain critical temperature ( $T_c$ ), the junction becomes superconducting and Cooper pairs can tunnel across the insulator. The voltage measure cross the junction can be defined in terms of a phase difference,  $\phi$ , between the wave functions of the Cooper pairs on either side of the insulator:

$$V = \frac{\hbar}{2e} \frac{\partial \phi}{\partial t}$$

## Modeling Josephson Junctions Motion

When studying the behavior of Josephson Junctions and superconductors in general, hysteresis is a concept that proves useful to understand. Hysteresis, the state of systems depending on history, can be observed in Josephson Junctions. Due to the size of the JJs, visible observation of its behavior is near impossible.

In order to better understand the JJs, we have constructed a flywheel pendulum, which is capable of being the states: “whirling/dynamic” and “static”. When switching bifurcation (a certain applied torque limit) is surpassed, the pendulum goes from static to whirling. When the applied torque is decreased, a return the static state expected, and a retrapping torque is recorded.

The differential equation below is used to calculate the torque of the pendulum, as you look at each term of the equation, there is a similarity to the equation used to describe the behavior of JJs:

$$T = I \frac{d^2 \phi}{dt^2} + D \frac{d\phi}{dt} + mg\ell \sin \phi$$

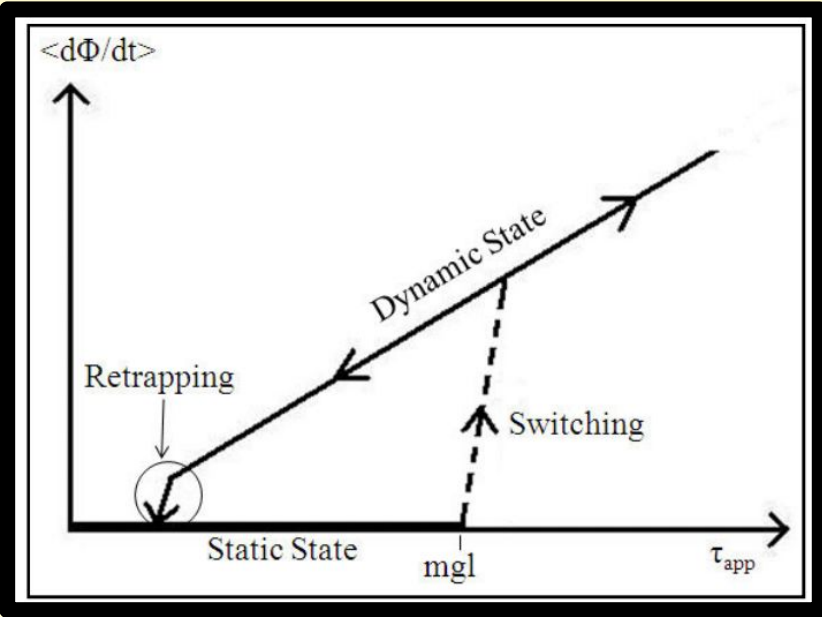


Fig. 1: Hysteresis Curve

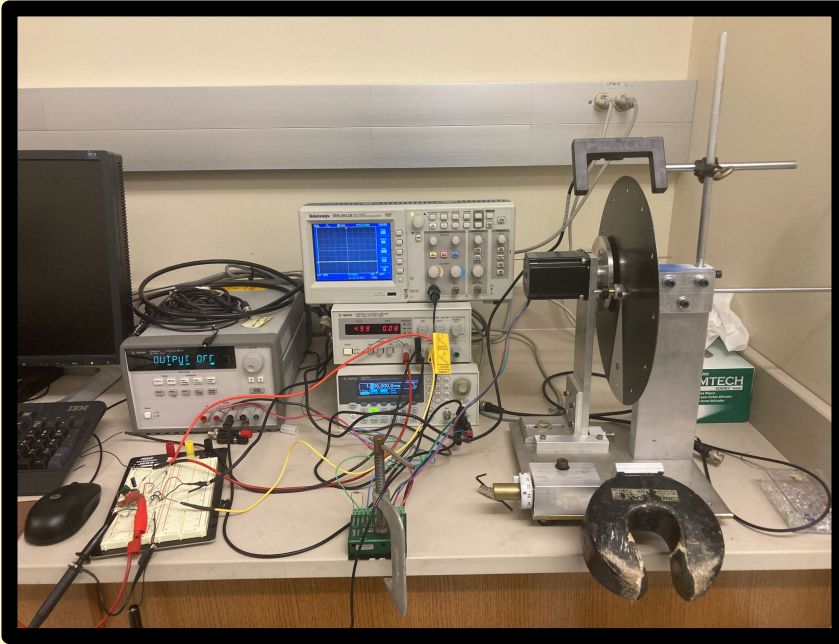


Fig. 2 Magnetic flywheel pendulum

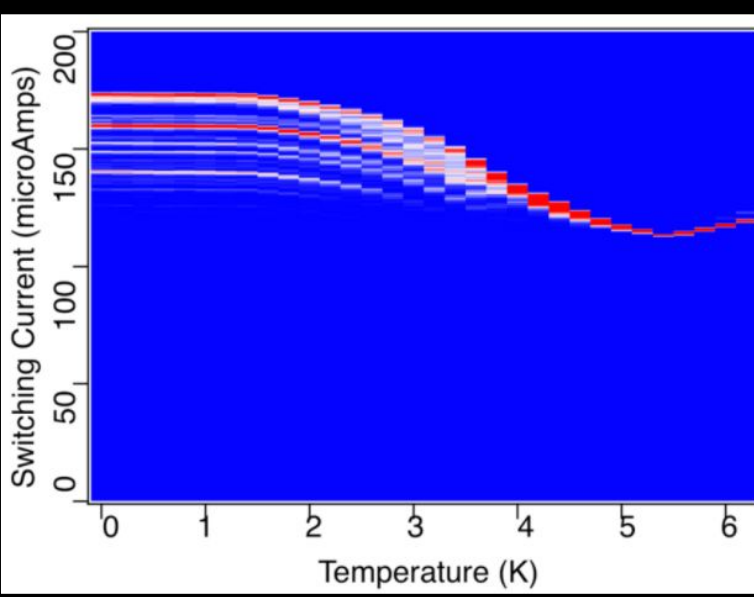
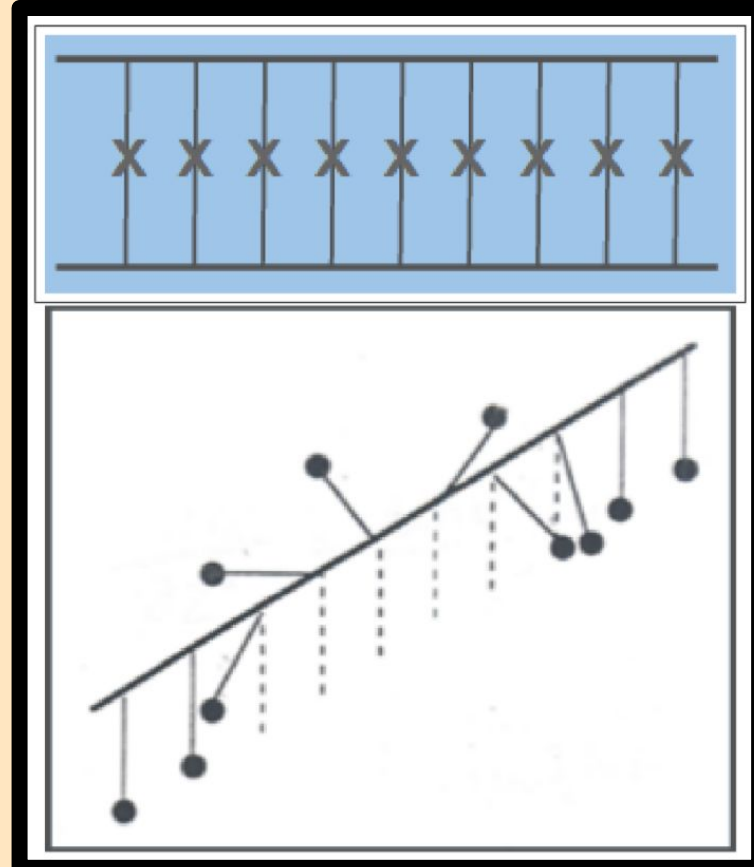
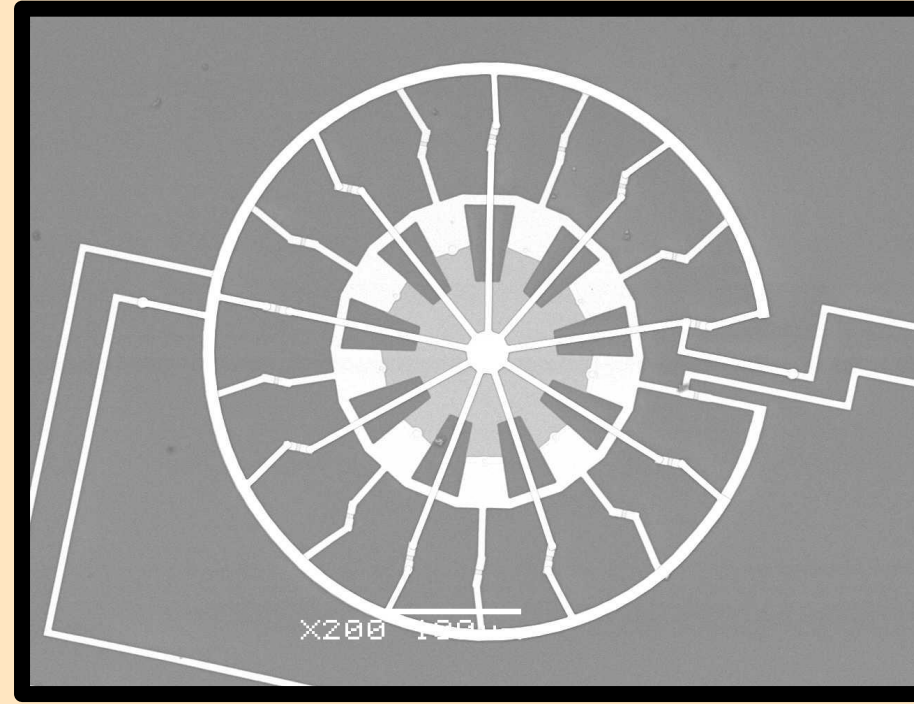
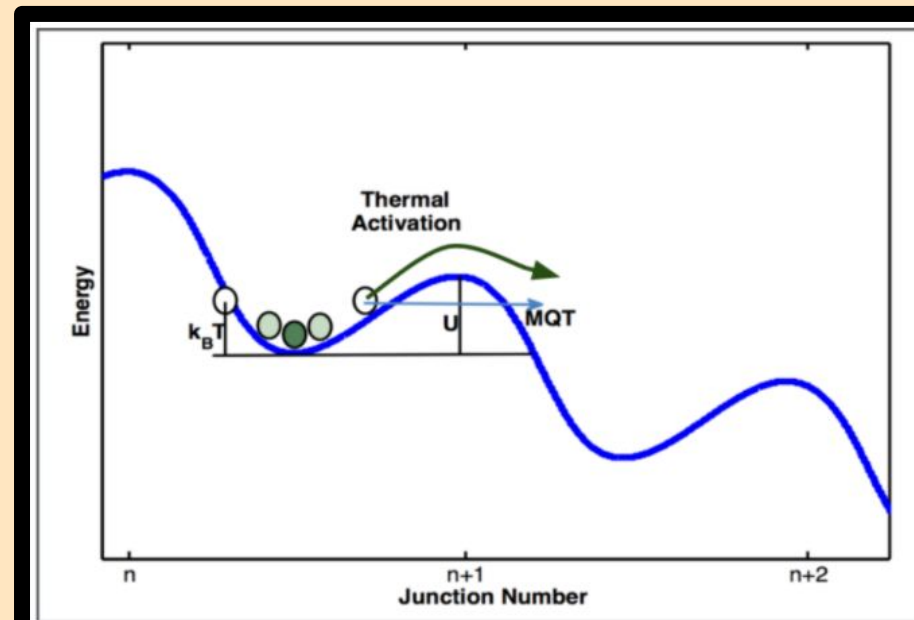
## Fluxon Dynamics and Parallel Josephson Arrays

