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Time Series Analysis and Forecasting of Mercedes-Benz Group Stock Price Development - Final Project -

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Contents

1 Introduction

Stocks, also called shares, are a type of security paper and represent ownership in a company. The stock market refers to the financial market where buyers and sellers can trade these securities. Investment in stocks has been crucial for a large quantity of individuals, companies and other organizations. Purposes include most prominently the potential of long-term capital appreciation, such as for retirement. Historically, the growth of the stock market has beaten cumulative inflation over a long period of time. [2] Thus, a persistent investment was able to circumvent the decrease of economic buying power caused by inflation. The intention of generating a regular income stream can be another goal. Some companies take a fraction out of their realized profits and pay them out as dividends to their owners or shareholders. Diversifying an investment portfolio is a legitimate strategy as well since certain stocks might not be correlated with other asset classes like bonds or real estate. Besides, volatility and sharp price movements make short-term speculation attractive and risk hedging important for certain market participants. For all these objectives, it is beneficial to analyze the available data and try to forecast price development into the future which may be used for making trading decisions. Taking into account that a stock price history resembles a time series, time series analysis can be applied to extract meaningful statistics and data characteristics and ultimately to predict future values by identifying patterns, seasonality and cyclical behavior.

The particular stock price development of interest in the following is Mercedes-Benz Group, a Germany-based automaker (formerly known as Daimler AG until a major company restructuring took place in 2022). With a total of more than 2.5 million cars, the company ranked 13th worldwide in 2017 in terms of produced vehicles per year. [3] However, with respect to revenue, they reached 185 million US dollars and 3rd rank. It shows a comparably high revenue per car and manifests Mercedes-Benz's position in the luxury car market segment. [4] The company's vehicles are world-renowned for their reliability, comfort and high quality manufacturing. Moreover, the brand Mercedes-Benz is well-recognized and perceived to be prestigious, making it the 15th most valuable brand worldwide and the most valuable German brand in 2022 according to Brands Finance's Global 500 annual report. [5] Interbrand puts the brand even higher on rank 8. [6] Mercedes-Benz Group shares primarily trade on the Frankfurt stock exchange in Germany denominated in Euro (ticker symbol: MBG.DE). They are included in the DAX (Deutscher Aktien Index), the preeminent stock market index in Germany which consists of the 40 major German publicly traded companies. As of June 2022, the stock is weighted in the DAX with 4.2 percent, making it the 8th most influential stock in the calculation of the index. [7] Further, it is noteworthy that Mercedes-Benz Group (then: Daimler AG) was the most popular single stock traded by retail investors in Germany in 2015. [8] These considerations give the motivation for a time series analysis and a forecast of stock prices of Mercedes-Benz Group.

2 Methodology

2.1 ARIMA Model

Autoregressive Integrated Moving Average - ARIMA(p,d,q) - is a model proposed by Box and Jenkins in 1976 and combines p autoregressive terms, d differencing terms and q moving averages. The model can be expressed as follows:

$$\left(1 - \sum_{i=1}^{p} \phi_i L^i\right) (1-L)^d y_t = \left(1 + \sum_{i=1}^{q} \theta_i L^i\right) e_t$$
$$e_t \sim \mathcal{N}(0, \sigma^2)$$

where y_t is the (adjusted) closing price of Mercedes-Benz Group in Euro, e_t is white noise and L is the lag operator.

2.2 GARCH Model

The generalized Autoregressive Conditional Heteroskedasticity - GARCH(p,q) model - is used to describe time series with time-dependent (=heteroscedastic) variance. It extends the ARCH(p) model and allows for both autoregressive (AR) and moving average (MA) components in the variance term. The model equations are:

$$\begin{aligned} y_t &= x_t' b + \epsilon_t \\ \epsilon_t | \psi_{t-1} &\sim \mathcal{N}(0, \sigma_t^2) \\ \sigma_t^2 &= \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \end{aligned}$$

where y_t is the (adjusted) closing price of Mercedes-Benz Group in Euro, σ_t is the non-constant variance and L is the lag operator.

2.3 EGARCH Model

To capture asymmetric effects between positive and negative asset returns, meaning impacts of negative shocks persist longer than of positive ones, Nelson introduced Exponential GARCH in 1991. An EGARCH(m,s) can be written as:

$$\ln \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i g(\epsilon_{t-i}) + \sum_{j=1}^{s} \beta_i \ln \sigma_{t-j}^2$$

2.4 FGARCH Model

Hentschel's fGARCH model, also known as Family GARCH, is an omnibus model that nests a variety of other popular symmetric and asymmetric GARCH models. Here, the asymmetric power GARCH (APARCH) is used. In Power-GARCH model, the logarithm function of the EGARCH model is replaced by a power.

