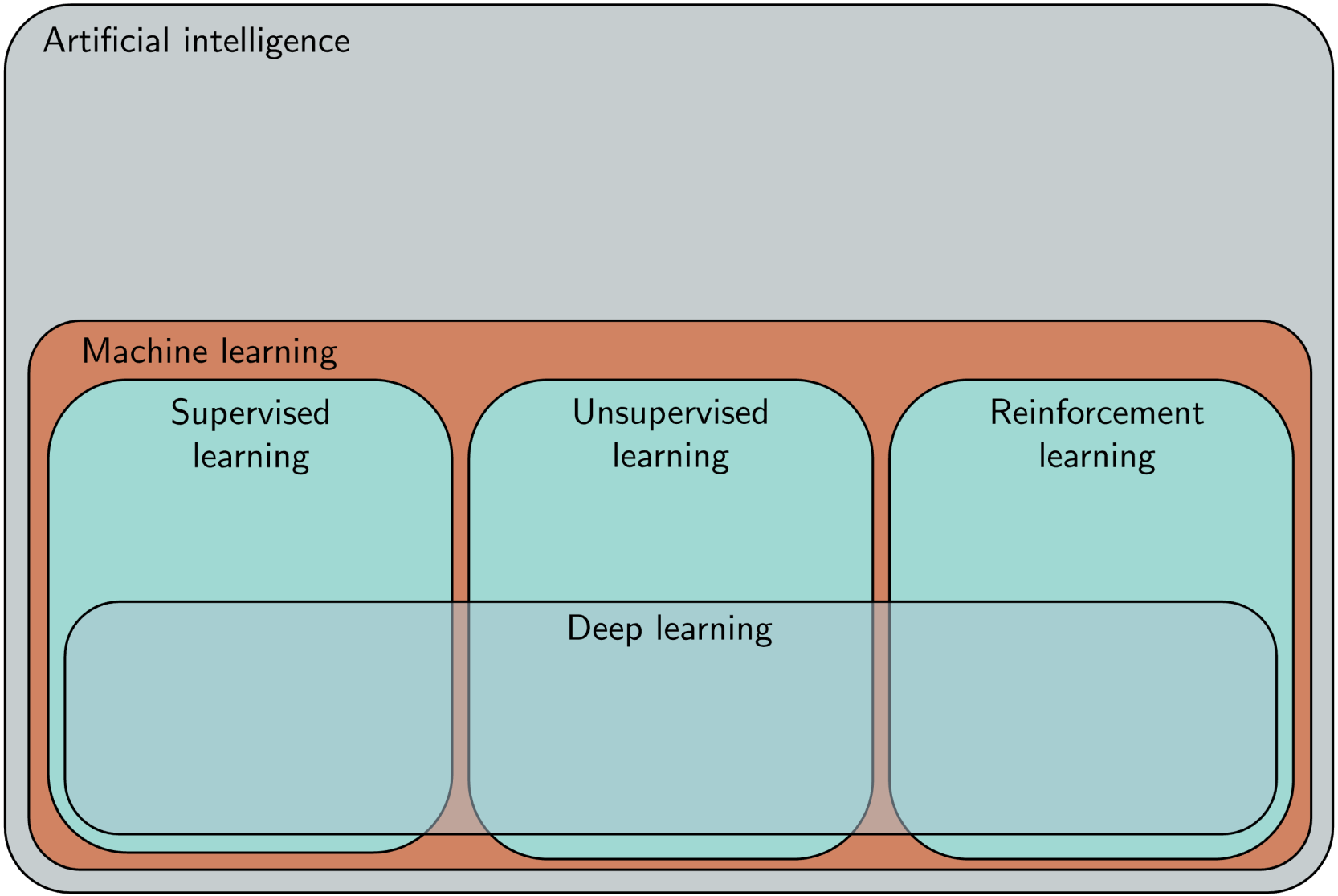
UNIVERSITY OF BATH

CM20315 - Machine Learning

Prof. Simon Prince

2. Supervised learning


Artificial intelligence
Machine learning
Supervised learning
Unsupervised learning
Reinforcement learning
Deep learning

Supervised learning

Bicycle

Apple

Aardvark

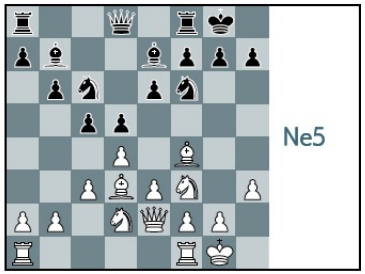
Supervised learning

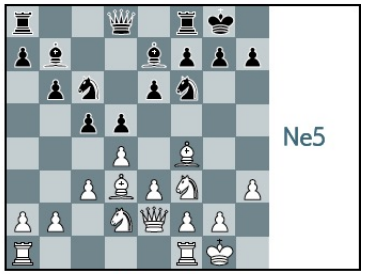
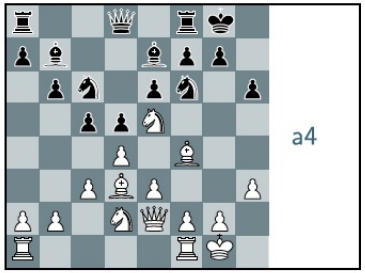
Bicycle

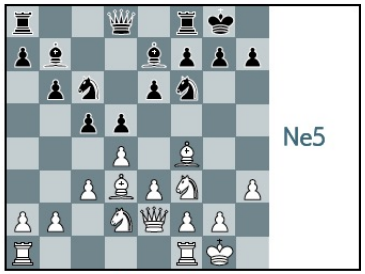
Apple

Aardvark

Unsupervised learning

Supervised learning	Unsupervised learning	Reinforcement learning
 Bicycle		 Reward = 0
 Apple		
 Aardvark		

Supervised learning	Unsupervised learning	Reinforcement learning
 Bicycle		 Reward = 0
 Apple		 Reward = -1
 Aardvark		

Supervised learning	Unsupervised learning	Reinforcement learning
		
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		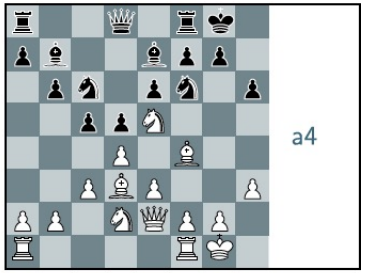
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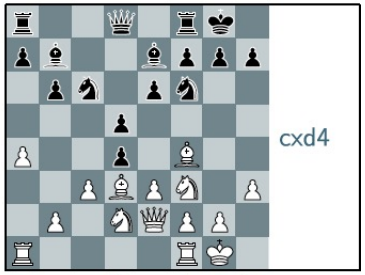
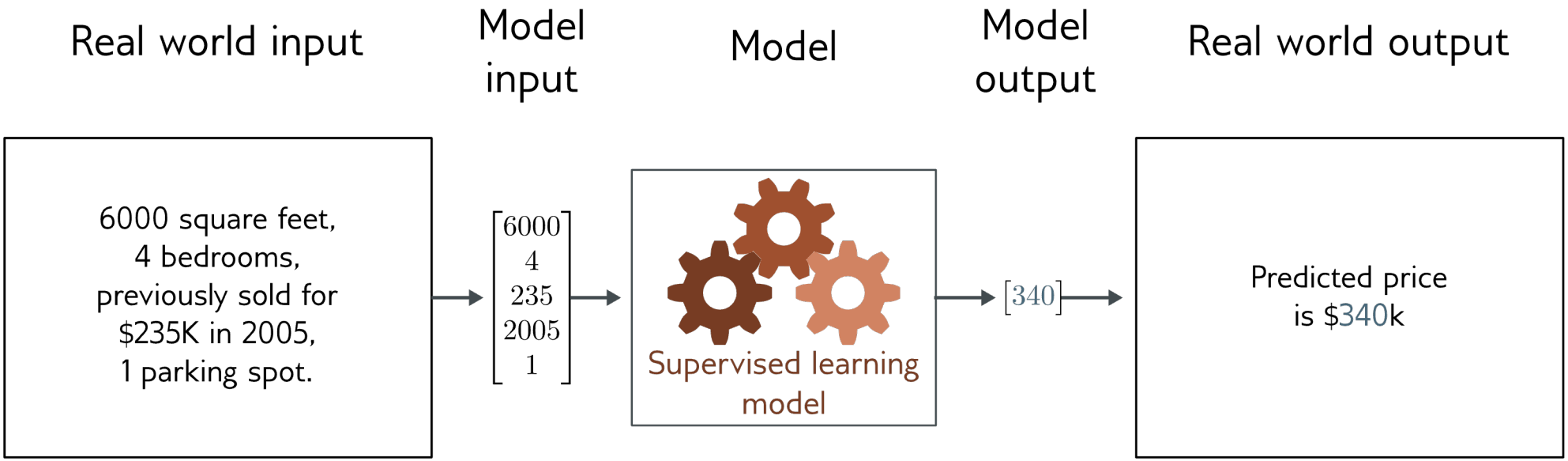
		
Aardvark		Reward = +1

Regression



- Univariate regression problem (one output, real value)

Supervised learning

- Overview
- Notation
  - Model
  - Loss function
  - Training
  - Testing
- 1D Linear regression example
  - Model
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- Where are we going?

