Analsysis of the Nearest Neighbor Heuristic for the Online Time Dependant Freeze-Tag Problem

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Abstract

In the Online Time Dependant Freeze-Tag Problem (OTDFTP) we are given a swarm of hibernating robots and a single awake robot. Our goal is to awaken the entire swarm in the least amount of time possible. Hibernating robots enter sleep mode once they have reached an internal release time. Awake robots can then awaken a sleeping robot by traveling to it's location and touching it. Using the Nearest Neighbor Heuristic to choose awakening targets, we reach a competitive ratio of $\frac{5}{2} - \epsilon$ for any $\epsilon > 0$. We also provide empirical analysis using random geometric datasets.

I. Introduction

The Freeze-Tag Problem (FTP) [1] is a problem in the field of swarm robotics in which a strategy to awaken a swarm in the minimum makespan must be found. In the original problem as proposed in [1], robots have two states, sleeping and awake. Sleeping robots are awakened when touched by an awake robot.

At the beginning of the problem we are given a set of sleeping robots and a single awake robot known as the *source robot*. The source robot then awakens sleeping robots which then help awaken the other robots.

The Online Time Dependant Freeze-Tag Problem (OTDFTP) is a variation of FTP in which each robot has a release time associated with it. The OTDFTP was first proposed by Hammar et al in [2]. In the OTDFTP robots have an additional state we will refer to as *hibernation mode*. When A robot is in hibernation mode, it is not visible to other robots. Once a hibernating robot's release time has been reached, it will enter sleep mode and become visible to awake robots.

Related Work. Hammer et al. [2] found the Offline Time Dependant Freeze Tag Problem has a lower bound of $7/3 - \epsilon$, for any $\epsilon > 0$.

In [3], Sztainberg et al. have proven a lower bound for the FTP using the nearest neighbor heuristic in which robots do not claim targets is at least $4 - \epsilon$ times optimal for any $\epsilon > 0$[3, Theorem]. The authors go on to find the strategy also has a tight approximation bound of $\Theta(\sqrt{\log n})$ for points on a plane and $\Theta((\log n)^{1-\frac{1}{d}})$ for points in d dimensions.

Preliminaries. Let $R = \{r_0, r_1, r_2, \ldots, r_n\} \subset M$ be the set of n robots in some continuous metric space M. M is a d-dimensional Euclidean space with distances measured according to an L_p metric.Unless otherwise stated, the robot r_0 is the *source robot* and is the only awake robot at the beginning of the problem.

Robots have three modes: *awake mode*, *sleep mode*, and *hibernation mode*. In awake mode, a robot is fully functional and is free to move. In hibernation mode a robot is completely inactive and can not move nor be awakened by any external means. In sleep mode, a robot is inactive but can be awakened by other awake robots by being touched.

Robots have a *release time*. Once a robot's release time has passed, it will exit hibernation mode and enter sleep mode. This would be comparable to the real world example of a robot charging it's batteries and only becoming available to awaken once it's battery is full.

In order to avoid robots from traveling in a single pack, awake robots can claim a target and no other robot will target a claimed robot. [3]

A solution to the OTDFTP is considered *rational* if:

1. Each robot with no target will immediately choose an unclaimed robot in wait mode and begin moving toward it.

2. A robot does not move other than to advance on it's target. If a robot has no target, it will not move.

This definition is similar to the one given in [1] with slight modification for release times.

Summary of Results. We find that the competitive ratio of the Nearest Neighbor heuristic is 5/2.

II. The Algorithm

The Nearest Neighbor Algorithm is a *rational* wake-up strategy for the OTDFTP. Each robot chooses the closest unclaimed robot in wait mode. Once a robot has chosen a target it will not choose another target until it has reached it's current target.

Algorithm 1 Returns the nearest unclaimed sleeping robot.

Precondition: A is the set of sleeping robots and t be the robot's current target.

```
function NEAREST NEIGHBOR(A, t)
    if t ≠ NULL then
        return t
    if A.size = 0 then
        return NULL
    m ← NULL
    for robot in A do
        if ¬robot.claimed then
            if m = NULL then
                m ← robot
            else
                a ← dist(self, robot)
                b ← dist(self, m)
                if a < b then
                    m ← robot
    return m
```

III. Analysis

Theorem 1. *For any $\epsilon > 0$, there exists an instance of the OTDFTP for which the Nearest Neighbor Algorithm results in a makespan less than $\frac{5}{2} - \epsilon$ times optimal.*

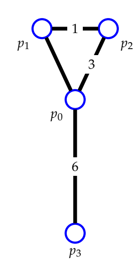
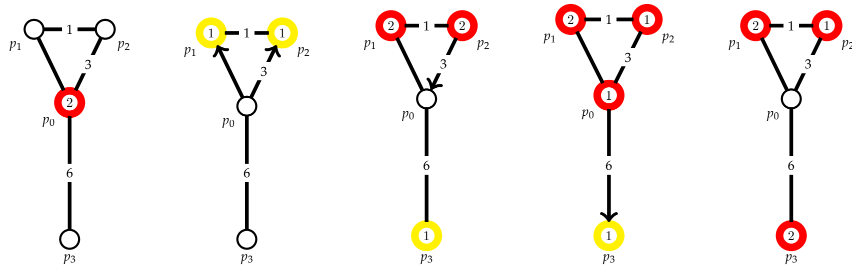
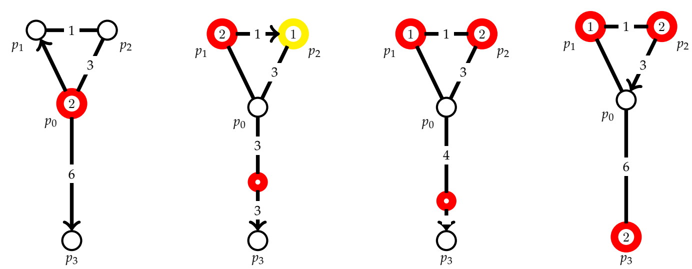