2.5 Model Evaluations

The performance of forecasting models are evaluated using the following measures: Mean Square Error (MSE):

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2$$

Root Mean Square Error (RMSE):

$$\mathrm{RMSE} = \sqrt{\mathrm{MSE}}$$

Mean Absolute Percentage Error (MAPE):

$$\mathrm{MAPE} = \frac{100\%}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right|$$

3 Data Analysis

3.1 Data Set

As a more than a century old company, the stock price history of Mercedes-Benz Group is extensive. To get a first impression on how the stock behaves we can inspect the development of the last 30 years in Figure 1. It is apparent that the stock price generally moves in a range between a lower and an upper bound. The lower bound was reached in 1993, 2003, 2008/09 and 2020. The stock moved near the upper bound in 1998/99, 2007, 2015 and 2021. Most noticeable is no visibility of a clear long-term positive trend. In comparison, broad stock market indices like the S&P 500 tend to appreciate in value in a large timeframe. [9] Different factors may have contributed to this behavior. Being in the automotive sector, Mercedes-Benz Group as well as its competitors have revenues and profits in tandem with the state of the economy and tied to the business cycle. Peter Lynch, an American investor and mutual fund manager calls stocks of likewise companies "cyclical". [10] Moreover, as a well-established company, Mercedes-Benz Group might not have opportunities for business expansion anymore. When its core market - the new car market - is near to saturation, the management may struggle to reinvest the profits and decide to distribute them as high dividends. A simulation of an investment in the stock as if all paid dividends would be directly reinvested in the stock itself on payday gives a much better idea of the development. It can be shown that reinvested dividends made a long-term investment in Mercedes-Benz Group profitable.

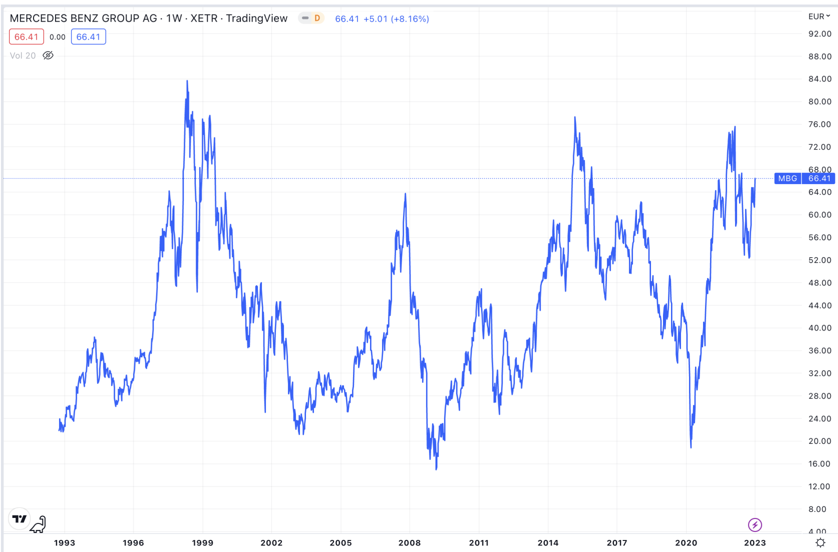
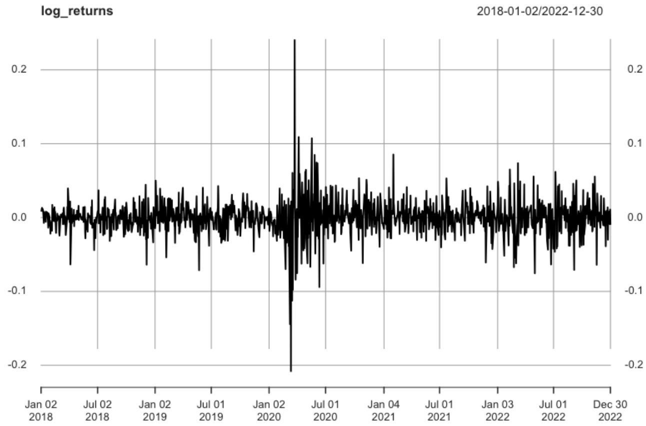