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Supervised learning overview

- Supervised learning model = mapping from one or more inputs to one or more outputs
- Model is a mathematical equation

- Computing the inputs from the outputs = inference

Supervised learning overview

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- Example:
  - Input is age and milage of secondhand Toyota Prius
  - Output is estimated price of car

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- Parameters affect outcome of equation

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Supervised learning overview

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- Computing the inputs from the outputs = inference
- Model also includes parameters
- Parameters affect outcome of equation
- Training a model = finding parameters that predict outputs “well” from inputs for a training dataset of input/output pairs

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Appendix A

Notation

This appendix details the notation used in this book. This mostly adheres to standard conventions in computer science, but deep learning is applicable to many different areas, so it is explained in full. In addition, there are several notational conventions that are unique to this book, including notation for functions and the systematic distinction between parameters and variables.

Scalars, vectors, matrices, and tensors

Scalars are denoted by either small or capital letters a, A, α. Column vectors (i.e., 1D arrays of numbers) are denoted by small bold letters $\mathbf{a}, \boldsymbol{\phi}$, and row vectors as the transpose of column vectors $\mathbf{a}^T, \boldsymbol{\phi}^T$. Matrices and tensors (i.e., 2D and ND arrays of numbers, respectively) are both represented by bold capital letters $\mathbf{B}, \boldsymbol{\Phi}$.

Variables and parameters

Variables (usually the inputs and outputs of functions or intermediate calculations) are always denoted by Roman letters $a, \mathbf{b}, \mathbf{C}$. Parameters (which are internal to functions or probability distributions) are always denoted by Greek letters $\alpha, \boldsymbol{\beta}, \boldsymbol{\Gamma}$. Generic, unspecified parameters are denoted by $\boldsymbol{\phi}$. This distinction is retained throughout the book except for the policy in reinforcement learning, which is denoted by π according to the usual convention.

Sets

Sets are denoted by curly brackets, so $\{0, 1, 2\}$ denotes the numbers 0, 1, and 2. The notation $\{0, 1, 2, \ldots\}$ denotes the set of non-negative integers. Sometimes, we want to specify a set of variables and $\{\mathbf{x}_i\}_{i=1}^{I}$ denotes the I variables $\mathbf{x}_1, \ldots \mathbf{x}_I$. When it's not necessary to specify how many items are in the set, this is shortened to $\{\mathbf{x}_i\}$. The notation $\{\mathbf{x}_i, \mathbf{y}_i\}_{i=1}^{I}$ denotes the set of I pairs $\mathbf{x}_i, \mathbf{y}_i$. The convention for naming sets is to use calligraphic letters. Notably, $\mathcal{B}_t$ is used to denote the set of indices in a batch at iteration t during training. The number of elements in a set $\mathcal{S}$ is denoted by $|\mathcal{S}|$.

The set $\mathbb{R}$ denotes the set of real numbers. The set $\mathbb{R}^+$ denotes the set of non-negative real numbers. The notation $\mathbb{R}^D$ denotes the set of D-dimensional vectors containing real

Notation:

- Input:

$$\mathbf{x}$$

- Output:

$$\mathbf{y}$$

- Model:

$$\mathbf{y} = \mathbf{f}[\mathbf{x}]$$

Variables always Roman letters

Normal = scalar
Bold = vector
Capital Bold = matrix

Functions always square brackets

Normal = returns scalar
Bold = returns vector
Capital Bold = returns matrix

Notation example:

- Input:

$$\mathbf{x} = \begin{bmatrix} \text{age} \\ \text{mileage} \end{bmatrix}$$

Structured or tabular data

- Output:

$$y = [\text{price}]$$

- Model:

$$y = \text{f}[\mathbf{x}]$$

Model

- Parameters:

$$\boldsymbol{\phi}$$

- Model :

$$\mathbf{y} = \mathbf{f}[\mathbf{x}, \boldsymbol{\phi}]$$

Parameters always Greek letters

Loss function

- Training dataset of I pairs of input/output examples:

$$\{\mathbf{x}_i, \mathbf{y}_i\}_{i=1}^{I}$$

- Loss function or cost function measures how bad model is:

$$L\Big[\underbrace{\boldsymbol{\phi}, \mathrm{f}[\mathbf{x}, \boldsymbol{\phi}]}_{\text{model}}, \underbrace{\{\mathbf{x}_i, \mathbf{y}_i\}_{i=1}^{I}}_{\text{train data}}\Big]$$

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or for short:

$$L\left[\boldsymbol{\phi}\right]$$

Returns a scalar that is smaller when model maps inputs to outputs better

Training

- Loss function:

$$L\left[\boldsymbol{\phi}\right]$$

Returns a scalar that is smaller when model maps inputs to outputs better

- Find the parameters that minimize the loss:

$$\hat{\boldsymbol{\phi}} = \underset{\boldsymbol{\phi}}{\operatorname{argmin}}\Big[\mathrm{L}\left[\boldsymbol{\phi}\right]\Big]$$

Testing

- To test the model, run on a separate test dataset of input / output pairs

- See how well it generalizes to new data

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Example: 1D Linear regression model

- Model:

$$y = \mathrm{f}[x, \boldsymbol{\phi}]$$
$$= \phi_0 + \phi_1 x$$

- Parameters

$$\boldsymbol{\phi} = \begin{bmatrix} \phi_0 \\ \phi_1 \end{bmatrix}$$

ϕ_0 ← y-offset

ϕ_1 ← slope

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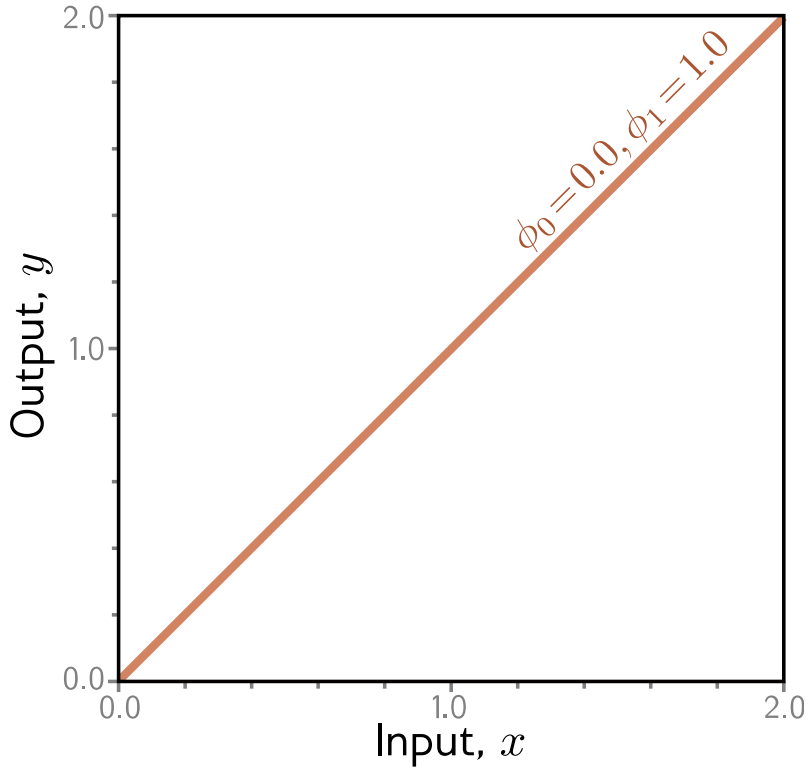

