Leveraged Portfolios Risk Analysis

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November 9, 2025

Abstract

An investment delivers twice the market's return. Should you invest? The answer depends on a question most overlook: how much additional risk did you take?

This study investigates leveraged Exchange-Traded Funds (ETFs) tracking the S&P 500 Index, specifically Direxion Daily S&P 500 Bull 3X Shares (SPXL), ProShares UltraPro S&P 500 (UPRO), and ProShares Ultra S&P 500 (SSO). Financial theory suggests that leverage should proportionally amplify both returns and risk. However, our empirical analysis reveals a critical inefficiency: the actual returns of leveraged ETFs significantly underperform their theoretical targets relative to the associated increase in volatility.

Using daily data from 2010 to 2025, we apply the Capital Asset Pricing Model (CAPM) with exponential weighting to estimate beta coefficients, and employ rolling analysis, factor attribution, and variance decomposition to understand the sources of performance deviation. Our results show that leverage magnifies systematic risk almost linearly: beta coefficients of 2.02 for SSO and approximately 3.04 for both SPXL and UPRO closely match their nominal leverage ratios. Yet cumulative returns grow at a substantially slower rate: SSO delivers only 1.60x the benchmark returns (not 2x), while SPXL and UPRO deliver approximately 2.03x (not 3x). This represents only 67% of the returns expected for the risk taken, a massive inefficiency.

Factor attribution reveals that while CAPM-predicted returns follow theoretical leverage multiples, actual ETF performance consistently falls short, with idiosyncratic factors systematically reducing returns over time. This performance gap suggests structural inefficiencies in ETF construction beyond simple market exposure.

These findings demonstrate that leveraged ETFs, though designed to mechanically scale market exposure, behave as imperfect amplifiers. They expose investors to disproportionate risk without proportional reward, highlighting the critical need to analyze investment opportunities through rigorous risk models rather than accepting backtested returns at face value. The study also reinforces that CAPM serves not merely as a return prediction tool but as an essential framework for estimating and decomposing risk exposures, a lesson crucial for aspiring quantitative professionals.

Contents

1 Introduction

Debt financing can be defined as the use of debt instruments to fund investments, thereby increasing (or multiplying) an investor's exposure to certain assets (Modigliani and Miller, 1958). Also called leverage, it is a technique employed by many investors to amplify the returns that a single asset or portfolio can generate. The leverage ratio, then, is the multiple of the debt acquired for the strategy relative to the value of the equity (Brealey and Myers, 2003).

Modern Portfolio Theory, introduced by Markowitz, 1952, established the fundamental trade-off between risk and return in investment decisions. Within this framework, investors seek to enhance expected returns without proportionally increasing risk, which naturally leads to the use of leverage as a tool to optimize portfolio performance. Building on this idea, debt financing can be defined as the use of debt instruments to fund investments, thereby increasing (or multiplying) an investor's exposure to certain assets (Modigliani and Miller, 1958).

Financial theory suggests that the returns of a leveraged investment are equivalent to the returns of the underlying asset multiplied by the leverage ratio, net of the cost of debt (the credit rate). Nevertheless, empirical evidence indicates that leveraged portfolios may suffer from inefficiencies, causing the actual returns to be lower than expected. Hence, the returns are amplified, but by a factor smaller than the corresponding increase in volatility.

In this study, we analyze the effects of leverage on certain leveraged ETFs. An Exchange-Traded Fund (ETF) is a relatively recent investment instrument that serves as a tracking vehicle for a specific benchmark (Deville, 2008). Many ETFs are designed to track entire markets (such as the U.S. S&P 500) or particular industries. To study the effects of leverage, we focus on leveraged versions of the SPDR S&P 500 ETF (SPY), which tracks the widely followed index representing the U.S. economy.

Our goal is to test the inefficiencies implied by these ETFs. Specifically, we aim to analyze whether the returns generated by these leveraged ETFs are indeed lower than those expected based on their leverage ratios. To conduct our analysis, we employ the Capital Asset Pricing Model (CAPM) proposed by Treynor, 1999, Sharpe, 1964, and Lintner, 1965. We calculate the beta coefficient and examine whether its values serve primarily as a measure of risk or as a return multiplier.

In the following section, we describe the methodologies used for this analysis, detailing the techniques applied in this study. In the third section, we present our results and discuss their implications. Finally, the conclusion summarizes our findings and outlines potential directions for future research.

