Kineret Sea Level Time Series

Statistical Analysis

Project Report

Part 1

Introduction

In this project, we analyzed the time series data of the Kineret sea level. As we all know, the subject of the Kineret level used to be a common discussion point, due to the water shortage in Israel. However, as a result of water desalination efforts, it is less commonly discussed nowadays. Despite that, in our eyes, it is a subject that is still relevant to each and every citizen. This is because this topic affects several, diverse domains, such as environmental, ecological, social, health and more. As a result, it would be fascinating to perform a broad spectrum of analyses on it. In particular, we explored the data, and fit a wide variety of models to our time series to find which one is most suitable for it. Lastly, we experimented with the incorporation of various exogenous variables and presented their association with our time series.

For the GitHub repository of the entire project (containing all data and code), please follow the link.

Data Selection and Preprocessing

The raw data was taken from the Kineret Authority's website. This website contains daily records of the Kineret sea level from 1/1/1969 until present day. Since the Kineret Authority is a government-controlled entity, it is reasonable to assume that the data is highly reliable.

In order for our data to be as diverse and rich as possible, we would like to include all of the information available. However, since the records are daily, this would result in an overly large dataset (roughly 18k records). To deal with this issue, we chose to focus on months as our time periods (instead of days). In addition to solving the said issue, this change also allows us to analyze the data in a more condense form. For those reasons, we chose to extract daily records of a time interval which only consists of full months. Therefore, we extracted all daily records between 1/1/1969 (first available date) and 31/3/2024 (March is the last full month available by the time of writing this report).

After receiving the daily records, we aggregated them to receive the desired monthly values. We did so by averaging across all corresponding days, for each month. From the aggregation mentioned, we received our final dataset, which contains **663 records** (indeed over 150, as requested).

Note that there is some missingness in the raw data (some daily records are missing). Thus, the number of days in each month might vary, which could add some noise to our transformed time series. However, upon checking, over 90% of the months in the dataset contain at least 20 daily records. Hence, the mentioned issue will not have too great of an impact.

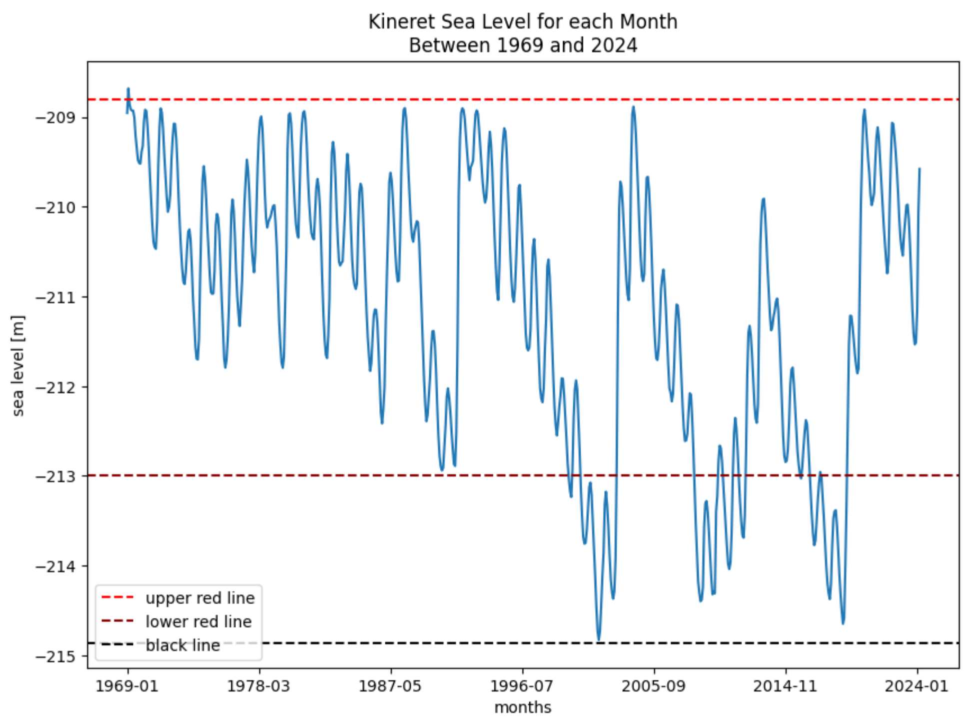
The raw and final datasets are provided in the following link.

Data Exploration and Visualization

After obtaining the final dataset, we wanted to explore it.

First, let us display the Kineret sea level time series below. We also plot some meaningful, official sea levels:

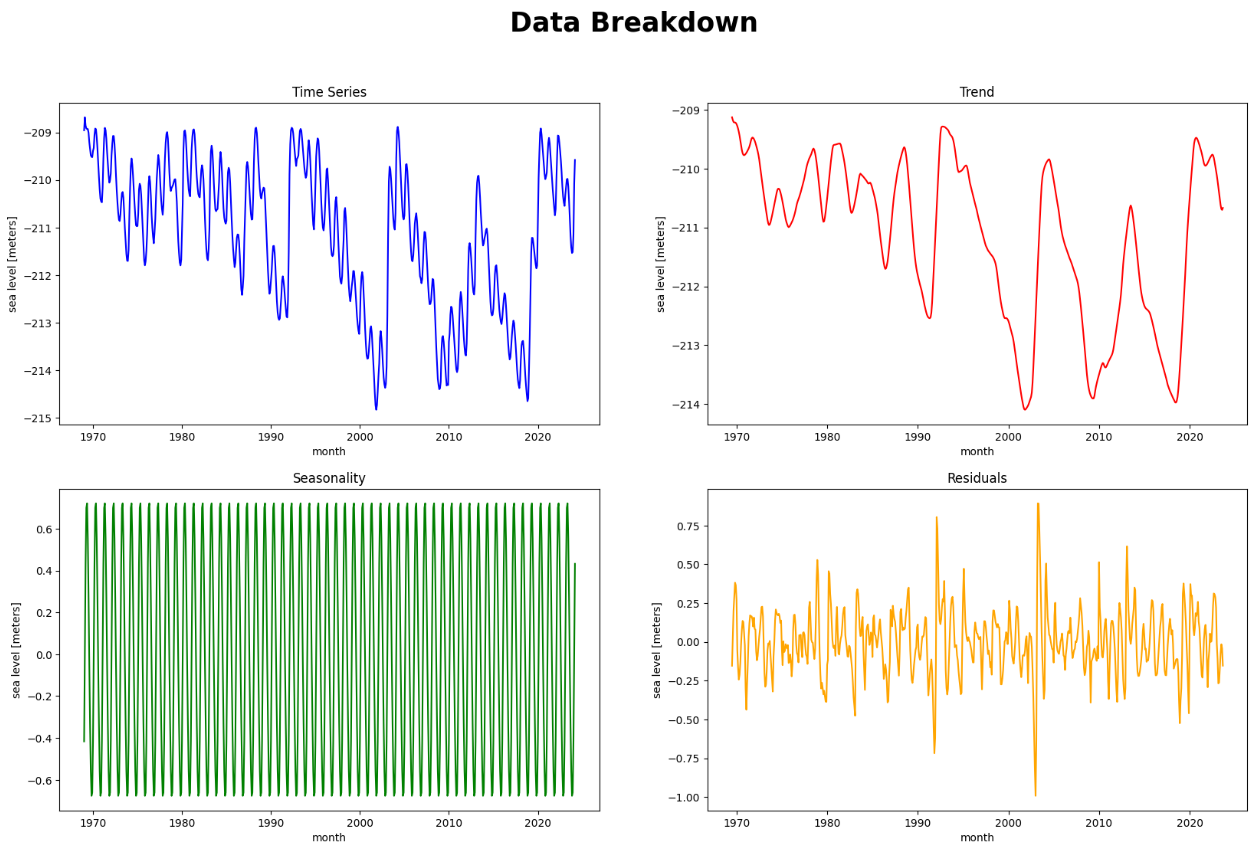
- The "upper red line" (-208.8m): the maximum sea level that the Kineret should not pass in order to prevent flood damage to nearby sites in case of a tide.
- The "lower red line" (-213m): the minimum sea level that the Kineret should not go under, in order to prevent damage to the ecological system, as well as to water quality. Unfortunately, the line was changed during drought periods (in order to legally extract water from the Kineret), which might have caused some ecological damage. We chose to display its most updated level.
- The "black line" (-214.87m): the sea level which water extraction is prohibited below it, under any circumstances. This line was determined in December 2001, when the Kineret level reached the lowest sea level ever recorded.