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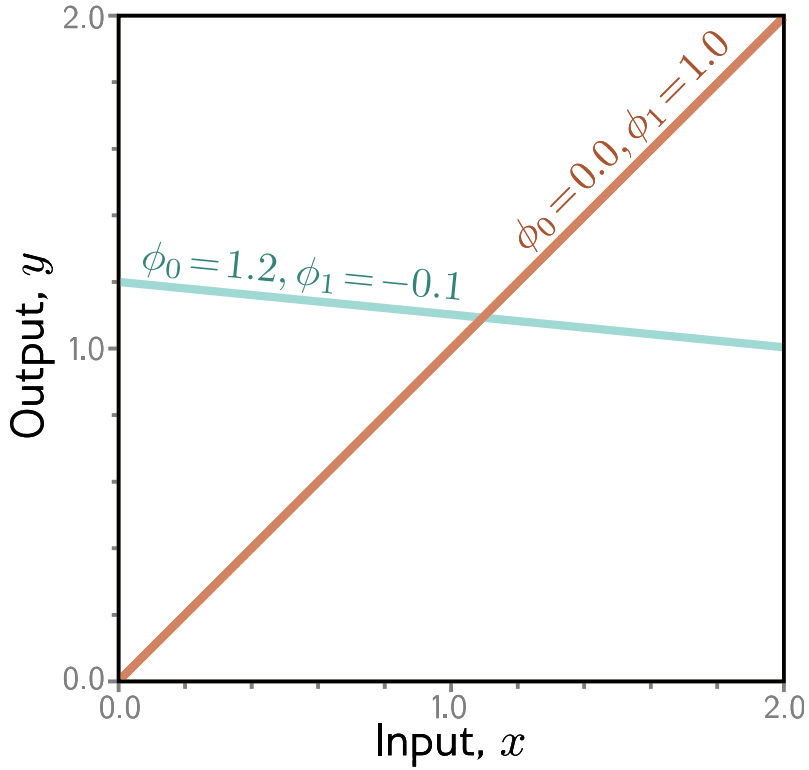
← slope



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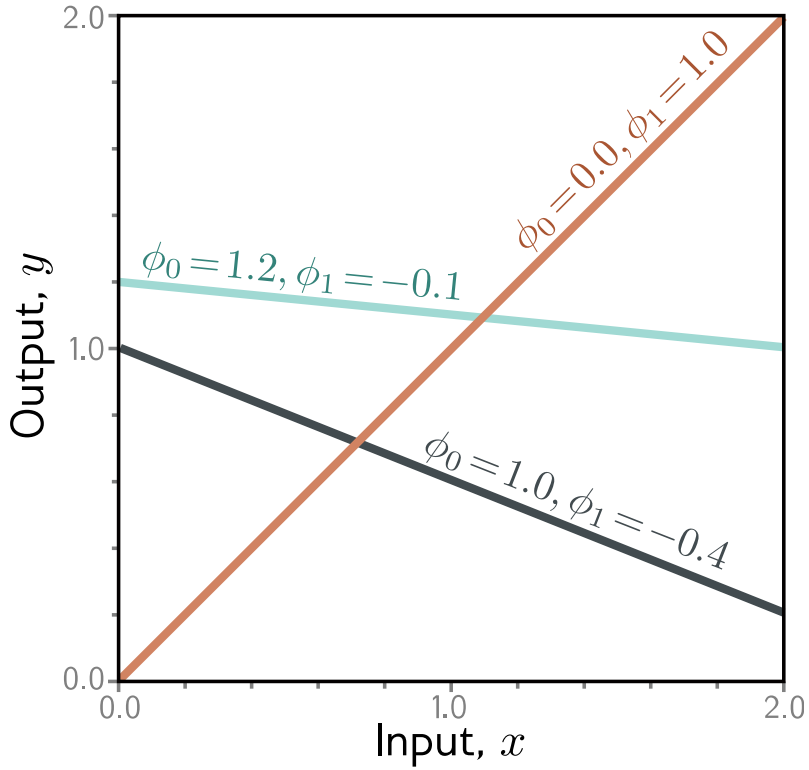
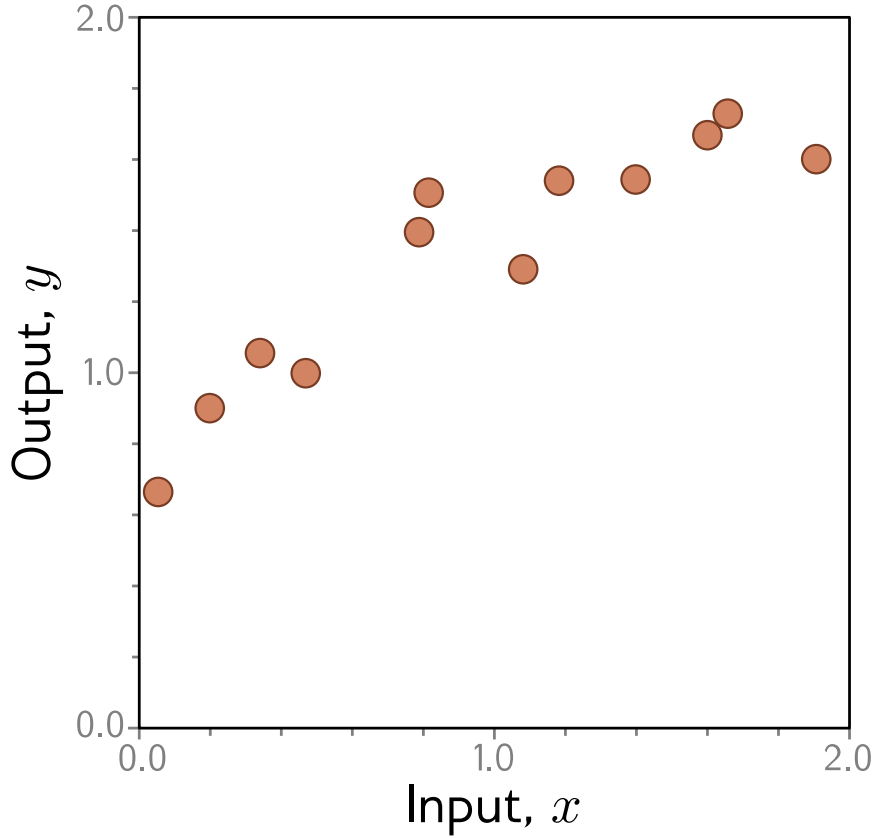


Example: 1D Linear regression training data



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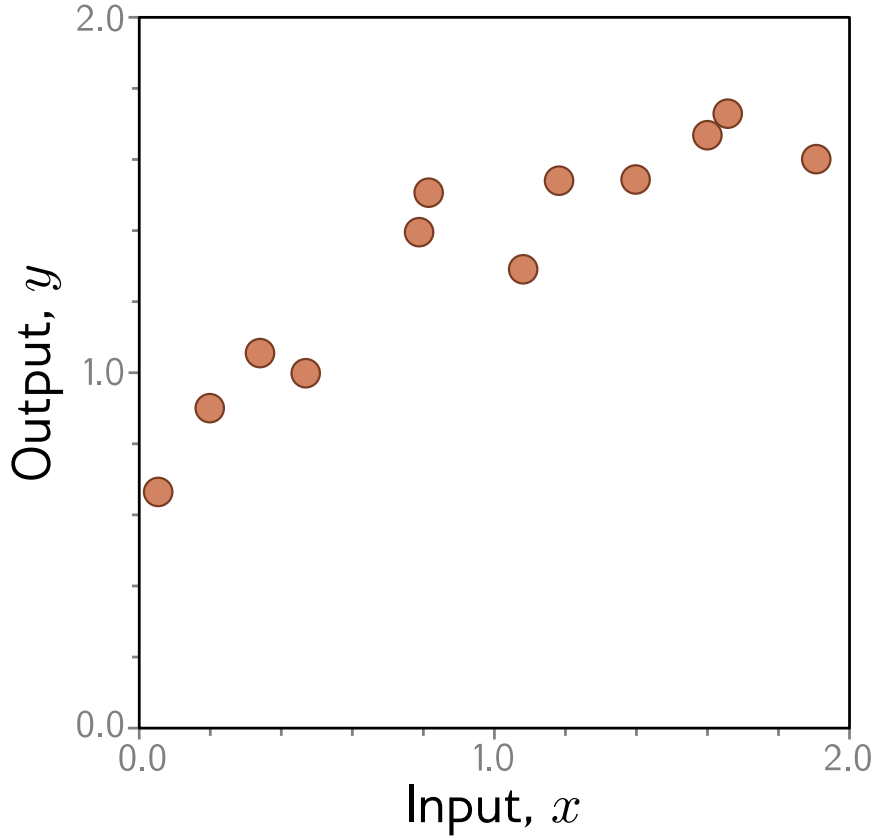