Figure 1: 30 year stock price history

For time series analysis, the focus is set to the stock price history of the last four years - i.e. from January 2018 to December 2022. It can be argued that the recent price history is likely more significant for market activity opposed to older data. Company specific conditions

(a) Adjusted stock price 2018-2022



(b) Log returns 2018-2022

Figure 2

like the automotive market, new competitors, management, changing customer preferences could have changed substantially over a long period. General conditions affecting multiple sectors like economic situation, inflation, interest rate environment, currency exchange rates, politics, market sentiment and participants including automated trading algorithms possibly differ a lot from the past as well. Closing price data was obtained from Yahoo Finance and was analyzed using R. For predicting future values and comparing them to actual ones, the data set is split into a training set (in-sample) and a test set (out-of-sample). The training set includes stock prices from 2018/01/02 to 2022/11/30 and consists of 1248 data points. The 21 remaining values from 2022/12/01 until 2022/12/30 are used in the test set to later measure the accuracy of the model's prediction.

The initial retrieved time series exhibits a discontinuity between 2021/11/26 and 2021/11/29. The share price decline of 17 percent is not likely to occur without any company news or broad market volatility. On this day, Daimler AG published the prospectus about the separation of the Daimler Truck subsidy as a part of a major company restructuring announced in February 2021. [11] Shareholders received one new Daimler Truck share for every two shares of Daimler AG. As a result, shares of Daimler AG dropped on that day to compensate. Since no monetary value was lost for investors, a realistic time series is retained by adjusting the price history. Closing prices prior to the prospectus publication on 2021/11/26 are divided by a factor of 1.2045. The renaming to Mercedes-Benz Group and the corresponding ticker change from DAI to MBG took place in February 2022 without any major effect on share price.

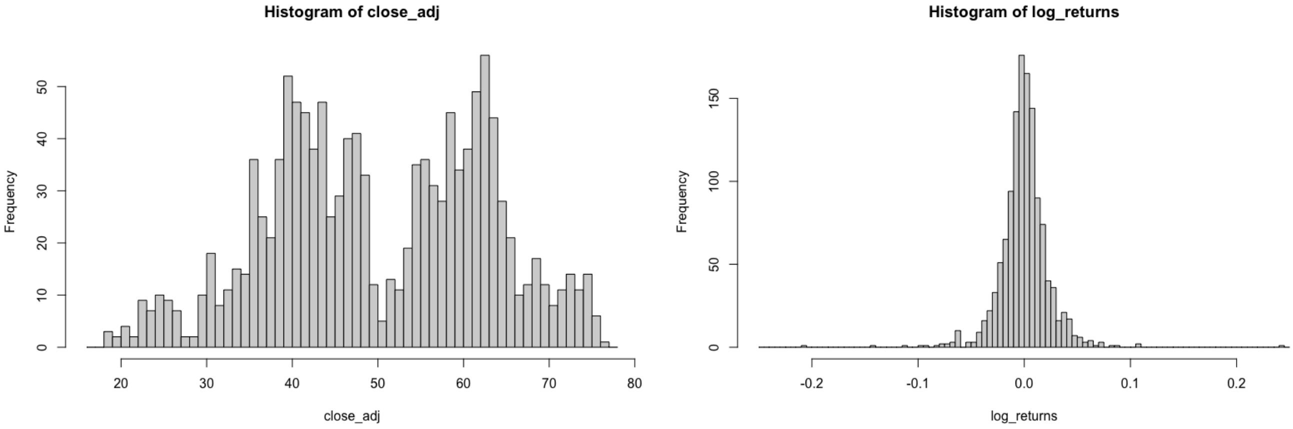
The stock's pattern over the chosen time frame can be visually divided into three main sections seen in Figure 3a. Starting in 2018, the price is in a declining trend lasting until March 2020, when it cratered sharply in the midst of the COVID-19 outbreak [12] and reached its low of 18.13 EUR (adjusted) on 2020/03/20. Following, the stock quickly reversed and commenced an upward trend of about one year duration. From March 2021 to December 2022, Mercedes-Benz Group found itself trading in a sideways pattern without large movements in either direction. A maximum closing price of 76.06 EUR was observed on 2022/02/17, just seven days prior to the Russian invasion in Ukraine. [13]

The discussed characteristics clearly do not give an impression of stationarity as the series wanders up and down for long periods. With the help of the Augmented Dickey-Fuller (ADF) unit root test, stationarity is checked. A p-value of 0.4336, which is larger than 0.05, does not reject the null hypothesis - the existence of a unit root. The series is not stationary and thus needs to be differenced for further analysis. Return data is calculated out of the original time series by dividing the price at time t by its predecessor and applying the natural logarithm. Differences of log prices represent the daily return in stock price in percent. It is chosen over a simple return as the relative change is generally of higher interest compared to pure change in price. The ADF test for the return series now rejects the null hypothesis with a p-value of 0.01, it is stationary. Further differencing does not need to be applied.

