Evolutionary Computing
Lecture 3
Evolution Strategies. Memetic Algorithms

Danylo Tavrov

Outline

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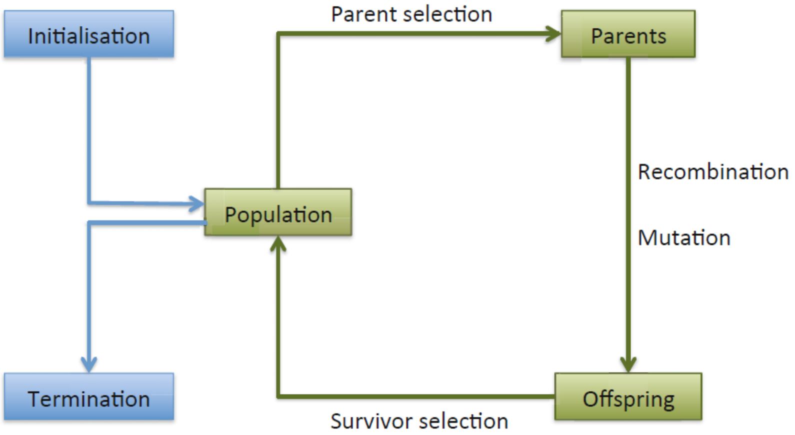
1. Evolution Strategies

2. Memetic Algorithms



Eiben, A.E., Smith, J.E.: Introduction to Evolutionary Computing. Springer-Verlag, Berlin, Heidelberg (2015), Fig. 3.2

- As was mentioned in Lecture 1, there exist several different dialects of evolutionary algorithms, differing mostly in representation of individuals
- *Genetic algorithms*: strings over a finite (typically binary) alphabet
- *Evolution strategies*: real-valued vectors
- *Evolutionary programming*: originally finite state machines, but then real-valued vectors as well
- *Genetic programming*: parse trees
- In previous lectures, we focused on genetic algorithms
- In this lecture, we will discuss evolution strategies

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Motivation

- If the functional relation between the variables and the objective function is unknown, one is forced to experiment[1]
- This is sometimes called **experimental**, or **blind**, **optimization**
- Systematic investigation of all possible states of the system is costly
- On the other hand, random sampling of various combinations is too unreliable
- A search procedure must be *significantly more effective* if it systematically exploits information about preceding attempts
- Experiments are characterized by unavoidable effect of (stochastic) disturbance on the measured results

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Original Two-Membered ES

1. **Initialization**: $t = 0$, create a random vector $\mathbf{x}^t = \left(x_1^t, \ldots, x_n^t\right)^\top \in \mathbb{R}^n$
2. Test **termination condition**. If it holds, stop, continue otherwise
3. **Mutation**: draw $z_i \overset{\text{i.i.d.}}{\sim} N(0, \sigma)$ and create $y_i^t = x_i^t + z_i$, $i = 1, \ldots, n$
4. **Selection**[2]: if $f(\mathbf{x}^t) \leq f(\mathbf{y}^t)$, choose $\mathbf{x}^{t+1} = \mathbf{x}^t$, otherwise choose $\mathbf{x}^{t+1} = \mathbf{y}^t$
5. Set $t = t + 1$ and go to step 2

2. For minimization problems

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Some Analysis

- Let us discuss how mutation works in this ES[3]
- Normal distribution is chosen to make small mutations more likely than large ones
- Mean zero is chosen also to make mutations neutral on average
- If each $z_i \sim N(0, \sigma_i)$ and all are independent, we can write the probability density function of a (column) vector $\mathbf{z}$ as follows:

$$p(z_1, \ldots, z_n) = \prod_{i=1}^{n} \phi(z_i) = \frac{1}{(2\pi)^{\frac{n}{2}} \prod_{i=1}^{n} \sigma_i} \exp\left(-\frac{1}{2}\sum_{i=1}^{n}\left(\frac{z_i}{\sigma_i}\right)^2\right)$$

