

Elliptic Billiard: Surprising New Properties



D. Reznik – Upper West Soluções

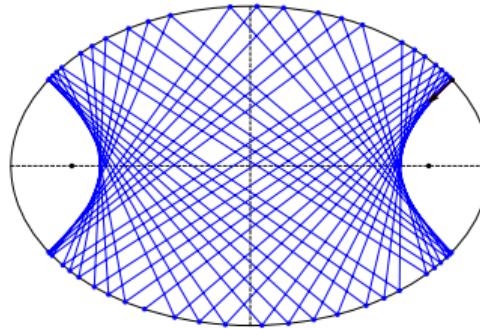
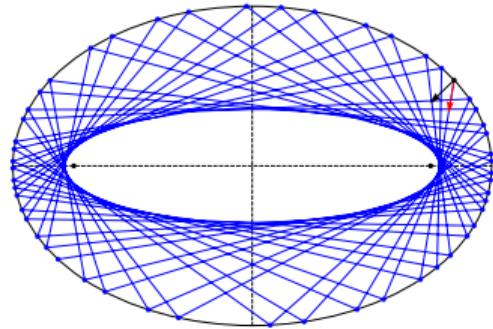
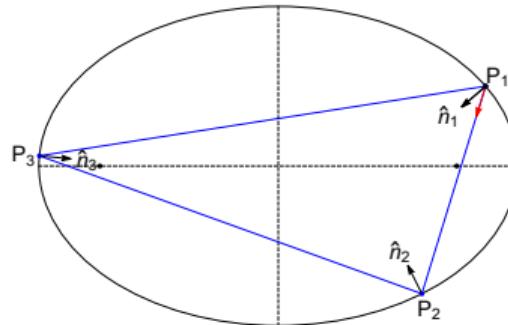
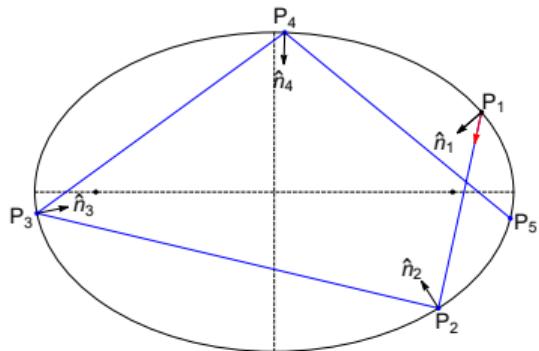
R. Garcia – Univ. Federal de Goiás

J. Koiller – Univ. Federal de Juiz de Fora

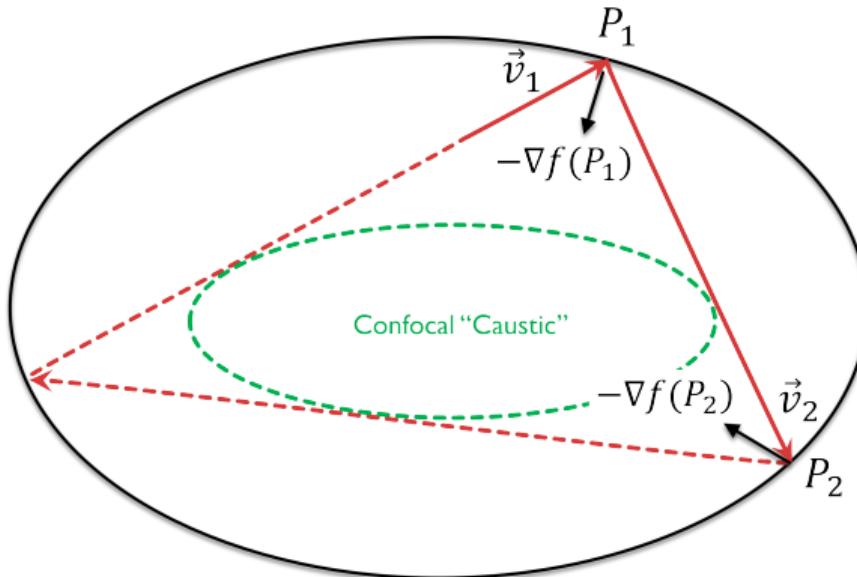
Section 1

Preliminaries

Billiard Trajectories [video]



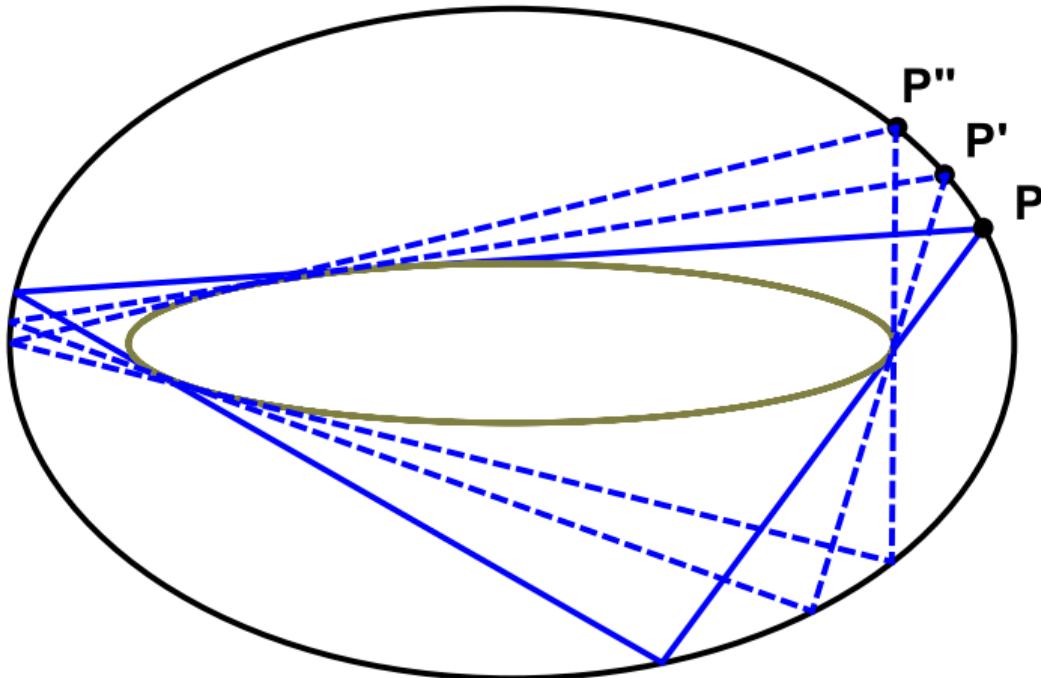
Integrability & Conservation



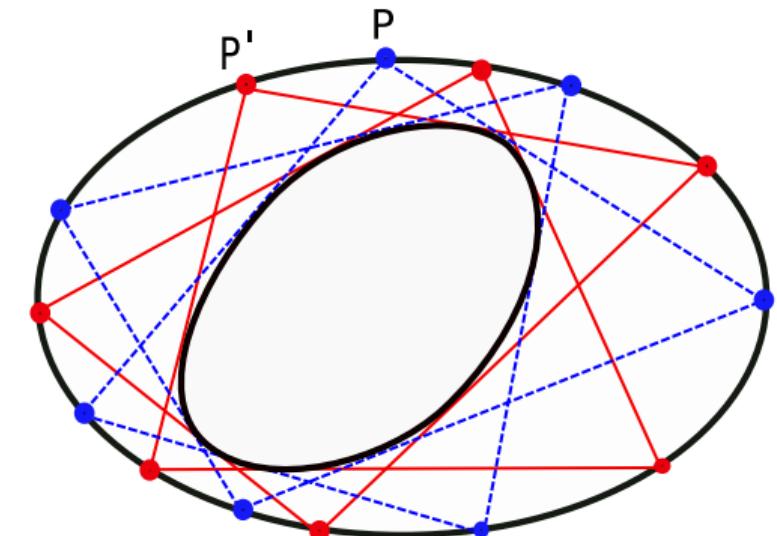
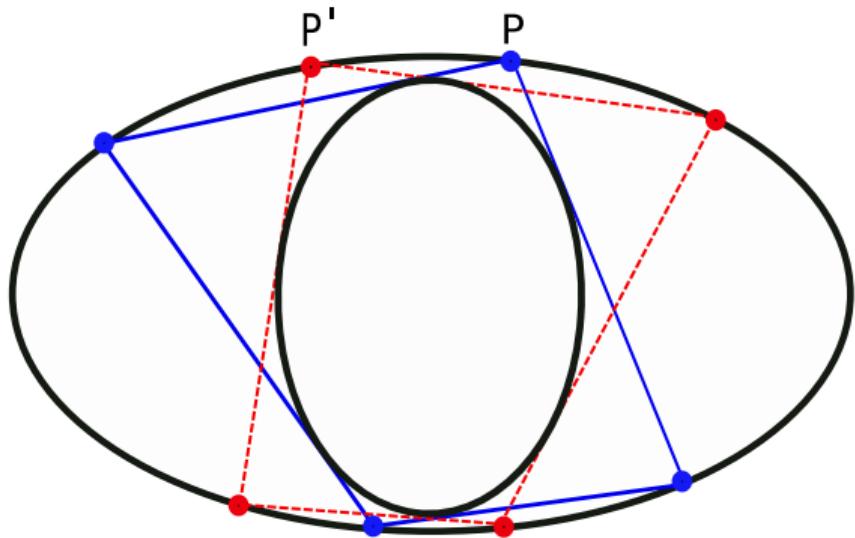
Energy $\Rightarrow L = \text{const.}$

Joachmishthal's $\Rightarrow \gamma = \frac{1}{2} \hat{v} \cdot \nabla f(P) = \text{const.}$

$N = 3$ Caustic [video]



