In the previous section we have identified the type of goods present in our model. The identification and specification of financial products to be included in the model has the same importance and is performed in this section.

This task is not trivial and we perform it by using a stocks matrix approach that allows us to identify the type of financial assets needed to have a consistent framework.

As we will explain later on in this section, using this approach we can also identify mechanisms to manage coherently situations of financial stress. The latter are important to study the functioning of the economy in pathological cases.

To avoid inaccuracies in performing this thorny task, we start from a trivial representation where consumers and firms are present. We then gradually enrich the framework to obtain a configuration where government and banking sectors are also considered.

Before starting we recall that the stock matrix we will use hereafter reports the balance sheets of economic agents. Stock consistency implies that the sum of elements in each column and row of the matrix is zero.

Let's consider an island-barter economy (money does not exist) populated by a single agent named Robinson Crusoe, whose main activity is to find food. The food in the island is perishable and inventories cannot be carried to the following periods. To perform his activity, Robinson establishes the Crusoe&co to which he bestows his tools (K). From an accounting point of view we can distinguish two subjects: Robinson as a consumer and the Crusoe&co. The balance sheets of the two subjects in the economy are represented as follows:

	firm	
real (production capital)	K	$+$
financial	EF	$-$
	$\sum$	0

	household	
financial	EF	$+$
counterbalance to financial (wealth)	WH	$-$
	$\sum$	0

note that setting up an accounting framework creates two "artificial" categories: the counterpart to the production capital in Crusoe&co balance sheet, EF, that we will call equity capital and the counterpart to equity capital in Robinson's balance sheet, WH, that measures his wealth. The signs are used to specify whether the component of the balance sheet is an asset $+$, or a liability $-$.

This reasoning allows us to identify a first category of financial assets: equity capital that in modern economies materializes in *shares*. With a change of perspective we say that both K and WH counterbalance the financial aspect so

that the balance sheet of the two subjects can be summarized in the following stock matrix:

	household	firm
EF	$+$	$-$
counterbalance to financial	WH	K
$\sum$	0	0

It is worth emphasizing that the rows of the matrix relate to the types of assets that are accounted for in the economy. In the simple case presented in this section, we consider only shares as financial assets.

As stated above, stock consistency implies that the rows of the matrix also sum to zero:

	household	firm	$\sum$
EF	$+$	$-$	0
counterbalance to financial	WH	K	0
$\sum$	0	0	0

Let's consider a richer (and more populated) economy where someone invented a new contract. The Bank&co is created to implement a business based on the newly created contract named bank account (BA). Both households and firms can sign a bank account contract with the Bank&co. Households, firms and bank can be either lenders or borrowers. The lender records the BA amount with a positive sign while the borrower with a negative sign. Note that the BA amounts are usually referred to as either *deposit*, if the household or the firm act as lenders, or *loan* if they act as borrowers.

Introducing this agent (the bank), two new financial activities are to be considered in the economy: its equity (EB) and the bank account. Concerning the latter, instead of using two different rows, one for deposits and one for loans, we think it is more coherent to use one line because the BA is the unique contract an agent can sign with banks. This is of convenience because we will deal with a dynamic context in which a bank account changes its sign from period to period so that considering one entry rather than two will make the accounting easier.

After considering the BA, the agents' balance sheets are:

	household	firm	bank	$\sum$
BA	$+$	$-$	$BA_h + BA_f$	0
EB	$+$		$-$	0
EF	$+$	$-$		0
counterbalance to financial	WH	K	0	
$\sum$	0	0	0	

Table 1: stock matrix with $BA_h > 0$, $BA_f < 0$, $BA_h > -BA_f$ and shares are held exclusively by households

	household	
BA	$+/-$	
EB	$+$	
EF	$+$	
counterpart to fin.	WH	$-$
$\sum$	0	

	firm	
BA	$+/-$	
EB	$+$	
EF	$-$	
counterpart to fin.	K	$+$
$\sum$	0	

	bank	
$\sum BA$	$+$	
EB	$-$	
EF	$+$	
counterpart to fin.	0	
$\sum$	0	

Let us now build a stock matrix for an economy populated by one household, one firm and one bank. We consider the case in which $BA_h > 0$, $BA_f < 0$ and $BA_h > -BA_f$ and shares are held exclusively by households. The stock matrix is reported in table 1.