Josephson Junctions can be connected in parallel with one another, much like other circuit elements. This is physically analogous to a system of coupled pendulums as seen in the image below, with neighboring junctions acting like neighboring pendulums. These parallel arrays are often connected in circular rings of junctions to make a continuous loop, with our arrays typically consisting of nine or fifteen junctions.

When cooled down below  $T_c$ , current loops in the array can cause quantized bits of magnetic flux - called fluxons - to become trapped between to junctions in the array. At a certain current,  $I_{sw}$ , or thermal energy level, a fluxon will begin to move around the array and a voltage is detectable.  $I_{sw}$ , however, can vary significantly. It is strongly suspected that this variation is caused by production uncertainty in the size of the junctions, akin to a particle moving over a hill.

The flux is trapped by cooling own the sample in a magnetic field, and as the chip passes through its critical current, the superconductors become perfect diamagnets, and these fluxons act like a "kink" or a displacement in the system of pendulums.

Seen in the bottom right is an image of a nine junction array taken on a scanning electron microscope. There are leads to measure both voltage and current, as both are needed to attain an accurate picture of what is happening in the chip. The goal of the project is to confidently observe Macroscopic Quantum Tunneling (MQT), which would describe the behavior of the flux in the arrays. It is thought that MQT occurs when the fluxon tunnels from one side of a junction to the other without sufficient energy from an applied current or from heat energy.



## Frequency Synchronization in Coupled Josephson Junctions

Collective synchronization is a phenomenon observed in nature in which individuals in a system are driven to a common state without a leader. The Kuramoto model describes the synchronization of large coupled oscillators through a mathematical model. Single Josephson Junctions (JJs) mimic physical oscillators that operate at their own natural frequencies (proportional to their areas), and can exhibit frequency synchronization through the Kuramoto model when coupled.

The sync chip holds two circular arrays, each with 10 vertical JJs that are coupled by SQUIDS. Each of the vertical JJs have slight variations in area, therefore varying their natural frequencies. Using WR Spice, we ran current ramp simulations to observe how the input current affects synchronization, and at what points bifurcations occur (Figure 3).

In the following weeks, we will be running current ramps on the sync chip to look at how experimental results compare to our simulated results.