- When all standard deviations are the same, we get

$$p(z_1, \ldots, z_n) = \frac{1}{(2\pi)^{\frac{n}{2}} \sigma^n} \exp\left(-\frac{\mathbf{z}^\top \mathbf{z}}{2\sigma^2}\right)$$

- The **length** of $\mathbf{z}$ is given by $S = \sqrt{\mathbf{z}^\top \mathbf{z}}$, and we know from statistics that $S^2 \sim \sigma\chi_n^2$ (χ^2-distribution with n degrees of freedom)
- It is not hard to derive the distribution of S as the square root of S^2 (it is the so called χ-distribution with n degrees of freedom)

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- From the central limit theorem, as n becomes "large," we know that a random variable with χ_n^2 distribution has approximate normal distribution $N(n, 2n)$
- Using the so-called delta method, we can show that χ_n has approximate normal distribution $N(\sqrt{n}, 0.5)$
- Finally, we see that as n is "large," $\mathbb{E}[S] = \sigma\sqrt{n}$, $\mathrm{Var}(S) = \frac{\sigma^2}{2}$, and the coefficient of variation is

$$\frac{\sqrt{\mathrm{Var}(S)}}{\mathbb{E}[S]} = \frac{1}{\sqrt{2n}}$$

- In practice, this means that the most probable value for $\|\mathbf{z}\|$ at constant σ increases as $\sqrt{n}$
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- The geometric locus of equally likely changes in variation is an n-dimensional hyperellipsoid with the equation

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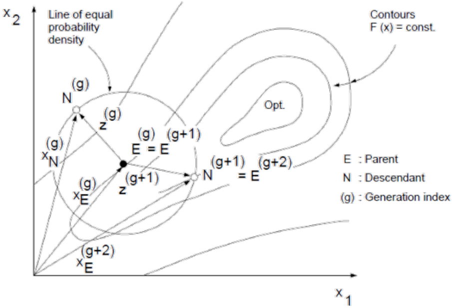

Schwefel, H.-P.: Evolution and Optimum Seeking. John Wiley & Sons, Inc., New York (1993), Fig. 5.1

- Evolutionary algorithms operate locally, by generating offspring in a certain neighborhood of their parents
- Therefore, evolution takes place only within a very narrow band of mutation step size, which is sometimes called **evolution window**
- The only parameter that influences ES performance is the mutation step size σ: it determines the extent, to which given gene alleles are perturbed by the mutation operator
- In original ES, this parameter was changed with time according to the 1/5 success rule[4]:

$$\sigma' = \begin{cases} \frac{\sigma}{c}\,, & p_s > 0.2 \\ \sigma \cdot c\,, & p_s < 0.2 \\ \sigma\,, & p_s = 0.2 \end{cases}$$

- Here p_s is the relative frequency of successful mutations over a number of generations
- c was typically chosen as 0.85
- In **modern ES**, however, this rule is not used
- Instead, mutation step size is *incorporated* into the genotype

4. Rechenberg, I.: Evolutionstrategie: Optimierung technischer Systeme nach Prinzipien der biologischen Evolution. Frommann-Holzboog Verlag (1973)

Adaptation in Original ES

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- c was typically chosen as 0.85
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2. Test **termination condition**. If it holds, stop, continue otherwise
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   - For $l = 1, \ldots, \lambda$ do:

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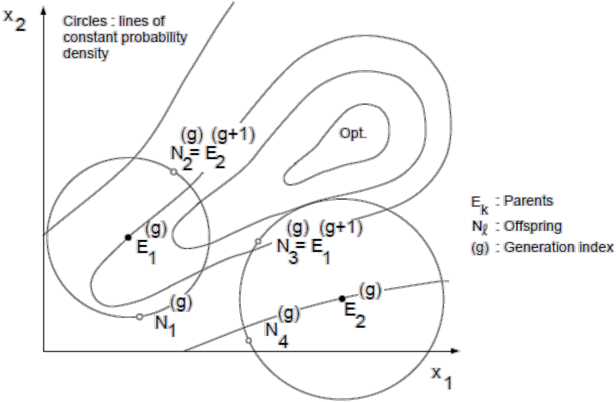
Illustration for a $(2, 4)$ ES



