Dan Seol - 260677676 MATH 525 Assignment 2

#3.24 Show that the variance is minimized for a fixed cost with the cost function in (3.12) when $n_h \propto \frac{N_h S_h}{\sqrt{c_h}}$ We want to minimize

$$C = c_0 + \sum_{h=1}^{H} c_h n_h$$

for a fixed variance

$$V(\hat{t}_{str}) = \sum_{h=1}^{H} (1 - \frac{n_h}{N_h}) N_h^2 \frac{S_h^2}{n_h}$$

We use Lagrange multipliers.

First, for a given strata, with samples $\vec{n} = (n_1, n_2, ..., n_H)$ construct a function $L(\vec{n})$ such that

$$L(\vec{n}) = c_0 + \sum_{h=1}^{H} c_h n_h - \lambda \{ \sum_{h=1}^{H} (1 - \frac{n_h}{N_h}) (\frac{N_h}{N})^2 (\frac{S_h^2}{n_h}) \}$$

such that

$$\nabla L(\vec{n}) = (\frac{\partial L}{\partial n_1}, \frac{\partial L}{\partial n_2}, ..., \frac{\partial L}{\partial n_h}) = \vec{0}$$

Remark $\forall i \in \{1, ..., H\}$

$$\frac{\partial L}{\partial n_i} = c_i - \lambda \frac{\partial}{\partial n_i} \left\{ (1 - \frac{n_i}{N_i}) (\frac{N_i}{N})^2 \frac{S_i^2}{n_i} \right\} = 0$$

$$\implies c_i - \lambda (\frac{N_i}{N})^2 \frac{\partial}{\partial n_i} \left\{ \frac{S_i^2}{n_i} - \frac{S_i^2}{N_i} \right\} = c_i n_i - \lambda (\frac{N_i}{N})^2 \frac{\partial}{\partial n_i} (\frac{S_i^2}{n_i}) = 0$$

$$\implies c_i + \lambda (\frac{N_i}{N})^2 \frac{S_i^2}{n_i^2} = 0$$

$$\implies n_i^2 c_i = -\lambda \frac{N_i^2}{N^2} S_i^2 = \frac{-\lambda}{N^2} N_i^2 S_i^2 \implies$$

$$n_i^2 = \frac{-\lambda}{N^2} \frac{N_i^2 S_i^2}{c_i} \implies n_i = \sqrt{\frac{-\lambda}{N^2}} \frac{N_i S_i}{\sqrt{c_i}} = r \frac{N_i S_i}{\sqrt{c_i}}$$

where

$$r:=\sqrt{\frac{-\lambda}{N^2}}$$

Showing

 $\forall h \ n_h \propto \frac{N_i S_i}{\sqrt{c_i}}$

#3.37

#Import necessary packages

library(tidyverse)

-- Attaching packages ------ tidyverse 1.2.1 -

```
v purrr
## v ggplot2 3.1.0
                                0.3.0
## v tibble 2.0.1
                               0.7.8
                   v dplyr
                     v stringr 1.3.1
## v tidyr
          0.8.2
            1.3.1
## v readr
                      v forcats 0.3.0
## -- Conflicts ------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()
                    masks stats::lag()
library(survey)
## Loading required package: grid
## Loading required package: Matrix
##
## Attaching package: 'Matrix'
## The following object is masked from 'package:tidyr':
##
##
      expand
## Loading required package: survival
##
## Attaching package: 'survey'
## The following object is masked from 'package:graphics':
##
##
      dotchart
library(srvyr)
##
## Attaching package: 'srvyr'
## The following object is masked from 'package:stats':
##
##
      filter
library(knitr)
library(kableExtra)
#Discovered a function: strata?
#svydesign turned out to be a better function since it allows a complex survey design
```

##(a)