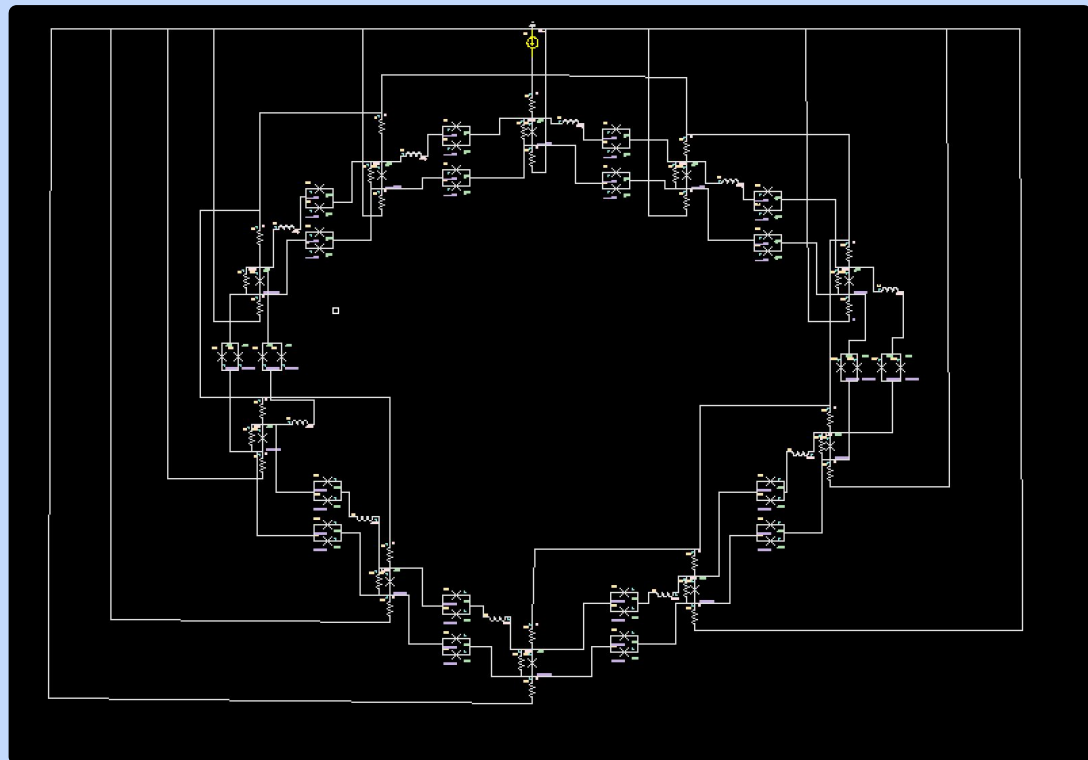


Figure 1. Electrical layout of 10 parallel JJ array

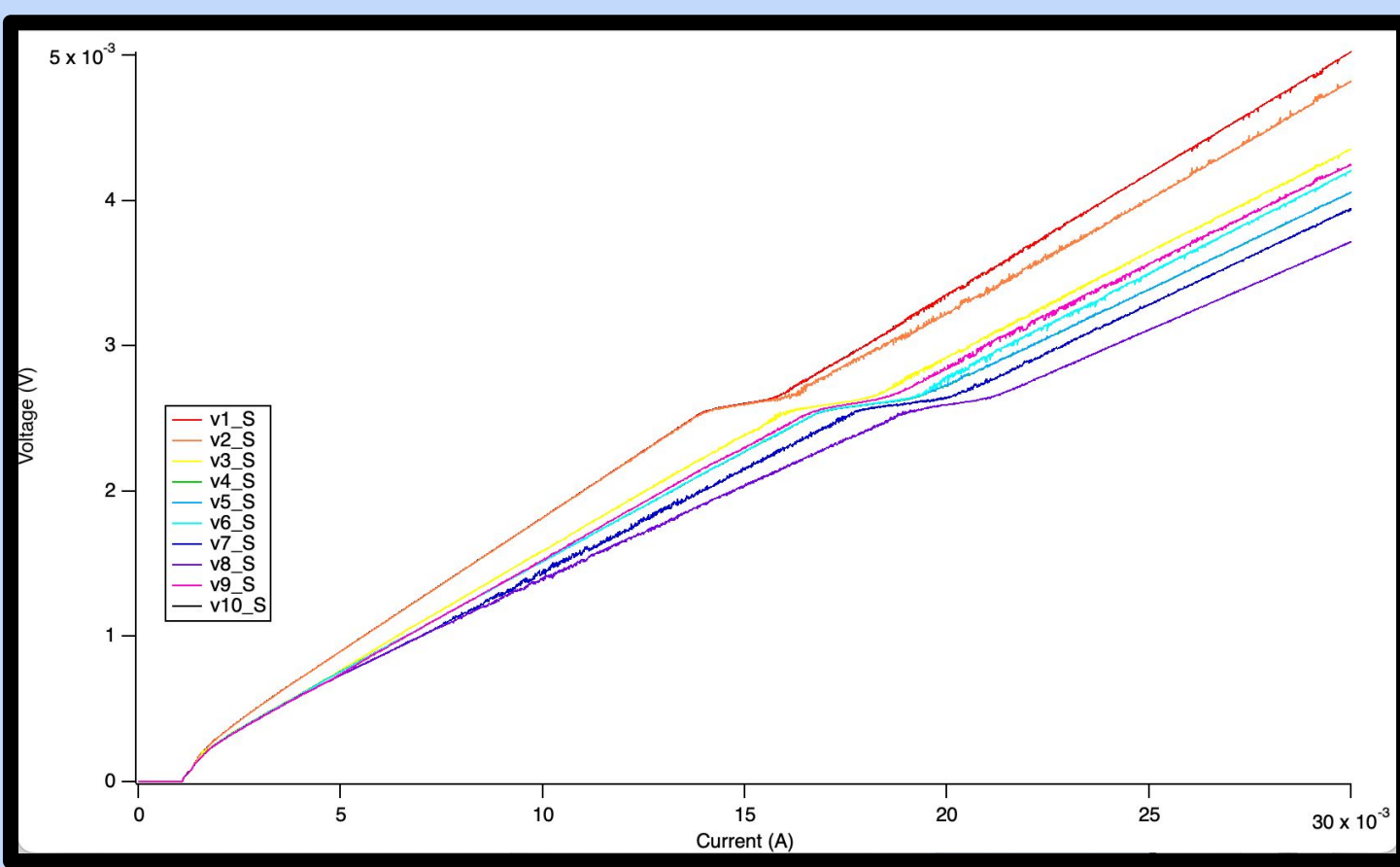


Figure 3. Simulated current ramp of 10 JJ array. Average V taken at each I,  $V \propto f$