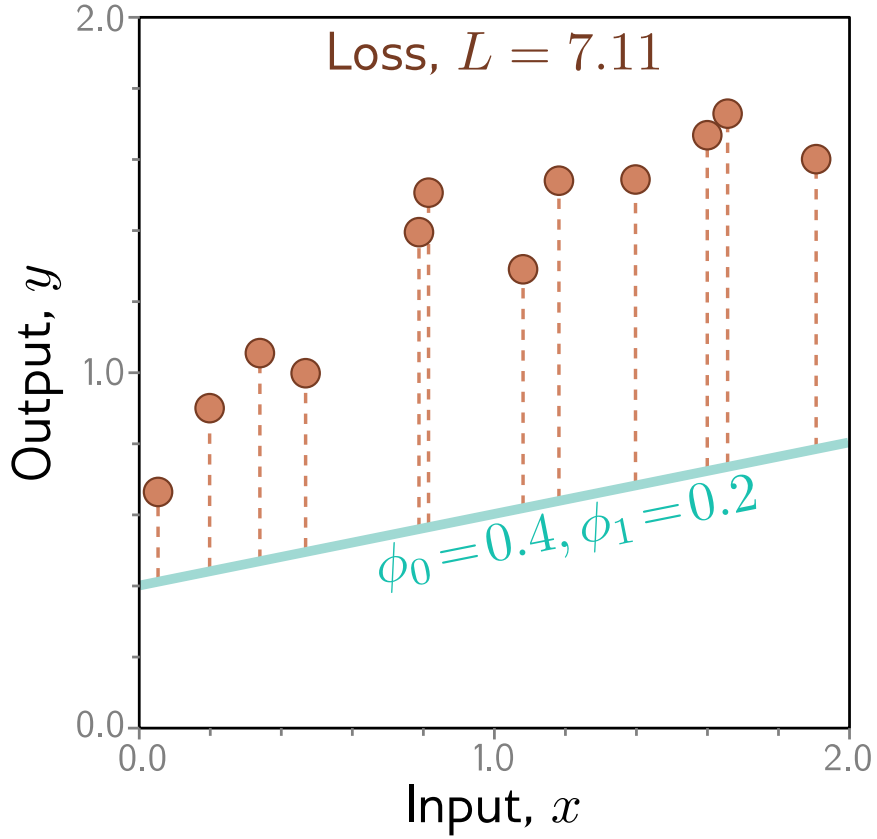
Loss function:

$$L[\boldsymbol{\phi}] = \sum_{i=1}^{I} (\mathrm{f}[x_i, \boldsymbol{\phi}] - y_i)^2$$

$$= \sum_{i=1}^{I} (\phi_0 + \phi_1 x_i - y_i)^2$$

"Least squares loss function"

Example: 1D Linear regression loss function



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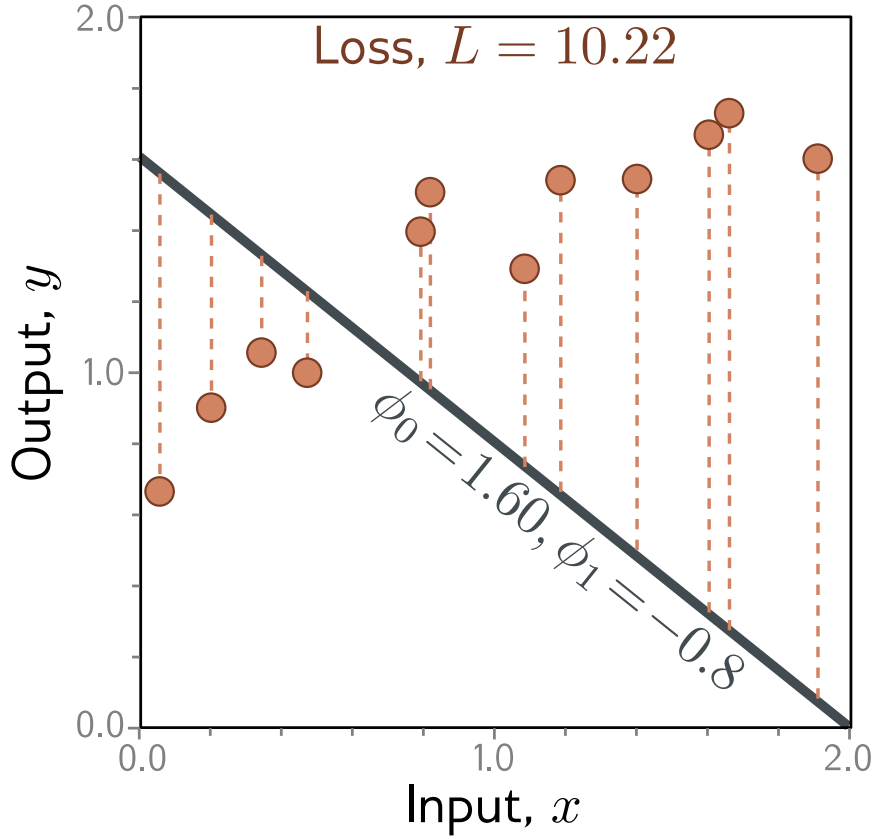

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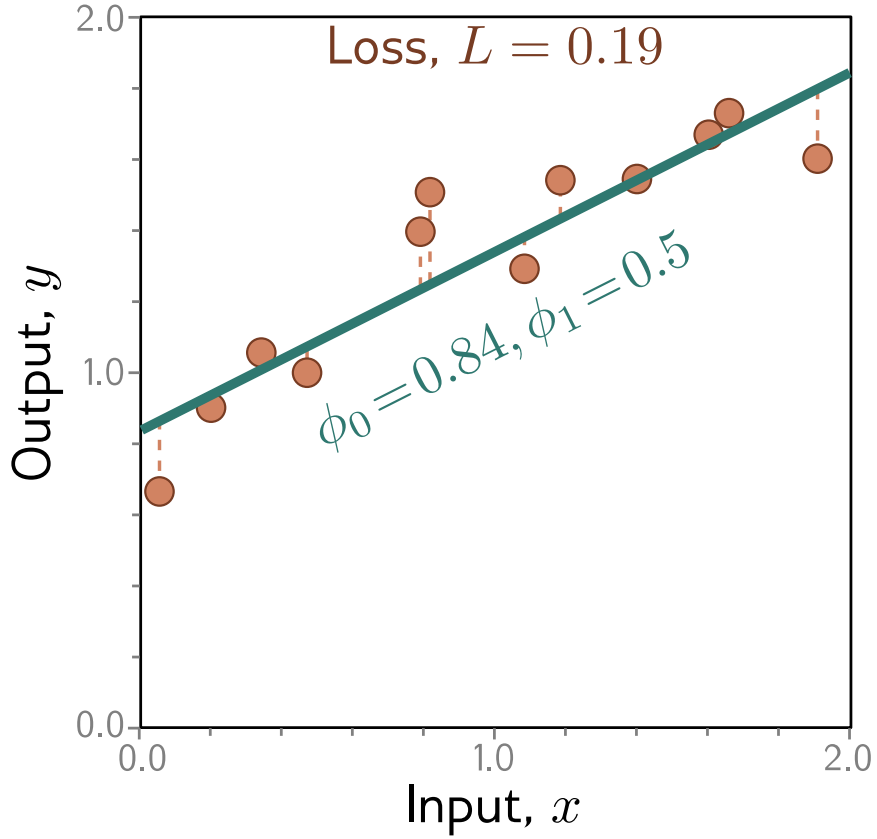

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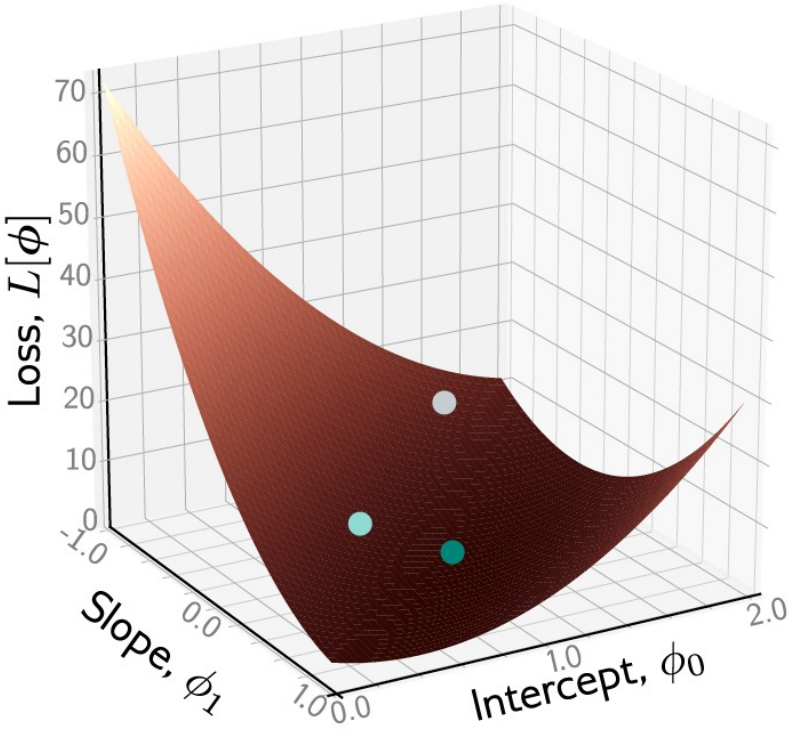