From the graph above, we can conclude the following:

- The maximum sea level (since recording began) is -208.682m, and was received on 1969-02.
- The minimum sea level (since recording began) is -214.829m, and was received on 2001-11.
- The duration of the Kineret sea level being under the "lower red line" is about 14.78% of the total time duration recorded.
- Fortunately, the Kineret sea level met the "black line" only once (2001-11).

Now, we broke the time series down into its key features: trend, seasonality and noise. We did that using the seasonal decomposition function from the “statsmodels” Python library. Below is the plot received:



From the plots above, we can conclude the following:

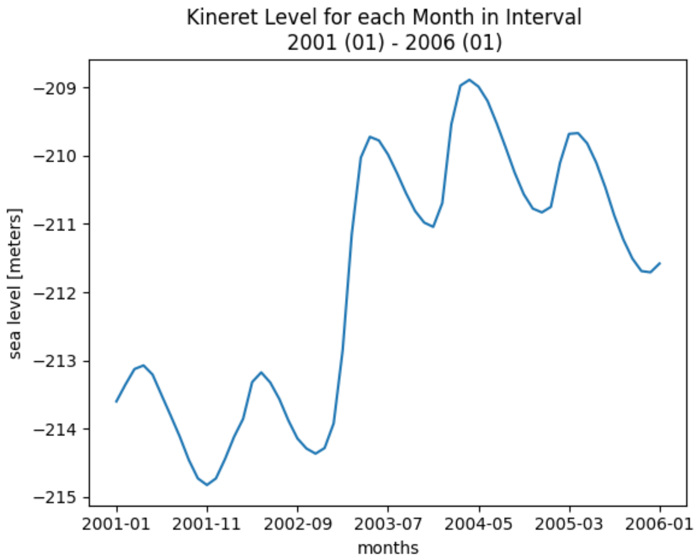
- The trend seems not to be constant. That is, the Kineret sea level seems to be regularly changing, in both directions.
- The seasonality plot appears to be highly periodic. Thus, it greatly indicates that our time series data has a significant seasonality component. This insight does not surprise us, since the Kineret sea level is largely affected by the weather (specifically, precipitations), which is seasonal.
- The residuals seem to be symmetrically distributed around the line $y = 0$. This highly indicates that the residuals’ mean is 0.

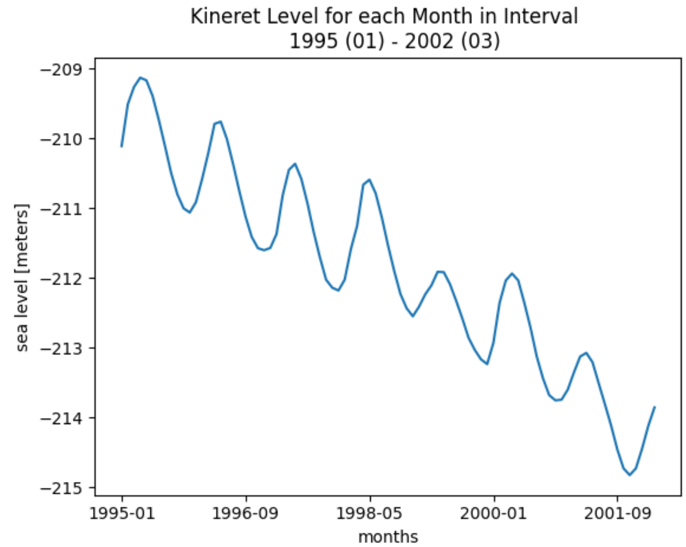
In the previous plots, we displayed the entire time series data at once. However, since we have a large number of data points, the graph is condensed. Therefore, small-scale patterns in small intervals are hard to notice.

Thus, to be able to fully see such patterns in any given interval, we created a simple user interface. This interface takes an interval as input, and displays the time series data in the corresponding interval. Instead of attaching plots of numerous time intervals, we invite you to try out this user interface to fully immerse yourself in our time series. It allows you to examine the time intervals which you find the most interesting.

To access said user interface, please follow the link.

Let us display below two patterns we found interesting in specific time intervals, using our user interface:

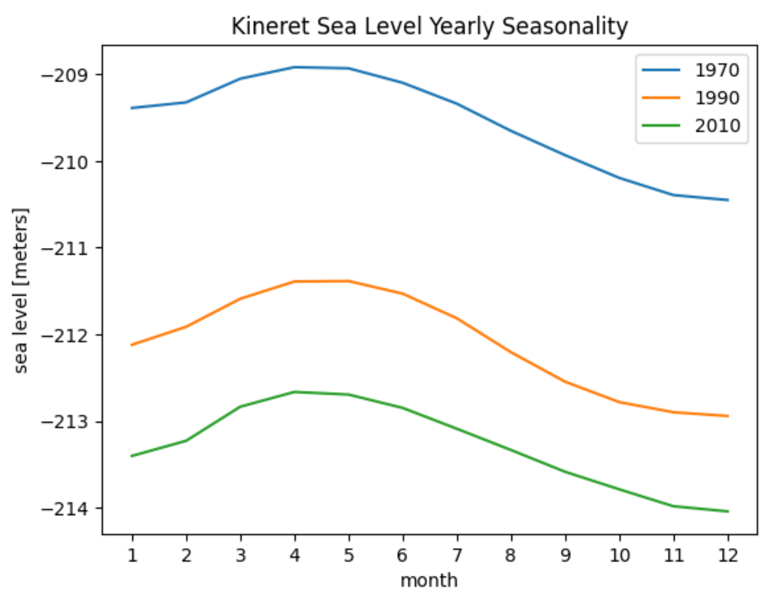




In the left graph, we can see a significant increase in the Kineret sea level, in a very short time period – roughly 5 meters in about 6 months (!).

In the right graph, we can see a steady decrease in the Kineret sea level, roughly 6 meters in about 6 years – an average of 1 meter per year (!).

Now, let us display the seasonality of our time series data, in a more intuitive way. In order to visually capture it, we displayed the Kineret sea level throughout the year, for three completely different years (20 years apart). We received the plot below:



As we can see, although the years are greatly far apart, the sea level pattern throughout each one appears to be nearly identical. In addition, there appears to be significant difference of the Kineret level between 1970 and 2010.

Part 2

After exploring the time series, we wanted to experiment with several models, in order to find which one best fits our data. We chose a variety of models, as follows:

- SARIMA
- Exponential Smoothing
- Prophet

For each model, we performed tests on a wide range of values for their parameters, in order to find the optimal ones. We evaluated the different parameter values using the following measures:

- BIC score
- MSE – we split the data into train and test sets, where the train set consists of the **first** 85% of the records, and the test set consists of the **last** 15% of the records[1]. Then, we trained our models on the train set, created a forecast for the test set, and calculated the MSE between the forecasted values and the true values.