Using one or more following variables: age, sex, race or marstat, divide the population into strata. Explain how you decided upon your stratification variable and how you chose the number of strata to us. (Note: It is NOT FAIR to use the values of incott in the population to choose your strata! However, you may draw a pilot sample of size 200 using an SRS to aid you in constructing your strata.

```
ipums <- read_csv('ipums.csv')

## Parsed with column specification:
## cols(
## Stratum = col_double(),
## Psu = col_double(),
## Inctot = col_double(),
## Age = col_double(),</pre>
```

```
##
     Sex = col double(),
##
     Race = col_double(),
##
     Hispanic = col_double(),
     Marstat = col_double(),
##
##
     Ownershg = col_double(),
##
     Yrsusa = col double(),
##
     School = col double(),
     Educrec = col_double(),
##
##
     Labforce = col_double(),
##
     Occ = col_double(),
     Classwk = col_double(),
##
     VetStat = col_double()
## )
head(ipums)
## # A tibble: 6 x 16
##
     Stratum
               Psu Inctot
                                    Sex Race Hispanic Marstat Ownershg Yrsusa
                             Age
##
       <dbl> <dbl>
                     <dbl> <dbl>
                                  <dbl> <dbl>
                                                  <dbl>
                                                          <dbl>
                                                                    <dbl>
                                                               5
## 1
           1
                      4105
                              18
                                      1
                                            2
                                                      0
                                                                        0
                                                                                0
                                                                        2
                                                                                0
## 2
           1
                  1
                      7795
                               20
                                      1
                                            1
                                                      0
                                                               5
## 3
           1
                  1
                     16985
                               24
                                      1
                                            1
                                                      0
                                                               1
                                                                        1
                                                                                0
## 4
           1
                  1
                      7045
                               21
                                      1
                                            1
                                                      0
                                                               1
                                                                        2
## 5
           1
                  1
                      2955
                               23
                                      1
                                            1
                                                      0
                                                               5
                                                                        2
           1
                  1
                         0
                              17
                                      1
                                            1
                                                      0
                                                               5
                                                                        1
## # ... with 6 more variables: School <dbl>, Educrec <dbl>, Labforce <dbl>,
       Occ <dbl>, Classwk <dbl>, VetStat <dbl>
dim(ipums)
## [1] 53461
ipums complete = ipums %>% filter(!is.na(Inctot))
#Verify that none of the data is missing
dim(ipums_complete)
## [1] 53461
summary(ipums_complete)
##
       Stratum
                          Psu
                                          Inctot
                                                             Age
##
    Min.
           :1.000
                            : 1.00
                                             :-9995
                                                               :15.00
                     Min.
                                      Min.
                                                       Min.
    1st Qu.:2.000
                     1st Qu.:19.00
                                      1st Qu.: 1505
                                                       1st Qu.:25.00
    Median :6.000
                     Median :51.00
                                      Median: 6005
                                                       Median :37.00
##
    Mean
          :4.977
                     Mean :45.21
                                      Mean
                                            : 9194
                                                       Mean
                                                              :41.17
##
    3rd Qu.:7.000
                     3rd Qu.:68.00
                                      3rd Qu.:13260
                                                       3rd Qu.:56.00
##
    Max.
           :9.000
                     Max.