Schwefel, H.-P.: Evolution and Optimum Seeking. John Wiley & Sons, Inc., New York (1993), Fig. 5.4

- **First**, the strategy parameters are mutated
- We get $\boldsymbol{\sigma}' = (\sigma'_1, \ldots, \sigma'_n)$
- **Then**, object variables are changed:

$$\mathbf{x}' = \mathbf{x} + \Delta\mathbf{x}$$

- Here the probability density function of $\Delta\mathbf{x}$ depends on strategy parameters: $p(\Delta\mathbf{x}) = p(\Delta\mathbf{x}, \boldsymbol{\sigma}')$
- This sequence is **obligatory**, otherwise we will not have self-adaptation

Mutations in Modern ES

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- Typically, the strategy parameters $\sigma_1, \ldots, \sigma_n$ are mutated according to the following general scheme[5]:

$$\sigma_i' = \sigma_i \cdot W$$

- Random variable W should possess the following properties
- The *median* should be 1 (no deterministic drift)
- *Increase* and *decrease* in step length should occur with the same frequency
- Small changes should occur more often than large ones
- All these requirements can be satisfied by the **log-normal** distribution

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Uncorrelated Mutation With One Step Size

- The same distribution is used for each object variable: $\sigma_1 = \ldots = \sigma_n = \sigma$
- This σ is mutated as $\sigma' = \sigma \cdot W$, where $\ln W \sim N(0, \tau^2)$
- Here τ is the learning rate, which is proportional to $\frac{1}{\sqrt{n}}$
- The constant of proportionality a depends on λ/μ (basically selection pressure)
- For a $(10, 100)$ ES, it should be set to 1[6]
- For higher λ, we can set a sublinearly according to $\lambda \sim \sqrt{a}e^{a}$
- To prevent too small mutations, we set σ' to ε_0 if $\sigma' < \varepsilon_0$, for some user-specified constant ε_0
- Finally, the object variables are then mutated as

$$\mathbf{x}' = \mathbf{x} + \mathbf{z} \; ,$$

where $\mathbf{z}$ is the multivariate normal distribution with zero mean and covariance matrix $(\sigma')^2 \cdot \mathbf{I}$

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- The constant of proportionality a depends on λ/μ (basically selection pressure)
- For a $(10, 100)$ ES, it should be set to 1[6]
- For higher λ, we can set a sublinearly according to $\lambda \sim \sqrt{a}e^a$
- To prevent too small mutations, we set σ' to ε_0 if $\sigma' < \varepsilon_0$, for some user-specified constant ε_0
- Finally, the object variables are then mutated as

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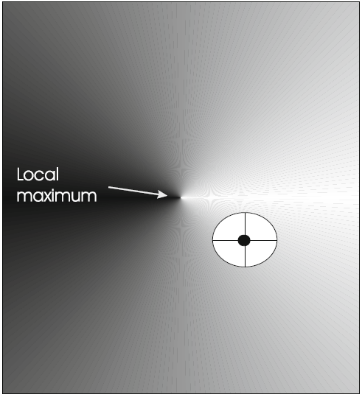
6. Schwefel, H.-P.: Evolution and Optimum Seeking. John Wiley & Sons, Inc., New York (1993)

Illustration



Eiben, A.E., Smith, J.E.: Introduction to Evolutionary Computing. Springer-Verlag, Berlin, Heidelberg (2015), Fig. 4.4