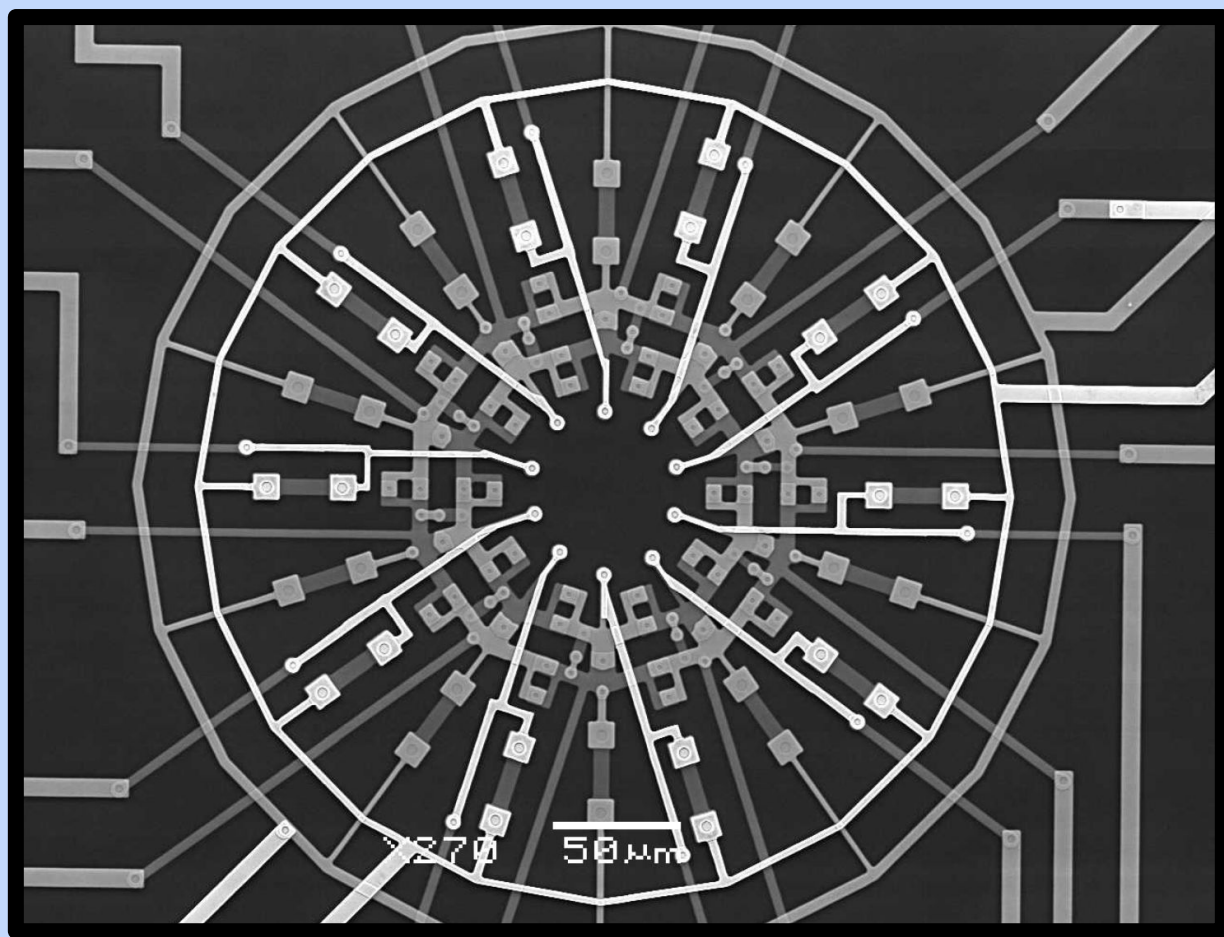


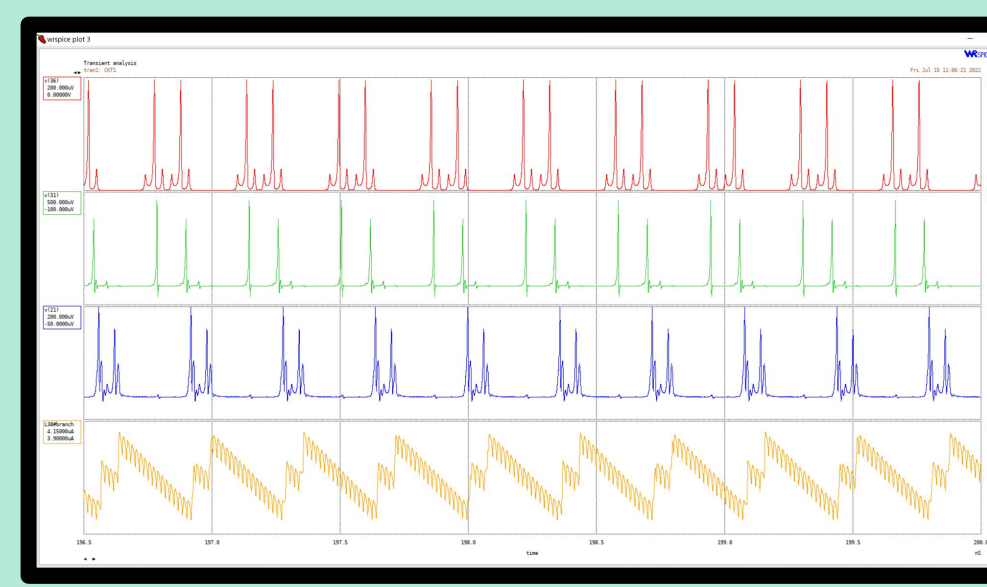
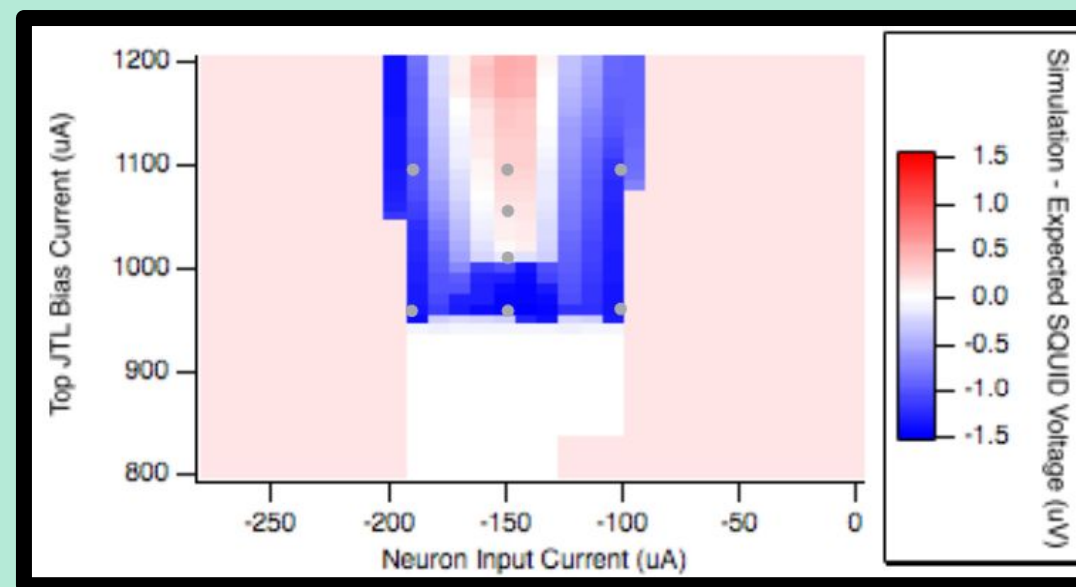
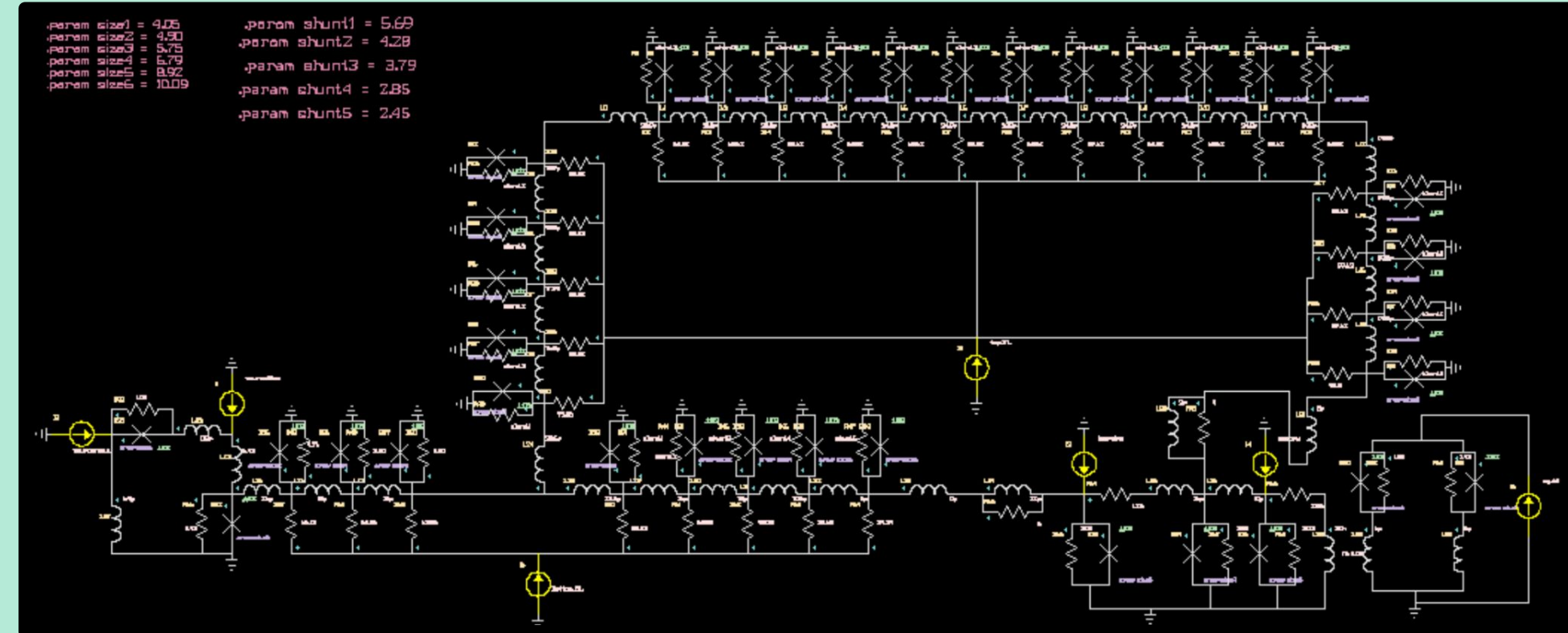
Figure 2. SEM image of sync chip