Table 1: Results of the Augmented Dickey-Fuller (ADF) tests for stationarity

	Dickey Fuller	Lag order	p-value
Closing price	-2.3419	10	0.4336
Log returns	-10.507	10	0.01

Figure 3b depicts the stationary log return series. Volatility clustering or persistence is visible as the daily returns experience varying variance during certain market phases. The histogram of da Most evident are price swings up to 20 percent in both directions during the crash following the COVID-19 outbreak in March 2020. Pre-pandemic trading in late 2019 and early 2020 appears to be more tranquil. In 2021, daily changes started to settle down until volatility picked up again in early 2022 amid the ongoing Russo-Ukrainian war and the recent inflation surge in several countries. [14] Descriptive statistics of the return series (Table 2) support this observation. High positive excess kurtosis of 13.612824 is present and implies a leptokurtic or fat–tailed distribution. It differs significantly from a normal (Gaussian) distribution in terms of the number of outliers it produces. Hence, the risk for investors is increased and it could lead to worrisome situations. An example of leptokurtic distributions is the Laplace distribution which has been used for modeling stock price returns and has shown to be more suitable compared to Gaussian. [15] A mean slightly differing from zero is indicative of asymmetry and is expected as the total return over the given time is positive. The skewness is near zero and shows that the data is farily symmetrical. Next to the log returns, the descriptive statistics of the original data is shown as well. The negative kurtosis corresponds to a platykurtic distribution, one with a flatter peak and thinner tails compared to a normal distribution. It is visible in Figure 3a where the histogram of the closing prices is plotted.



(a) Closing prices (b) log returns

Figure 3: Histograms of the data

It is characterized by a lack of clustering around the mean, as it would be expected for normal distributions. Instead, two price areas developed where the stock traded for more days. The skewness is also close to zero.