2 Materials and Methods

In this section, we describe the dataset and present key descriptive statistics for the leveraged ETFs under study. We detail the sources, sample period, and calculation methods used to obtain daily returns for each fund, as well as the benchmark and the risk-free asset. By examining basic statistical measures, such as mean, standard deviation, and correlations, we provide an initial understanding of the risk-return profile of these ETFs. This descriptive analysis serves as a foundation for the subsequent application of the Capital Asset Pricing Model (CAPM), where we decompose returns into systematic and idiosyncratic components and evaluate the impact of leverage on both risk and performance.

2.1 Data and Descriptive Statistics

We downloaded data from Yahoo Finance for three different ETFs: Direxion Daily S&P 500 Bull 3X Shares (SPXL), ProShares UltraPro S&P 500 (UPRO), and ProShares Ultra S&P 500 (SSO). These funds are leveraged versions of the S&P 500 Index with leverage ratios of 3:1, 3:1, and 2:1, respectively. The sample covers the period from January 1, 2010, to January 1, 2025, allowing us to capture complete business cycles, including the major economic crisis the U.S. economy experienced in 2020. Our decision was also based on data availability, as UPRO ETF began trading in June 2009.

Additionally, we downloaded data for the SPDR S&P 500 ETF (SPY), whose objective is to track the performance of the 500 largest publicly traded companies in the United States. The data for SPY covers the same period as the other ETFs, and we use it as the benchmark in the Capital Asset Pricing Model (CAPM) developed to estimate betas and perform risk attribution. We then calculate the logarithmic returns for all four ETFs.

We obtained 3,773 trading days for each ETF. What we can observe from Table 1 is that, even though the leverage ratios are 2:1 for SSO, and 3:1 for both SPXL and UPRO, the average daily returns are not proportionally doubled or tripled. For instance, SSO's average daily return is only 1.66 times higher than SPY's, while SPXL's and UPRO's are 2.08 and 2.1 times higher, respectively. Nevertheless, the risks are indeed amplified, as the standard deviations are 2.02 times higher for SSO, 3.03 for SPXL, and 3.04 for UPRO. This may suggest that the additional risk is not fully compensated by the incremental returns provided by leverage.

It is important to note that Table 1 presents the descriptive statistics of the excess returns of each ETF (the corresponding figures for raw data are shown in the Appendix). The subtraction of the risk-free rate is required because the CAPM framework is built not on the raw performance of each asset, but on their excess returns relative to the risk-free

Table 1: Descriptive Statistics for ETFs' Excess Returns

Statistic	**SSO**	**SPXL**	**UPRO**	**SPY**
Count	3773	3773	3773	3773
Mean	0.0729	0.0916	0.0925	0.0440
Std. Dev.	2.1735	3.2706	3.2769	1.0773
Min	-26.6048	-41.3597	-42.9974	-11.5907
25%	-0.7515	-1.1296	-1.1253	-0.3777
50% (Median)	0.1234	0.1972	0.1954	0.0591
75%	1.1142	1.6875	1.6793	0.5703
Max	17.3030	24.5323	24.6569	8.6709

Source: Yahoo Finance Data.

asset (Sharpe, 1964). Therefore, we subtracted the yields of the 10-Year Treasury Bond from all selected ETFs, including the Benchmark, replying the methodology of Frazzini and Pedersen, 2014. This adjustment ensures that the estimated betas and expected returns capture only the market-related risk premium, consistent with the theoretical formulation of the model.

Finally, we conducted a correlation analysis, which is also insightful as it reveals that all ETFs are almost perfectly correlated with the benchmark (SPY). This finding allows us to conclude that when the benchmark exhibits positive returns, the probability of the leveraged ETFs also recording gains is nearly one. Such behavior is expected, since leveraged ETFs are essentially scaled versions of the benchmark; in other words, leverage alters the magnitude of performance, not its direction. Figure 1 presents the correlation matrix.

We can note that the least correlated ETF with the SPY is the UPRO which, although the difference is very small, has a Pearson correlation coefficient of 0.9975—slightly lower than 0.9981 for the SPXL and 0.9989 for the SSO. Interestingly, the UPRO is also the ETF with the highest standard deviation (volatility), as shown in Table 1. This might represent an interesting finding that will gain more relevance later in this paper.