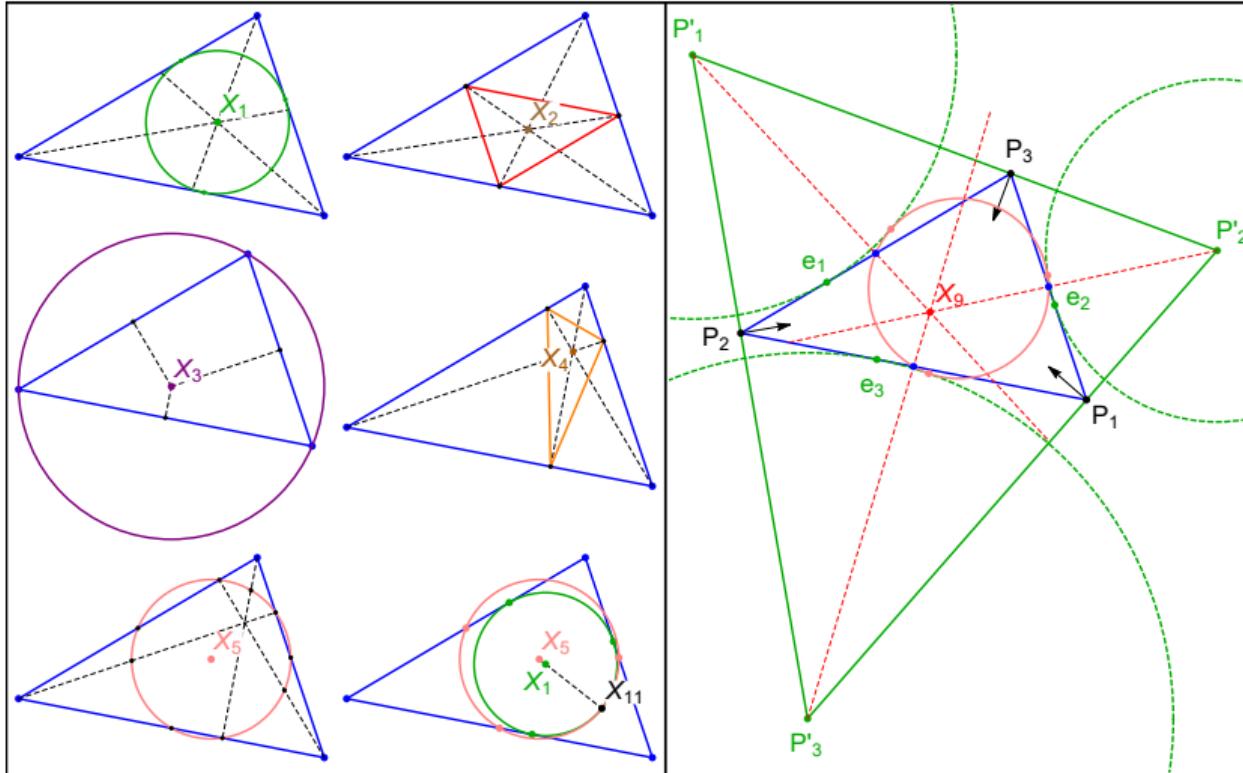
Poncelet's Porism



Section 2

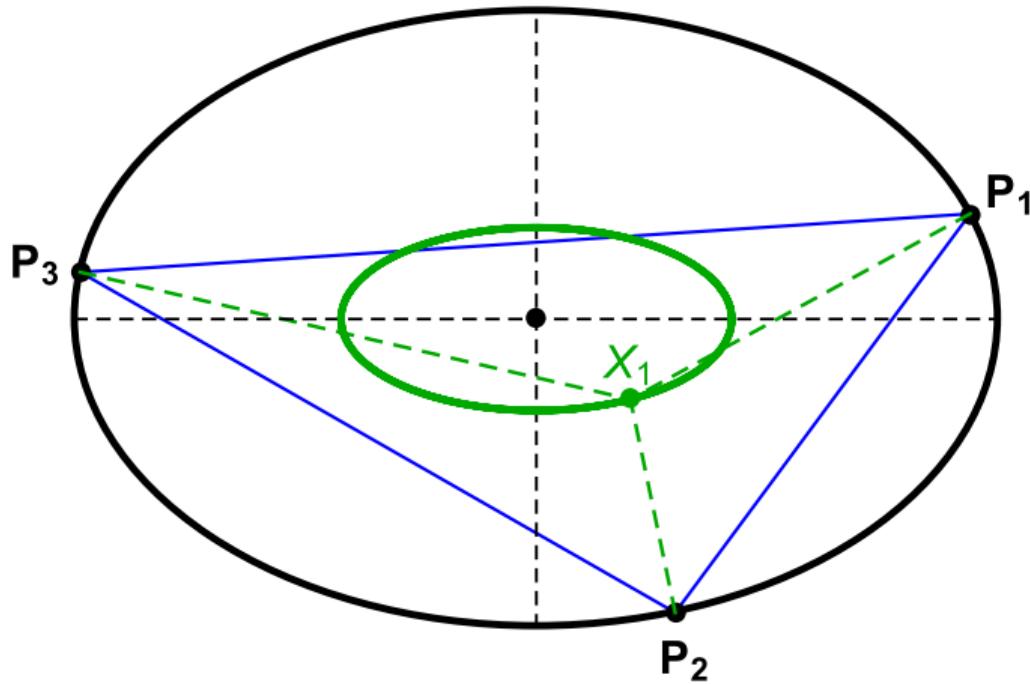
Locus Pocus

Triangle Geometry

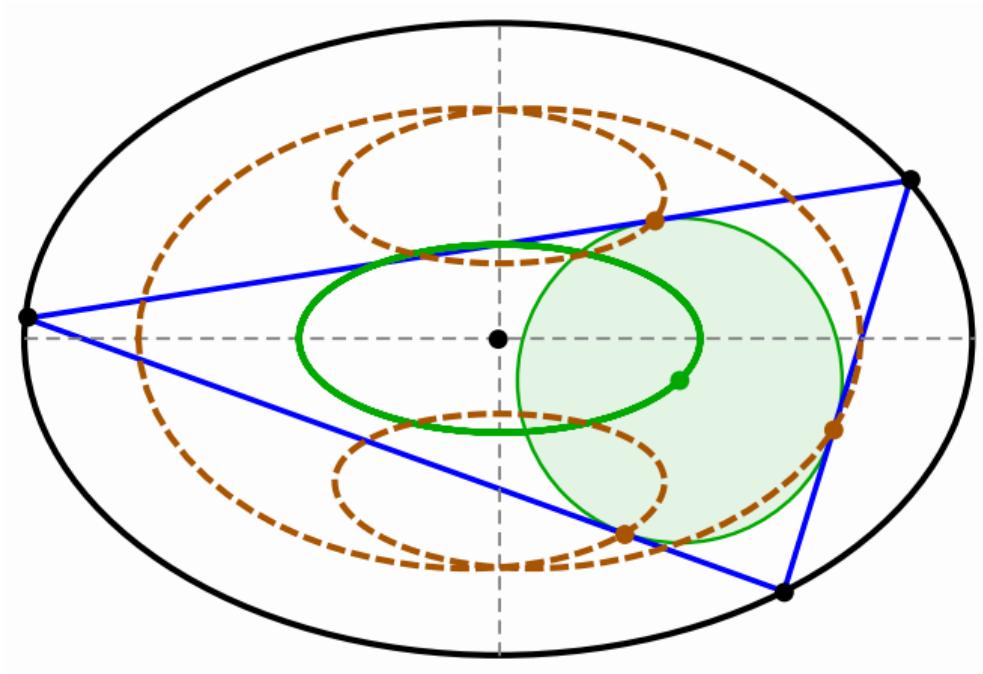


- | | |
|----------|-----------------|
| X_1 | Incenter |
| X_2 | Barycenter |
| X_3 | Circumcenter |
| X_4 | Orthocenter |
| X_5 | 9-Point center |
| X_9 | Mittenpunkt |
| X_{11} | Feuerbach Point |

Locus of Incenter is Elliptic [video]



Locus Intouch Points is Sextic [video]



Interactive Applet

>>> Billiard Aspect Ratio
 1.1 1.25 1.352 1.5 2.0 3.0

>>> Elliptic Loci
 X1* X2* X3* X4*
 X5* X7* X8
 X10* X11* X12
 X40* X88* X100*
 X142* X144* X173