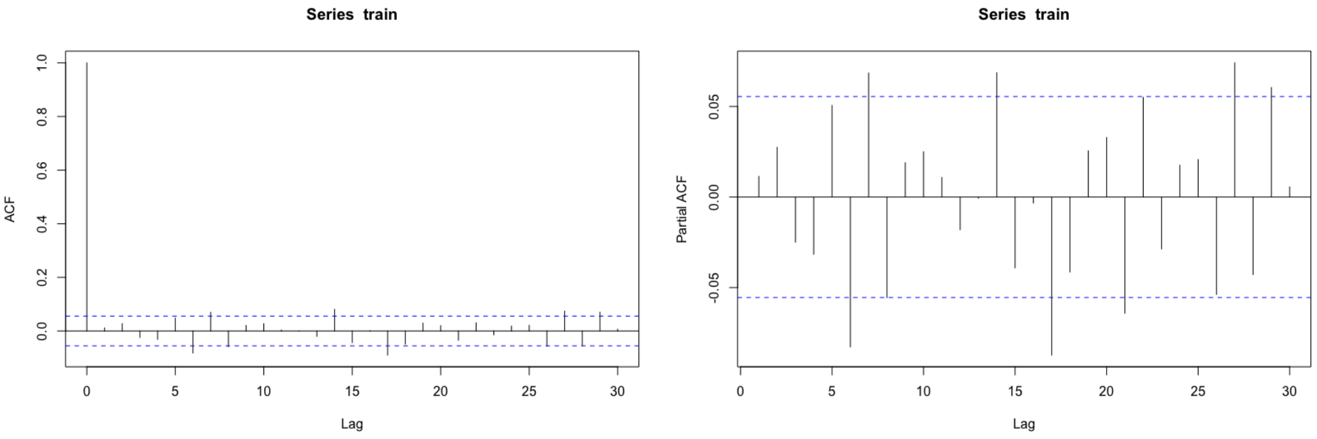
3.2 ARIMA Modeling

To select possible candidates for fitting an ARIMA(p,d,q) model to the given data, the autocorrelation function (ACF) and partial autocorrelation function (PACF) for the log returns are plotted at first (Figure 4). The blue dotted lines serve as thresholds for statistical significance - values inside the threshold band cannot be distinguished from zero. The ACF shows no major peaks except of course for a lag of 0, indicating that the log return series shows no correlation with previous values. In the PACF, there are no salient peaks present either. These characteristics are suggesting an ARIMA(0,0,0) model for the log returns. Since the price data is already differenced, this corresponds to an ARIMA(0,1,0) model for the price data. This process is well-known as a random walk process and frequently adopted for modeling financial data. In "The Pricing of Options and Corporate Liabilities", Black and Scholes published a model which assumes the log return of the stock price to be a random walk with drift and prices to follow a geometric Brownian motion with a constant drift and volatility. [16] Since its publication in 1973, their model has been regularly used by market participants.

Based on the corrected Akaike Information Criterion (AICc), the most optimal models for log returns are ARIMA(2,0,2) and the already suggested ARIMA(0,0,0) as they produce the lowest AICc values. Both models rank high when either a zero mean is assumed or not. The candidates can now be examined for their residuals in Figure 5. It is distinct that the residuals are not white noise. The volatility cluster remains visible in all selected models. Checking the residuals with the Ljung-Box test results in the rejection of the null-hypothesis of uncorrelated residuals. Instead, they

Table 2: Descriptive Statistics

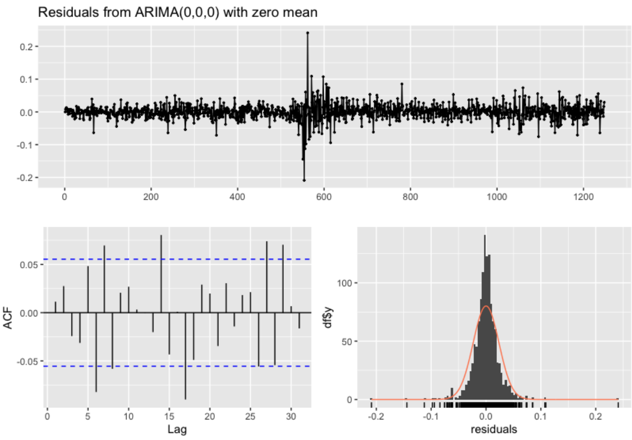
	Log returns	Closing prices
Minimum	-0.208896	18.132005
Maximum	0.241193	76.059998
1. Quartile	-0.010110	40.415110
3. Quartile	0.010271	60.946454
Mean	0.000036	50.053083
Median	0.000000	48.443336
Sum	0.046161	63517.362056
SE Mean	0.000656	0.356203
LCL Mean	-0.001250	49.354271
Variance	0.001323	161.011324
Standard Deviation	0.000546	12.689024
Skewness	0.076940	-0.104632
Kurtosis	16.612824	-0.786334



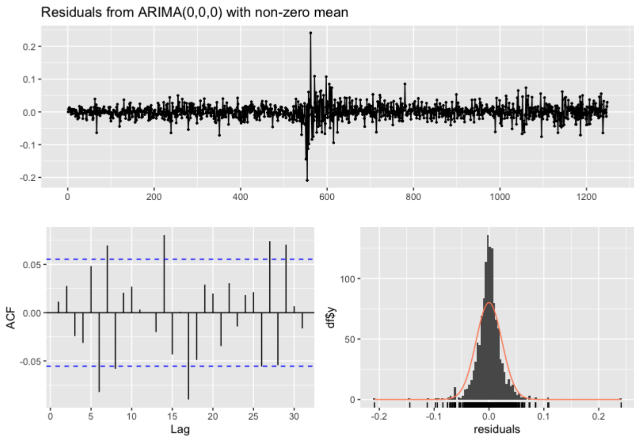
(a) ACF

(b) PACF

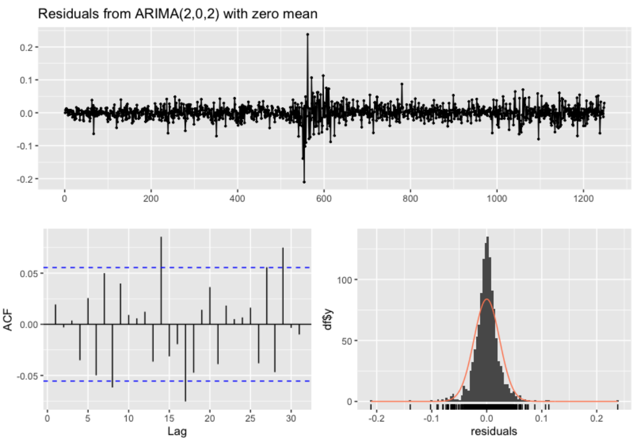
Figure 4: Correlation functions for log returns