                            :90.00
                                      Max.
                                             :75000
                                                       Max.
                                                               :90.00
         Sex
##
                          Race
                                         Hispanic
                                                            Marstat
##
    Min.
           :1.000
                     Min.
                            :1.000
                                      Min.
                                              :0.00000
                                                         Min.
                                                                :1.000
##
    1st Qu.:1.000
                     1st Qu.:1.000
                                      1st Qu.:0.00000
                                                         1st Qu.:1.000
    Median :2.000
                     Median :1.000
                                      Median :0.00000
                                                         Median :1.000
##
    Mean
          :1.522
                     Mean
                           :1.186
                                      Mean
                                             :0.05425
                                                         Mean
                                                                :2.402
##
    3rd Qu.:2.000
                     3rd Qu.:1.000
                                      3rd Qu.:0.00000
                                                         3rd Qu.:5.000
           :2.000
##
    Max.
                            :5.000
                                                         Max.
                                                                 :5.000
                     Max.
                                      Max.
                                              :1.00000
##
       Ownershg
                         Yrsusa
                                           School
                                                           Educrec
                                                                :1.000
##
           :0.000
                            :0.0000
                                              :1.000
   \mathtt{Min}.
                     Min.
                                       Min.
                                                        Min.
    1st Qu.:1.000
                                                        1st Qu.:5.000
                     1st Qu.:0.0000
                                       1st Qu.:1.000
```

```
## Median :1.000 Median :0.0000
                                   Median :1.000 Median :7.000
## Mean :1.256 Mean :0.2548
                                  Mean :1.148 Mean :6.343
## 3rd Qu.:2.000 3rd Qu.:0.0000
                                  3rd Qu.:1.000 3rd Qu.:8.000
## Max.
          :2.000 Max.
                          :5.0000
                                  Max. :2.000 Max.
                                                          :9.000
##
      Labforce
                        Осс
                                     Classwk
                                                   VetStat
## Min.
         :0.000
                                  Min. : 0.00 Min.
                                                        :0.000
                 Min. : 0.00
## 1st Qu.:1.000 1st Qu.: 0.00
                                  1st Qu.: 0.00
                                                  1st Qu.:1.000
## Median :2.000 Median :18.00
                                  Median :22.00
                                                  Median :1.000
## Mean :1.588 Mean :28.77
                                  Mean :16.34
                                                  Mean :1.132
## 3rd Qu.:2.000
                   3rd Qu.:48.00
                                  3rd Qu.:22.00
                                                  3rd Qu.:1.000
## Max.
          :2.000 Max.
                          :96.00
                                  Max.
                                         :29.00
                                                  Max.
                                                        :2.000
#Pick age, sex, race or Marstat
#It seems that everything besides age is semantically a factor variable
#Make sure R recognizes those columns as factors
lapply(list(ipums_complete$Marstat, ipums_complete$Age, ipums_complete$Race, ipums_complete$Sex), is.fa
## [[1]]
## [1] FALSE
##
## [[2]]
## [1] FALSE
##
## [[3]]
## [1] FALSE
##
## [[4]]
## [1] FALSE
ipums_complete$Marstat = as.factor(ipums_complete$Marstat)
ipums_complete$Race = as.factor(ipums_complete$Race)
ipums_complete$Sex = as.factor(ipums_complete$Sex)
#Let's discretize ipums_complete$Age
ipums_complete$Agecat<-cut(ipums_complete$Age, c(0, 20, 40, 60, 80, 100),labels= c("S1", "S2", "S3", "S4",
is.factor(ipums_complete$Agecat)
## [1] TRUE
counts1 = ipums_complete %>% count(Race)%>% mutate(prop=n/sum(n),
                                                  prop_alloc = round(prop*660))
counts2 = ipums_complete %>% count(Sex)%>% mutate(prop=n/sum(n),
                                                  prop_alloc = round(prop*660))
counts3 = ipums_complete %>% count(Marstat)%>% mutate(prop=n/sum(n),
                                                  prop_alloc = round(prop*660))
countsAge = ipums_complete %>% count(Agecat)%>% mutate(prop=n/sum(n),
                                                  prop_alloc = round(prop*660))
#for part b
counts1
## # A tibble: 5 x 4
##
              n
                 prop prop_alloc