- The motivation is to treat dimensions differently
- Each σ_i is mutated as
$$\sigma_i' = \sigma_i(W + V_i) \ ,$$
where $\ln W \sim N\left(0, (\tau')^2\right)$, $\ln V_i \sim N(0, \tau^2)$
- Here τ' is proportional to $\frac{1}{\sqrt{2n}}$, and τ is proportional to $\frac{1}{\sqrt{2\sqrt{n}}}$
- The first part allows for an overall change of the mutability (the preservation of all degrees of freedom)
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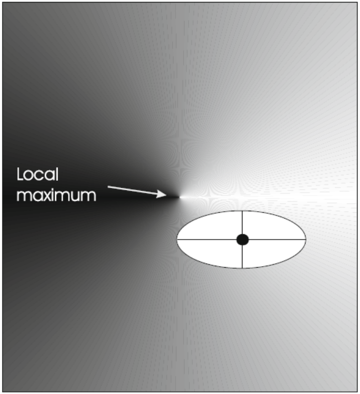

Eiben, A.E., Smith, J.E.: Introduction to Evolutionary Computing. Springer-Verlag, Berlin, Heidelberg (2015), Fig. 4.5

Recombination in Modern ES

- In ES, recombination can be **panmictic** ($\rho = \mu$) or **sexual** ($\rho = 2$)
- In the latter case, all individuals can be selected as parents with uniform probabilities
- Recombination can be:
  - **Discrete (dominant)**: coordinate-wise random selection from the ρ corresponding coordinate values of the parent family
  - **Intermediate**: arithmetic mean of the ρ parents (in practice, used more often)

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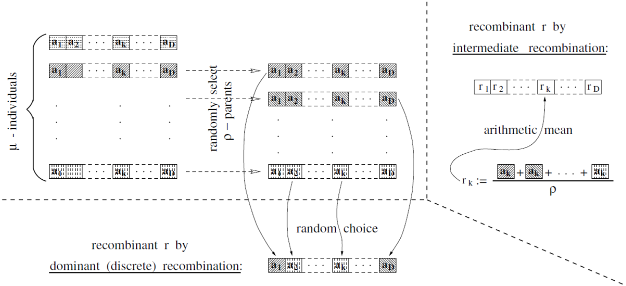

Beyer, H.-G., Schwefel, H.-P.: Evolution Strategies. A Comprehensive Introduction. Natural Computing, 1, 3–52 (2002), Fig. 6

- The best μ individuals for the next generation are always chosen **deterministically**
- In the (μ, λ) selection, μ fittest individuals are selected among λ children
- Since all the parents are discarded, it is possible to leave local optima
- The suggested ratio[7] is $7\mu = \lambda$
- In the $(\mu + \lambda)$ selection, μ fittest individuals are selected among μ parents and λ offspring
- This selection guarantees the survival of the best individual found so far
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- In general, it seems to be better to use the (μ, λ) selection for unbounded search spaces

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Initialization, Termination, Constraint Handling

- Initial population is obtained by mutating the starting point μ times
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Evolution Strategy Jupyter Notebook

Covariance Matrix Adaptation ES

- There is another version of ES, in which not only mutation step sizes are self-adapted, but rotation angles as well
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- The idea is to enable mutation of the whole covariance matrix (not only its diagonal elements σ_i)

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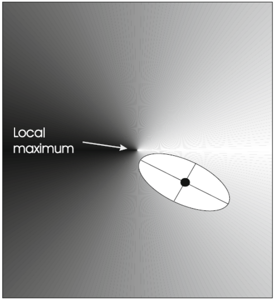

Eiben, A.E., Smith, J.E.: Introduction to Evolutionary Computing. Springer-Verlag, Berlin, Heidelberg (2015), Fig. 4.6

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Outline

- As we have seen, EAs are very good at *exploration*, but are less good at fine-tuning of solutions (*exploitation*)
- It seems to be a good idea to incorporate problem-specific knowledge in EAs
- The concept of memes was introduced by Richard Dawkins in 1976 in *The Selfish Gene*
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Genes vs. Memes