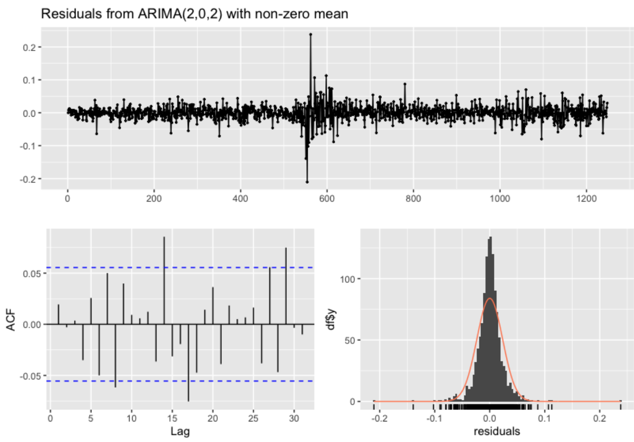
(a) ARIMA(0,0,0) with zero mean



(b) ARIMA(0,0,0) with non-zero mean



(c) ARIMA(2,0,2) with zero mean



(d) ARIMA(2,0,2) with non-zero mean

Figure 5: Residuals for ARIMA candidates

Table 3: Tested corrected Akaike Information Criterion (AICc) on ARIMA orders

	Non-zero mean	Zero mean
ARIMA(0,0,0)	-5929.711	-5931.714
ARIMA(0,0,1)	-5927.805	N/A
ARIMA(1,0,0)	-5926.810	N/A
ARIMA(1,0,1)	Inf	Inf
ARIMA(1,0,2)	-5923.998	-5926.009
ARIMA(1,0,3)	-5923.045	-5925.058
ARIMA(2,0,1)	-5922.885	-5924.910
ARIMA(2,0,2)	-5934.273	-5936.200
ARIMA(2,0,3)	Inf	Inf
ARIMA(3,0,1)	-5921.499	N/A
ARIMA(3,0,2)	Inf	Inf
ARIMA(3,0,3)	-5930.061	N/A

exhibit serial correlation. The model can still be used for forecasting, but the prediction intervals may not be accurate due to the correlated residuals.

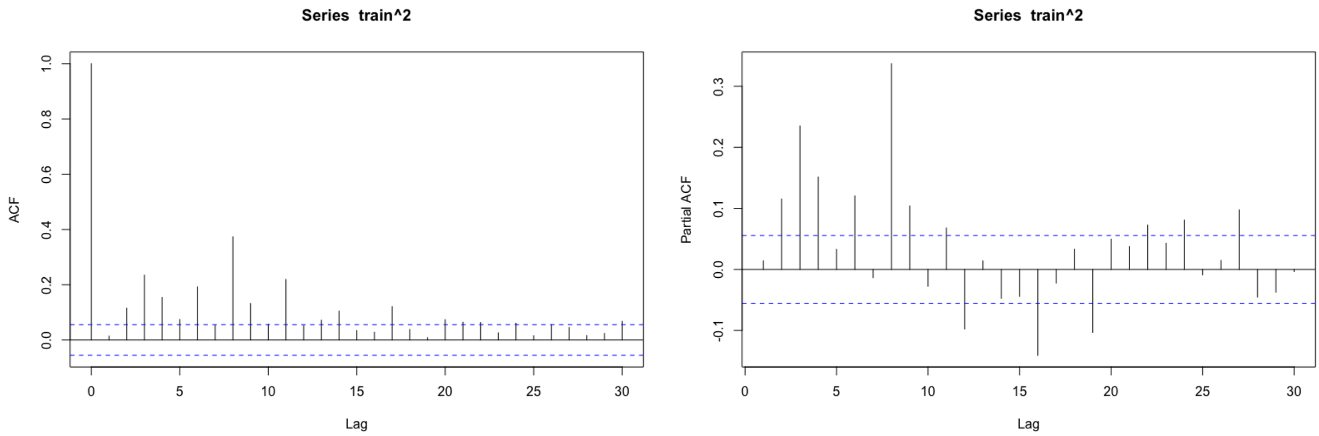
Due to the presence of volatility persistence, it is of interest to screen possible Autoregressive heteroscedastic (ARCH) effects. For that, the ACF and PACF of the squared returns can be examined in Figure 7. The various significant values in both functions show the existence of conditional heteroscedasticity. To confirm, ARCH effects are tested with the Lagrange Multiplier (LM) test (Table 4). The null hypothesis is rejected with a p-value of $2.2 \cdot 10^{-16} < 0.05$, hence the log returns are heteroscedastic and the variance of returns are non-constant.