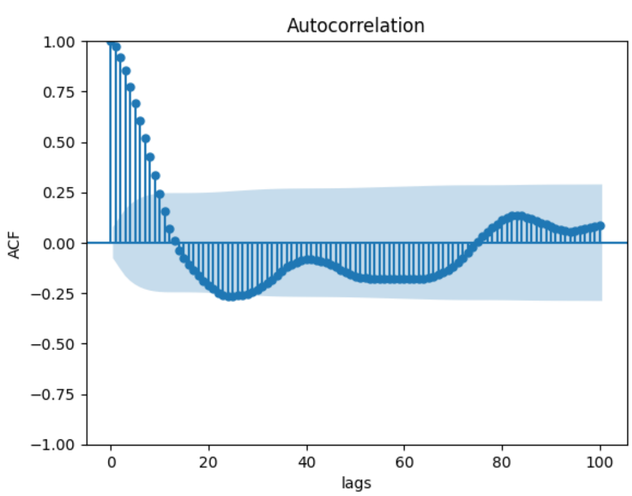
These measures will guide us to choose the best parameters for each of the three models.

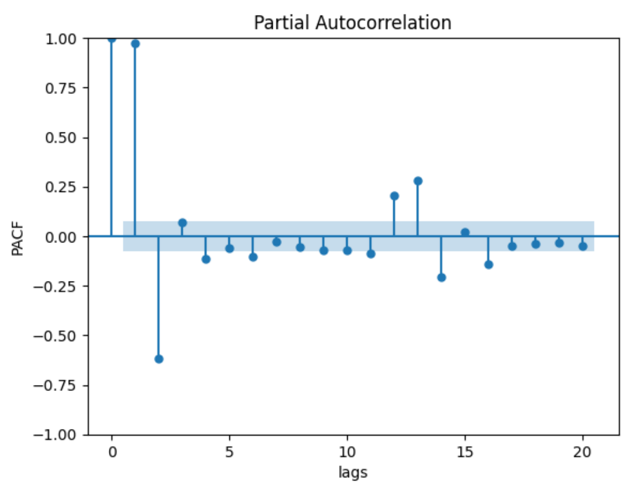
SARIMA Model

First, we wanted to receive several SARIMA models that are potential "candidates" for our time series.

As we saw Part 1, the Kineret sea level is largely affected by the seasons. Hence, it is not stationary. In order to transform it into one, we performed differentiation of 12 time periods (months). Formally, we created a new time series $Y_t = X_t - X_{t-12}$. The transformed time series visually appears to be stationary (for the transformed time series plot, please refer to the Model Comparison Appendix, SARIMA Model section). In order to further verify this assumption, we performed the ADF test, which supported stationarity with high statistical significance ($PV < 10^{-4}$).

Hence, we could display the ACFs and PACFs of the transformed time series (as they are well defined):





[1] We first considered window splitting the data into 4 sets, using a 40%/10%/40%/10% split, where the larger sets (of size 40%) are train sets and the smaller ones (of size 10%) are test sets. However, due to factors that drastically affect the data in recent years (e.g. water desalination), the results on the earlier test set are likely not to be indicative of the model's performance on recent data. Thus, we chose not to do so.

From the graphs above, we can observe the following:

- The ACFs decay to zero.
- The first and second PACFs are clearly not zeros. In addition, PACFs 12, 13 and 14 are also not zeros.
- It is reasonable to assume that the rest of the PACFs are zeros.

Hence, we can conclude that the models which are probably suitable for our transformed time series are $AR(1)$ and $AR(2)$.

Therefore, models that might fit the original time series are:

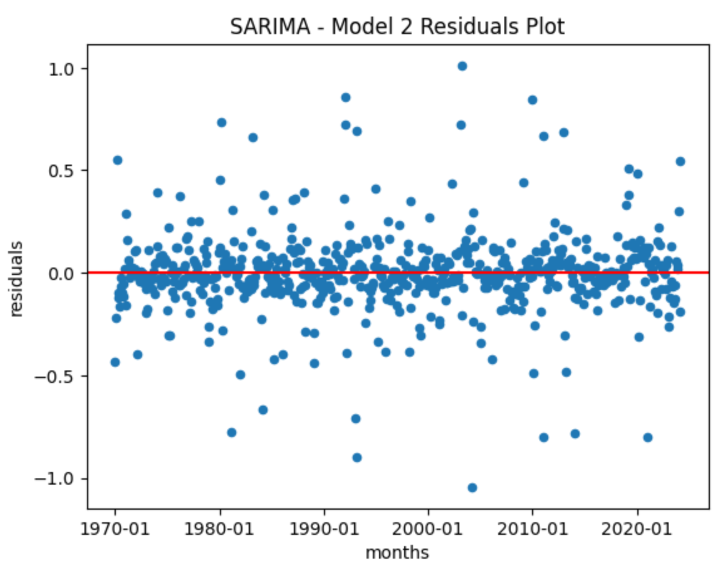
- Model 1 - $SARIMA(1,0,0)(0,1,0)_{12}$.
- Model 2 - $SARIMA(2,0,0)(0,1,0)_{12}$.

In addition, models that take into account higher non-zero PACFs are:

- Model 3 - $SARIMA(1,0,0)(1,1,0)_{12}$.
- Model 4 - $SARIMA(2,0,0)(1,1,0)_{12}$.

After finding the values of the measures for all four mentioned models, we found that the one which yields the best results is model 2, with $BIC = -296.709$ and $MSE = 3.986$. For the full comparison between the four models, please refer to the [Model Comparison Appendix, SARIMA Model section](#).

In addition, in order to visually assess the SARIMA model's performance, we created a residual plot:



As we can see, the residual plot seems to be valid:

- The points are symmetrical around the line $y = 0$ (which indicates that the expected error is 0).
- It appears that the points are uniformly distributed around that line (which indicates that the variance is constant).

Exponential Smoothing Model

Simple Exponential Smoothing Model

First, for this model category, we initially wanted to fit a Simple Exponential Smoothing model for our time series. Let us be reminded that this model requires a single parameter α (smoothing level), which exists in the interval of [0, 1]. For those reasons, we decided to use the grid search method in order to find an optimal α value.

After finding the optimal α value, we wanted to visually assess the model's performance. Unfortunately, we saw that the model's forecasts on the test set form a horizontal line. This insight suggested that the model we used was overly simple for our time series. Therefore, we decided to utilize a more advanced Exponential Smoothing model – the Holt-Winters model.

For the full analysis, please refer to the Model Comparison Appendix, Exponential Smoothing section.

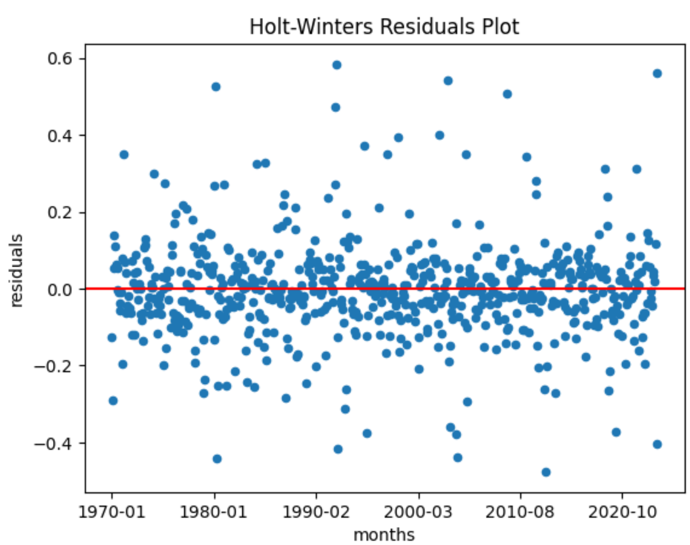
Holt-Winters Model

Let us be reminded that this model has three main parameters: α (smoothing level), β (smoothing trend), γ (smoothing seasonal). Ideally, we would like to perform the grid search method in order to fine-tune said parameters. However, since there are three different ones, it is computationally heavy to accurately find the optimum. As a result, and differently from the previous section, we had to use the model's built-in function in order to find the parameters' values to proceed with.

In addition, when defining the model, we set the trend and seasonality components to be additive, since it better aligns with the nature of the time series. Lastly, we set the seasonality periods to be 12 (as seen in the SARIMA Model section).