- Genes and memes differ strongly in the copying fidelity: memes mutate with a much higher rate
- Memes allow improvement: the individual adapts the meme as it sees best; memes can be improved by the individual holding it
- This idea can be used in genetic algorithms to improve individuals
- *Local heuristics* can be applied so that the space of possible solutions is reduced to the *subspace of local optima*

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- The term “memetic algorithm” was coined by Pablo Moscato in 1989
- Memetic algorithms (MAs) are “carefully orchestrated interplay between (stochastic) global search and (stochastic) local search algorithms”
- These algorithms are also frequently called hybrid genetic algorithms
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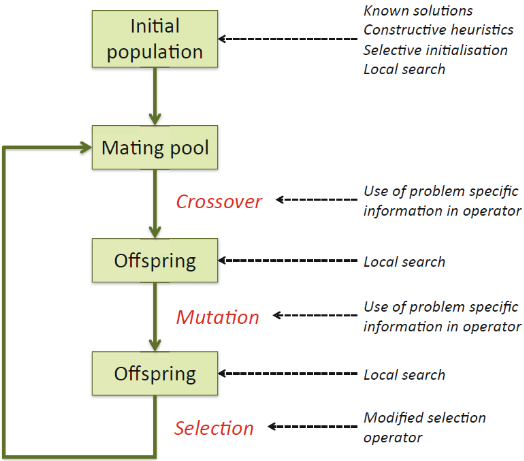
Places to Apply Hybridization



Eiben, A.E., Smith, J.E.: Introduction to Evolutionary Computing. Springer-Verlag, Berlin, Heidelberg (2015), Fig. 10.3

- Local search can be integrated within the evolutionary cycle in two ways[9]
- The first one is "lifetime learning": application of the local search to a candidate solution
- In this case the metaphor is cultural development of individuals, which is then transmitted to other solutions over subsequent generations
- The second one is application of the local search during solution **generation**, i.e. generation of a perfect child
- This class of memetic implementations aims at selecting the most convenient offspring among potential offspring solutions
- E.g., heuristic selection of the crossover point or the best bit to mutate

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- A neighborhood N over S is any function that associates to each solution s from S other solutions from $N(s)$
- Any solution s' from $N(s)$ is called a **neighbor** of s
- A solution s^l is a **local optimum** with respect to N if it is the best solution in $N(s^l)$ in terms of some objective function
- The notion of neighborhood can be explained in terms of the **move** operator
- Typically applying a move mv to solution s changes s slightly and leads to a neighbor solution s':

$$s' = s \oplus mv$$

- Then, the neighborhood can be defined as

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- Variants: steepest-descent (explore the full neighborhood), random (pick a random neighbor and decide)
- **Simulated annealing**: the same as hill climbing, but the replacement is possible if s' is worse, with a probability

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- **Simulated annealing**: the same as hill climbing, but the replacement is possible if s' is worse, with a probability

$$p = \begin{cases} 1 \ , & f(s) - f(s') > 0 \\ e^{-\frac{f(s)-f(s')}{T}} \ , & \text{otherwise} \end{cases} \ ,$$

  where T is a time-varying *temperature*, decreased using a *cooling schedule* (many exist in the literature)
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Some Local Search Procedures for Continuous Domains

- Any well-known optimization methods can be used
- **Zeroth-order methods** (direct search methods): methods that don't use derivatives (e.g., Nelder-Mead methods)
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Parametrization of Memetic Algorithms

- When designing memetic algorithms, the following issues must be tackled[12]
- How often should local search be applied?
- The more frequently you apply, the longer it takes, also, there is a risk to get stuck in local optima
- On which solutions should local search be used?
- The choice can be random or fitness-based
- How long should the local search be run?
- The more thorough the search, the longer it takes to compute
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