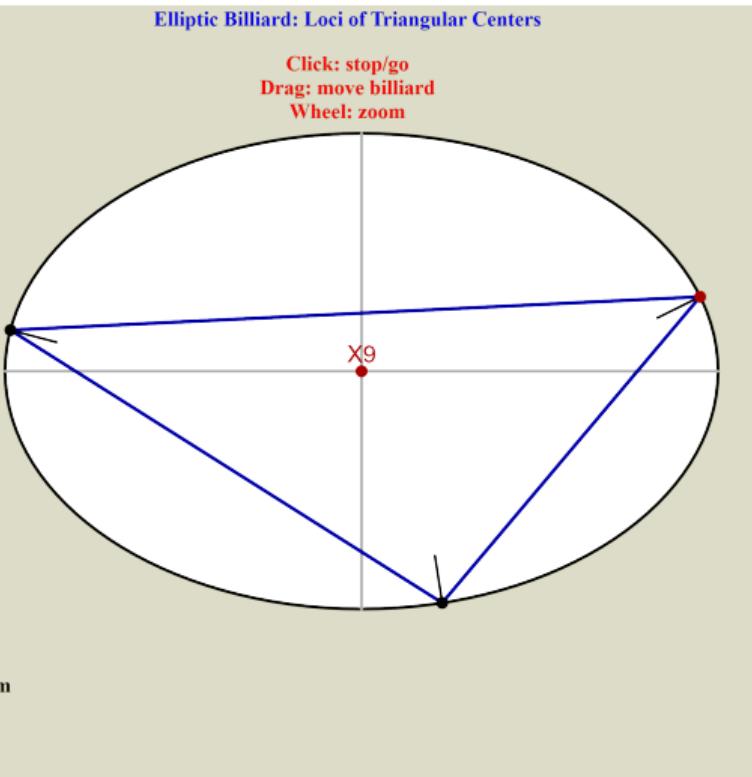
>>> Non-Elliptic Loci
 X6~ X15~ X99~

>>> Notable Circles
 Incircle Circum 9-Point
 Excircles Antinecircle CosCirc

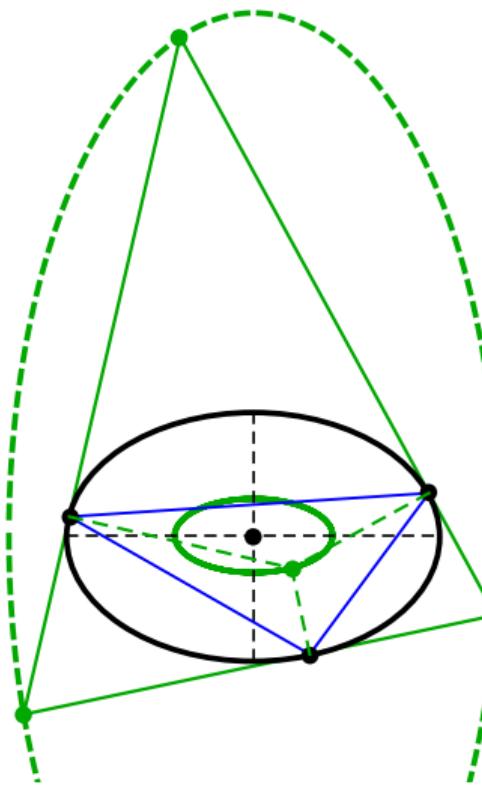
>>> Derived Triangles Show Locus
 ΔExctrl ΔIntouch ΔExtouch
 ΔMedial ΔAnticpl ΔFeuerb
 ΔOrthic OrthicInc ΔEuler
 ΔX15p ΔSymm ΔExcSymm

>>> Constructions
 EulerLin Alt Feet X9Ins ExcPerps

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Visit [MathWorld](#) and [ETC](#)



Locus of Excenters is Elliptic [video]



Encyclopedia of Triangular Centers [ETC]

Clark Kimberling's
ENCYCLOPEDIA OF TRIANGLE CENTERS - ETC

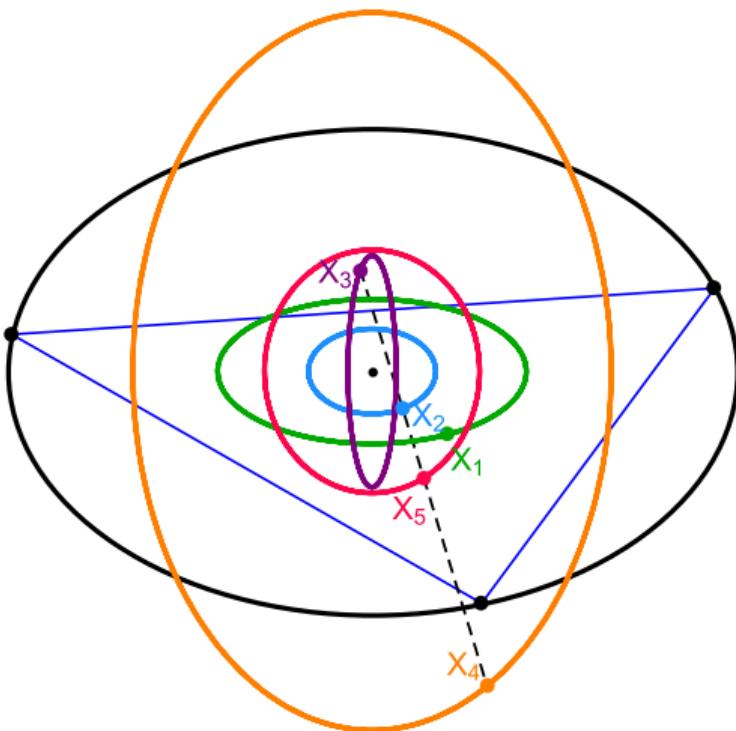
Tables Glossary Search Sketches Links Thanks

This is PART 1: Introduction and Centers X(1) - X(1000)

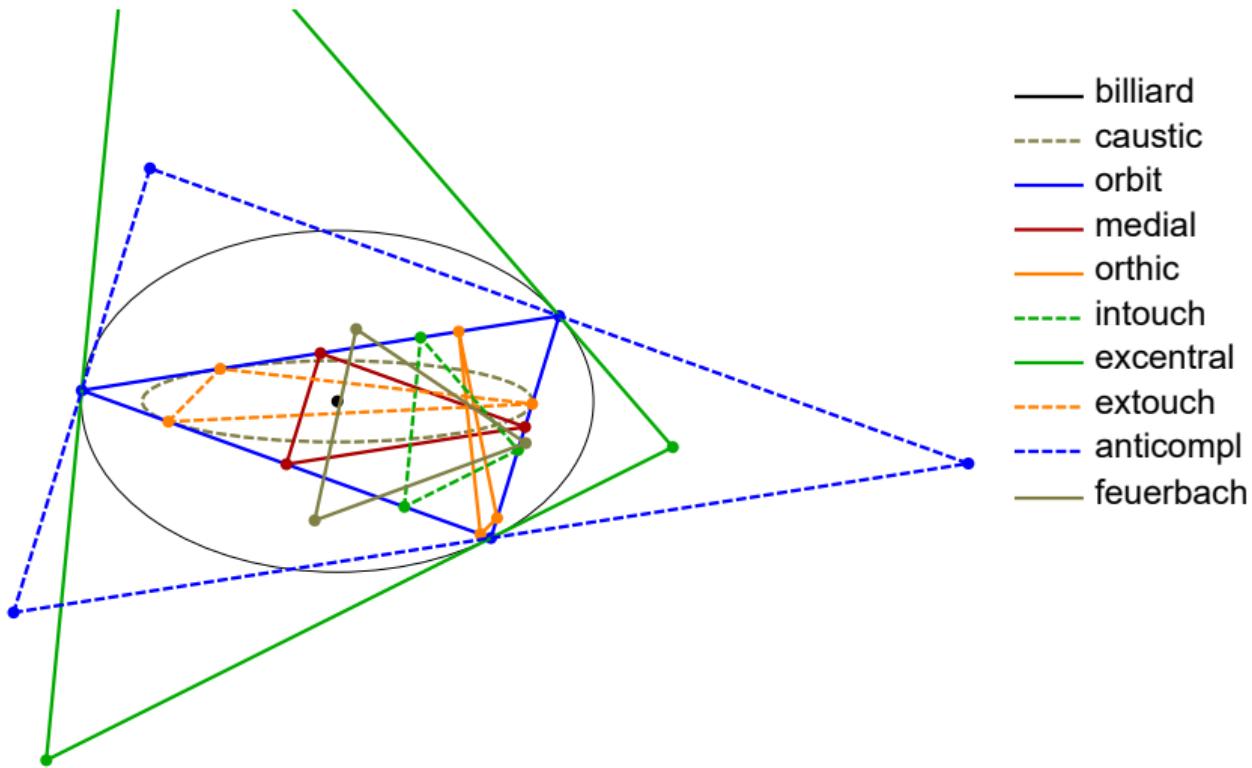


PART 1:	Introduction and Centers X(1) - X(1000)
PART 2:	Centers X(1001) - X(3000)
PART 3:	Centers X(3001) - X(5000)
PART 4:	Centers X(5001) - X(7000)
PART 5:	Centers X(7001) - X(10000)
PART 6:	Centers X(10001) - X(12000)
PART 7:	Centers X(12001) - X(14000)
PART 8:	Centers X(14001) - X(16000)
PART 9:	Centers X(16001) - X(18000)
PART 10:	Centers X(18001) - X(20000)
PART 11:	Centers X(20001) - X(22000)
PART 12:	Centers X(22001) - X(24000)
PART 13:	Centers X(24001) - X(26000)
PART 14:	Centers X(26001) - X(28000)
PART 15:	Centers X(28001) - X(30000)
PART 16:	Centers X(30001) - X(32000)
PART 17:	Centers X(32001) - X(34000)
PART 18:	Centers X(34001) - X(36000)
PART 19:	Centers X(36001) - X(38000)
PART 20:	Centers X(38001) - X(40000)

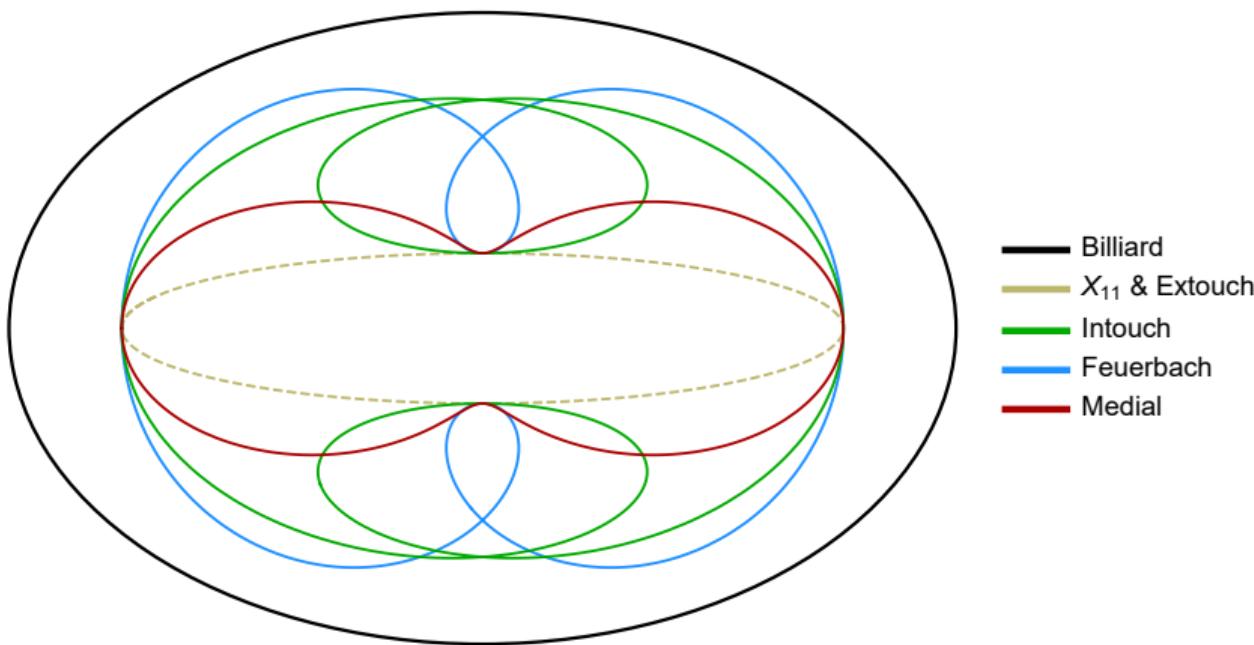
Elliptic Loci of Major Triangular Centers [video]