After initializing the model as described above, the said parameters received the following values: $\alpha = 1.0,\ \beta = 0.708,\ \gamma = 0.0$, which yielded the results $BIC = -2411.662$ and $MSE = 3.16$.

In order to visually assess the model's performance, we created a residual plot:



As we can see, the plot is valid for the same reasons mentioned in the SARIMA Model section.

Prophet Model

For this model category, we fit the Prophet model for our time series.

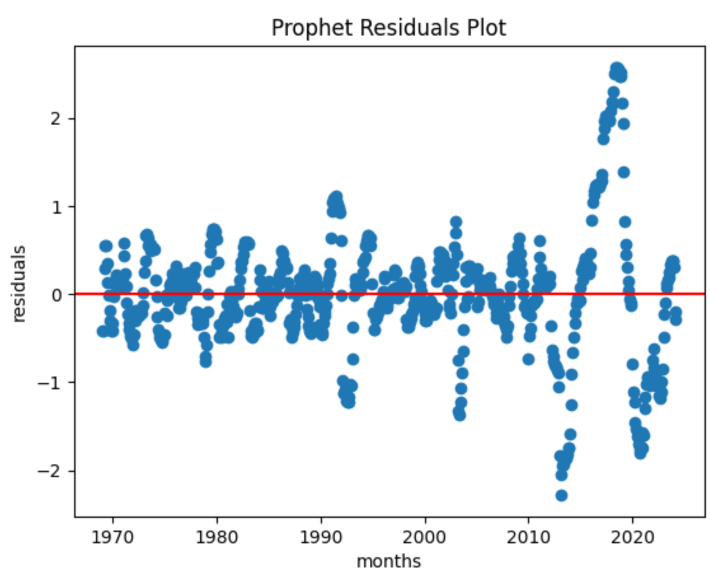
As we know, the Prophet model is extremely customizable, with a high number of parameters to choose from. However, in order to have a reasonable number of tests, we focused on fine-tuning only the "changepoint prior scale" and "seasonality prior scale" parameters. These parameters control the flexibility of the trend and seasonality, respectively. In other words, they determine how drastically the trend and seasonality could change at any given point. A higher value of these parameters allows for more flexibility, meaning that the model can adapt more quickly to changes in the data. Since our time series changes quite drastically, we hope that optimizing these parameters will help us capture it.

In addition, when defining the model, we set the seasonality mode to be additive and growth to be linear, due to the nature of our data. Moreover, we chose not to incorporate holidays into our model, since they are short-term, and we assume that they do not have any noticeable effect on the Kineret sea level during their time period.

In order to fine-tune the prior scale parameters, we performed the grid search method – iterating over **900** unique combinations of the parameters.

For the "changepoint prior scale" and the "seasonality prior scale" parameters, we received the optimal values of 0.31 and 44.1, respectively. This model yielded the result $MSE = 6.595$ (the BIC score is not available for this model). For the full details regarding the grid search procedure, please see the Model Comparison Appendix, Prophet Model section.

In order to visually assess the model's performance, we created a residual plot:



As we can see, the residual plot is quite valid:

The points are symmetrical around the line $y = 0$, which indicates that the expected error is 0. However, although the points up until 2010 appear to be uniformly distributed (to some degree) around the line, from that point on, it no longer seems like it. This indicates that the variance might not be constant.

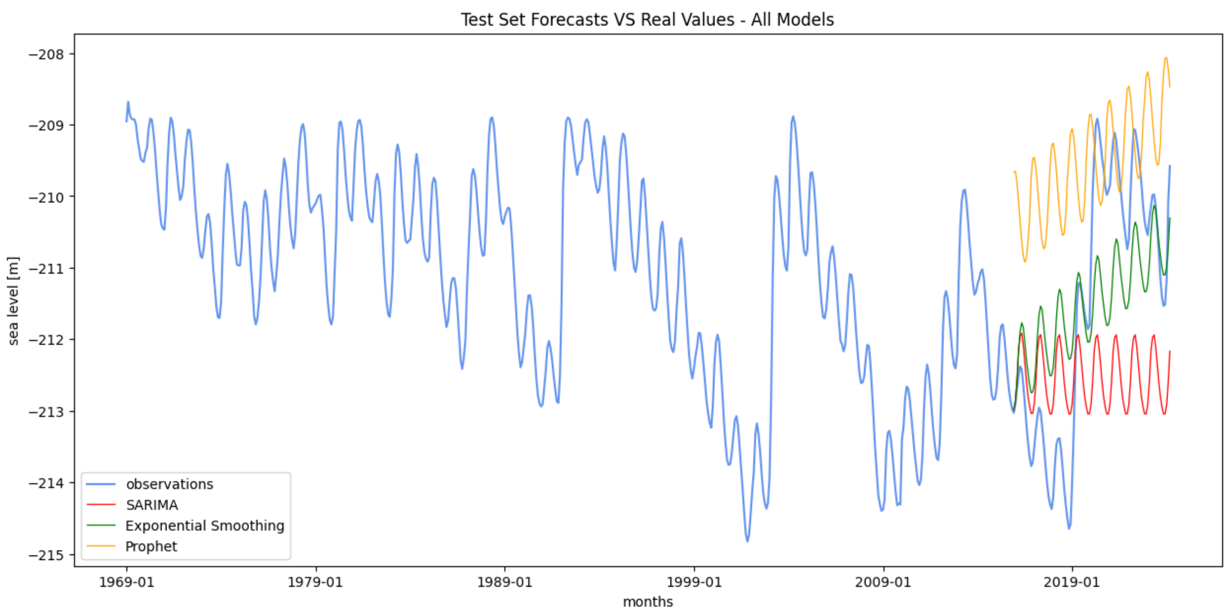
Final Model Selection

In the sections above, we evaluated different models on our time series data. Let us be reminded of the models we proceeded with, and their results:

Model	BIC	MSE
SARIMA(2, 0, 0)(0, 1, 0)$_{12}$	-296.709	3.986
Exponential Smoothing – Holt-Winters	-2411.662	3.16
Prophet	---	6.595

As we can see, the Exponential Smoothing model has both the best MSE and BIC score. In addition, notice that the MSE of the Prophet model is quite larger than the rest.

Now, we will display all of the models' forecasts (on the test set) in the same plot (along with the true values):



Let us discuss and compare the performances of the different models:

First, we can see that all three models capture the yearly seasonality of the Kineret sea level very well.

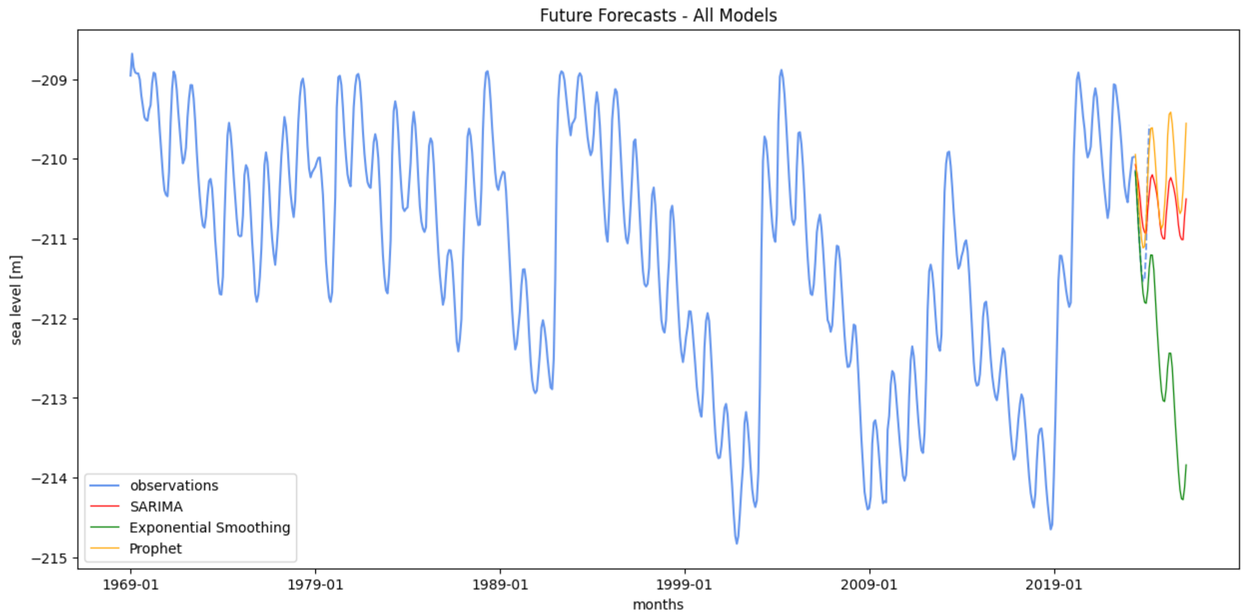
However, the Prophet model (yellow) does not seem to capture the true values at all. We can see that the first forecast highly overestimates the true value, a phenomenon which continues in most of its predictions.

