A Hypothesized Lower Bound for VC Dimension Based on Degrees of Freedom

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It is said that intuitively, the VC dimension measures the number of degrees of freedom of a set of hypotheses. However, there are cases where the number of degrees of freedom is clearly less than the VC dimension. We provide a topological definition of "degrees of freedom" that matches up with intuition in many common cases, and present the hypothesis that this is a lower bound for the VC dimension. We provide examples of cases where the intuitive number of degrees of freedom appears to be more than the VC dimension, and show that surprisingly these seem to be the cases where our topological definition disagrees with intuition. We prove our hypothesis in the case where the number of degrees of freedom is less than 3 (that is, 0, 1, or 2), provide a counterexample to a slightly stronger version of the hypothesis, and give some rather broad consequences that would easily follow.