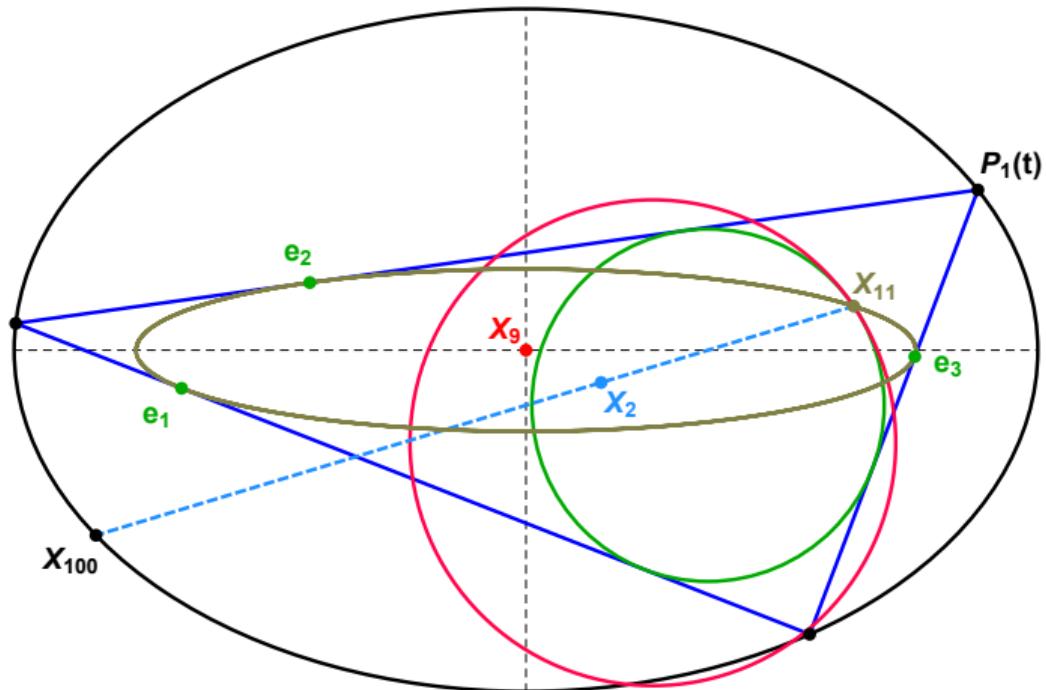
Derived Triangles [video]



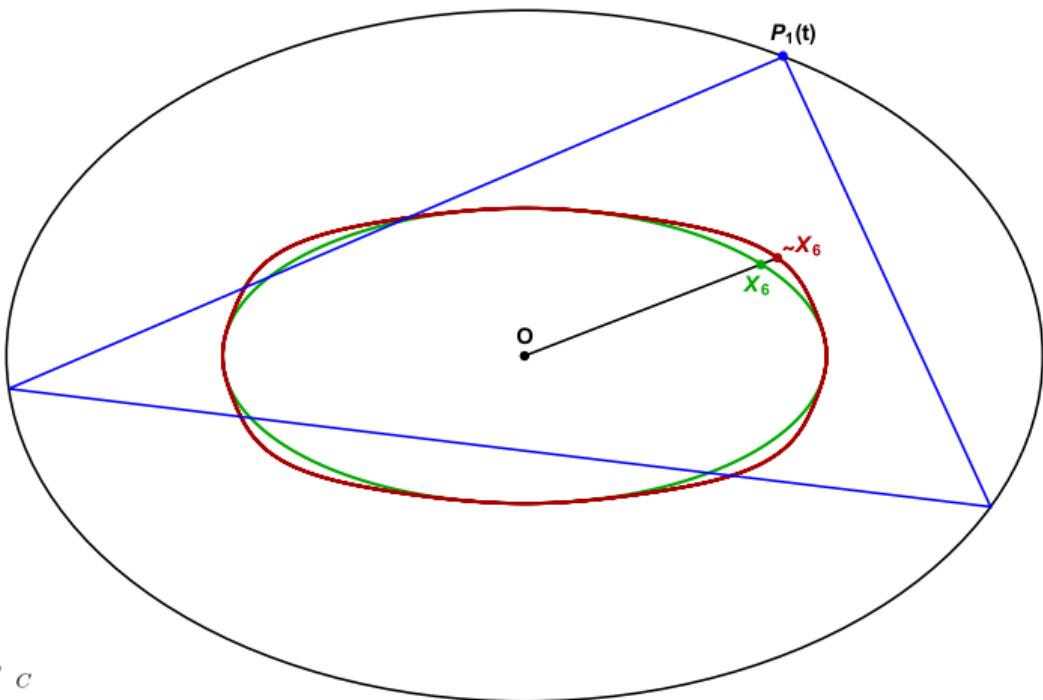
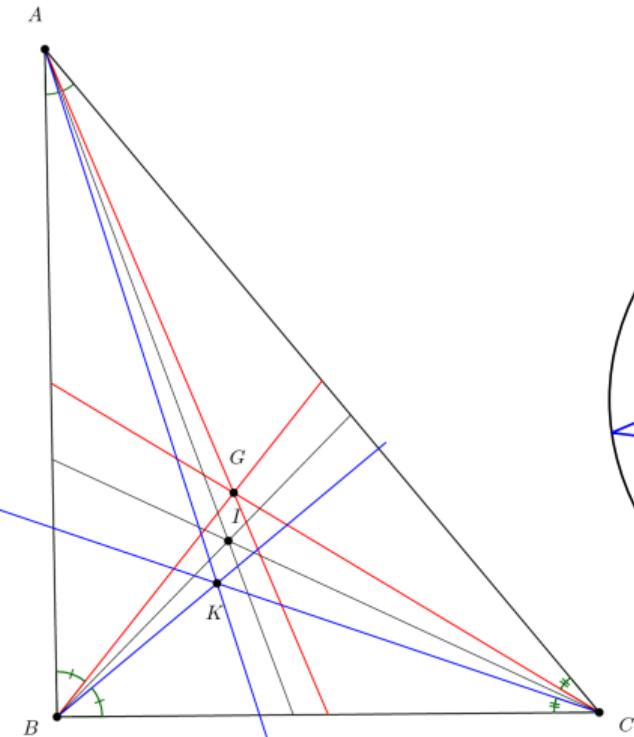
Loci of Vertices of Derived Triangles [video]



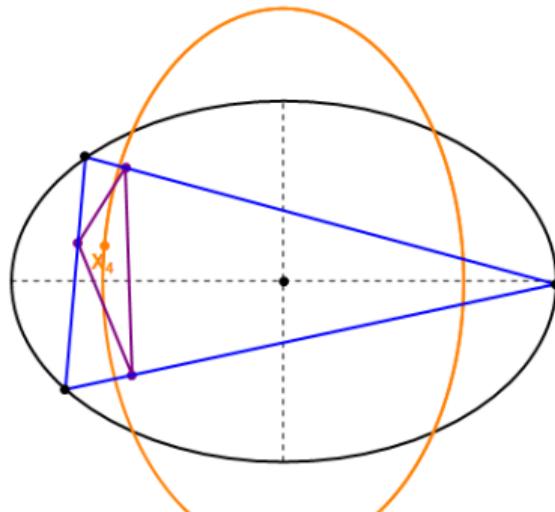
Feuerbach, Anticomplement, and Extouchpoints [video]



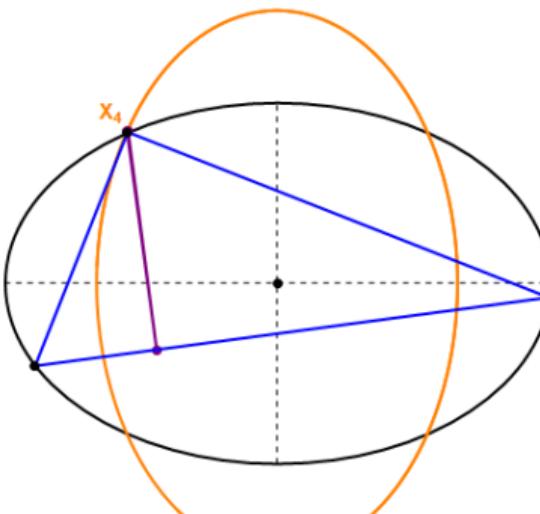
Symmedian Point: Quasi-Elliptic Convex Quartic