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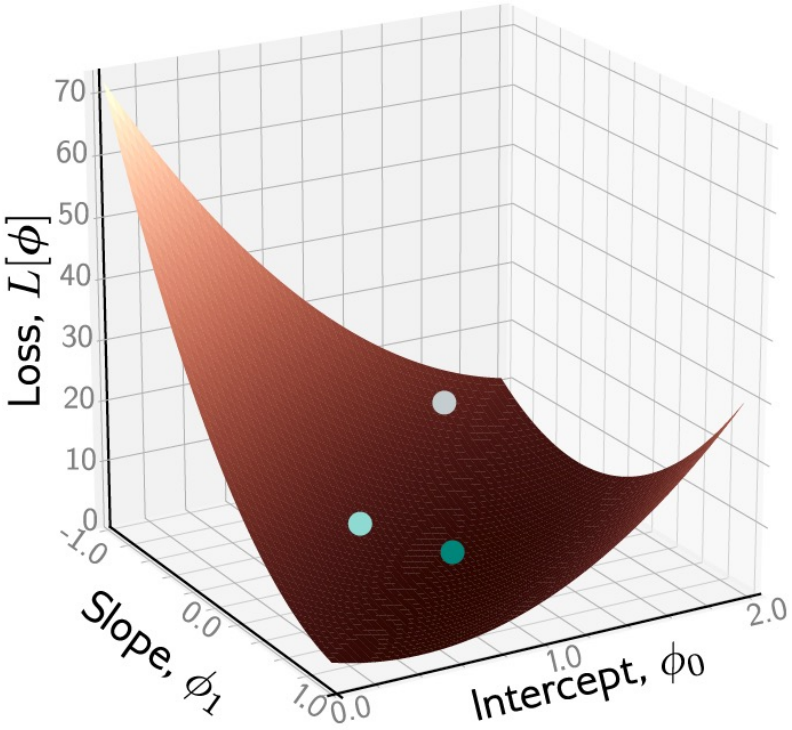

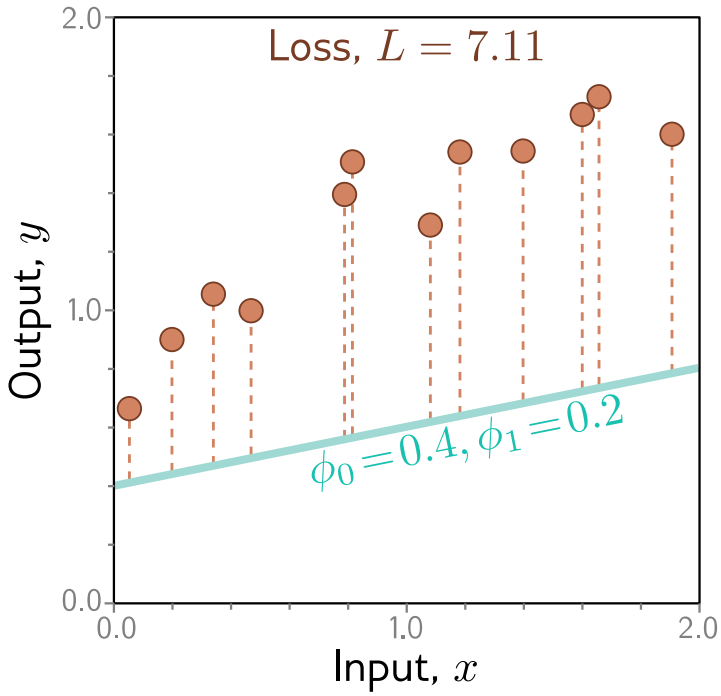

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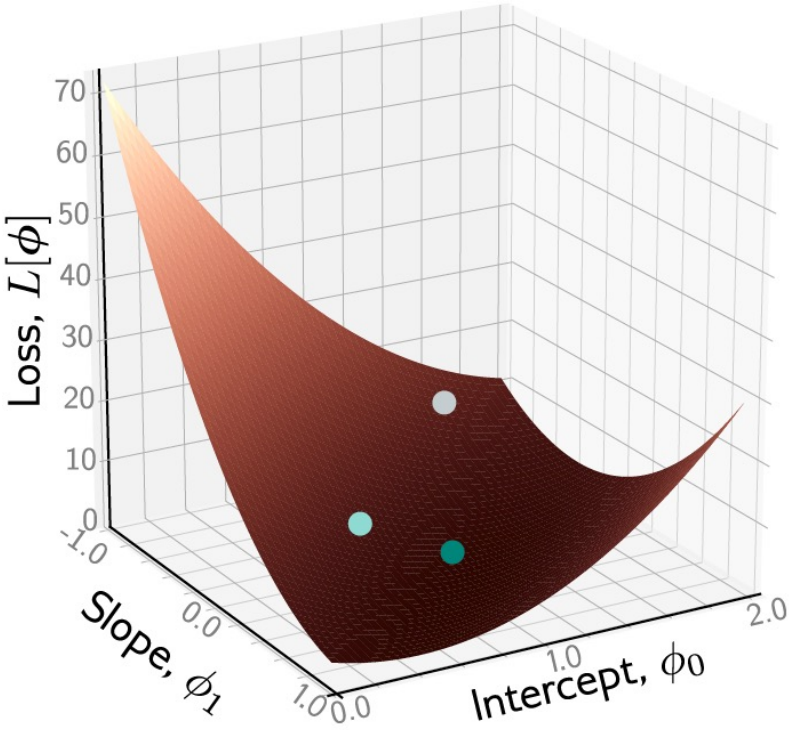

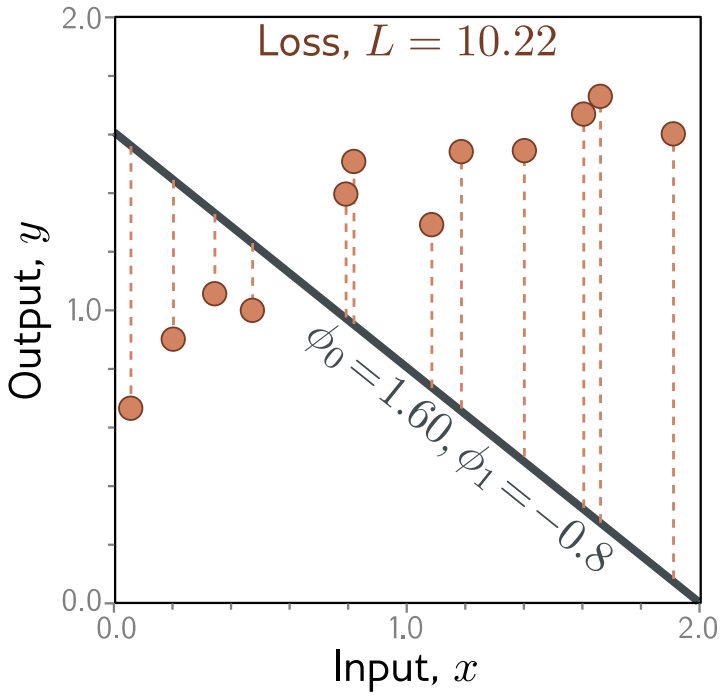

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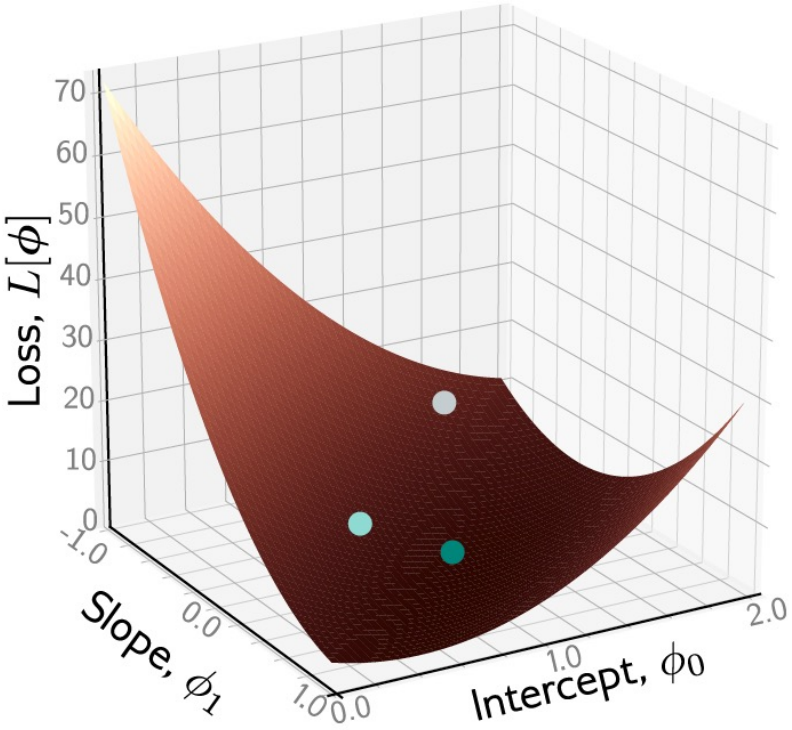