Regarding the SARIMA model (red), its first predictions are quite accurate. However, it remains monotone and does not reflect any trend in the time series.

Lastly, the Exponential Smoothing model (green) seems to perform quite decently. That is, the first and last few forecast points seem to capture the true values very well. This result is quite impressive, especially on the last forecast points (which are over 8 years ahead). In addition, although the model did not capture the extreme highs and lows in the Kineret sea level during this period, it indeed captured the overall trend.

As a result, in our eyes, the Exponential Smoothing model appears visually superior in the plot above.

Now, let us display future forecasts 24 months ahead[2]:



As we can see, all models provided decent forecasts. It appears that both the SARIMA and the Prophet models made relatively conservative forecasts, whereas the Exponential Smoothing model made bolder ones. However, we can see that those forecasts seemingly fit the time series in a natural manner.

From the above discussion, we conclude that in our eyes, the model which overall suits our time series best, is the **Exponential Smoothing (Holt-Winters) model**.

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[2] Note that we start forecasting 10 months before the last recorded time period. That is because there is a large spike towards the last records in our data, which misleads our models and negatively affects their predictions.

Part 3

In this part, we introduce two explanatory variables for our time series: natural and man-made. We chose this approach to better understand the factors that are associated with our time series.

Natural Explanatory Variable – Weather Conditions

For this variable, we chose to model it as a combination of the following attributes:

- Maximum Temperature (Monthly Average)
- Minimum Temperature (Monthly Average)
- Accumulated Rainfall (Monthly)

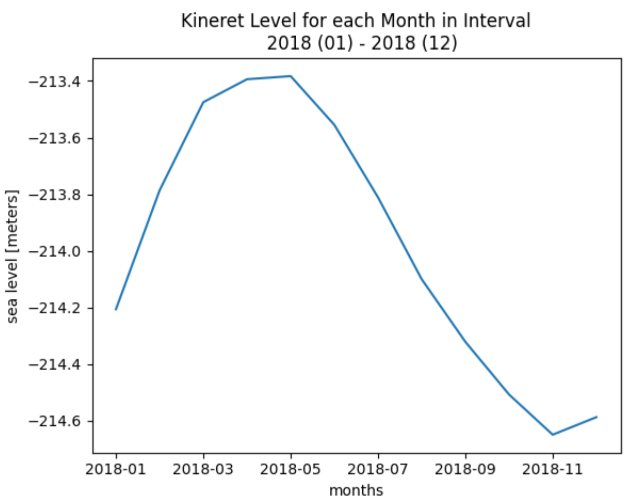
The data mentioned above was collected from the website of the Israel Meteorological Service. For further information and visualization, refer to the Data Extraction Appendix, Natural Variable section. In order to check for the correlation between this exogenous variable and our original time series, as well as the direction and magnitude of effect of each of the variable's attributes, we performed linear regression.

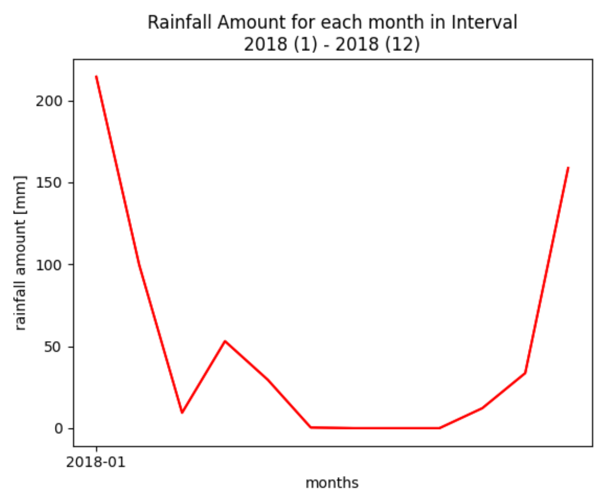
We received the following coefficients:

- $\beta_{\text{max-temp}} = 0.3983$ (positive effect)
- $\beta_{\text{min-temp}} = -0.4502$ (negative effect)
- $\beta_{\text{rainfall}} = 0.0067$ (positive effect)

In addition the model's $PV < 10^{-6}$, and $\mathrm{R}^2 = 0.149$. This result indicates that there is some correlation between the two time series, with high statistical significance.

Let us present below a visual example for the relationship found, by displaying the rainfall and Kineret level time series, in a one-year time interval, side by side:





As we can see, the Kineret level continues to rise, up until the rain stops (around June). In addition, in months when there is no rain, the Kineret level sharply declines. Lastly, although it starts raining again around September, the Kineret level starts to rise only a couple months later.

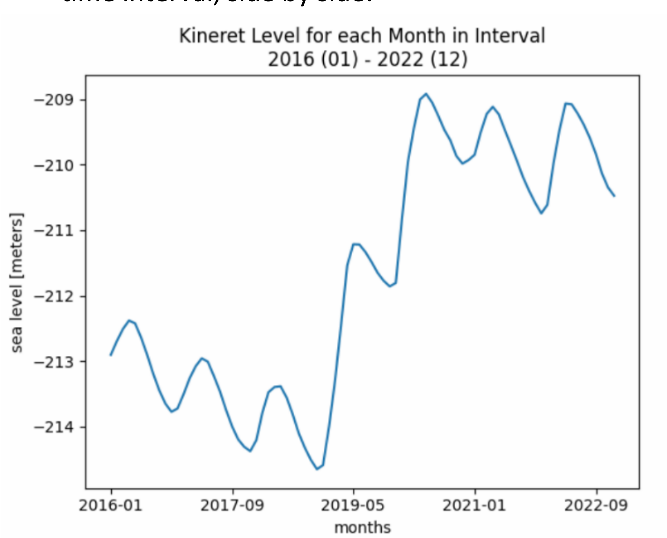
Man-made Explanatory Variable – Water Desalination

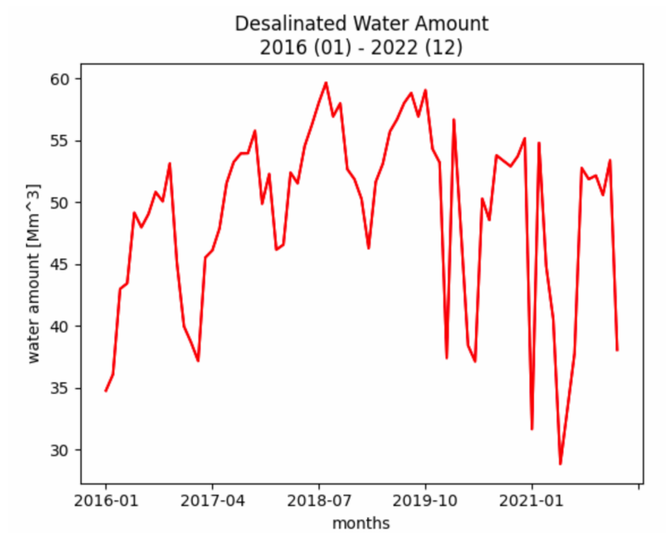
The data was collected from the "Data Gov" website. For further information and visualization, refer to the Data Extraction Appendix, Man-made Variable section. In order to check for correlation and direction of effect between this exogenous variable and our original time series, we performed linear regression.