    <fct> <int>
                  <dbl>
                             <dbl>
```

```
46186 0.864
                               570
## 1 1
## 2 2
         5874 0.110
                                73
## 3 3
           325 0.00608
                                4
## 4 4
            886 0.0166
                                11
## 5 5
            190 0.00355
                                 2
counts2
## # A tibble: 2 x 4
##
    Sex
              n prop prop_alloc
    <fct> <int> <dbl>
                           <dbl>
## 1 1
          25538 0.478
                             315
## 2 2
          27923 0.522
                             345
counts3
## # A tibble: 5 x 4
##
    Marstat n prop prop_alloc
##
    <fct> <int> <dbl>
                              <dbl>
## 1 1
           31160 0.583
                                385
## 2 2
            1186 0.0222
                                15
## 3 3
             3298 0.0617
                                 41
## 4 4
             4109 0.0769
                                 51
## 5 5
            13708 0.256
                                169
countsAge
## # A tibble: 5 x 4
    Agecat n prop prop_alloc
##
    <fct> <int> <dbl>
                             <dbl>
## 1 S1
           7536 0.141
                               93
## 2 S2
           21860 0.409
                               270
## 3 S3
           13950 0.261
                               172
## 4 S4
           8752 0.164
                               108
## 5 S5
            1363 0.0255
                               17
set.seed(329)
srs_200=ipums_complete %>% slice(sample(1:nrow(ipums_complete),size=200,replace=F))
dim(srs_200)
## [1] 200 17
srs_200 %>%
 summarise(SampleMean=mean(Inctot),
                         SampleVar = var(Inctot),
                         SampleSD = sd(Inctot)) %>%
 gather(stat,val) %>%
 kable(.,format="latex",digits=0) %>%
 kable_styling(.)
```

```
        stat
        val

        SampleMean
        8605

        SampleVar
        88327430

        SampleSD
        9398
```

```
#Pilot SRS of 200
sampleRace = srs_200 %>% count(Race)%>% mutate(prop=n/sum(n),
```

```
prop_alloc = round(prop*200))
sampleSex = srs_200 %>% count(Sex)%>% mutate(prop=n/sum(n),
                                                 prop_alloc = round(prop*200))
sampleMarstat = srs_200 %>% count(Marstat)%>% mutate(prop=n/sum(n),
                                                 prop_alloc = round(prop*200))
sampleAge = srs_200 %>% count(Agecat)%>% mutate(prop=n/sum(n),
                                                 prop_alloc = round(prop*200))
sampleRace
## # A tibble: 5 x 4
   Race
           n prop prop_alloc
   <fct> <int> <dbl>
## 1 1
           176 0.88
                           176
## 2 2
            19 0.095
                            19
## 3 3
            1 0.005
                             1
## 4 4
             3 0.015
                              3
## 5 5
              1 0.005
                              1
sampleSex
## # A tibble: 2 x 4
    Sex
             n prop prop_alloc
   <fct> <int> <dbl> <dbl>
## 1 1
            91 0.455
                             91
## 2 2
            109 0.545
                            109
sampleMarstat
## # A tibble: 5 x 4
## Marstat n prop prop_alloc
   <fct> <int> <dbl>
##
                          <dbl>
            121 0.605
## 1 1
                             121
## 2 2
              4 0.02
                               4
## 3 3
              9 0.045
                                9
## 4 4
              16 0.08
                               16
## 5 5
              50 0.25
                               50
sampleAge
## # A tibble: 5 x 4
    Agecat n prop prop_alloc
## <fct> <int> <dbl>
                           <dbl>
## 1 S1
             26 0.13
## 2 S2
            80 0.4
                              80
## 3 S3
             48 0.24
                              48
              38 0.19
## 4 S4
                              38
## 5 S5
              8 0.04
                              8
#Comparing pilot sample vs. sample in part b
sampleRace
## # A tibble: 5 x 4
    Race
            n prop prop_alloc
   <fct> <int> <dbl>
                        <dbl>
## 1 1
                           176
          176 0.88
## 2 2
            19 0.095
                             19
## 3 3
            1 0.005
                             1
```

```
## 4 4 3 0.015 3
## 5 5 1 0.005 1
counts1
## # A tibble: 5 x 4
## Race n prop_rop_alloc
## <fct> <int> <dbl> <dbl>
## 1 1 46186 0.864
                       570
       5874 0.110
                        73
## 2 2
## 3 3
        325 0.00608
                        4
## 4 4
        886 0.0166
                        11
2
sampleSex
## # A tibble: 2 x 4
## Sex n prop prop_alloc
## <fct> <int> <dbl> <dbl>
## 1 1 91 0.455 91
## 2 2 109 0.545 109
counts2
## # A tibble: 2 x 4
## Sex n prop prop_alloc
## <fct> <int> <dbl> <dbl>
## 1 1 25538 0.478
                      315
## 2 2 27923 0.522 345
sampleMarstat
## # A tibble: 5 x 4
## Marstat n prop prop_alloc
## <fct> <int> <dbl> <dbl>
## 1 1 121 0.605 121
## 2 2 4 0.02 4
          4 0.02
                        4
## 2 2
## 3 3
           9 0.045
           16 0.08 16
50 0.25 50
## 4 4
     50 0.25
## 5 5
counts3
## # A tibble: 5 x 4
## Marstat n prop_alloc
## <fct> <int> <dbl> <dbl>
## 1 1 31160 0.583
                        385
                        15
        1186 0.0222
3298 0.0617
## 2 2
## 3 3
                         41
## 4 4
         4109 0.0769
                         51
169
sampleAge
## # A tibble: 5 x 4
## Agecat n prop prop_alloc
## <fct> <int> <dbl> <dbl>
```