## Superconducting Learning Circuit

This circuit was inspired by neuron-synapse relationships. Within the circuit we mimic the action potential pulses as flips in the Josephson Junction (JJ) to represent the threshold firing. In this case the action potentials are similar to Single-Flux Quantum Pulses (SFQ). The JJs were utilized within the circuit to create a Josephson Transmission Line (JTL). The bottom and top JTL work similar to a neuron's axon and carried the pulses to other areas of the circuit.

Using the circuit, pictured on the right, simulations, in WR Spice, can be run to see if learning (or change in current) occurs in the system and whether memory is held.

By using a SQUID we determine the memory as the amount of flux in the circuit.



Each point in the plot above was ran to see whether learning occurred after each SFQ[1]. An example of the initial pulse, top JTL pulse, bottom JTL pulse, and learning current are presented in the transient graph above for neuron input of -150 uA and Top JTL current of 950 uA..

[1] W. Friend, Spike Timing Dependence in Josephson Junction Neuron-Synapse System, thesis, 2021.

## Fabrication and Experimental Devices

All of our research has - or one day will have - experimental results that aim to match simulations. To achieve the superconducting temperatures required for our experiments, we use helium cryostats capable of cooling down to 250 millikelvin. The operation of these require a variety of electrical devices and systems that have been, and currently are being, fabricated by students.