Figure 1



(a) $t = 0$ **(b)** $t = 3$ **(c)** $t = 6$ **(d)** $t = 9$ **(e)** $t = 15$

Figure 2: *Illustration of the competitive ratio proof for the Nearest Neighbor algorithm.*



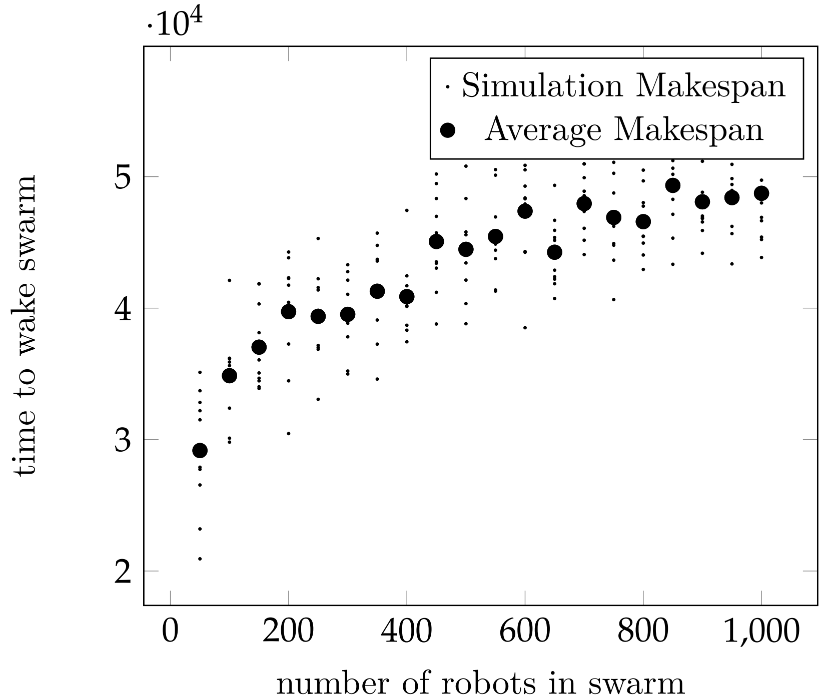
(a) $t = 0$ **(b)** $t = 3$ **(c)** $t = 4$ **(d)** $t = 6$

Figure 3: *Illustration of the optimum solution for the problem in Theorem 1.*

Proof. Let $G = V, E$ be the graph in Figure 1. The source robot r_0 activates at vertex v_0. At time $t = 0$ robot r_1 enters the sleep mode and is immediately awakened by r_0. At $t = 3$ r_2 enters sleep mode at v_1 and r_3 enters sleep mode at v_2. r_0 claims r_2 and begins moving down edge (v_0, v_1). r_1 claims r_3 and begins moving down edge (v_0, v_2). r_0 and r_1 arrive and awaken their respective targets at $t = 6$. At the same time r_4 enters sleep mode at v_3 and is claimed by r_0. r_0 travels down (v_1, v_0) to v_0 then travels down (v_0, v_3). r_0 arrives at v_0 at $t = 15$.

In the optimum solution, the source robot r_0 still starts at v_0 and awakens r_1 at $t = 0$. r_0 immediately moves down (v_0, v_3) and r_1 moves down (v_0, v_1). At $t = 3$, r_1 arrives at v_1 as r_2 and r_3 are entering sleep mode. r_1 immediately claims and awakens r_2 then claims r_2 and begins moving down (v_1, v_2). r_1 arrives at v_2 and awakens r_3 at $t = 4$. At $t = 6$ r_0 arrives at v_3 as r_4 is entering sleep mode and immediately awakens it.

This gives us a makespan of 15 for the Nearest Neighbor algorithm and a makespan of 6 for the optimal solution to Figure 1. This gives us a competitive ratio of $\frac{5}{2}$. □



IV. Experiment

I. Experiment Setup

The experiment is based on a Python 2.7.9 simulation of swarms of varying sizes where the robots are running an implementation of the Nearest Neighbor Algorithm. The simulations were performed on a 64-bit PC running Linux OS.

Dataset. The simulations are run with 210 different geometric datasets consisting of increasing numbers of robots randomly placed on a 10000×10000 unit two dimensional Euclidean plane. The release time of each robot is a random number between 0 and 5000 time units. The source robot is the first robot placed on the plane.

Parameters. Robots were given a movement speed of 1 unit of geometric space per 1 unit of time.

Performance Measure The performance of the algorithm on the datasets is based solely on the makespan.

II. Experiment Results

In our plot the horizontal axis corresponds to the number of robots in the swarm. The vertical axis corresponds with the makespan of the simulation. Small dots are the results of individual runs of the simulation and the large dots are the average of all runs for the given swarm size.

V. Future Work

It would be interesting to explore other heuristics for the OTDFTP as well as test other dataset types such as grid swarm arrangement using the Nearest Neighbor algorithm.

References

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[2] Mikael Hammar, Bengt J. Nilsson, and Mia Persson. The Online Freeze-Tag Problem. In *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, volume 3887 LNCS, pages 569–579. 2006.

[3] Marcelo O. Sztainberg, Esther M. Arkin, Michael A. Bender, and Joseph S. B. Mitchell. Analysis of Heuristics for the Freeze-Tag Problem. In *Algorithm Theory — {SWAT} 2002*, pages 270–279. 2002.