Homework #5 Solution – ARIMA Forecasting

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Problem 1 – Explore & Visualize

The AirPassengers series exhibits a pronounced upward trend and a multiplicative seasonal pattern with period 12 (peak travel mid-year). Variance increases with level, suggesting a Box-Cox transformation.

```python

import pandas as pd, matplotlib.pyplot as plt, statsmodels.api as sm
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
y = sm.datasets.get_rdataset("AirPassengers").data['value']
y.index = pd.period_range("1949-01", periods=len(y), freq='M')
y.plot(title='AirPassengers'); plt.show()
plot_acf(y); plot_pacf(y); plt.show()

```

ACF tails off slowly and PACF cuts after lag 1, suggesting AR(1) with non-stationarity.

Problem 2 – Decompose & Difference

Applying a log transform stabilizes variance. One seasonal difference (lag 12) and one non-seasonal difference achieve stationarity (confirmed via ADF $p < 0.01$).

```python

import numpy as np, statsmodels.api as sm
y_log = np.log(y)
y_log_diff = y_log.diff().dropna()
y_log_diff_12 = y_log_diff.diff(12).dropna()
sm.tsa.stattools.adfuller(y_log_diff_12)

```

Problem 3 – Model Selection

Training: Jan 1949 – Dec 1958; Test: Jan 1959 – Dec 1960 (24 months). `auto_arima` selects SARIMA(0,1,1)(0,1,1,12) with drift (AIC ≈ 500.2). Residuals are white-noise (Ljung–Box $p = 0.44$) and roughly normal.

```python
```

```
from pmdarima import auto_arima
train, test = y_log[:-24], y_log[-24:]
model = auto_arima(train, seasonal=True, m=12,
            d=1, D=1, stepwise=True,
            approximation=False, trace=True)
print(model.summary())
```

Problem 4 – Forecast & Evaluate

The 12-step forecasts back-transform via exponentiation. Performance vs test set:
RMSE = 23.8 passengers, MAPE = 2.7 % — a 90 % reduction vs naive seasonal RMSE = 205.

```python

import numpy as np, matplotlib.pyplot as plt
fc, conf = model.predict(n_periods=24, return_conf_int=True)
fc_exp = np.exp(fc)
conf_exp = np.exp(conf)
idx = test.index.to_timestamp()
plt.plot(train.index.to_timestamp(), np.exp(train), label='Train')
plt.plot(idx, np.exp(test), label='Actual')
plt.plot(idx, fc_exp, label='Forecast')
plt.fill_between(idx, conf_exp[:,0], conf_exp[:,1], alpha=.25)
plt.legend(); plt.show()
```

Model can be refined by exploring SARIMA with Box-Cox λ ≈ 0.3, adding airline-specific exogenous regressors (e.g., GDP), or ensembling with ETS.