We received the coefficient: $\beta_{desalination} = -0.0749$. Notice that the direction of effect is negative. This result surprised us at first, since we expected that when desalinating more water, less water would have to be extracted from the Kineret, resulting in higher sea levels. However, our data suggests otherwise. A reasonable explanation could be that water desalination capabilities could not fully fulfill (for now) the water demand in Israel. That is, the desalination production is probably planned according to the Kineret's water extraction constraints.

In addition, the model's $PV < 0.005$, and $R^2 = 0.094$. This result indicates that there is some correlation between the two time series, with high statistical significance. At first glance, it seems that the correlation is quite poor. However, let us remember that this variable contains a single attribute (compared to the three from the natural variable), and represents an action which is completely man-made. Hence, for it to receive above 60% of the natural variable's R^2 score, is quite impressive in our eyes.

Let us present below a visual example for the relationship found, by displaying the time series in a specific time interval, side by side:





As we can see from the graphs above, there was a steady decrease in the Kineret level from 2016 to 2019. In accordance to that phenomenon, we can see an upward trend in the production of desalinated water, during the same time period. In addition, after a major spike in the Kineret sea level, we can see a downward trend in the desalinated water production.

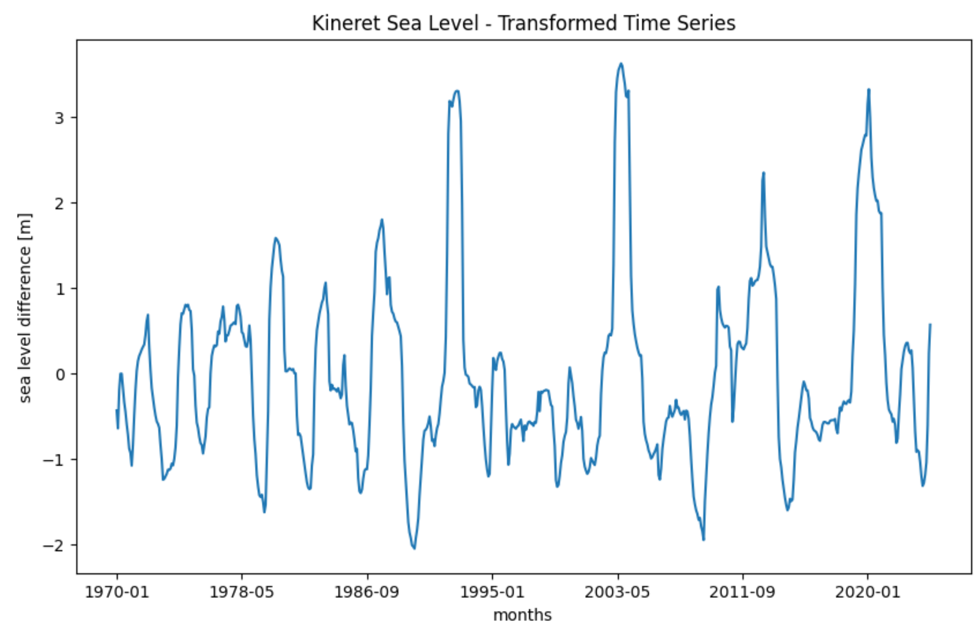
Conclusion

To summarize, in this project, we visually explored the Kineret sea level time series and discovered interesting phenomena. In addition, after performing a comprehensive model comparison procedure, we showed that the Holt-Winters model is the most suitable among the ones we tested. Lastly, we introduced two completely different exogenous variables and showcased their association with our time series.

Model Comparison Appendix

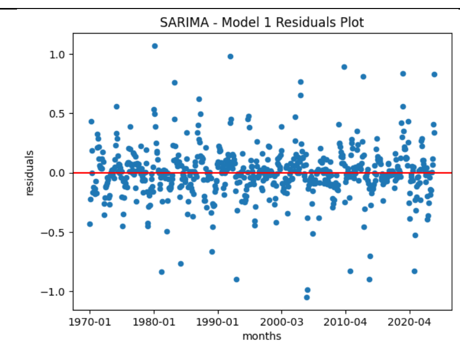
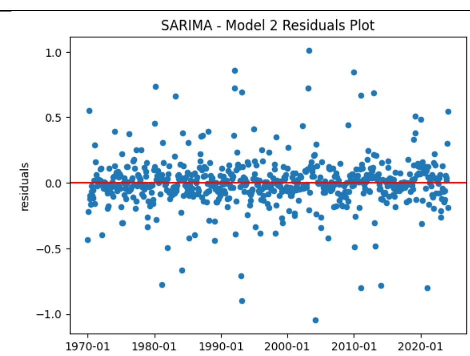
SARIMA Model

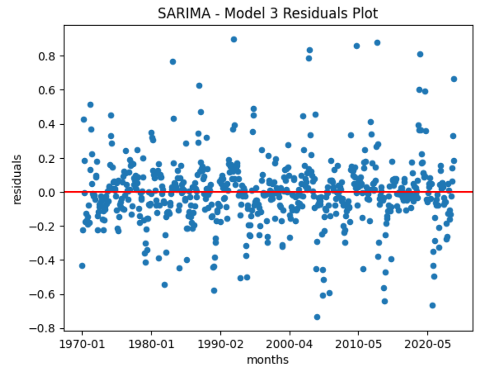
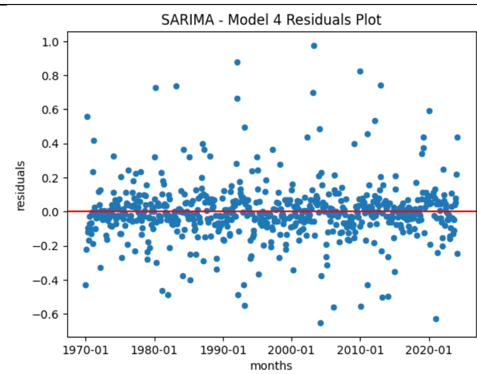
After performing the transformation descried in the report, we received the following time series:



As we can see, the seasonality is much less present than in the original time series.

Let us display the results for each model below:

Model	BIC score	MSE	
1	25.015	4.542	
2	-296.709	3.986	

3	-95.597	9.017	
4	-424.740	4.434	

Let us discuss the results:

It is clear to see that for all four models, the residuals are distributed as we would like to expect. However, for models 2 and 4, both the BIC score and the MSE are the best among them.

For model 2, the MSE is better than model 4's. For model 4, the BIC score is better than model 2's.

Since we prefer to choose our models based on practical measures, we decided to put more emphasis on the MSE measure. Thus, we chose model 2 as the SARIMA model to proceed with.

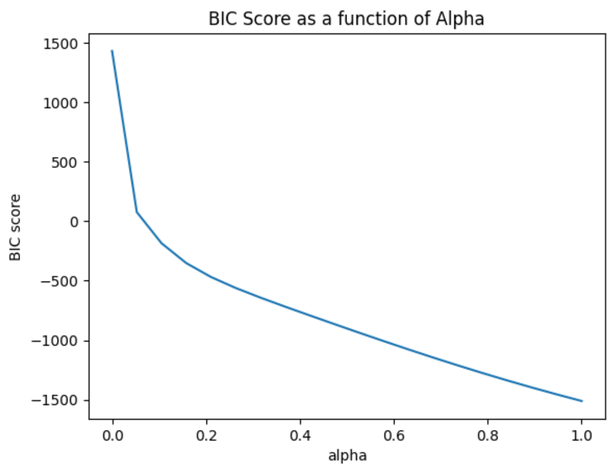
*Note that for models 1 and 3, the residual plots appear after we removed outliers from that. Those outliers consisted of less than 0.5% of the data. Thus, we chose to remove them in order to follow the “ink-paper ratio” principle.