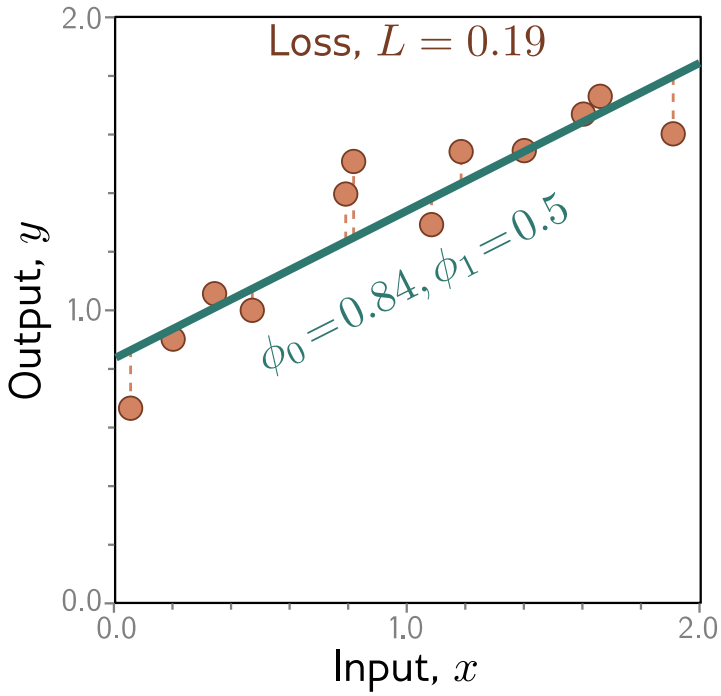

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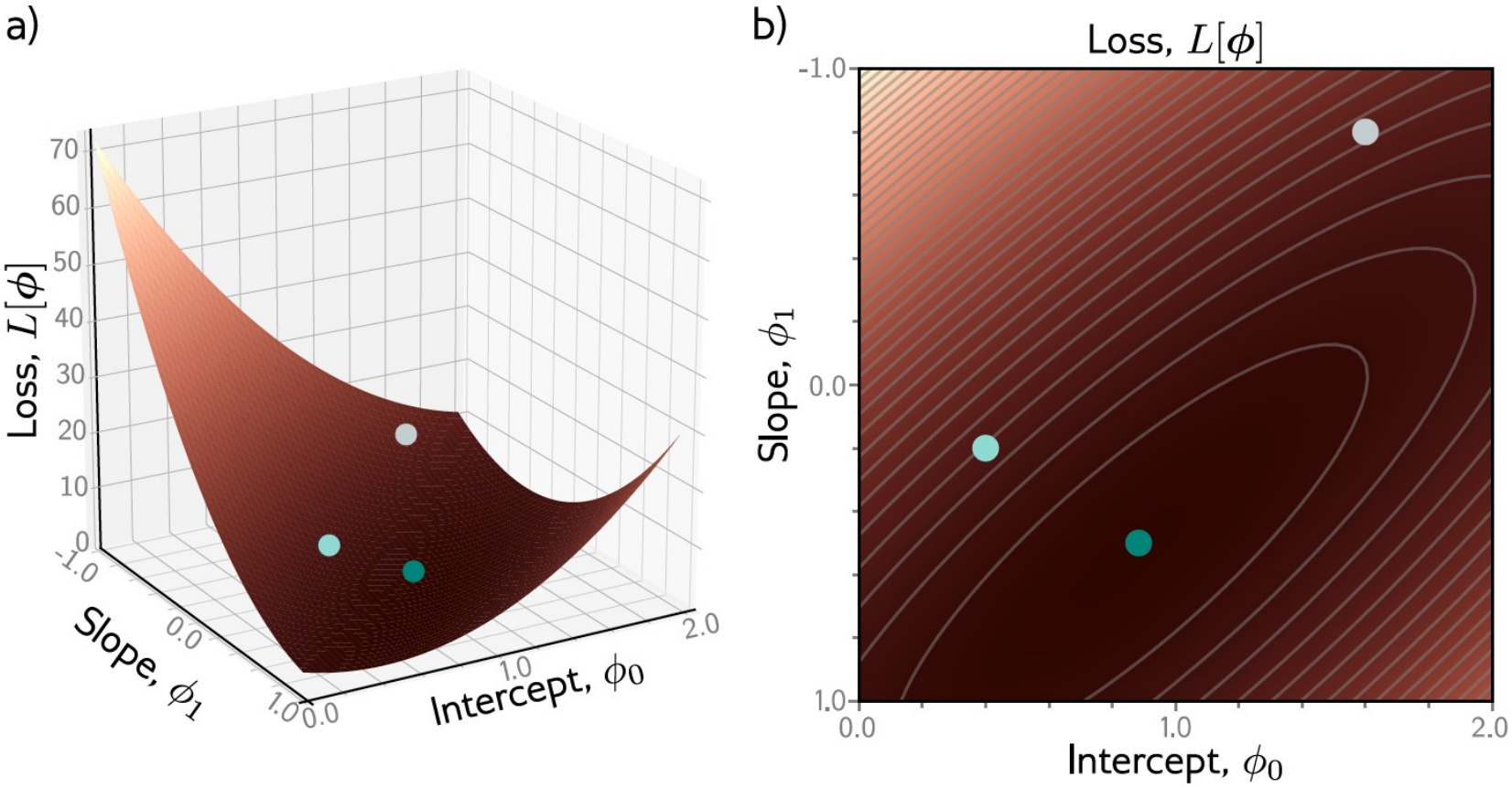

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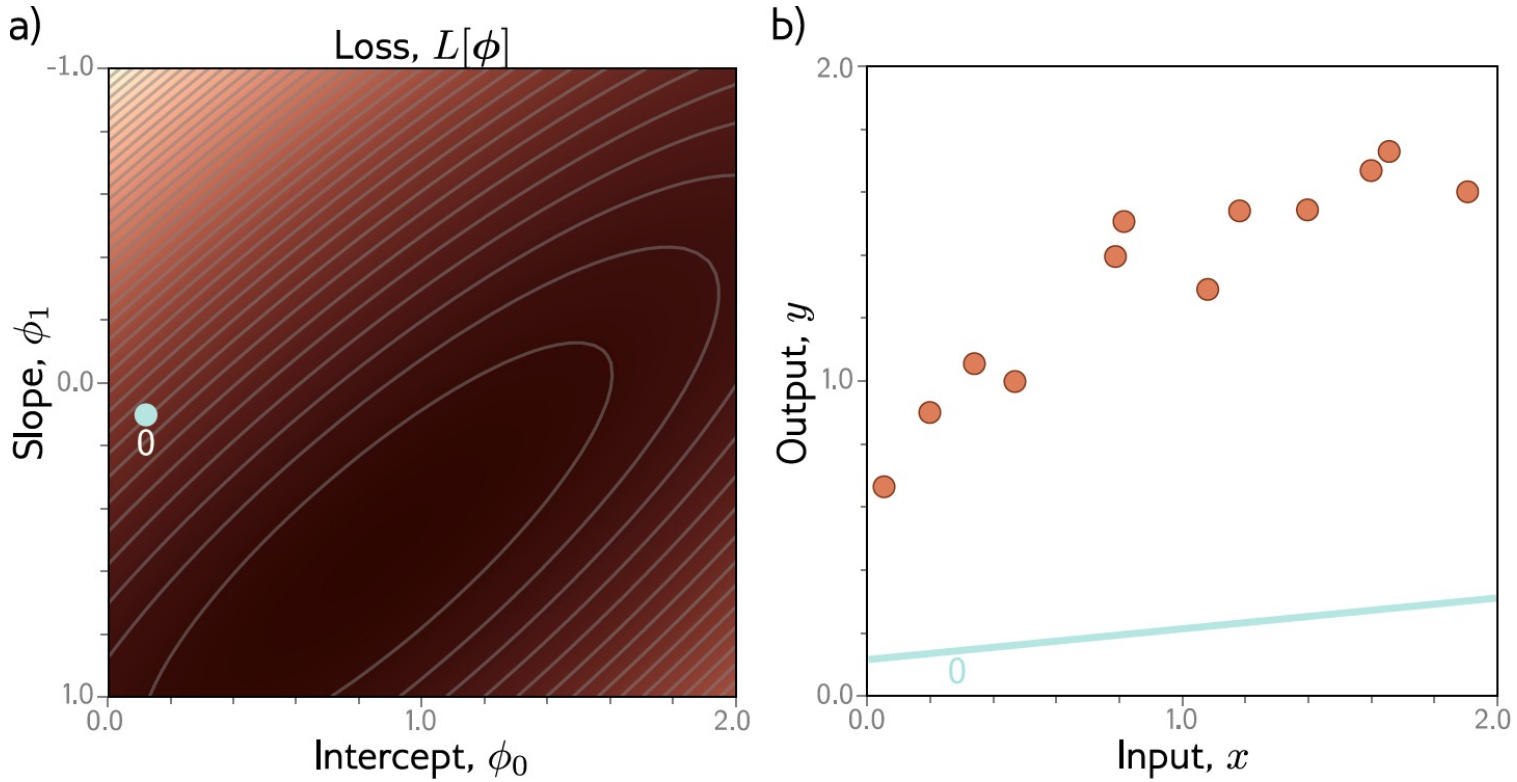

