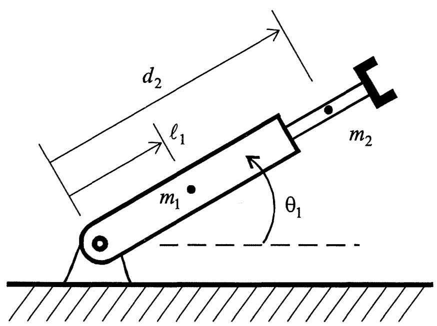
ME 531 Problem Set 1
Fall 2014 • Due October 20

Problem 1.1: Bay problem 1.11, shown below. Additionally, convert your linearized equations into state-space form.

1.11 The robot shown in Figure P1.11 has the differential equations of motion given. Symbols $m_1, m_2, I_1, I_2, \ell_1$, and g are constant parameters, representing the characteristics of the rigid body links. Quantities θ_1 and d_2 are the coordinate variables and are functions of time. The inputs are τ_1 and τ_2. Linearize the two equations about the operating point $\theta_1 = \dot{\theta}_1 = \ddot{\theta}_1 = 0$, $d_2 = 3$, and $\dot{d}_2 = \ddot{d}_2 = 0$.



P1.11

$$(m_1\ell_1^2 + I_1 + I_2 + m_2 d_2^2)\ddot{\theta}_1 + 2m_2 d_2 \dot{\theta}_1 \dot{d}_2 + (m_1\ell_1 + m_2 d_2) g \cos\theta_1 = \tau_1$$

$$m_2 \ddot{d}_2 - m_2 d_2 \dot{\theta}_1^2 + m_2 g \sin\theta_1 = \tau_2$$

Problem 1.2: Bay problem 1.10, shown below. Please provide a well-commented Matlab script that uses matrix multiplication to accomplish the change of basis. (It may be helpful to do Bay 2.8 as practice if this technique is unfamiliar to you)

1.10 For the state variable description of the system in terms of $x_i(t)$,

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 18 & 9 & 13 \\ 50 & 23 & 35 \\ -65 & -31 & -46 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 5 & -5 & 5 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

change the state variables and write new state equations for variables

$$\xi_1(t) = -4x_1(t) - 2x_2(t) - 3x_3(t)$$

$$\xi_2(t) = 15x_1(t) + 7x_2(t) + 10x_3(t)$$

and

$$\xi_3(t) = -5x_1(t) - 2x_2(t) - 3x_3(t)$$

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Problem 1.3: Bay 2.6, shown below. It may be helpful to think in terms of matrices and null spaces.

2.6 Prove or disprove the following claims: if $\boldsymbol{u}$, $\boldsymbol{v}$, and $\boldsymbol{w}$ are linearly independent in vector space V, then so are

a) $\boldsymbol{u}$, $\boldsymbol{u}+\boldsymbol{v}$, and $\boldsymbol{u}+\boldsymbol{v}+\boldsymbol{w}$.

b) $\boldsymbol{u}+2\boldsymbol{v}-\boldsymbol{w}$, $\boldsymbol{u}-2\boldsymbol{v}-\boldsymbol{w}$, and $4\boldsymbol{v}$.

c) $\boldsymbol{u}-\boldsymbol{v}$, $\boldsymbol{v}-\boldsymbol{w}$, and $\boldsymbol{w}-\boldsymbol{u}$.

d) $-\boldsymbol{u}+\boldsymbol{v}+\boldsymbol{w}$, $\boldsymbol{u}-\boldsymbol{v}+\boldsymbol{w}$, and $-\boldsymbol{u}+\boldsymbol{v}-\boldsymbol{w}$.

Problem 1.4: Bay 6.2, shown below. Please use Matlab to draw the vector fields, then add a representative set of trajectories showing the system's flow. For (h), please explain a simpler way to illustrate the phase portraits and include the resulting plots.

6.2 Draw phase portraits for the systems with the following A-matrices:

a) $\begin{bmatrix} -8 & -6 \\ 0 & -2 \end{bmatrix}$ b) $\begin{bmatrix} -8 & -6 \\ 0 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 4 & -4 \\ 4 & 4 \end{bmatrix}$ e) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ f) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

g) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ h) $\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$