2.2 Capital Asset Pricing Model

After the descriptive analysis of the ETFs and observing that their returns do not fully compensate for the additional risk introduced by leverage, we proceed to perform a risk decomposition using the beta coefficients obtained from the Capital Asset Pricing Model (CAPM) proposed by Treynor, 1999, Sharpe, 1964, and Lintner, 1965. The CAPM relationship can be summarized by the following equation:

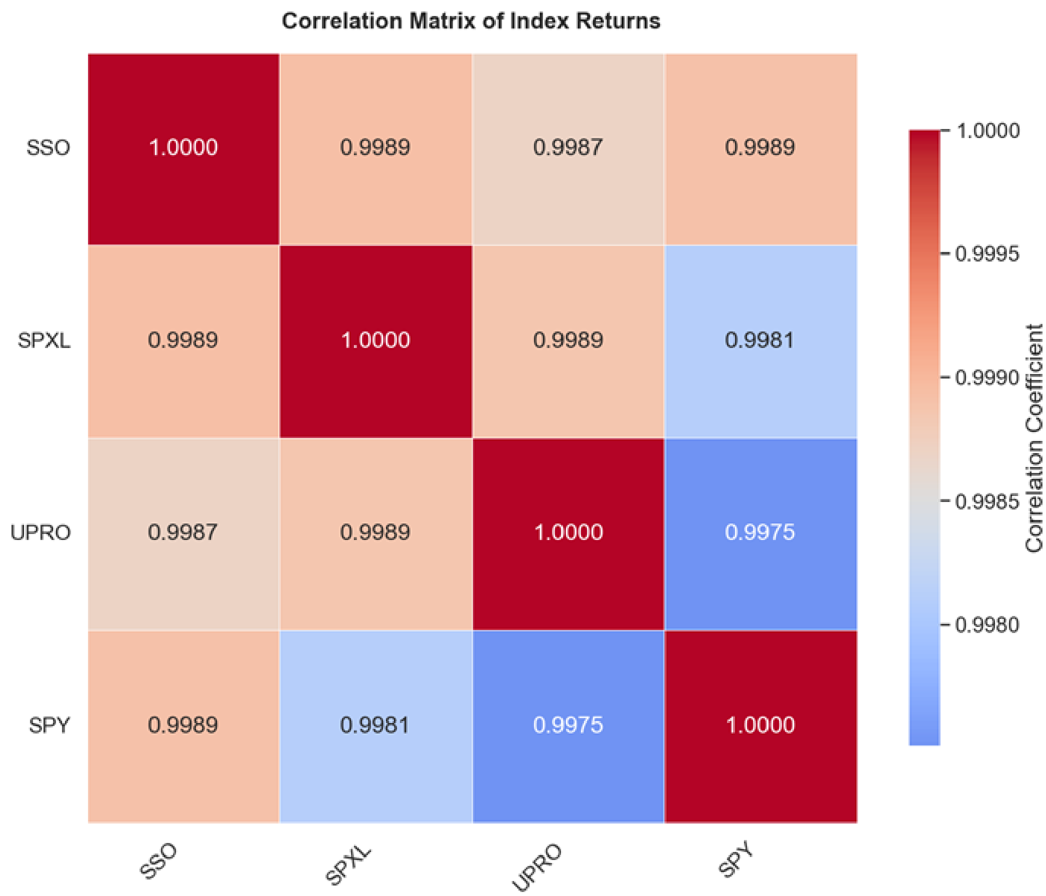

Figure 1: Correlation Matrix of ETFs.

Source: Own Creation based on Yahoo Finance Data.

$$\mu_i - r_f = \alpha_i + \beta_i \left(\mu_M - r_f\right) + \varepsilon_i \tag{1}$$

Here, μ_i represents the returns of a given ETF, and r_f is the risk-free rate, which in this case is the yield of the 10-Year Treasury Bond. The term μ_M denotes the returns of the benchmark, which is SPY in this example, while ε_i represents the idiosyncratic returns. The coefficients we aim to estimate are $[\alpha_i, \beta_i]$, where α_i captures the average returns explained by factors unrelated to the market (our selected benchmark), and β_i measures the sensitivity of ETF i to market movements.

For the estimation, we are going to use a Time-Series Version of the Weighted Least Squares Regression, using exponential increasing weights to give more importance to more recent observations relative to older observations. The method to calculate the weights is summarized in the following equations:

$$c = \frac{\log(1/2)}{h}, \qquad \omega_t = \frac{e^{ct}}{\sum_{k=0}^{T-1} e^{ck}} \tag{2}$$

Here, h represents the half-life of the weights, defined as the period after which the sum of the weights reaches exactly 50%. Typically, we set the half-life to be half of T, where T is the total number of available dates. The coefficient c represents the decay rate (in this case, an increasing rate) that governs the speed at which the weights grow exponentially. As can be observed, the weights are normalized so that their sum equals one.

