Untitled2

October 4, 2018

1 Question1

Let $\vec{Y} = (Y_1, Y_2, Y_3, Y_4) \sim multinomial(p_1, p_2, p_3, p_4)$ where (WLOG)

$$Y_1 = \begin{cases} 1 \text{ if the student is watching a movie} \\ 0 \text{ otherwise} \end{cases}$$

$$Y_2 = \begin{cases} 1 \text{ if the student is in COMP-551 class} \\ 0 \text{ otherwise} \end{cases}$$

$$Y_3 = \begin{cases} 1 \text{ if the student is playing} \\ 0 \text{ otherwise} \end{cases}$$

$$Y_4 = \begin{cases} 1 \text{ if the student is studying} \\ 0 \text{ otherwise} \end{cases}$$

under the constraint $\forall\ i \in \{1,2,3,4\}\ 1 \geq p_i \geq 0$ and $\sum_i p_i = 1$ From the random variable $V \sim_{\text{U}} (0,1)$ four random variables:

$$X_1 = \begin{cases} 1 \text{ if } V \in [0, p_1[\\ 0 \text{ otherwise} \end{cases}$$

$$X_2 = \begin{cases} 1 \text{ if } V \in [p_1, p_1 + p_2[\\ 0 \text{ otherwise} \end{cases}$$

$$X_3 = \begin{cases} 1 \text{ if } V \in [p_1 + p_2, p_1 + p_2 + p_3[\\ 0 \text{ otherwise} \end{cases}$$

$$X_4 = \begin{cases} 1 \text{ if } V \in [p_1 + p_2 + p_3, p_1 + p_2 + p_3 + p_4] \\ 0 \text{ otherwise} \end{cases}$$

As a pseudocode, given: - Random(a,b) returns any real number $x \in [a, b]$ with equal probability. - $N \in \mathbb{N}$:= # of samples - Array indices start from 1 - for $\{p_i\}_{i=1}^4$ $p_i \in [0,1]$ $\forall i$ and $\sum_i p_i = 1$

```
define SampleStudent(float $p_1,\ p_2,\ p_3,\ p_4$){

    for(int i = 1; i <= N ; i ++) {
    float x = Random(0, 1) # 0 <= x <= 1
```

```
    Array activities = Int[4]
    if (x < p_1) {

        activties[1]++;

    } else if ($p_1$ <= x < $p_1 + p_2$ ) {

        activities[2]++;

    } else if ($p_1+p_2$ <= x < $p_1+p_2+p_3$ ) {

        activities[3]++;

    } else { # $p_1+p_2+p_3 <=  x <= 1$

        activities[4]++

    } # conditional statement ended
}
```

}

2 Question 2

We will define MSE as below:

$$MSE = \frac{1}{N}(Y - \mathbb{X}\widehat{\beta})^T(Y - \mathbb{X}\widehat{\beta})$$

where: N is the number of sample points
Y is the target vector
$\mathbb{X}$ is our design matrix
β is our linear predictor assuming $\mathbb{E}$[Y | $\mathbb{X}$] = $\mathbb{X}\beta$
We create a matrix

$$X = \begin{pmatrix} 1 & x_1 & \cdots & x_1^{20} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & \cdots & x_n^{20} \end{pmatrix}$$

with the parameter estimates:

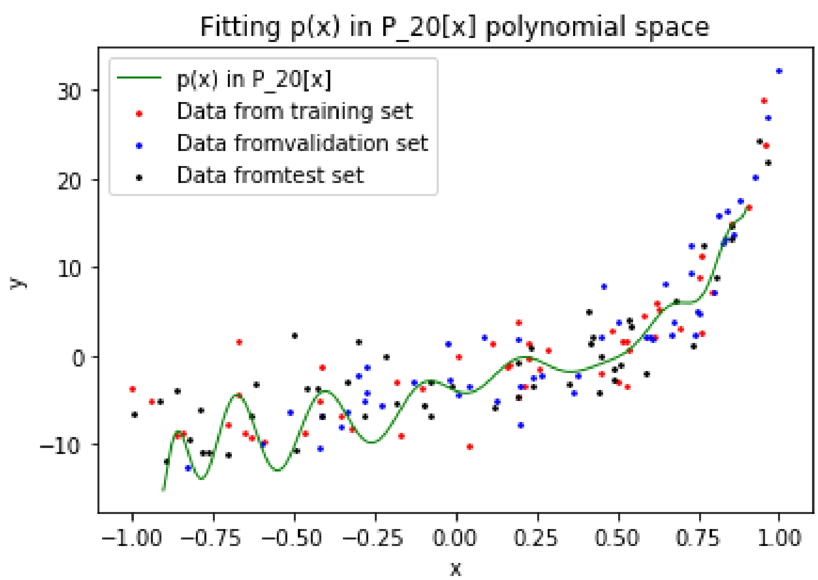
$$\hat{\beta} = (X^T X)^{-1} X y$$

We can have the estimate for our output with new $X^{(new)}$, which are our new observed data. Then we have:

$$\hat{y} = X^{(new)}\hat{\beta}$$

$$\hat{y} = X^{(new)}(X^T X)^{-1} X y$$

we now calculate beta


Fitting p(x) in P_20[x] polynomial space
p(x) in P_20[x]
Data from training set
Data fromvalidation set
Data fromtest set
30
20
10
0
−10
y
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x

alt text