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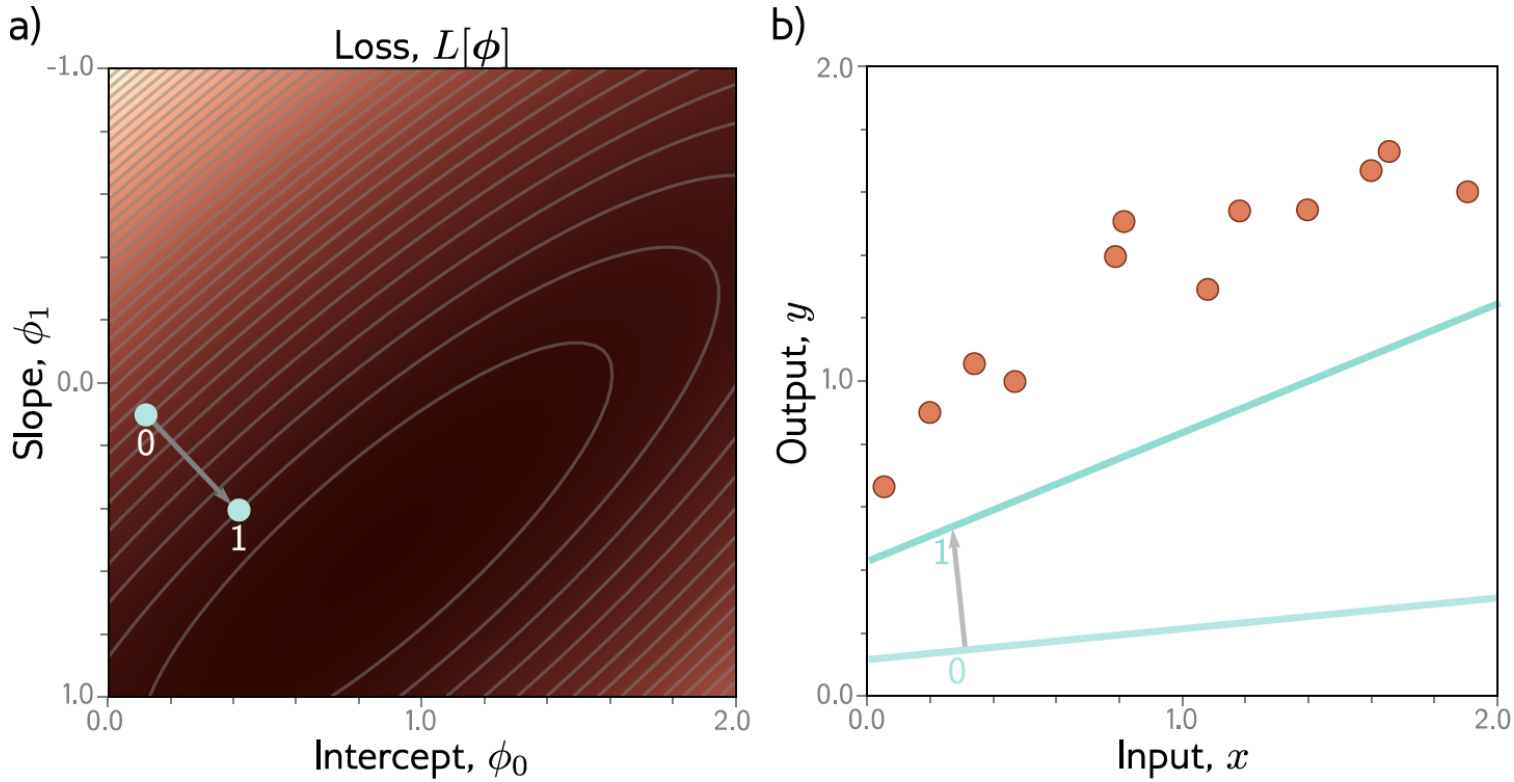

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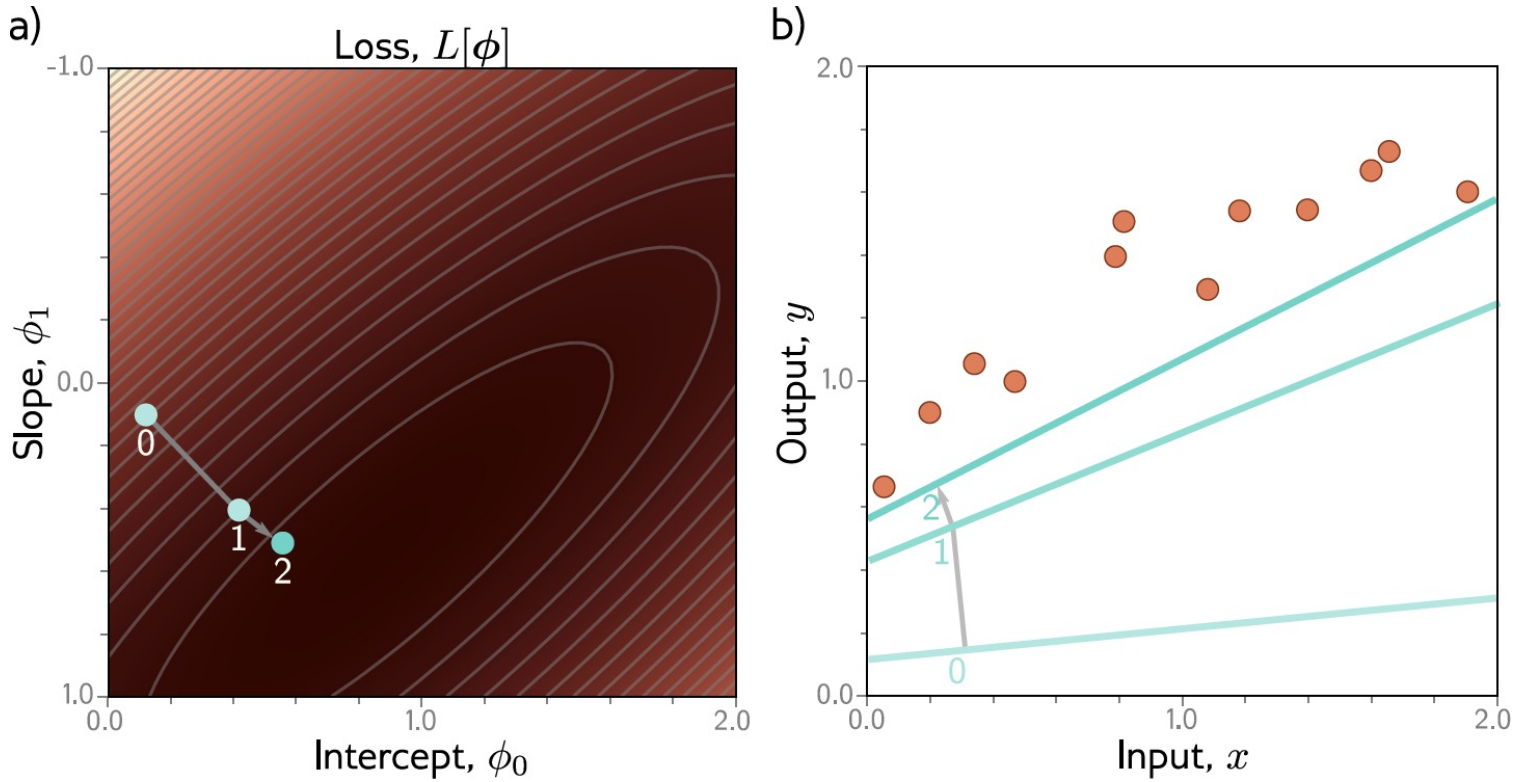

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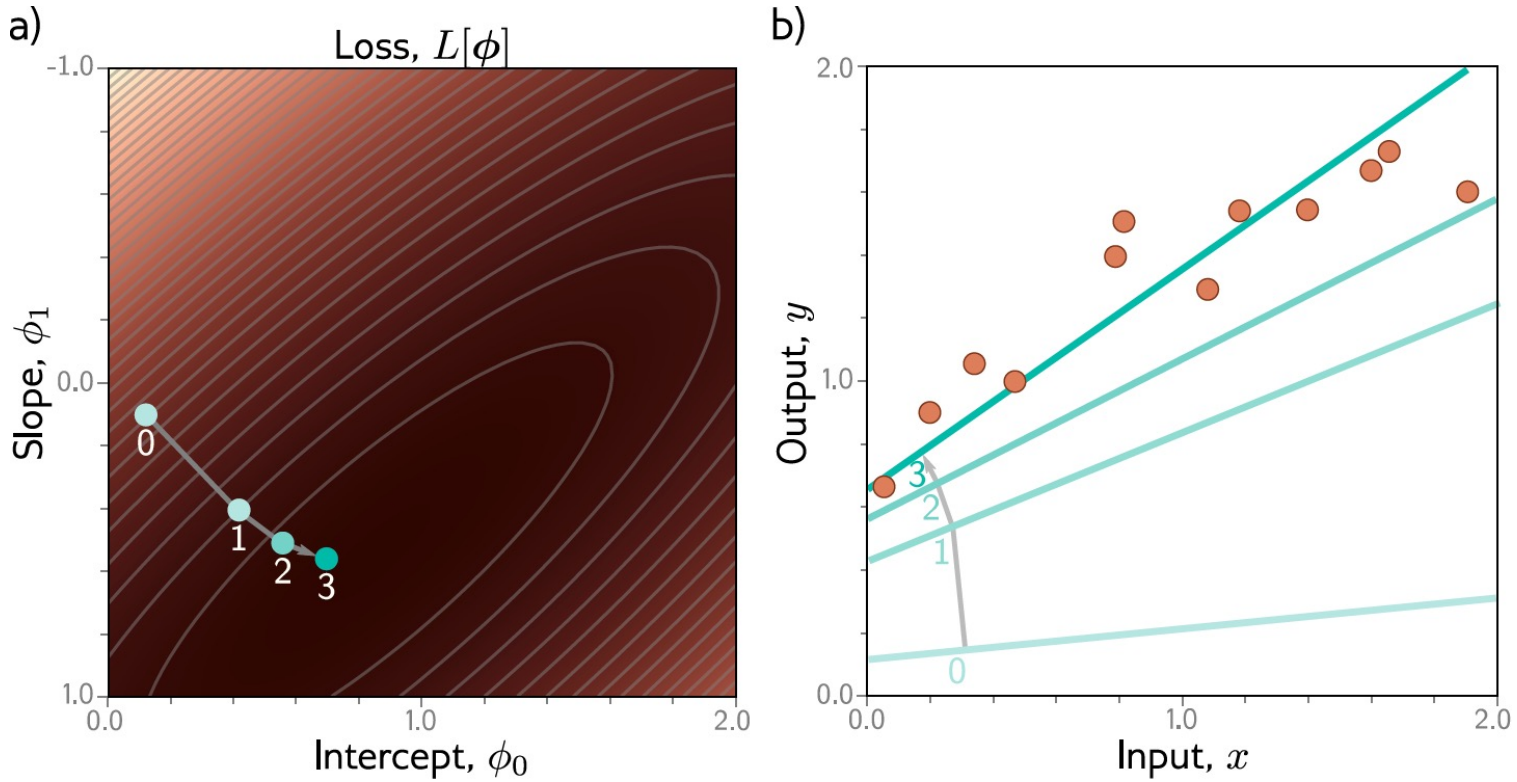

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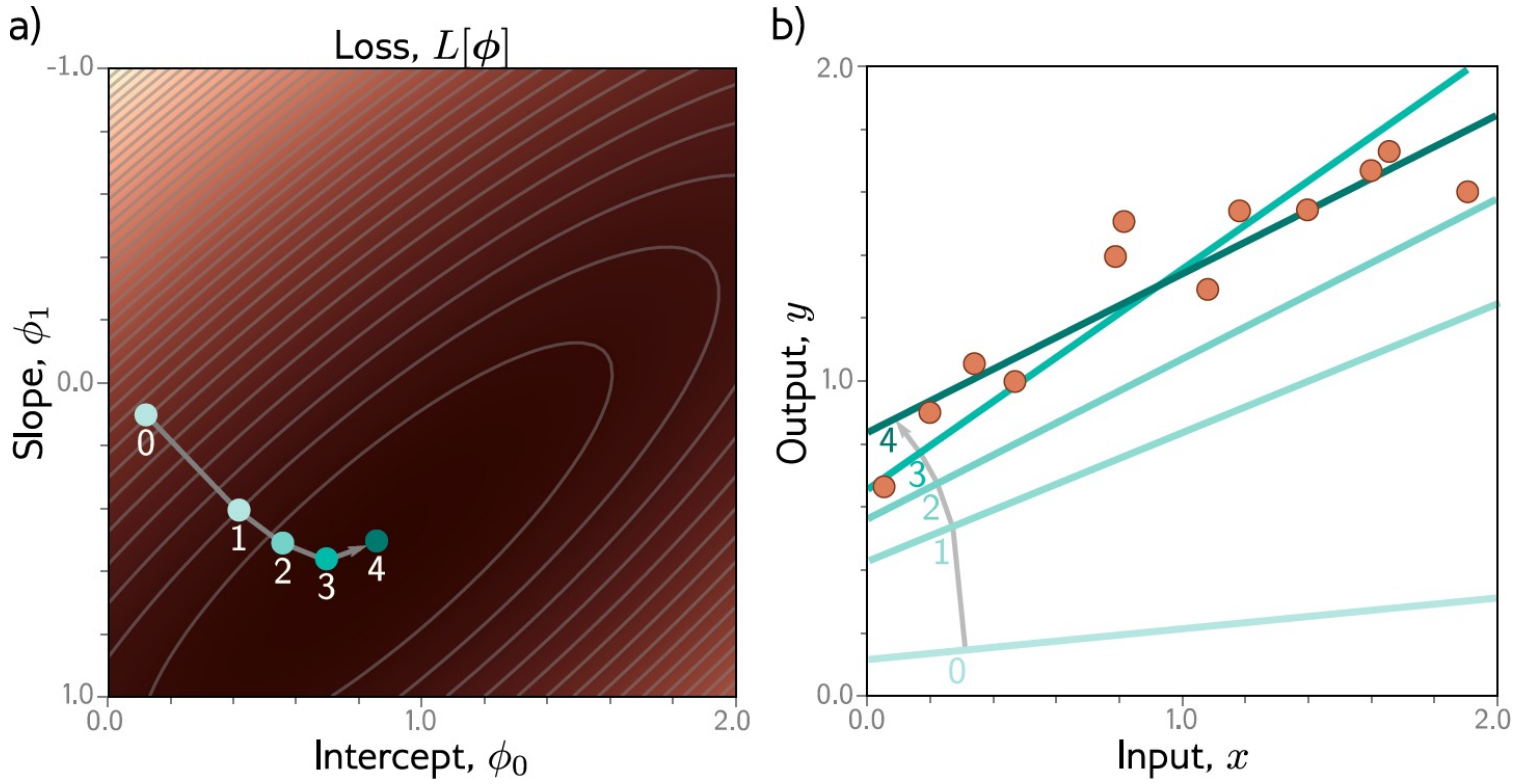


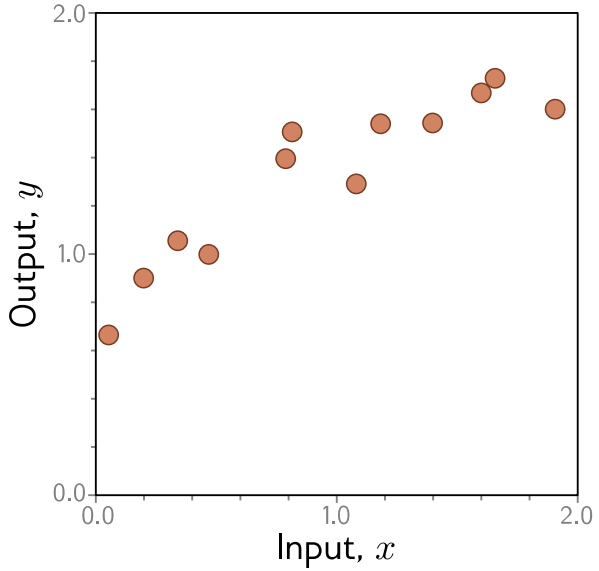
This technique is known as gradient descent

Possible objections

- But you can fit the line model in closed form!
  - Yes – but we won't be able to do this for more complex models
- But we could exhaustively try every slope and intercept combo!
  - Yes – but we won't be able to do this when there are a million parameters

Example: 1D Linear regression testing

- Test with different set of paired input/output data
  - Measure performance
  - Degree to which this is same as training = generalization
- Might not generalize well because
  - Model too simple
  - Model too complex
    - fits to statistical peculiarities of data
    - this is known as overfitting



Supervised learning

- Overview
- Notation
  - Model
  - Loss function
  - Training
  - Testing
- 1D Linear regression example
  - Model
  - Loss function
  - Training
  - Testing
- Where are we going?

Where are we going?

- Shallow neural networks (a more flexible model)
- Deep neural networks (an even more flexible model)
- Loss functions (where did least squares come from?)
- How to train neural networks (gradient descent and variants)
- How to measure performance of neural networks (generalization)