Table 4: LM test for log returns

LM	df	p-value
11097	1	$2.2 \cdot 10^{-16}$

3.3 GARCH Modeling

Volatility clustering in financial data can be sufficiently modeled by GARCH(1,1) according to Brooks and Burke. [17] Their suggestion is adopted here and GARCH(1,1) is employed. In Table 5, the performance of the tested models based on four different criteria (Akaike, Bayes, Shibata, Hannan-Quinn) are depicted.



(a) ACF

(b) PACF

Figure 6: Correlation functions for squared log returns

Table 5: Information Criteria for GARCH models

Model for log returns	Akaike	Bayes	Shibata	Hannan-Quinn
GARCH(1,1)	-4.985605	-4.969165	-4.985625	-4.979424
GARCH(1,1) std	-5.068600	-5.048100	-5.068600	-5.060900
GARCH(1,1) sstd	-5.067578	-5.042918	-5.067624	-5.058307
EGARCH(1,1) std	-5.079788	-5.055128	-5.079834	-5.070517
FGARCH(1,1) std	-5.085873	-5.057103	-5.085936	-5.075057
ARMA(2,2)-GARCH(1,1)	-4.984779	-4.951899	-4.984860	-4.972417

Table 6: Error Measures

Model	MSE	RMSE	MAPE
GARCH(1,1)	0.0002049847	0.01431729	34.13532
GARCH(1,1) std	0.0002042236	0.01429068	66.97411
GARCH(1,1) sstd	0.0002031196	0.01425201	127.1755
EGARCH(1,1) std	0.0002031153	0.01425185	125.6414
FGARCH(1,1) std	0.0002030656	0.01425011	110.5027
ARMA(2,2)-GARCH(1,1)	0.0001997410	0.01413298	30.59042
ARMA(0,0) with zero mean	0.0002034864	0.01426487	NaN
ARMA(0,0) without zero mean	0.0002038338	0.01427704	139.4128
ARMA(2,2) with zero mean	0.0002076897	0.01441144	41.2965
ARMA(2,2) without zero mean	0.0002080395	0.01442358	41.79123

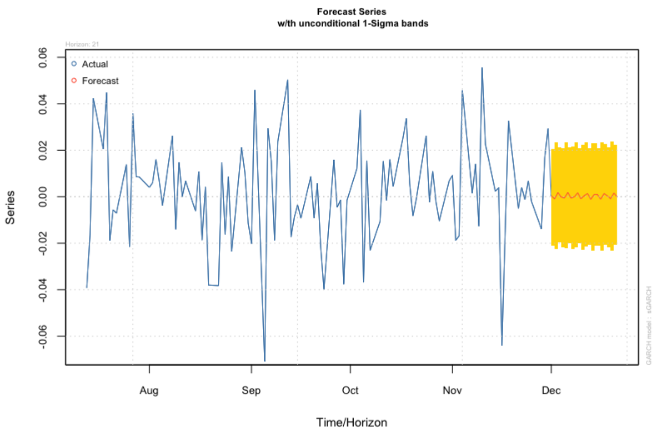
As previously mentioned, stock returns are far from normally distributed. The GARCH model in R uses a normal distribution by default. Changing to the Student-t-distribution (std) or the skew Student-t distribution (sstd) improves the information criteria measures and results in a better fit. The Student-t-distribution (std) shows to be the best. This distribution is now set to check modified models. A drawback of standard GARCH models is the assumption that positive and negative error terms have a symmetric effect on volatility. In practice, negative outliers exceed positive ones. By extending the standard model to an Exponential GARCH (EGARCH), this observed asymmetry can be factored in. EGARCH(1,1) increases the performance of the standard GARCH. All these GARCH types are modeled with a random walk - an ARMA(0,0) - as the mean model. Testing other ARMA models like the proposed ARMA(2,2) and combining them yields the worst result.