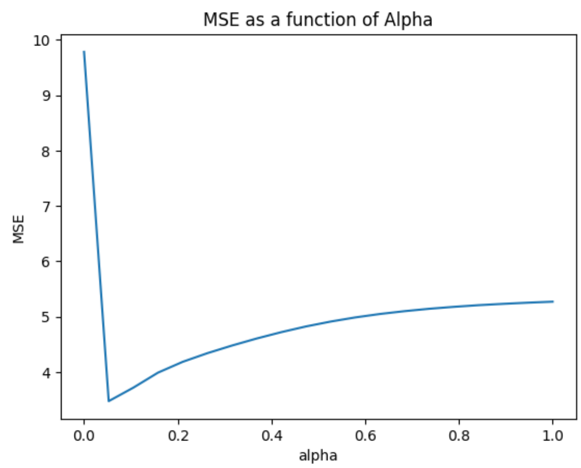
Exponential Smoothing Model

As mentioned, we wanted to fit a Simple Exponential Smoothing (SES) model for our time series. Thus, we performed the grid search method in order to find an optimal α value for the model.

First, we performed an initial search in order to find the potential interval of the optimal α value.

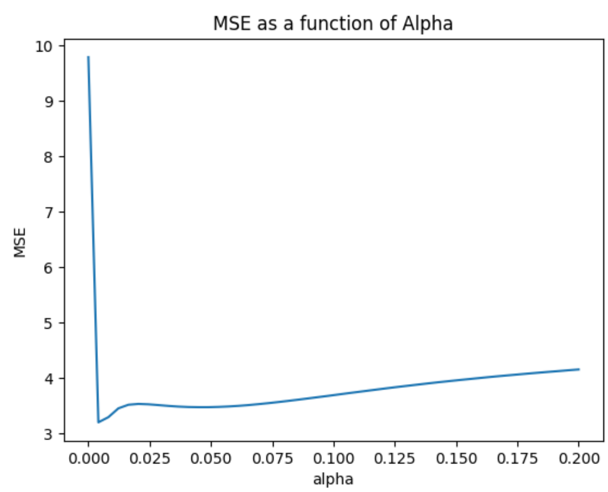
We examined a total of 20 points across the entire domain of α, and received the following plots for the MSE and BIC scores:





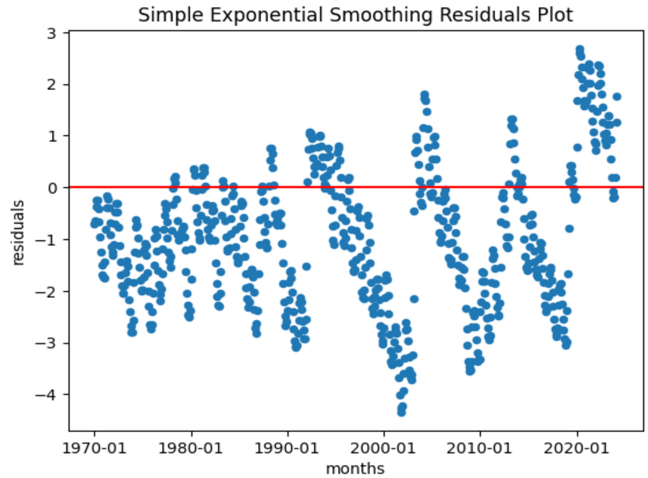
As we can see, the best BIC score is received at $\alpha = 1.0$. However, the best MSE is received for some α in the interval [0, 0.2]. Since we are interested in practical measures, we want to put more emphasis on the MSE measure.

Thus, we "zoomed in" in order to examine said interval:



The minimum MSE received is 3.199, and it is received for $\alpha = 0.004$. As a result, we chose to proceed with this value for the α parameter. The BIC score received for this α value is 693.652.

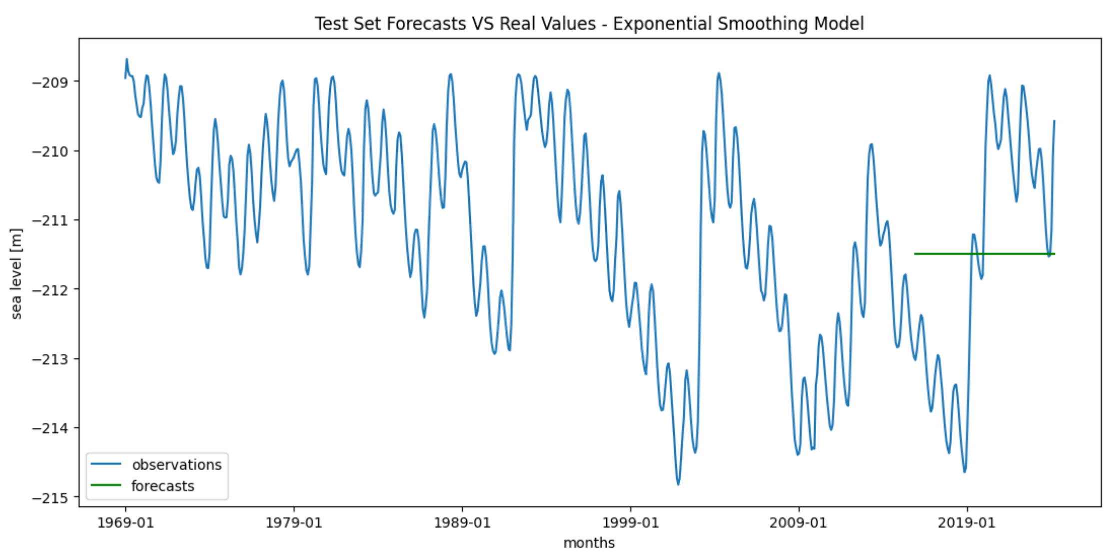
Then, we displayed the residual plot for our SES model (with said α):



As we can see, the residual plot is clearly not valid:

- The points are not symmetrical around the line $y = 0$, which indicates that the expected error is not 0. This insight demonstrates that there is a factor which affects the time series data, but our model does not capture.
- The points do not appear to be uniformly distributed around the line $y = 0$. This indicates that the variance is not constant.

To further explore the performance of this model, we displayed the model's forecasts on the test set (alongside the true values):



As we can see, the forecasted values form a simple, horizontal line. This indicates that our model is too simple for this time series. Hence, as mentioned in the report, we decided to utilize a more advanced Exponential Smoothing model – the Holt-Winters model.

*At first, the horizontal forecast line seems odd. As a result, we checked the plot of the model's forecasts for different α values. Yet, all α values resulted in horizontal forecast lines as well. This further indicates that the SES model is too simple for our time series.

Prophet Model

As mentioned, we wanted to perform grid search in order to optimize for the Prophet parameters "changepoint prior scale" and "seasonality prior scale". The parameter values we iterated through during the procedure are as follows:

