MATH 545 - Assignment 2 - 260677676

Dan Yunheum Seol

2018-10-14

# 2.2

Show that the process

where A and B are independent random variables with mean zero and variance 1. Show that it is stationary and thus function is non-negative definite.

## Solution.

###. X\_t is stationary. We have

and autocovariance

Using the identity

We obtain

Thus being independent of t. is stationary.

###. cos(wh) is non-negative definite. From class we learned that a real-valued function is non-negative definite if and only if the function is a autocovariance function of a stationary process. Since is the ACVF of the stationary process , the function is non-negative definite.

# 2.9

. #2.9

is the AR(1) with white noise, i. e.

and and

for all

## a. Prove is stationary.

We pick the unique stationary solution for

where then we have the mean zero

and autocovariance (i.e. the second moment)

if h = 0

and if h 0

Thus it is stationary.

## b.

Show that

is 1 correlated.

We have

and thus the autocovariance function would be the second moments:

if h= 0

[ \_Y(0)-2\_Y(1) + ^2 \_Y(0) = (1+^2)\_Y(0) -2\_Y(1)

] if h=1

by the linearity of expectation,

and finally if

Now, remember

and

if h is not zero. so

assuming h > 0 (WLOG),

Thus is 1 -correlated.

# 2.13

Given:

* We have 100 samples.
* and

Construct a 95% Confidence interval for

1. Assuming the data is from AR(1) and check whether it stays consistent with the assumption
2. Assuming the data is from MA(1) and check whether it stays consistent with the assumption We have a special case of Bartlett’s formula
3. ##a. It turns out that (from using the formula in textbook example 2.4.4) under the assumption
4. and

And now we use the The formula to construct the confidence interval

Under the assumption, , so the interval constructed for needs to include 0.8, the interval constructed for needs to include

#95% CI for AR(1) model  
rho\_1 = 0.438  
rho\_2 = 0.145  
phi = 0.8  
sqw11.a = sqrt(1- 0.8^2)  
sqw22.a = sqrt((1+0.8^2)^2- 4\* 0.8^4)  
rh1AR.CI <- c(rho\_1-1.96/10\*sqw11.a, rho\_1+1.96/10\*sqw11.a)  
rh2AR.CI <- c(rho\_2-1.96/10\*sqw22.a, rho\_2+1.96/10\*sqw22.a)  
rh1AR.CI

## [1] 0.3204 0.5556

rh2AR.CI

## [1] -0.05595497 0.34595497

(rh1AR.CI[1] < phi && phi < rh1AR.CI[2])

## [1] FALSE

(rh2AR.CI[1] < phi^2 && phi^2 < rh2AR.CI[2])

## [1] FALSE

It seems that neither of the assumed values is captured by the intervals, so we can conclude that the assumption does not stay consistent with the data. ##b. It turns out that

and

so that we can construct the intervals once again symmetrically. Remark that so the first interval needs to capture and since MA(1) is 1-correlated process, it follows that , and consequently the second interval needs to capture 0.

#95% CI for MA(1) model  
theta = 0.6  
rho <- theta/(1+theta^2)  
w11.b = (1-3\*rho\_1^2+4\*rho\_1^4)  
w22.b = (1+2\*rho\_1^2)  
rh1MA.CI <- c(rho\_1 - 1.96/10 \* sqrt(w11.b), rho\_1 + 1.96/10 \* sqrt(w11.b))  
rh2MA.CI <- c(rho\_2 - 1.96/10 \* sqrt(w22.b), rho\_2 + 1.96/10 \*sqrt(w22.b))  
rh1MA.CI

## [1] 0.2898048 0.5861952

rh2MA.CI

## [1] -0.08555533 0.37555533

(rh1MA.CI[1] < rho && rho < rh1AR.CI[2])

## [1] TRUE

(rh2MA.CI[1] < 0 && 0< rh2MA.CI[2])

## [1] TRUE

It seems that both the and value under the assumption are captured by the constructed intervals, thus we can conclude that our data stays consistent with the assumption on

# 2.15

Suppose that with t integers is a stationary process satisfying

where and for Show that

is the best linear predictor.

## Solution.

We look at the mean square error that we need to minimize

We decompose the MSE as follows:

if you differentiate the expression with respect to ’s and set them to 0, you obtain

and

It follows that [

E[(X\_{n+1} - *{i=1}^na\_i X*{n+1-i}) X\_{n+1-i}] =E[(X\_{n+1} - *{i=1}^na\_i X*{n+1-i})] E[X\_{n+1-i}]

]

implying that

If we pick the given solution,

which has mean zero, only leaving MSE of and since for , satisfying the conditions given above. Thus the given solution is the best linear predictor we have. #2.21 Let be observations from the MA(1) model, i.e.

where In class we learned:

and

where the vector is any solution for

with defined as in class. We also know

and

and

##a. find P We have

and

We must solve for that satisfies

Before solving the system, let us simplify the problem a bit

where are the with dropped. We have

so the answer would be

##b. find P We pretty much need to solve the same form of system with the identical and a different , so with a different :

so the so we would have

##c. find P((X\_3 |X\_1, X\_2, X\_4, X\_5) We would need to use a different and this time, dropping sigma,

with

So we have

##d. Since we know

We have

We know that

, thus

and symmetrically

.

Finally,