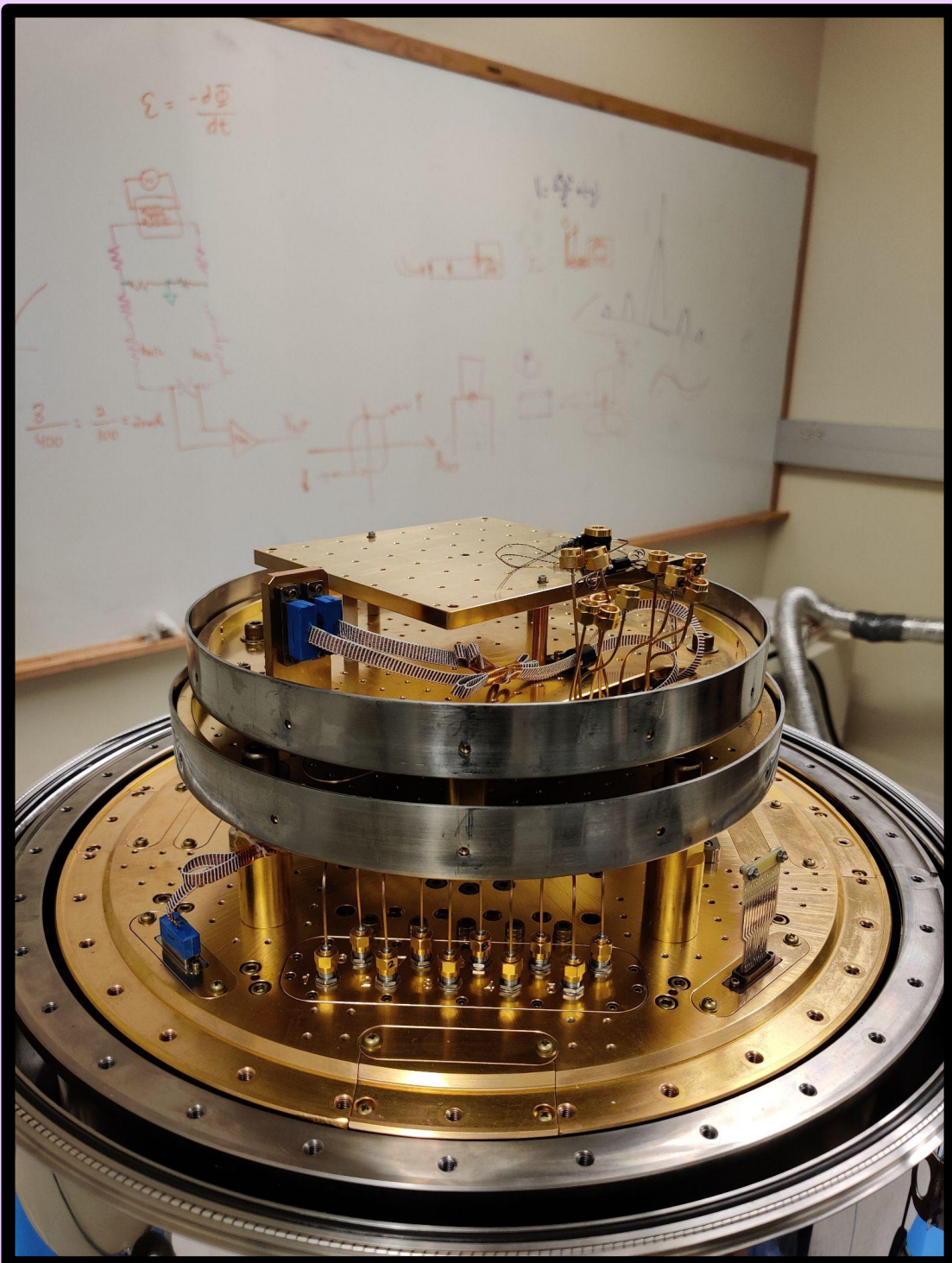


Fig. 1: Dry Cryostat

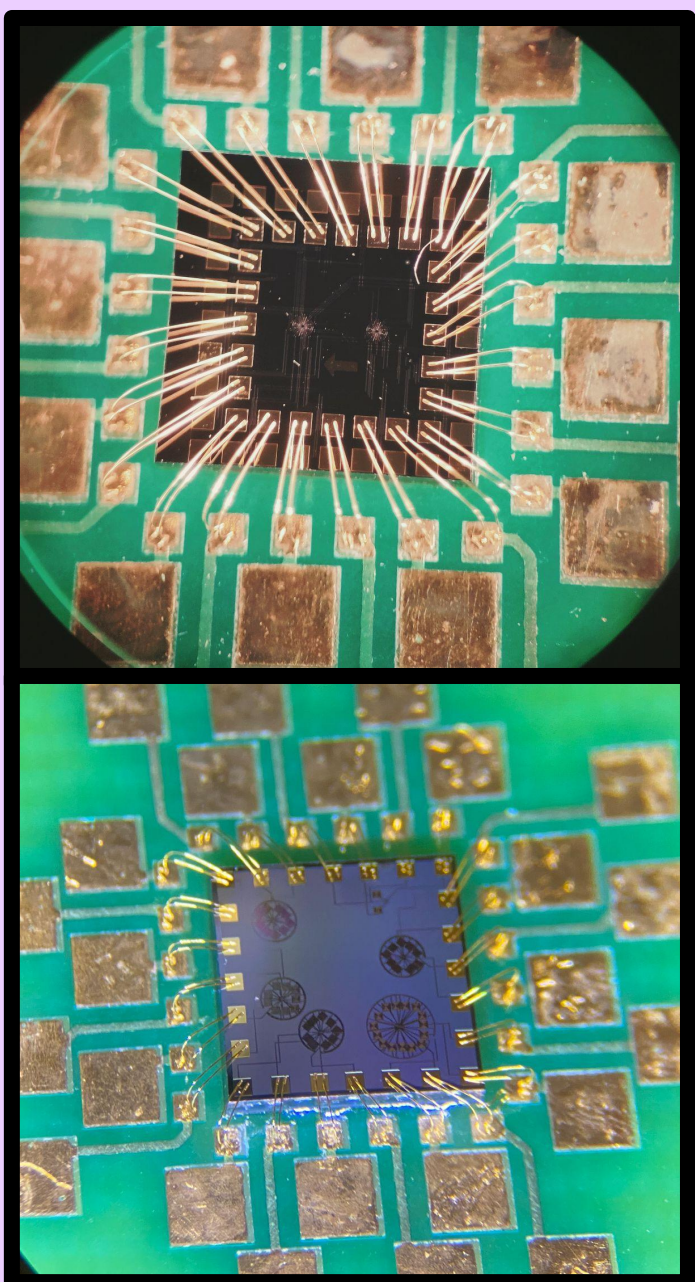


Fig. 2: Sync and Array Chips

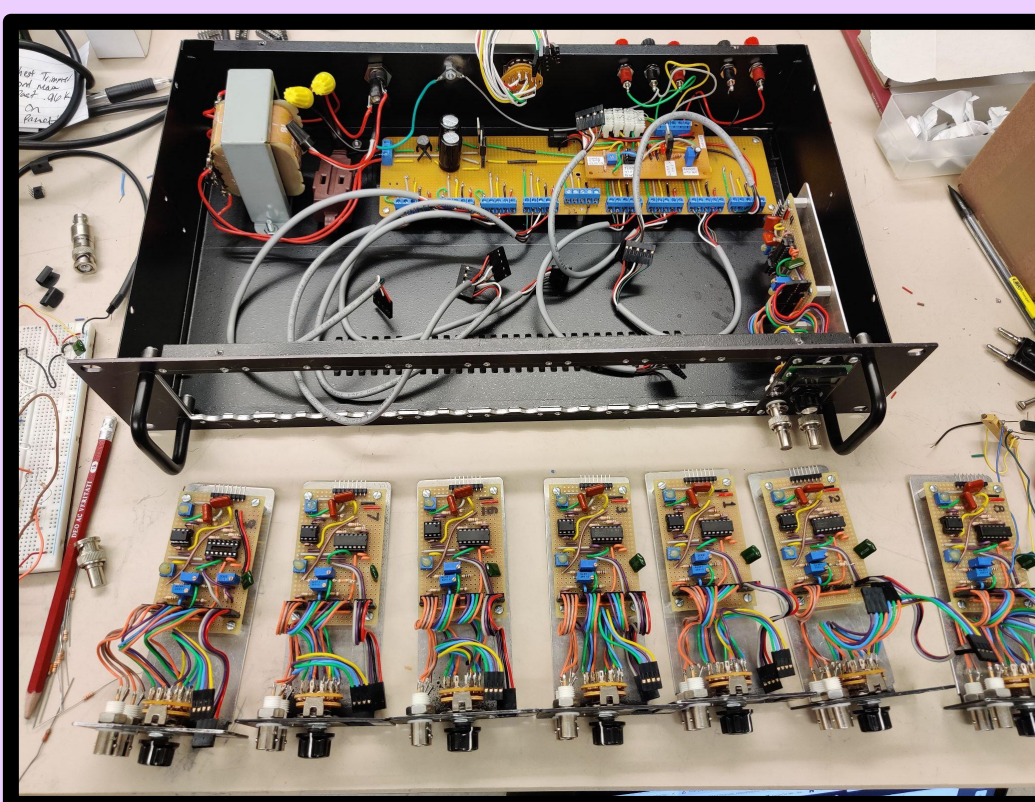