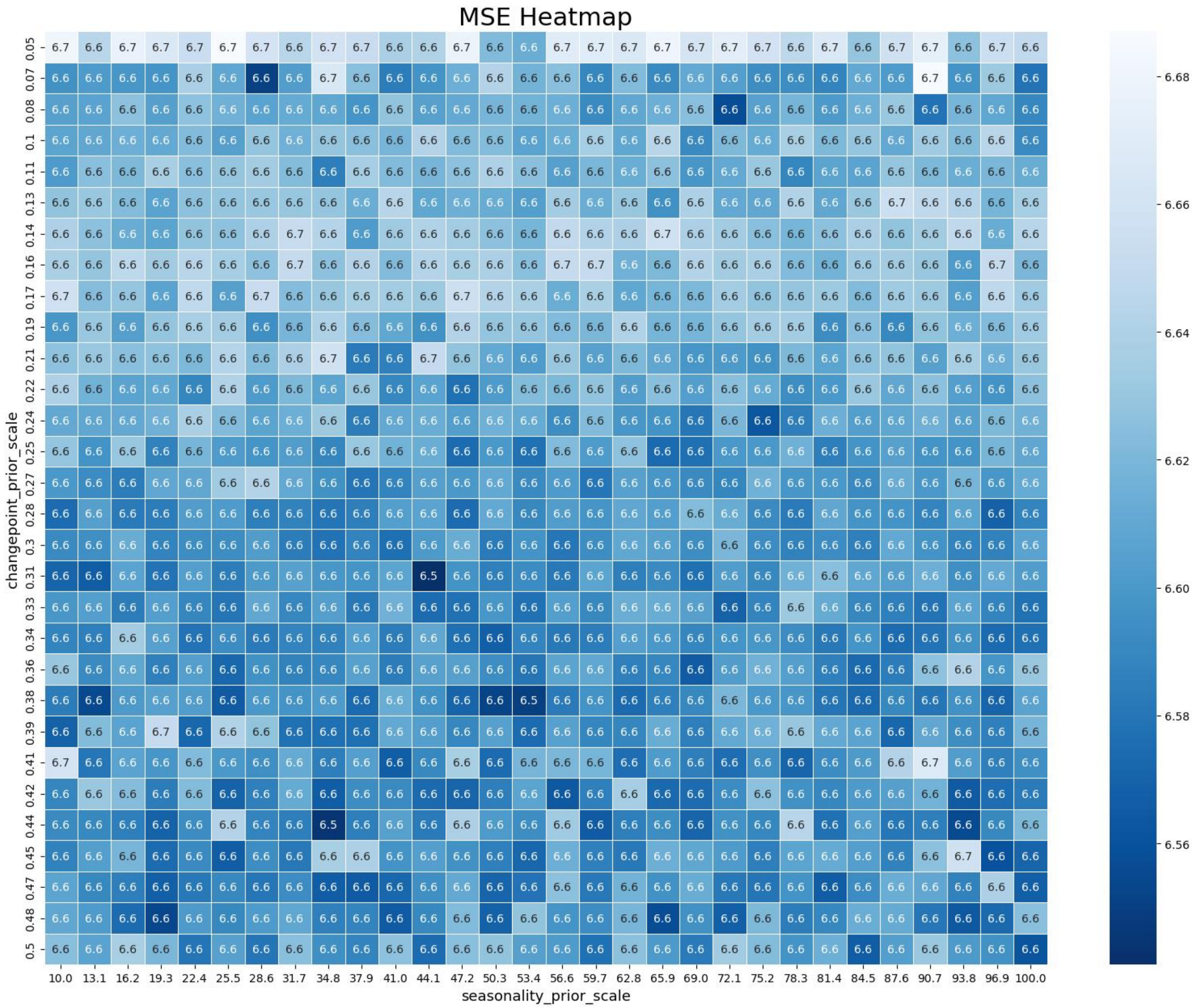
<u>Changepoint prior scale –</u> The domain of this parameter is the interval [0.001, 0.5]. Theoretically, the parameter's value can be set to be greater than 0.5. However, upon researching, it appears extremely not recommended to do so. In addition, its default value is 0.05. Thus, as a result of the mentioned flexibility in our time series, we examined different values for this parameter between the default value and the recommended upper bound (the interval [0.05, 0.5]).

<u>Seasonality prior scale –</u> The default value of this parameter is 10. Upon researching, the values that are considered "large" are about 100. Thus, we examined different values for this parameter between the default value and the suggested large value (the interval [10, 100]).

For each parameter, we examined 30 values, evenly spread across their corresponding interval. This results in a total of **900** unique combinations of the parameters, as mentioned in the report.

Lastly, for each such parameter combination, we fit a Prophet model and calculated its MSE on the test set. We displayed the results in a heatmap plot. For the plot, please refer to the next page.

Grid Search results heatmap:



As we can see, the best values for parameters are **changepoint_prior_scale=0.31** and **seasonality_prior_scale=44.1**.

Data Extraction Appendix

Natural Variable

As mentioned, the data was collected from the Israeli Meteorological Service (IMS) website. Since the IMS is a government-controlled entity, it is reasonable to assume that the data is highly reliable.

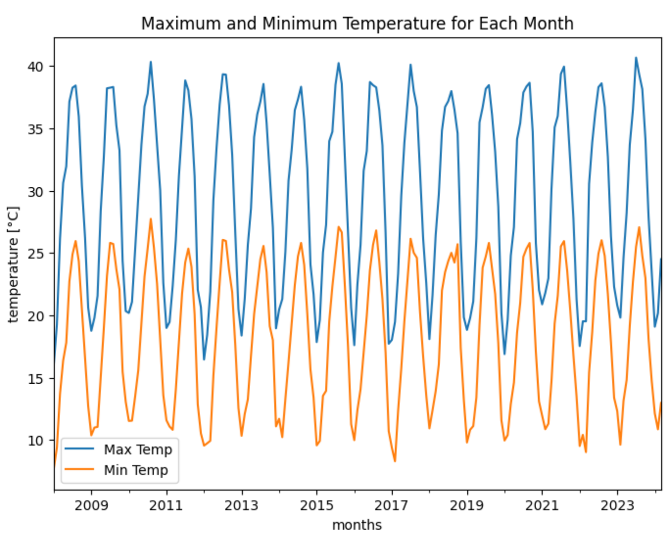
Notice that when extracting data from the website, one is required to choose a meteorological station, from which the data was collected. In order to receive weather conditions data as reliably as possible, we chose to use data from a meteorological station nearby the Kineret. The station chosen is "Kfar Nahum" (which is located on the northern Kineret shore). For the most accurate results, we extracted all records available for this station: January 2008 until March 2024.

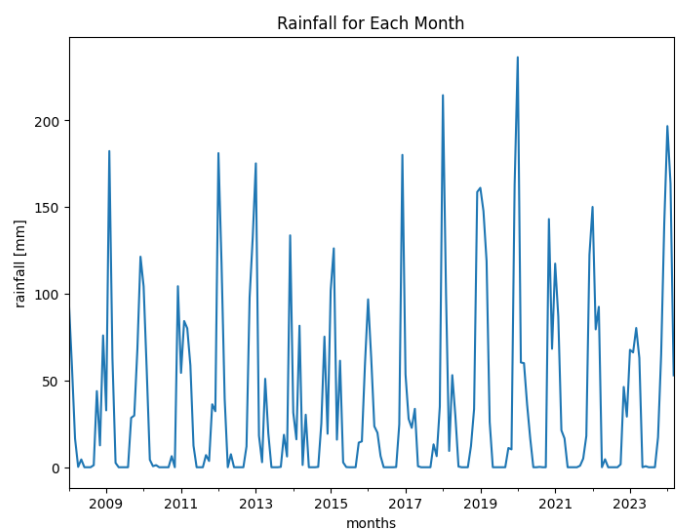
The website allows extracting data regarding accumulated rainfall throughout an entire month. However, it only contains data of daily maximum and minimum temperature records. Therefore, in order to receive monthly temperature data, we extracted said records and averaged across them for each month. From the mentioned procedure, we received **195 records** in total.

We will mention that several months contained missing values. In this case we averaged only across the existing records. In one case, all days throughout the month were missing (August 2018). We dealt with it by filling in the rainfall value of the previous year (August 2017).

For the datasets collected, follow the link.

Let us display the natural variable's attributes time series below:





Man-made Variable

As mentioned, the data was collected from the "Data Gov" website. Since that website is a government-controlled entity, it is reasonable to assume that the data is highly reliable.

The website contains data regarding the monthly production of all desalination facilities in Israel. It contains three separate datasets, which cover a time period between January 2016 and December 2022. For the most accurate results, we extracted all records available.

In order to receive the total desalinated water production in Israel, for each month, we summed all its values across all desalination facilities. In addition, the datasets contained no missing values.

For the datasets collected, follow the link.

Let us display the man-made variable's time series below: