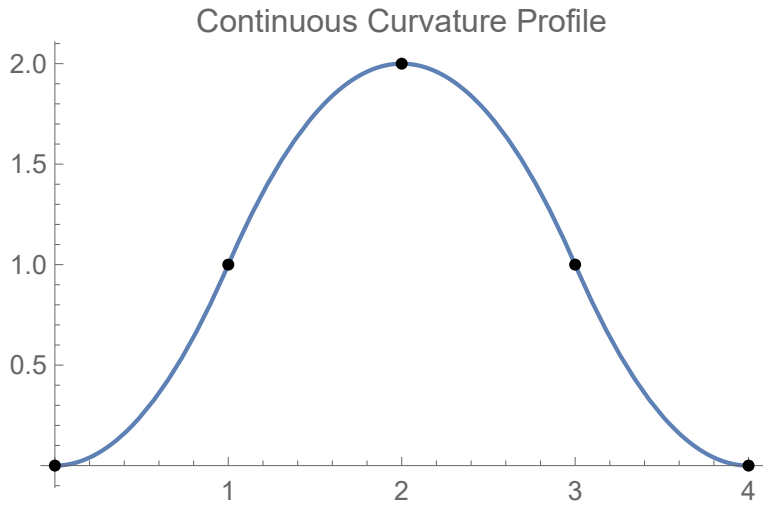
Continuous Curvature Profile w 3 Parabolic Segments

```
In[5]:= f[x_] := Piecewise[
          {{x^2, 0 <= x < 1},
           {-(x - 2)^2 + 2,
            1 <= x < 3},
           {(x - 4)^2, 3 <= x < 4}}]

In[70]:= cps = Module[{xs =
              {0, 1, 2, 3, 4}, ys},
            ys = f /@ xs;
            MapThread[
             {#1, #2} &, {xs, ys}]];
```

In[174]:=
```
Plot[f[x], {x, 0, 4},
 Exclusions → None,
 Epilog → {PointSize[
    Medium], Point[cps]},
 PlotLabel → "Continuous
    Curvature Profile"]
```

Out[174]=



Integrate Curvature to Get Plane Curve

If the curvature is parametrized by arc length, then

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 + \int_0^s \cos\left(\int_0^s \kappa(s)\, ds\right) ds \\ y_0 + \int_0^s \sin\left(\int_0^s \kappa(s)\, ds\right) ds \end{pmatrix}$$

In[19]:=
```
th[s_] =
  Integrate[f[x], {x, 0, s},
   Assumptions → s > 0]
```

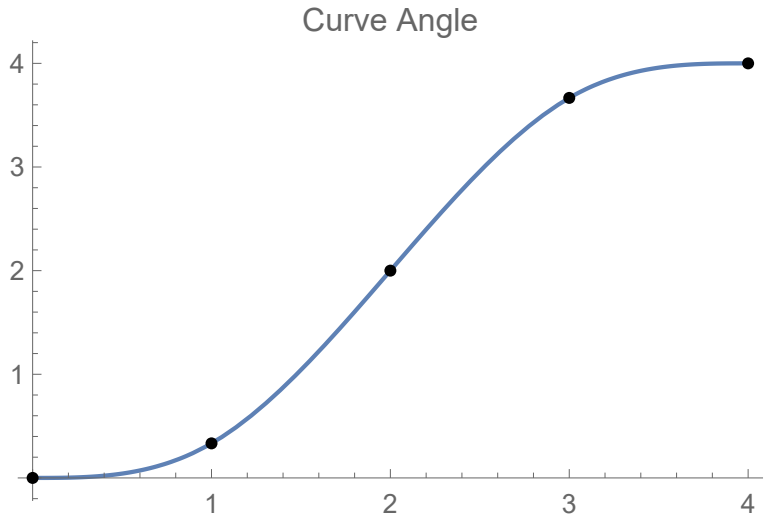
Out[19]=

$$\begin{cases} 4 & s > 4 \\ \frac{s^3}{3} & s \le 1 \\ \frac{1}{3}\left(2 - 6\,s + 6\,s^2 - s^3\right) & 1 < s \le 3 \\ \frac{1}{3}\left(-52 + 48\,s - 12\,s^2 + s^3\right) & \text{True} \end{cases}$$

In[68]:=
```
ss = Module[{xs =
      {0, 1, 2, 3, 4}, ys},
   ys = th /@ xs;
   MapThread[
    {#1, #2} &, {xs, ys}]];
```

In[173]:=
```
Plot[th[s], {s, 0, 4},
 Exclusions → None,
 Epilog →
  {PointSize[Medium],
   Point[ss]}, PlotLabel →
  "Curve Angle"]
```

Out[173]=



note :

to compute curve angle, curvature will be scaled

by constant "a" :

$$k(s) = a * f(x)$$

In[95]:=
```
Clear[thexy];
thexy[a_, s_] :=
 Module[{t},
  NIntegrate[t = a th[x];
   {Cos[t], Sin[t]},
   {x, 0, s}]];
```

Draw and Interact w/ Integrated Plane Curve, 0<s<4, varying curvature scale factor "a"

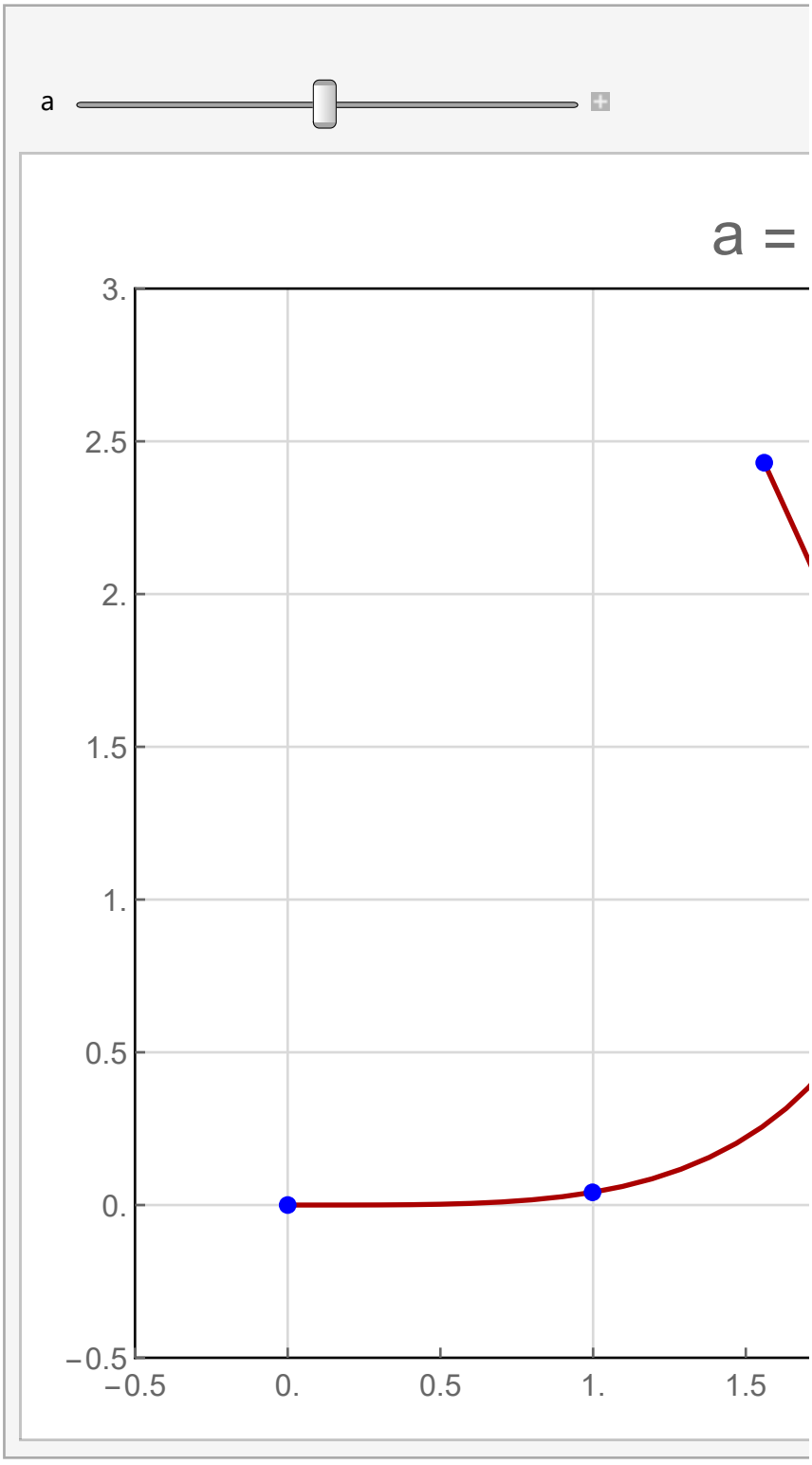
In[191]:=
```
oneFr[a_] := Module[
```

```
{xys, lab, xmin = -.5,
 xmax = 4, ymin = -.5,
 ymax = 3, xtks, ytks,
 curveStep = .1},
xys = Table[thexy[a, s],
    {s, 0, 4,
     curveStep}] //
  Chop;
lab = Style[
  "a = " <> ToString[
    NumberForm[a,
     {3, 2}]], Large];
xtks = Table[x,
  {x, xmin, xmax, .5}];
ytks = Table[x,
  {x, ymin, ymax, .5}];
Graphics[{
  {Thick, Darker[Red],
   Line[xys]},
  {Blue, PointSize[
    Large], Point[
    thexy[a, #] & /@
```

```
      {0, 1, 2, 3, 4}]}
},
ImageSize → Large,
PlotRange →
 {{xmin, xmax},
  {ymin, ymax}},
Frame → True,
FrameTicks →
 {{ytks, None},
  {xtks, None}},
FrameStyle → Medium,
AspectRatio →
 Automatic,
GridLines →
 Automatic,
GridLinesStyle →
 LightGray,
PlotLabel → lab]];
```

```
In[187]:= Manipulate[Module[{gr},
            gr = oneFr[a];
            gr],
           {{a, .5}, 0.01, 1, .01}]
```

Out[187]=



Export Animated Frames

In[192]:=
```
frs = Table[oneFr[a],
        {a, .01, 2, .01}];
```

In[193]:=
```
Export["integrated
    curvature.gif",
  Join[frs, Most[
    Reverse[frs]]],
  Options → {"Loop" → True}]
```

Out[193]= integrated curvature.gif