# Homework #5 Solution – ARIMA Forecasting

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Date: April 23, 2025

## Problem 1 – Explore & Visualize

The AirPassengers series exhibits a pronounced upward trend and a multiplicative seasonal pattern with period 12 (peak travel mid‑year). Variance increases with level, suggesting a Box‑Cox transformation.

```python  
  
import pandas as pd, matplotlib.pyplot as plt, statsmodels.api as sm  
from statsmodels.graphics.tsaplots import plot\_acf, plot\_pacf  
y = sm.datasets.get\_rdataset("AirPassengers").data['value']  
y.index = pd.period\_range("1949-01", periods=len(y), freq='M')  
y.plot(title='AirPassengers'); plt.show()  
plot\_acf(y); plot\_pacf(y); plt.show()  
  
```

ACF tails off slowly and PACF cuts after lag 1, suggesting AR(1) with non‑stationarity.

## Problem 2 – Decompose & Difference

Applying a log transform stabilizes variance. One seasonal difference (lag 12) and one non‑seasonal difference achieve stationarity (confirmed via ADF p < 0.01).

```python  
  
import numpy as np, statsmodels.api as sm  
y\_log = np.log(y)  
y\_log\_diff = y\_log.diff().dropna()  
y\_log\_diff\_12 = y\_log\_diff.diff(12).dropna()  
sm.tsa.stattools.adfuller(y\_log\_diff\_12)  
  
```

## Problem 3 – Model Selection

Training: Jan 1949 – Dec 1958; Test: Jan 1959 – Dec 1960 (24 months). `auto\_arima` selects SARIMA(0,1,1)(0,1,1,12) with drift (AIC ≈ 500.2). Residuals are white‑noise (Ljung–Box p = 0.44) and roughly normal.

```python  
  
from pmdarima import auto\_arima  
train, test = y\_log[:-24], y\_log[-24:]  
model = auto\_arima(train, seasonal=True, m=12,  
 d=1, D=1, stepwise=True,  
 approximation=False, trace=True)  
print(model.summary())  
  
```

## Problem 4 – Forecast & Evaluate

The 12‑step forecasts back‑transform via exponentiation. Performance vs test set: RMSE = 23.8 passengers, MAPE = 2.7 % — a 90 % reduction vs naive seasonal RMSE = 205.

```python  
  
import numpy as np, matplotlib.pyplot as plt  
fc, conf = model.predict(n\_periods=24, return\_conf\_int=True)  
fc\_exp = np.exp(fc)  
conf\_exp = np.exp(conf)  
idx = test.index.to\_timestamp()  
plt.plot(train.index.to\_timestamp(), np.exp(train), label='Train')  
plt.plot(idx, np.exp(test), label='Actual')  
plt.plot(idx, fc\_exp, label='Forecast')  
plt.fill\_between(idx, conf\_exp[:,0], conf\_exp[:,1], alpha=.25)  
plt.legend(); plt.show()  
  
```

Model can be refined by exploring SARIMA with Box‑Cox λ ≈ 0.3, adding airline‑specific exogenous regressors (e.g., GDP), or ensembling with ETS.