26

80

1 S1 26 0.13

2 S2 80 0.4

## 3	S3	48	0.24	48
## 4	S4	38	0.19	38
## 5	S5	8	0.04	8

countsAge

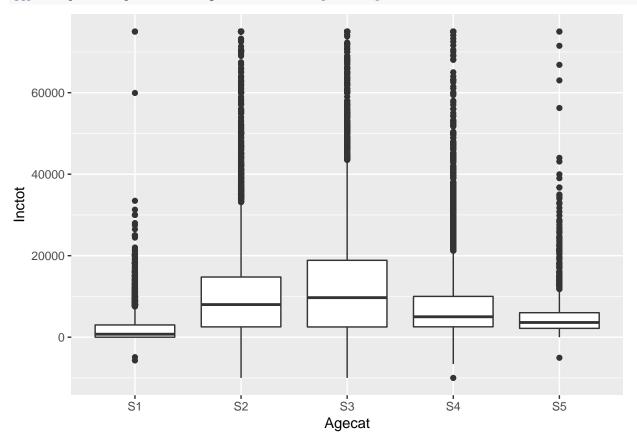
```
## # A tibble: 5 x 4
##
     Agecat
                 n
                     prop prop_alloc
##
     <fct>
                    <dbl>
                                <dbl>
             <int>
  1 S1
             7536 0.141
                                   93
  2 S2
             21860 0.409
                                  270
##
## 3 S3
             13950 0.261
                                  172
                                  108
## 4 S4
             8752 0.164
## 5 S5
             1363 0.0255
                                   17
```

It seems that partitioning data by age is the most promising option here, since there is a seemingly valid explanation why age might affect income (The higher position one gets with respect to career, the more income one would earn).

As we have seen from the table output given by the R chunk above, I have constructed five strata, namely

$$S_1 := (0, 20]$$
 $S_2 := (20, 40]$ $S_3 := (40, 60]$ $S_4 := (60, 80]$ $S_5 := (80.100]$

ggplot(ipums_complete, aes(Agecat, Inctot))+geom_boxplot()



##(b)

Using the strata you constructed, draw a stratified random sample using proportional allocation. Use the same overall sample size you used for your SRS in Exercise 37 of Chapter 2. Explain how you calculated the sample size to be drawn from each stratum.

```
#We have obtained 660 or 661 as optimal sample size in Assignment 1
set.seed(20171015)
strat 660 = inner join(ipums complete,countsAge,by="Agecat") %>% group by(Agecat) %>% slice(sample(1:n
## Warning in 1:n: numerical expression has 7536 elements: only the first used
## Warning in 1:n: numerical expression has 21860 elements: only the first
## used
## Warning in 1:n: numerical expression has 13950 elements: only the first
## used
## Warning in 1:n: numerical expression has 8752 elements: only the first used
## Warning in 1:n: numerical expression has 1363 elements: only the first used
dim(strat_660)
## [1] 660 20
ipums.stratdesign = svydesign(~1,strata=~Agecat,data=strat_660,fpc=~n)
#We find the sampling proportion
sampleProp <- 660/dim(ipums)[1]</pre>
sampleProp
```

[1] 0.01234545

Since we do proportional allocation with $\frac{n}{N} = 0.0123 = 1.24\%$, we sample 1.23% of each population stratum.

##(c)

##(d)

Using the stratified sample you selected with proportional allocation, estimate the total income for the population, along with 95% CI.

```
svytotal(~Inctot,ipums.stratdesign)
```

```
## total SE
## Inctot 484751317 20326751

confint(svytotal(~Inctot,ipums.stratdesign))

## 2.5 % 97.5 %
## Inctot 444911617 524591018
```

Using the pilot sample of size 200 to estimate the within-stratum variances, use optimal allocation to determine sample stratum sizes. Use the same value of n as in part 37b, which is the same n from the SRS in Exercise 37 of Chapter 2. Draw a stratified random sample from the population along with a 95% CI.

Since we have no associated cost function with it, we will assume every sample unit has the same cost. Then the optimal allocation becomes Neyman allocation where for strata h = 1, ...H

$$n_h \propto N_h S_h$$

$$n_h = n \frac{N_h S_h}{\sum_{i=1}^{N} N_i S_i}$$

#strat_200

```
S_i = inner_join(srs_200,sampleAge,by="Agecat") %>% group_by(Agecat) %>% summarise(SampleSD = sd(Inctot
S_i
## # A tibble: 5 x 2
    Agecat SampleSD
##
     <fct>
               <dbl>
## 1 S1
               3373.
## 2 S2
               8101.
## 3 S3
              12841.
## 4 S4
               7892.
## 5 S5
               4556.
N_i <- as.numeric(unlist(countsAge[2]))</pre>
s_i <- as.numeric(unlist(S_i[2]))</pre>
np_i = N_i*s_i/sum(N_i*s_i)
NeymanAge = ipums_complete %>% count(Agecat)%>% mutate(size = N_i, si=s_i, Neyman=np_i, neyman_prop_all
NeymanAge
## # A tibble: 5 x 6
##
     Agecat
                n size
                            si Neyman neyman_prop_alloc
     <fct> <int> <dbl> <dbl> <dbl>
                                                   <dbl>
            7536 7536 3373. 0.0556
## 1 S1
                                                      37
## 2 S2
            21860 21860 8101. 0.388
                                                     256
          13950 13950 12841. 0.392
## 3 S3
                                                     259
## 4 S4
            8752 8752 7892. 0.151
                                                     100
## 5 S5
             1363 1363 4556. 0.0136
                                                       9
set.seed(20171015)
#Due to rounding we are taking one more population unit in our sample
neyman_661 <- inner_join(ipums_complete,NeymanAge,by="Agecat") %>% group_by(Agecat) %>% slice(sample(1::
## Warning in 1:n: numerical expression has 7536 elements: only the first used
## Warning in 1:n: numerical expression has 21860 elements: only the first
## used
## Warning in 1:n: numerical expression has 13950 elements: only the first
## used
## Warning in 1:n: numerical expression has 8752 elements: only the first used
## Warning in 1:n: numerical expression has 1363 elements: only the first used
dim(neyman_661)
## [1] 661 22
ipums.Neymandesign = svydesign(~1,strata=~Agecat,data=neyman_661,fpc=~n)
svytotal(~Inctot,ipums.Neymandesign)
##
              total
                          SF.
## Inctot 493391266 19465182
confint(svytotal(~Inctot,ipums.Neymandesign))
              2.5 %
                       97.5 %
## Inctot 455240209 531542322
```

##(e) Under what conditions can optimal allocation be expected to perform much better than proportional allocation?

Do these conditions exist for this population? Comment on the relative performance you observed between these two allocations.

Optimal allocation (in our case it would be Neyman allocation) is guaranteed to perform better when all the standard deviations are known.

Yes, the conditions do exist since we can find population stratum-within variances:

```
S_perf <- inner_join(srs_200,sampleAge,by="Agecat") %>% group_by(Agecat) %>% summarise(PopulationSD = s
S_perf
## # A tibble: 5 x 2
           Agecat PopulationSD
##
            <fct>
                                              <dbl>
## 1 S1
                                              3373.
## 2 S2
                                              8101.
## 3 S3
                                            12841.
## 4 S4
                                              7892.
## 5 S5
                                              4556.
set.seed(20171015)
s_perf <- as.numeric(unlist(S_perf[2]))</pre>
np_perf = N_i*s_perf/sum(N_i*s_i)
NeymanAge_known = ipums_complete %>% count(Agecat)%>% mutate(size = N_i, si=s_perf, Neyman=np_perf, neyman=np_man=np_perf, neyman=np_man=np_perf, neyman=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_man=np_
NeymanAge_known
## # A tibble: 5 x 6
           Agecat
                                                                    si Neyman neyman prop alloc
                                     n size
##
            <fct> <int> <dbl> <dbl> <dbl>
                                                                                                                           <dbl>
## 1 S1
                              7536 7536 3373. 0.0556
                                                                                                                                  37
## 2 S2
                            21860 21860 8101. 0.388
                                                                                                                               256
## 3 S3
                            13950 13950 12841. 0.392
                                                                                                                               259
## 4 S4
                               8752 8752 7892. 0.151
                                                                                                                               100
## 5 S5
                               1363 1363 4556. 0.0136
neyman_perf <- inner_join(ipums_complete, NeymanAge_known, by="Agecat") %>% group_by(Agecat) %>% slice(sa
## Warning in 1:n: numerical expression has 7536 elements: only the first used
## Warning in 1:n: numerical expression has 21860 elements: only the first
## used
## Warning in 1:n: numerical expression has 13950 elements: only the first
## used
## Warning in 1:n: numerical expression has 8752 elements: only the first used
## Warning in 1:n: numerical expression has 1363 elements: only the first used
dim(neyman_perf)
## [1] 661 22
ipums.Neymanperf = svydesign(~1,strata=~Agecat,data=neyman_perf,fpc=~n)
svytotal(~Inctot,ipums.Neymanperf)
```

```
## total SE
## Inctot 493391266 19465182
confint(svytotal(~Inctot,ipums.Neymanperf))
## 2.5 % 97.5 %
## Inctot 455240209 531542322
```

Returning identical results with our estimated standard deviations with Neyman allocations, and doing significantly better than proportional allocations

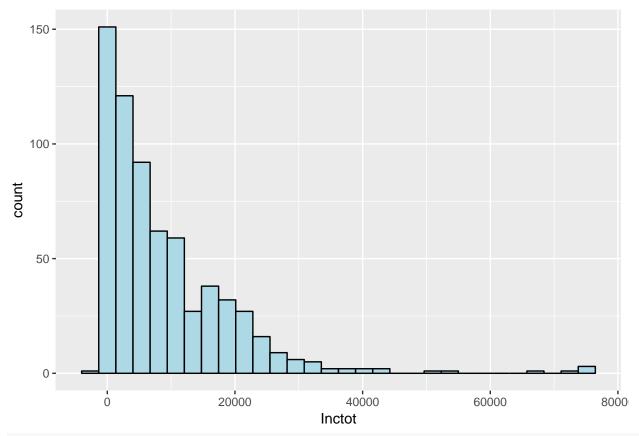
```
SE(Neyman) = 19465182 << 20326751 = SE(proportional)
```

##(f)

Overall, do you think your stratification was worthwhile for sampling from this population? How did your stratified estimates compare with the estimate from the SRS you took in Chapter 2? If you were to start over on the stratification, what would you do differently?

Let us compare the results:

```
For SRS,
set.seed(20171015)
srs_661 = ipums_complete %>% slice(sample(1:nrow(ipums_complete),
                                                size=661, replace=F))
srs_design = survey::svydesign(id=~1,data=srs_661, fpc=rep(53461,661))
svytotal(~Inctot,srs_design)
##
             total
                         SE
## Inctot 4.75e+08 21475362
confint(svytotal(~Inctot, srs_design))
              2.5 %
                       97.5 %
##
## Inctot 432906005 517087877
ggplot(srs_661,aes(x=Inctot)) + geom_histogram(fill="lightblue",col="black")
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



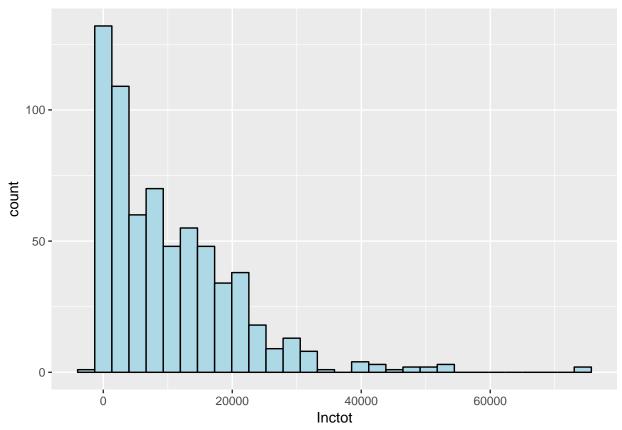
```
svytotal(~Inctot,ipums.Neymanperf)
```

```
confint(svytotal(~Inctot,ipums.Neymanperf))
```

```
## 2.5 % 97.5 %
## Inctot 455240209 531542322
```

ggplot(neyman_perf,aes(x=Inctot)) + geom_histogram(fill="lightblue",col="black")

`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.



Overall, I do think the stratification was worth done. Stratified sample returns a bit higher estimate of total population income compared to SRS

For options of alternative stratification I could have done might be:

- Having combinations of factors to construct strata; an example would be a combination of Sex and Age.
- I might have partitoned the strata with narrower intervals.