MATH 447 - Assignment 1

Dan Yunheum Seol 260677676

## 1.17

We have

and we need to obtain

We use the Law of Iterated Expectation

It follows that

R gives the result of

round((3-3\*exp(-3)\*(1+3))/(1-(exp(-3)+3\*exp(-3)+4.5\*exp(-3))), 5)

## [1] 4.16525

## 1. 28

We have

Then by the Laws of Iterated Expectation and Iterated Variance,

## 2.2

We have as our Markov chain with transition probability matrix , and initial distribution We also have that (the blkarray package had some errors.)

$$

P =

$$

###(a)

###(b)

P1 = matrix(c(0, 0.5, 0.5, 1, 0, 0, 1/3, 1/3, 1/3), nrow=3, ncol=3, byrow=TRUE)  
P1

## [,1] [,2] [,3]  
## [1,] 0.0000000 0.5000000 0.5000000  
## [2,] 1.0000000 0.0000000 0.0000000  
## [3,] 0.3333333 0.3333333 0.3333333

alpha = c(0.5, 0, 0.5)  
alpha

## [1] 0.5 0.0 0.5

alpha %\*% P1

## [,1] [,2] [,3]  
## [1,] 0.1666667 0.4166667 0.4166667

((alpha %\*% P1)[3]\*P1[3][1])

## [1] 0.1388889

5/36

## [1] 0.1388889

###(c)

### Code for computing from textbook

##### Matrix powers ###############################  
# matrixpower(mat,k) mat^k  
#  
matrixpower = function(X, n){  
   
 if(dim(X)[1] != dim(X)[2]){  
 throw("Dimensions of the matrix do not match: ", dim(X))  
 }  
 if (n==0){return(diag(dim(X)[1]))}  
 else if(n >0) {return(X %\*% matrixpower(X, (n-1)))}  
 else {return(matrixpower(solve(X), -n))}  
}

P1sq = matrixpower(P1, 2)  
P1sq

## [,1] [,2] [,3]  
## [1,] 0.6666667 0.1666667 0.1666667  
## [2,] 0.0000000 0.5000000 0.5000000  
## [3,] 0.4444444 0.2777778 0.2777778

alphaP1sq.1 = (alpha %\*% P1sq)[1]  
alphaP1sq.1

## [1] 0.5555556

60/108

## [1] 0.5555556

(5/36)/alphaP1sq.1

## [1] 0.25

###(d)

by Markov property.

P1sq[2][1]

## [1] 0

##2.7

We have , a Markov chain with TPM P.

We need to show that is a Markov chain and show its TPM.

We show that the property $P(Y\_{n+1}=j |Y\_n=i , … Y\_0 = i\_0) = P(Y\_{n+1} = j| Y\_n = i) $ for by induction on n.

###Base case For n= 0

holds trivially.

###Inductive step

Suppose our claim holds for some i.e.

for

by Markov property of .

By this we have shown that is a Markov chain. TPM for will be .

###2.13

$$ $$

where $ = $ (probability of book i being returned). If p\_i = for , we will have

$$ $$