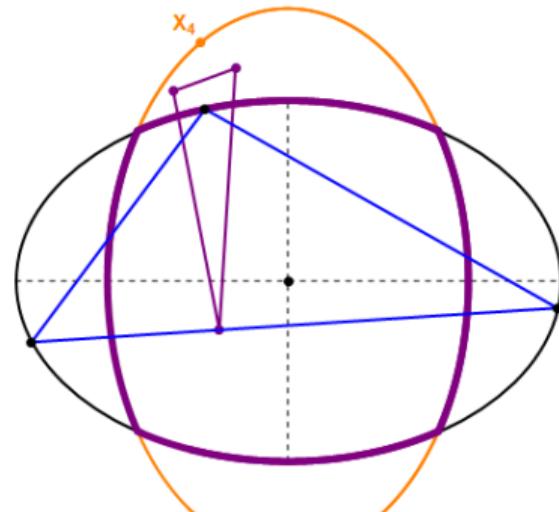
Orthic Incenter: Four Kinks [video]



(a)

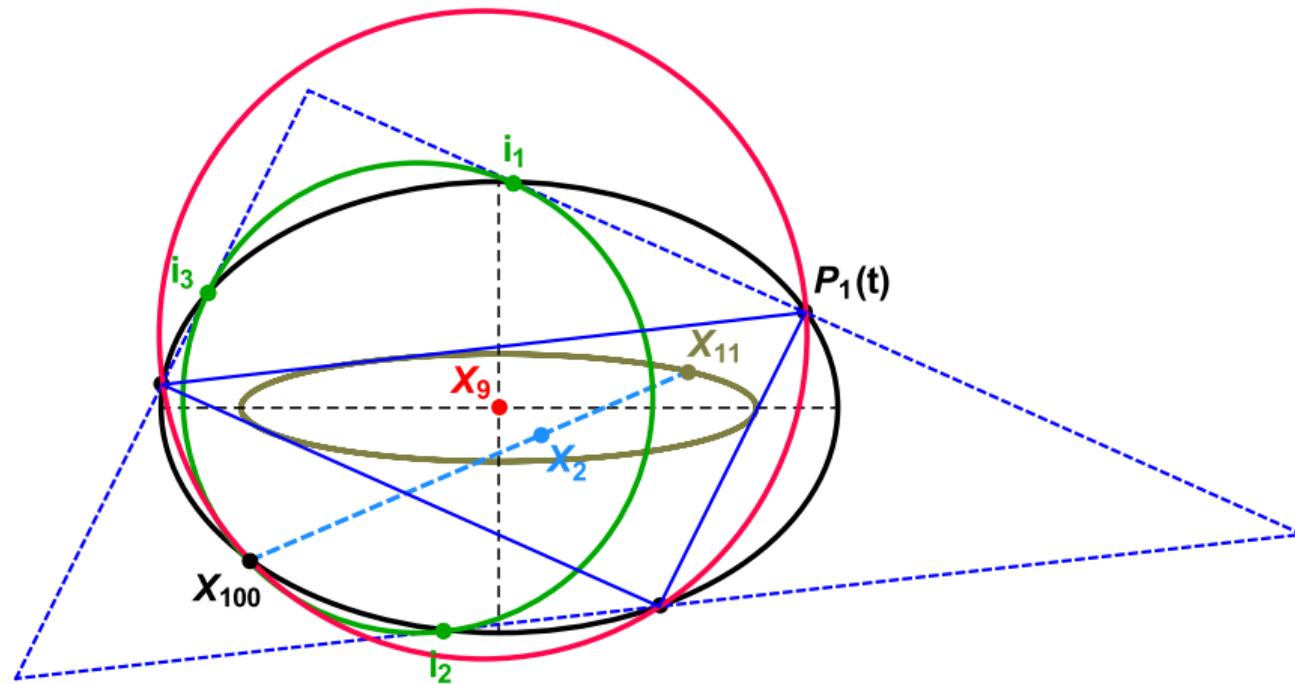


(b)

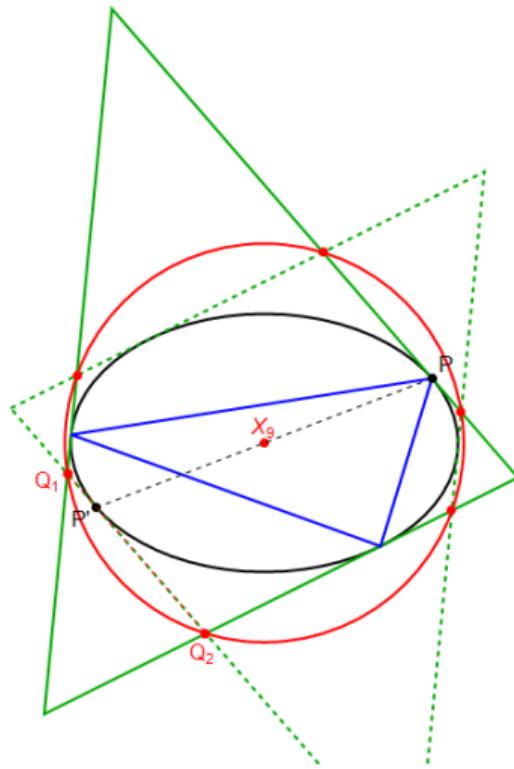


(c)

Intouchpoints of Anticomplementary Track Billiard [video]



A Circular Locus! [video]

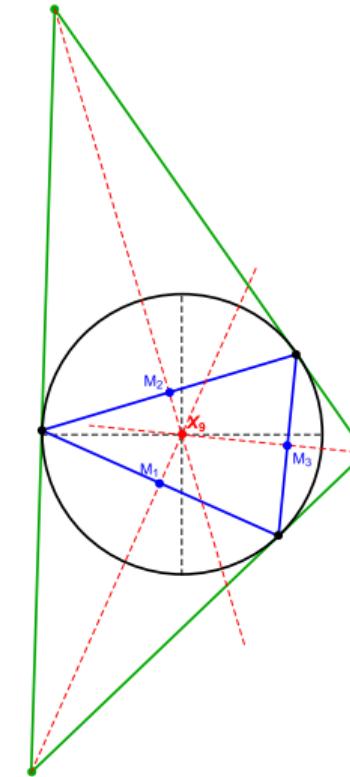
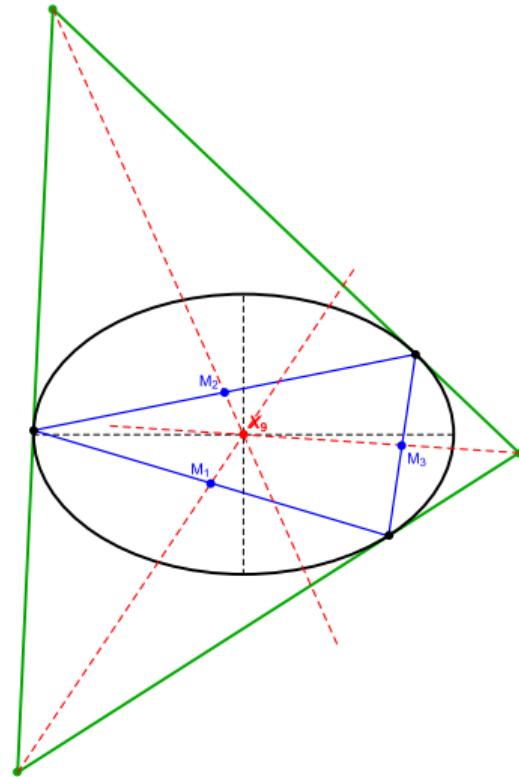


$$r^* = \frac{1}{\gamma}$$

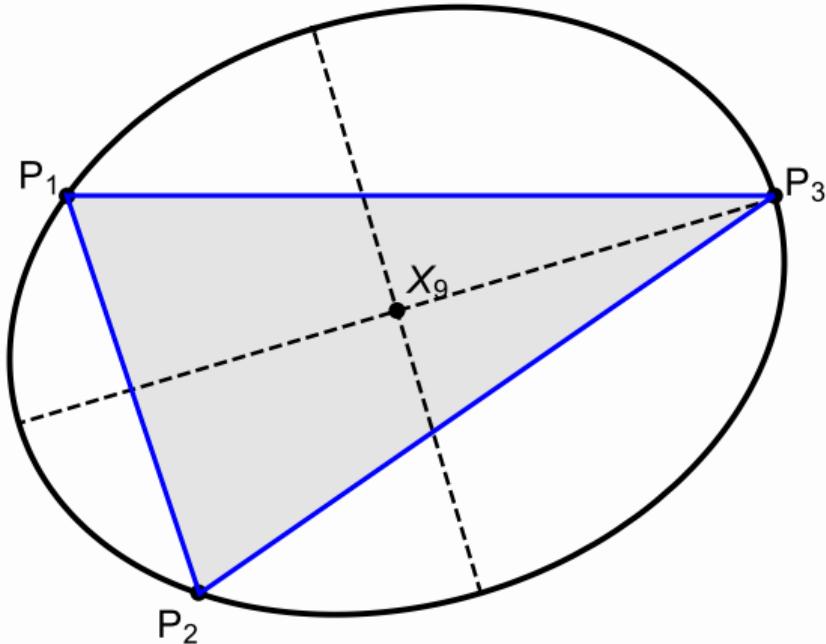
Section 3

A Point Locus!

Mittenpunkt is Stationary [video]



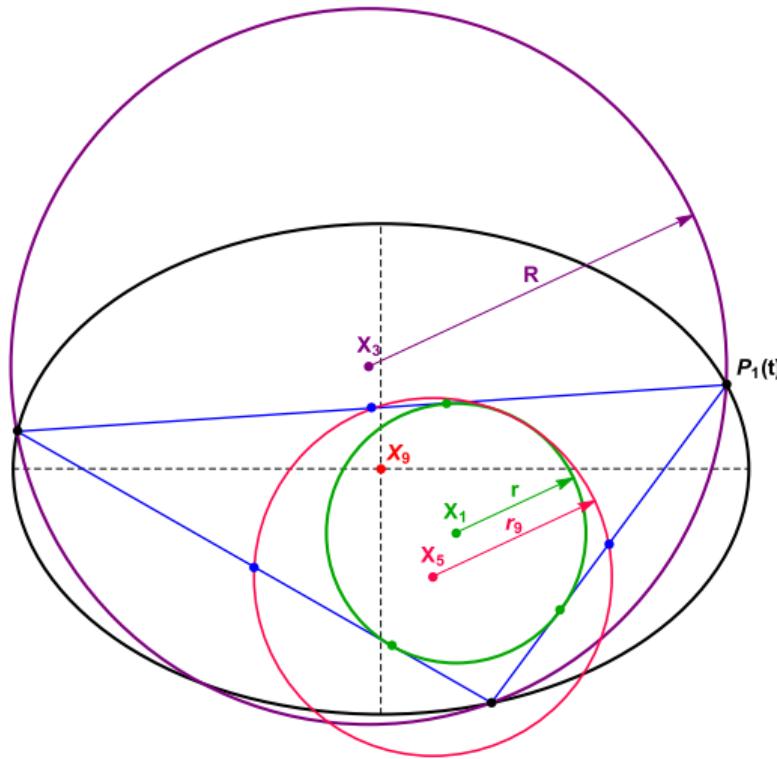
Circumbilliard [video]



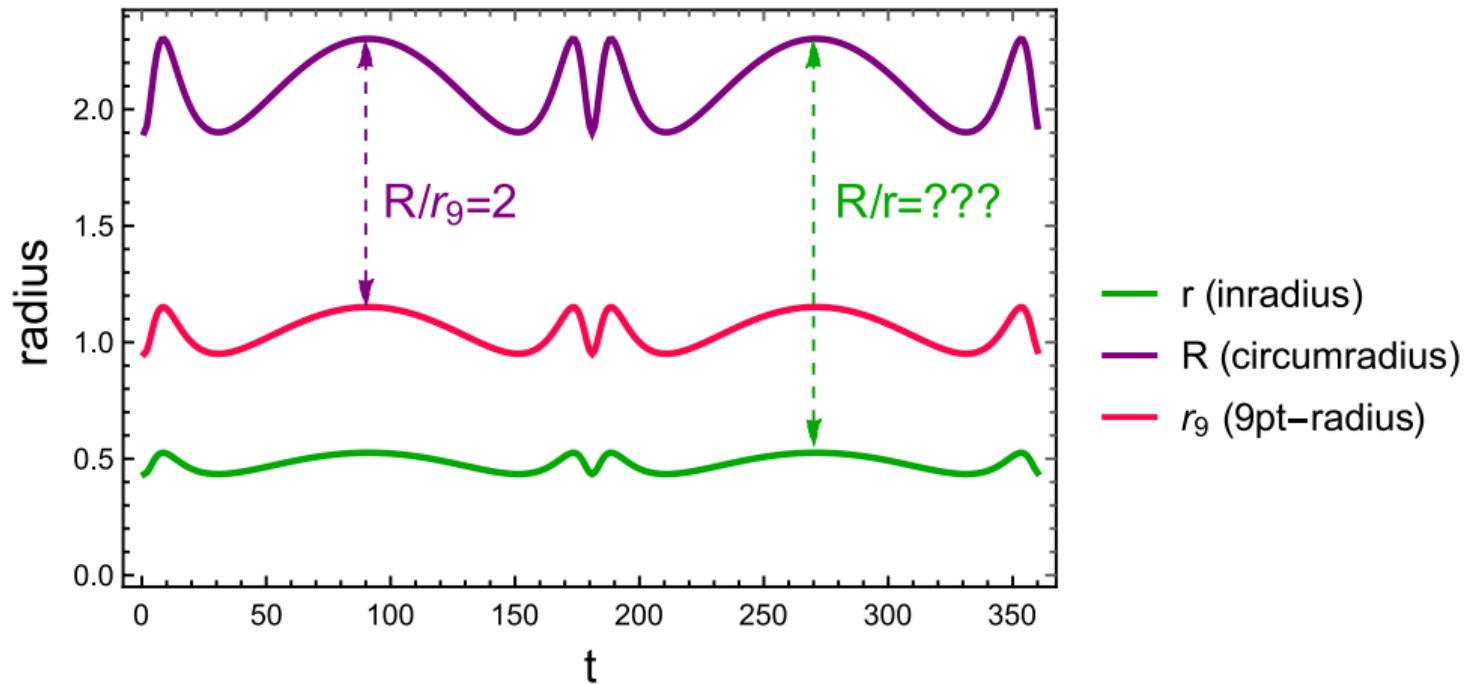
Section 4

The Beautiful Ratio

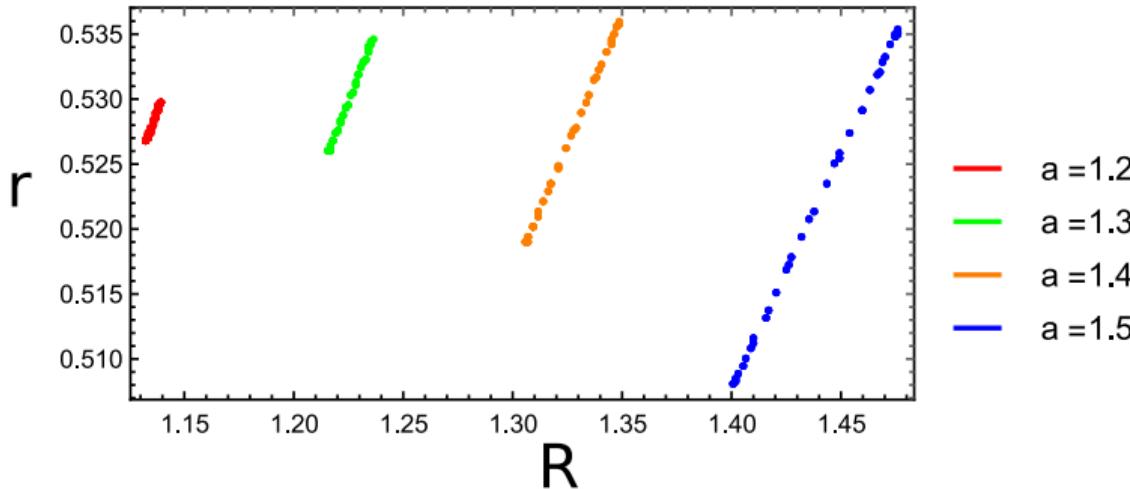
Three Little Radii [video]



Plotting Radii



A New Constant of Motion!



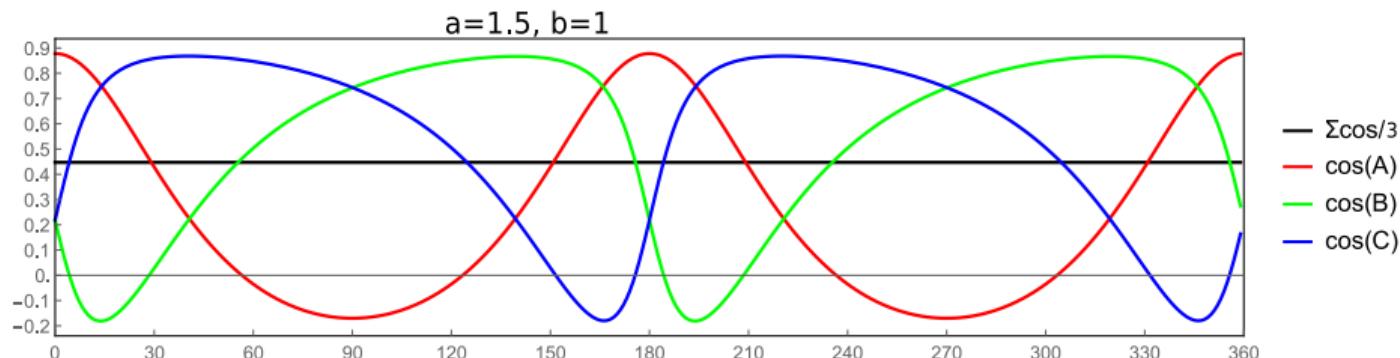
$$\frac{r}{R} = \gamma L - 4$$

Section 5

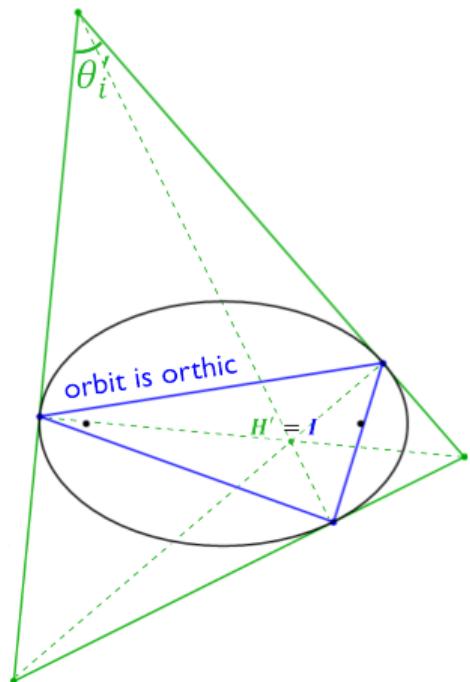
Three Amazing Corollaries

Sum of Orbit Cosines is Conserved!

$$\sum_{i=1}^3 \cos \theta_i = 1 + \frac{r}{R}$$

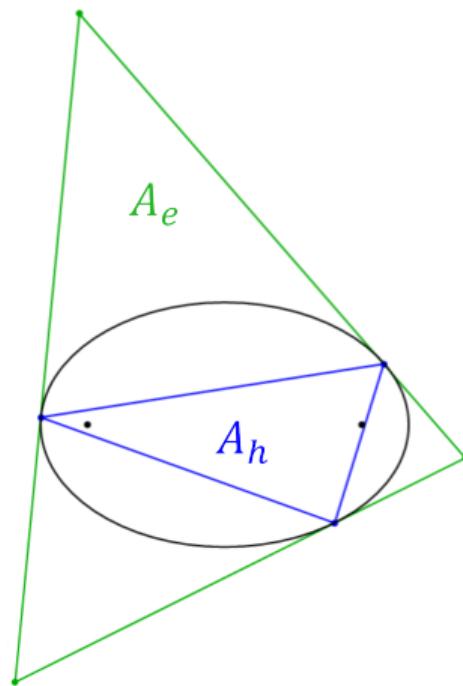


Product of Excentral Cosines is Conserved! [video]



$$\prod_{i=1}^3 |\cos \theta'_i| = \frac{1}{4} \frac{r_h}{R_h}$$

Orbit-to-Excentral Area Ratio is Conserved!