Fig. 3: Voltage Measurement Box



Fig. 4: Current Regulation Box

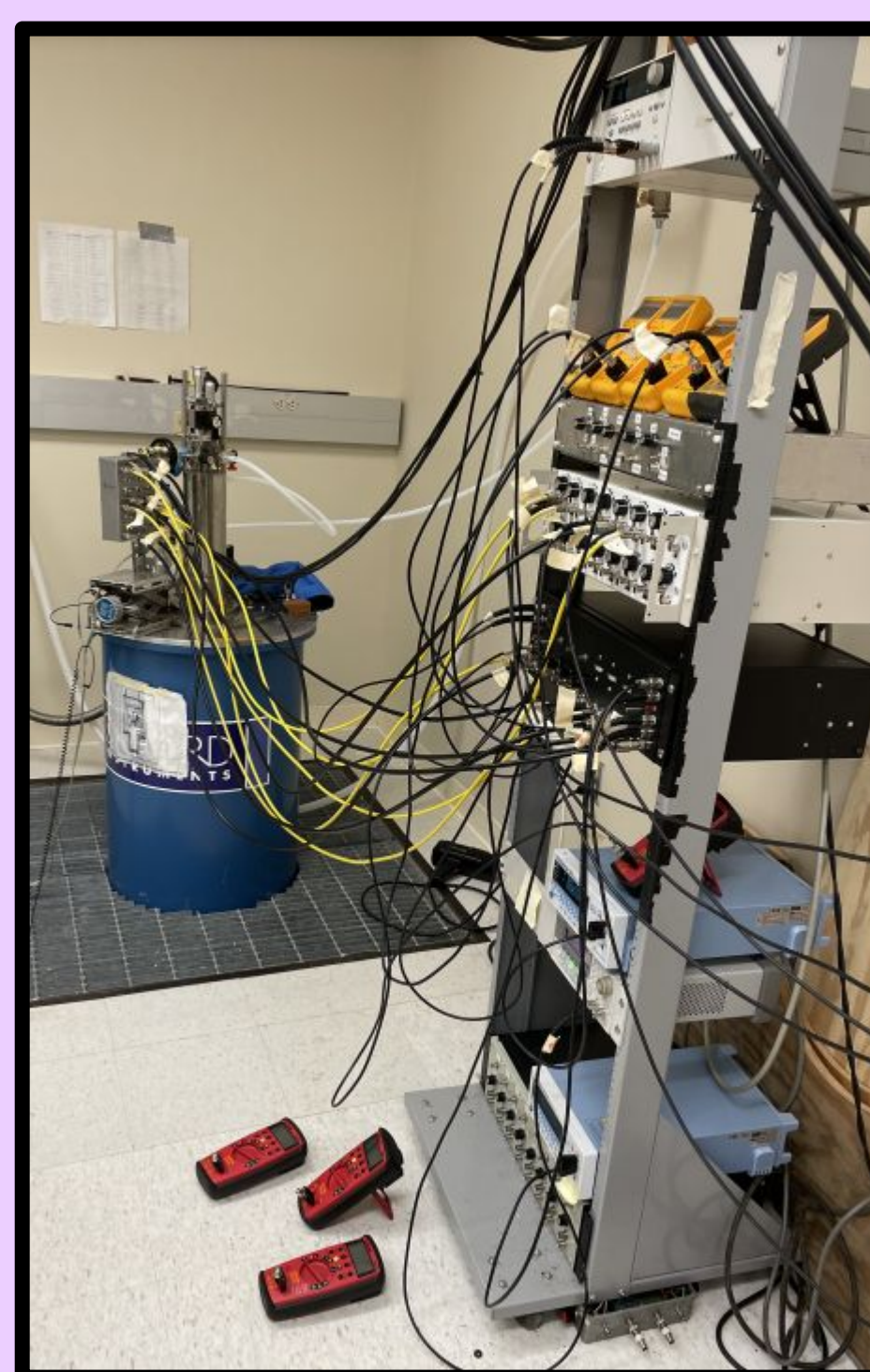


Fig. 5: Wet Cryostat



Fig. 6: Wet Sample Stage