A visual representation of the matrix is given in figure 1. It shows the

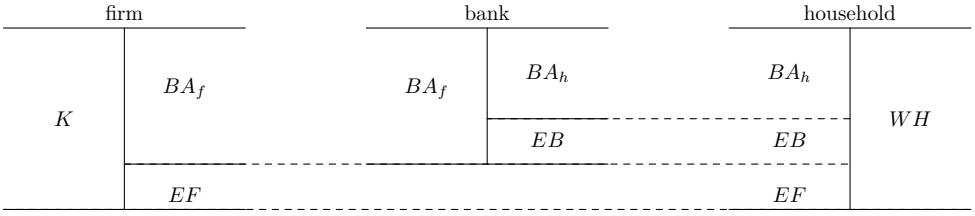

Figure 1: visual representation of the stock matrix

result often highlighted in macroeconomic textbook that in this kind of economy, wealth equates production capital:

$$WH = K$$

but it also shows less straightforward relationships such as

$$K = BA_h + EB + EF$$

	household	firm	bank	$\sum$
BA	$+$	$-$	$BA_h + BA_f$	0
EB	$+$	$+$	$-$	0
EF	$+$	$-$	$+$	0
counterbalance to financial	WH	K	0	
$\sum$	0	0	0	

Table 2: stock matrix with $BA_h > 0$, $BA_f < 0$, $BA_h > -BA_f$ and shares are held by all the agents

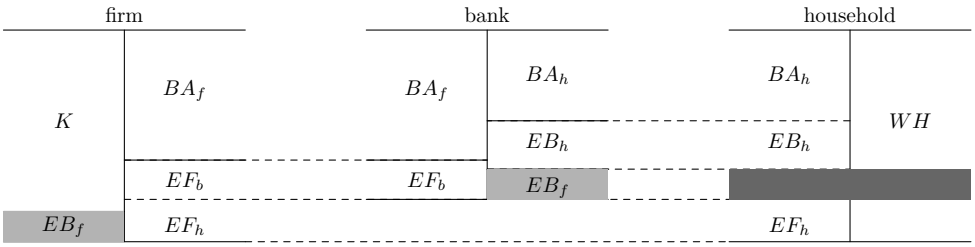

Figure 2: visual representation of the stock matrix

or

$$W = BA_f + EF$$

A slightly more sophisticated case in obtained by letting banks and firm holding each other shares. The stock matrix is now reported in table 2 and visually represented in figure 2.

The figure shows some gray area. They are introduced to find our relationships among balance sheet variables by visual inspection. In particular, the darker gray area signal areas that have to be discarded from the considered balance sheet while lighter ones are balance sheet items that are important for deducing relationships. Consider for example firm assets. Their amount is given by

$$K + EB_f$$

looking now to household's liabilities we deduce that we have to add the dark gray area to wealth to reach the same amount of firm assets. But the dark gray area is equal to EB_f. So we can write

$$K + EB_f = WH + EB_f$$

that brings us to state that

$$WH = K$$

holds again. As we did above, we can specify less straightforward relationships such as

$$K = BA_h + EB_h + EF_h$$

$$WH = BA_f + EF_h + (EF_b - EB_f)$$

that generalizes what written above.

We can now complete our framework by introducing the government. This also introduces an additional financial asset: government bonds (GB). Table 3 reports the balance sheets, table 4 the stock matrix and figure 3 the visual representation of the new situation. We can again deduce the economic equations.

$$W + EB_f - GB_f = K + GB_b + GB_f + EB_f$$

$$WH = K + GB_b + GB_f + GB_f$$

$$WH = K + WG$$

	household	
	BA	$+/-$
	EB	$+$
	EF	$+$
	GB	$+$
counterpart to fin.	WH	$-$
	$\sum$	0

	bank	
	$\sum BA$	$+$
	GB	$+$
	EB	$-$
	EF	$+$
counterpart to fin.		0
	$\sum$	0

	firm	
	BA	$+/-$
	EB	$+$
	GB	$+$
	EF	$-$
counterpart to fin.	K	$+$
	$\sum$	0

	government	
	$\sum GB$	$-$
counterpart to fin.	WG	$+$
	$\sum$	0

Table 3: balance sheets with government bonds

	household	firm	bank	gov	$\sum$
BA	$+$	$-$	$BA_h + BA_f$		0
EB	$+$	$+$	$-$		0
EF	$+$	$-$	$+$		0
GB	$+$	$+$	$+$	$-$	0
counterpart	WH	K	0	WG	0
$\sum$	0	0	0	0	

Table 4: stock matrix with $BA_h > 0$, $BA_f < 0$, $BA_h > -BA_f$ and shares are held by all the agents

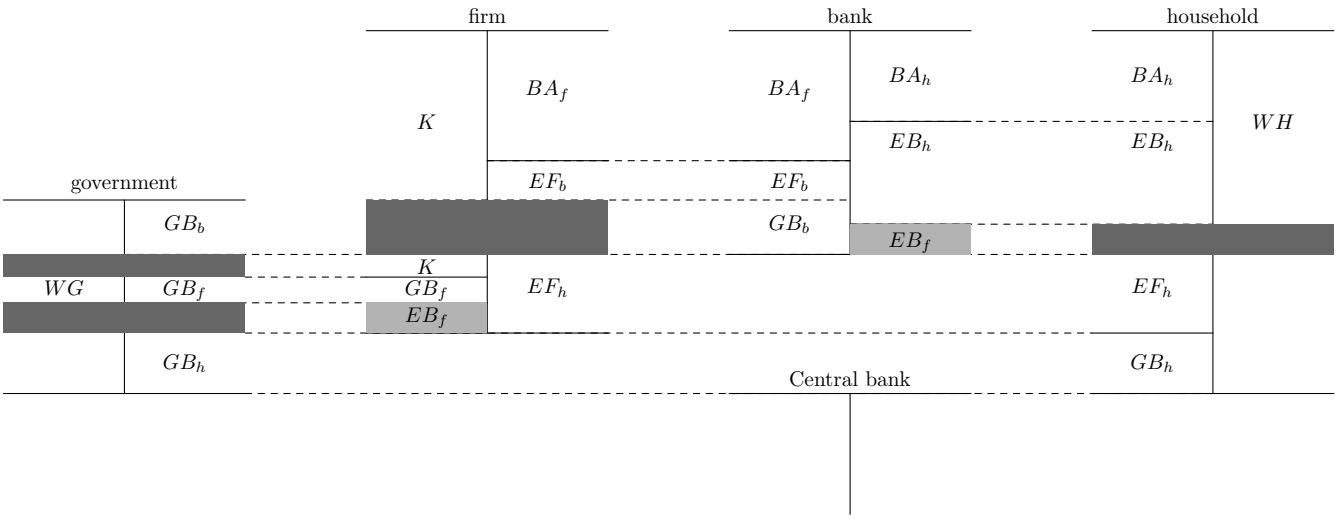

Figure 3: visual representation of the stock matrix

Figure 3 presents a rather complicate situation. We now simplify it to reach a starting point for considering situation of agents' financial stress. The simplifications is: shares abd bonds are hold exclusively by households.

Figure 4 shows the simplified situation.

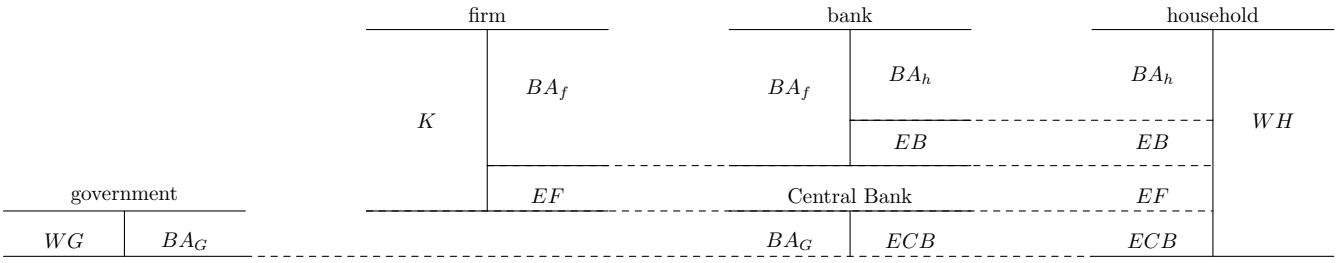

Figure 4: visual representation of the stock matrix

The aggregate stock matrix is obtained by summing over individual values and is reported in table 5.

	Household	Firm	Bank	$\sum$
BA	$\sum_h BA$	$\sum_f BA$	$-\sum_b(\sum_h BA + \sum_f BA)$	0
E_f	$\sum_h E_f$	-$\sum_f E_f$	$\sum_b E_f$	0
E_b	$\sum_h E_b$	$\sum_f E_b$	-$\sum_b E_b$	0
counterbalance to financial assets	$\sum_h W$	$\sum_f K$	0	0
$\sum$	0	0	0	0

Table 5: A simplified bottom-up stock matrix with a banking sector

In each balance sheet, one variable is determined as the complement to the others:

$$W_h = BA_h + E_b + E_f$$

$$E_f = K_f - BA_f + E_b$$

$$E_b = \sum BA + E_f$$

Economic models usually account for the "normal" case in which the three variables reported above have positive values. However, it is worth considering "pathological" cases in which these variables get negative. This section shed light on how these cases should be treated.