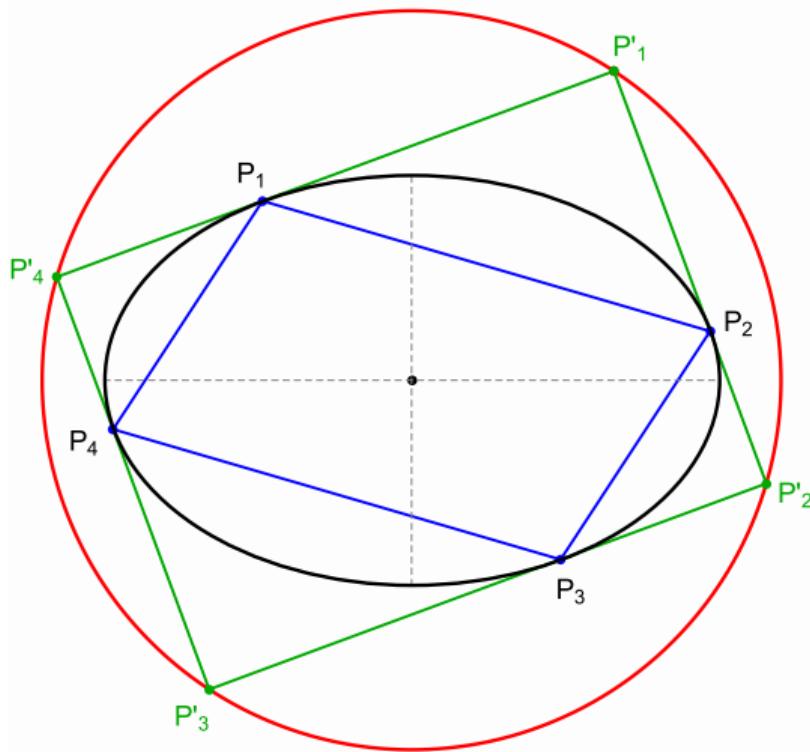


$$\frac{A_h}{A_e} = \frac{1}{2} \frac{r_h}{R_h}$$

Section 6

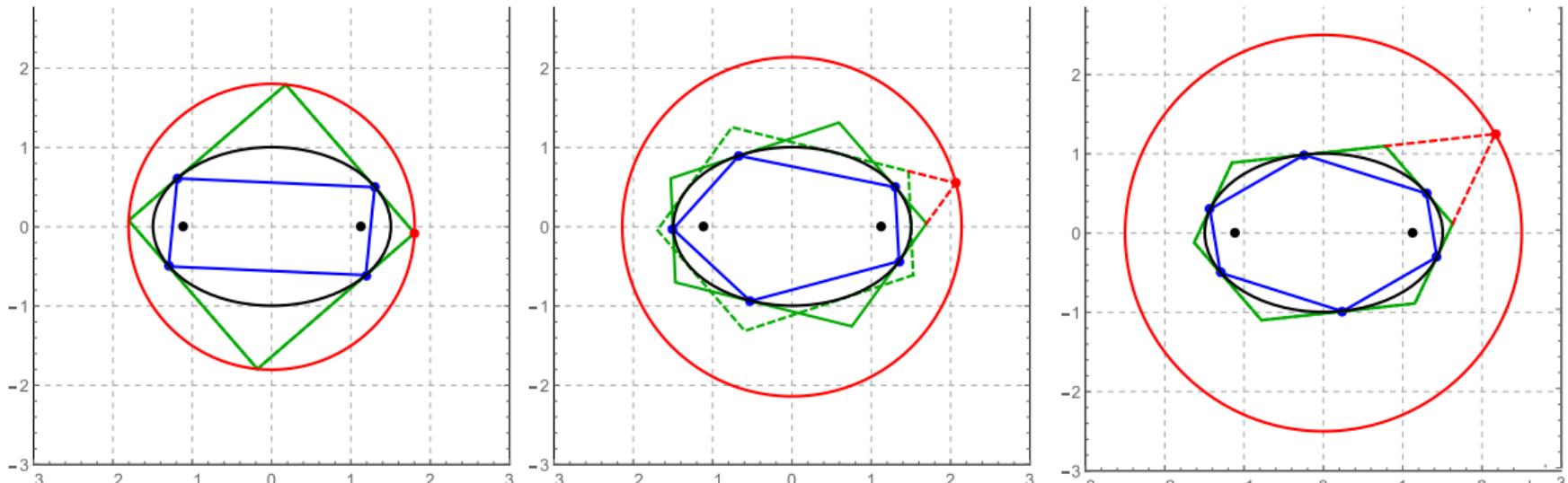
Generalize $\forall N$

Monge's Orthoptic Circle [video]



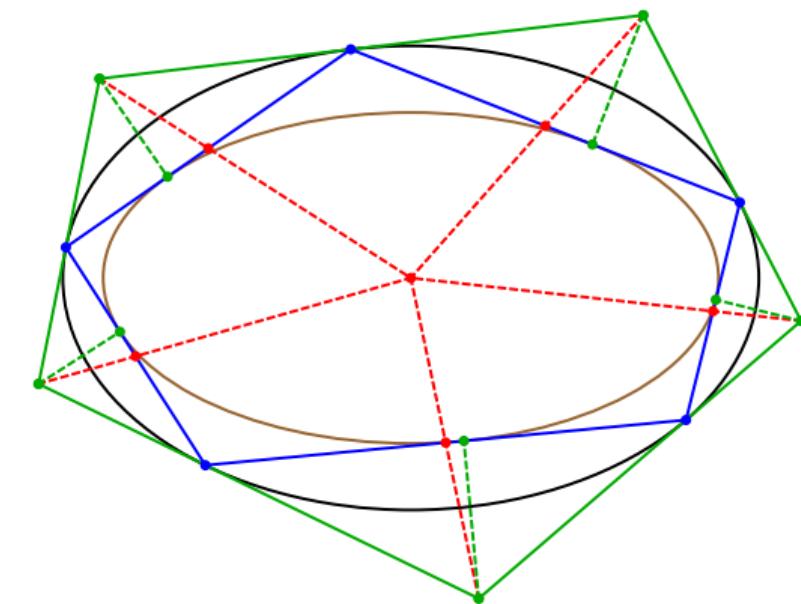
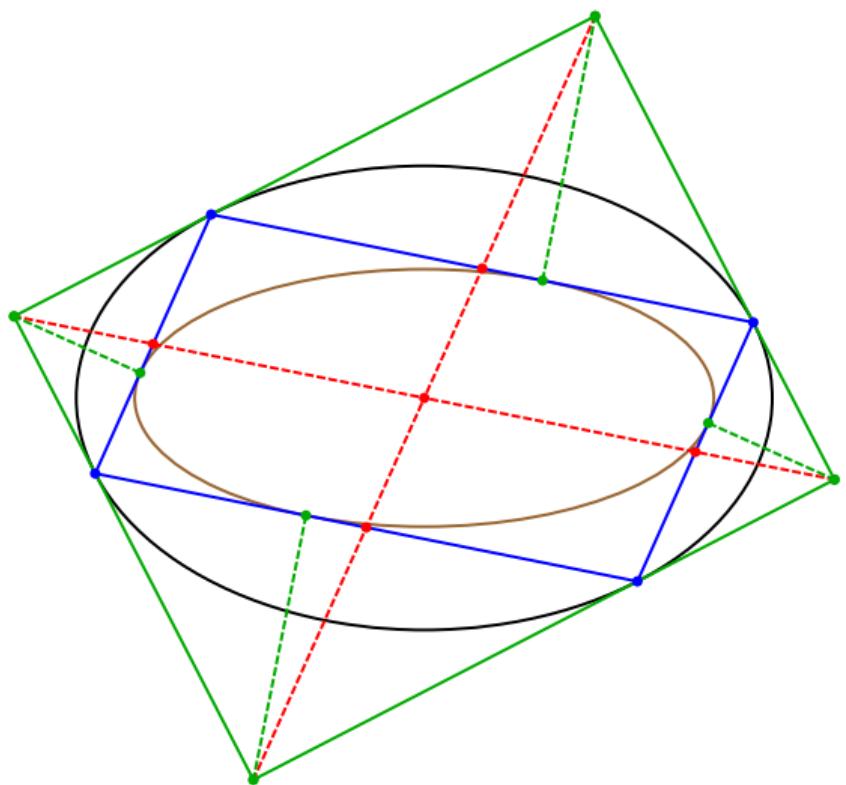
- * Stationary Circle
- * Rectangle diagonals.
- * $\sum \cos \theta_i = 0$.
- * $\prod \cos \theta'_i = 0$.