3.4 Forecasting

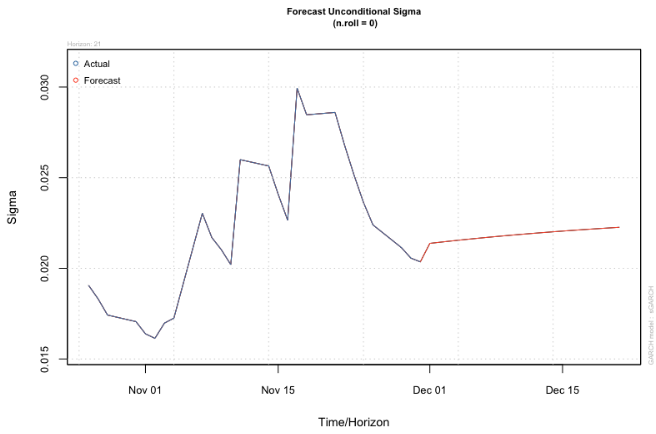
The selected models are trained on in-sample data and are now used to predict the out-of-sample stock returns. To measure the precision of the forecast the Mean Square Error (MSE), Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE) are evaluated in Table 6. In terms of minimizing the error between the predicted values and the actual observed price data it is suggestive to select the combined ARMA(2,2)-GARCH(1,1) model as it minimizes all error functions. It contradicts the findings from inspecting the information criteria. Table 8 shows the specific model parameters. The sum of ARCH and GARCH terms $\alpha + \beta$ is 0.97 which is close to 1 and indicates that volatility shocks are quite persistent. Moreover, it indicates that a unit root could exist, an IGARCH model would be another possibility. It In Table 7 the predicted values can be compared to the actual values from the test set.

Table 7: Comparison between predictions from ARMA(2,2)-GARCH(1,1) and test set

Date	Actual return	Predicted return
2022-12-01	-0.0037336383	0.00042155024
2022-12-02	-0.0010917428	-0.00098073479
2022-12-05	0.0026491017	0.00177077073
2022-12-06	-0.0074977389	-0.00014067318
2022-12-07	-0.0104019852	-0.00056567923
2022-12-08	0	0.00182829795
2022-12-09	0.0261136252	-0.00061888906
2022-12-12	-0.0114872107	-0.00004984023
2022-12-13	0.0087051379	0.00168230564
2022-12-14	-0.0007742201	-0.00095282358
2022-12-15	-0.0388490362	0.00049396482
2022-12-16	-0.0011278339	0.00136053549
2022-12-19	0.0096262896	-0.00110387792
2022-12-20	-0.0101100275	0.00099216154
2022-12-21	0.0137756622	0.00091301189
2022-12-22	-0.0300361831	-0.00105938608
2022-12-23	0.0057212744	0.00138024951
2022-12-27	0.0086019033	0.00040447047
2022-12-28	-0.0089278896	-0.00083324494
2022-12-29	0.0108652328	0.0016110238
2022-12-30	-0.0097245173	-0.00009486428
2022-12-29	0.0108652328	0.0016110238
2022-12-30	-0.0097245173	-0.00009486428



(a) Forecast of log returns



(b) Forecasts of σ

Figure 7: ARMA(2,2)-GARCH(1,1) forecasts

Table 8: Model Parameters

Parameter	ARMA(2,2)-GARCH(1,1)
μ	3.179696e-04
ar1	-1.197312e+00
ar2	-9.862242e-01
ma1	1.205159e+00
ma2	9.977815e-01
ω	1.620842e-05
α	1.137521e-01
β	8.563396e-01

4 Conclusion

To model the stock price development of Mercedes-Benz Group, different models were selected as candidates. Among them are ARIMA models with autoregressive and moving average terms. A random walk was also chosen as it is popular for modeling financial processes. Analyzing the characteristics of the time series lead to the observation of heteroscedasticity, the variance of the log return is time dependent. To take this into account, GARCH model and variations of it were proposed as they respect persistence. Although modified GARCH models first indicated to be better suitable regarding the information criteria, other models did better at minimizing the error term between the predicted and actual data for the selected train/test split. ARMA(2,2)-GARCH(1,1) is chosen and the predictions and actual values of the time series and changing variance were compared. Despite the fact that the mean error is the lowest, the predictions still differ from the observations. Ultimately, stock returns are dependent on a lot of variables and highly chaotic, which is why they cannot be predicted reliably. A possible future research could focus on the usage of Laplace distributions when modeling GARCH.

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