Generalized Stationary Circle [video 1] and [video 2]

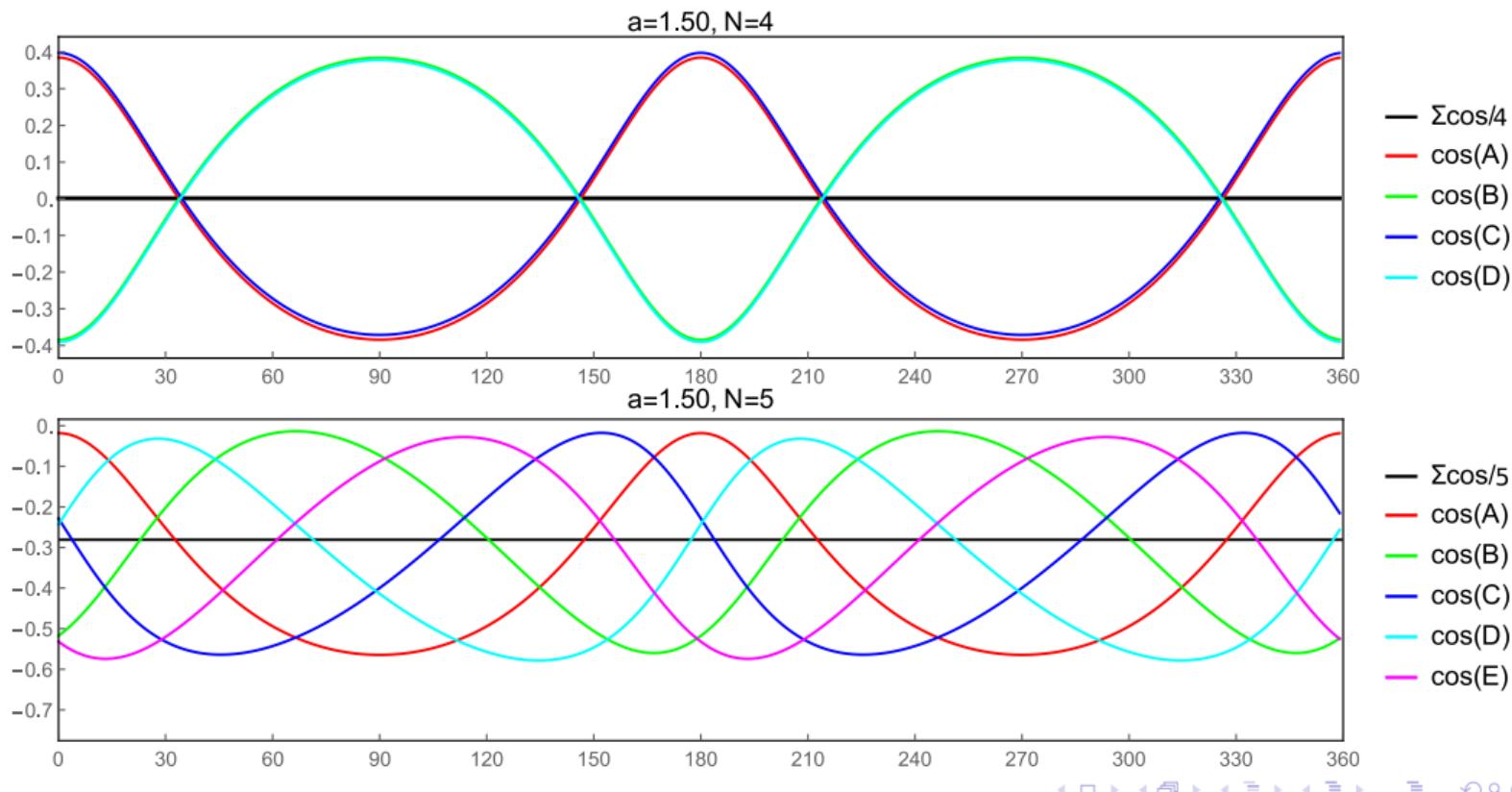


$\Rightarrow E'_i \cap -E'_{i+1}$ is a stationary circle of $r^* = 1/\gamma$

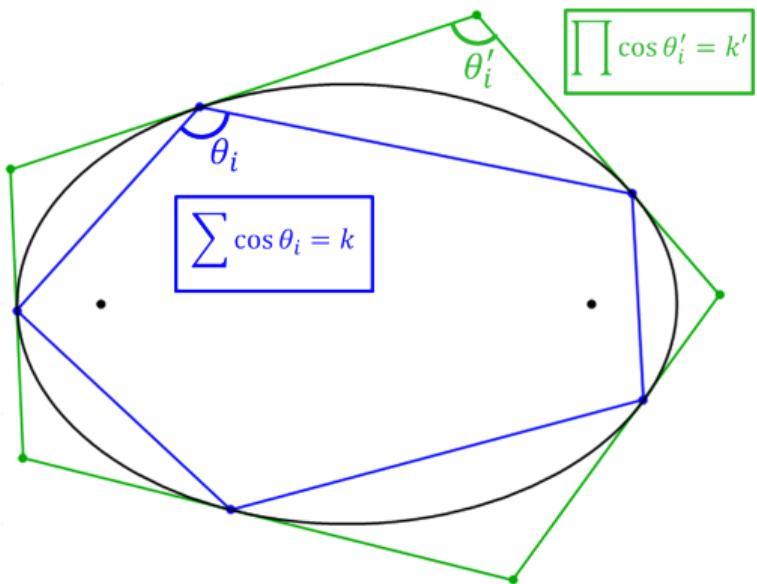
$N > 3$ Mittenpunkt [video 1] and Extouchpoints [video 2]



Cosine Sums for $N > 3$



Sum, Product, Area Ratios are Conserved!

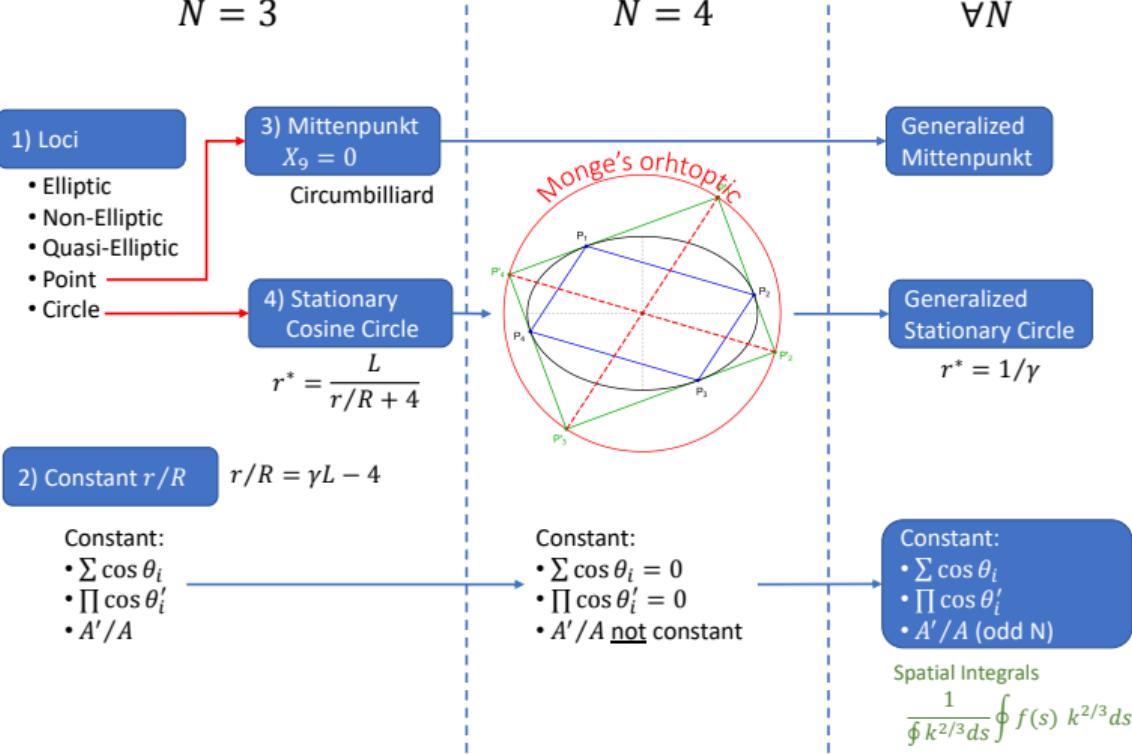


$\Rightarrow \sum_{i=1}^N \cos \theta_i$ conserved $\forall N$.
 $\Rightarrow \prod_{i=1}^N \cos \theta'_i$ conserved $\forall N$.
 $\Rightarrow A'/A$ conserved $\forall N$ **odd**.

Summary

0) Integrability constants

- Perimeter L
- Angular Mom. γ



Project's [Homepage]

The screenshot shows a web browser window with the title bar "New Properties of Triangular Orb". The address bar contains the URL "dan-reznik.github.io/Elliptical-Billiards-Triangular-Orbits/". The page content is as follows:

- 1 Introduction** (highlighted in blue)
- 2 Summary of Main Results
- 3 Results
- 4 Conclusion
- 5 Media by the Authors
- 6 Glossary of Terms
- 7 Appendix
- References

New Properties of Triangular Orbit in Elliptic Billiards

Dan Reznik, Ronaldo Garcia, Jair Koiller

Last update: 2019-09-04 14:42:15

Assets and Downloads

- Videos, applets, images: [here](#)
- Interactive [applet](#).
- IMPA talk on July 29, 2019. You can [enable English subtitles](#) on YouTube.
- IMPA [slides](#).
- Mathematical Intelligencer [draft](#), in preparation.

Questions

- * $N = 3$: why is a locus is elliptic or non?
- * Invariants for Ellipsoidal (3d) Billiard?
- * Invariants in self-intersecting orbits? ($N=4$ and $N=5$)
- * Invariants in non-billiard (Poncelet) orbit families?
- * Invariants for orbits on sphere?

Thanks!



dreznik[@]gmail[.]com
ragarcia[@]ufg[.]br
jairkoiller[@]gmail[.]com