We can calculate the beta coefficients using the matrix formulation of the linear regression model. The main inputs are two matrices: the first one of dependent variables, containing three columns (one for each ETF) and T rows, and another with two columns, benchmark returns and a vector of ones, which is used to estimate the constant (alpha) coefficient; this matrix also has T rows. The main output is a 2×3 matrix of estimated coefficients, consisting of three alphas and three betas.

$$\boldsymbol{\beta} = \left(\mathbf{X}^\top \mathbf{W} \mathbf{X}\right)^{-1} \left(\mathbf{X}^\top \mathbf{W} \mathbf{Y}\right) \tag{3}$$

The matrix $\mathbf{X}$ represents the data matrix, containing the benchmark returns and the vector of ones. The matrix $\mathbf{W}$ is a square diagonal matrix that holds the regression weights, with the diagonal elements containing the weights and all off-diagonal elements equal to zero. Finally, $\mathbf{Y}$ is a matrix that contains the series of returns for the three ETFs studied.

Once the betas are obtained, we can conduct an insightful analysis to better understand the characteristics of these ETFs.

2.3 Rolling Beta

Following the findings of Blume, 1971, we assess whether the calculated betas are constant over time. The previously defined Capital Asset Pricing Model is used to estimate a long time series of beta coefficients through a rolling window of 252 trading days, which corresponds to the standard length of a trading year (excluding weekends and holidays). The beta coefficients are also calculated using the exponential weighting scheme described in the previous section, giving greater importance to more recent observations. Our objective is to analyze the patterns and behavior of the estimated beta coefficients to better understand the evolution of market risk experienced by the ETF and compare them with the risk ratios derived from the standard deviation of each ETF relative to that of the market.

2.4 Factor Attribution

After estimating the beta coefficients, we computed the factor and idiosyncratic returns for each ETF. This enabled a risk attribution analysis to identify the specific drivers of risk affecting each fund. Subsequently, we constructed time series for the total ETF returns, the CAPM-implied (systematic) returns, and the inefficiency drift (negative alpha). Through this decomposition, we gain a clearer understanding of the inefficiencies observed in the leveraged versions of the SPY. The CAPM and inefficiency drift returns were calculated as follows:

$$\mu_{\beta i} = \beta_i \mu_M \tag{4}$$

$$\mu_{\varepsilon i} = \mu_i - \beta_i \mu_M \tag{5}$$

Here, $\mu_{\beta i}$ denotes the component of returns associated with the beta coefficient of each ETF, while $\mu_{\varepsilon i}$ corresponds to the portion attributable to factors unrelated to the market premium (SPY), which in this context may reflect the inefficiencies specific to each leveraged ETF. The behavior of these two series reveals how each ETF's returns can be decomposed and highlights the nature of their underlying sources.

2.5 Variance Decomposition

Using the CAPM as a risk model, we can decompose the sources of variance (risk) in the analyzed asset (i.e., UPRO). This approach allows us to better understand the composition of the portfolio's risk and to identify more clearly which components are not being compensated by the market. Following Fama and MacBeth, 1973 in their seminal paper on risk, return, and equilibrium, we performed the variance decomposition of each leveraged ETF. Starting from the CAPM equation, we applied the variance operator to derive the expression for an asset's total variance and to show how it can be separated into its constituent components:

$$\mathrm{Var}(\mu_i - r_f) = \mathrm{Var}\big(\alpha_i + \beta_i(\mu_M - r_f) + \varepsilon_i\big) \tag{6}$$

Applying the linear property of the variance, we can arrive at the following equation:

$$\mathrm{Var}(\mu_i - r_f) = \mathrm{Var}(\alpha_i) + \mathrm{Var}\big(\beta_i(\mu_M - r_f)\big) + \mathrm{Var}(\varepsilon_i) \tag{7}$$

The variance of a constant is essentially zero, and when a constant appears within a variance operator alongside a non-constant term, it can be factored out and squared. Thus, we obtain the following expression, substituting the variance operator with the Greek letter σ^2:

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_\varepsilon^2 \tag{8}$$

By expressing the beta coefficient in its alternative form, using the correlation between the market and the individual ETF, we obtain:

$$\sigma_i^2 = (\rho_{i,M} \frac{\sigma_i}{\sigma_M})^2 \sigma_M^2 + \sigma_\varepsilon^2 \tag{9}$$

After canceling terms and rearranging the expression, we can obtain a simplified version to obtain the inefficiency drift variance:

$$\sigma_\varepsilon^2 = (1 - \rho_{i,M}^2)\, \sigma_i^2 \tag{10}$$

After computing the variances, we can perform a risk attribution analysis to determine how each factor contributes to the total variance of the ETF. The goal is to quantify the portion of variance explained by the market (the benchmark ETF) versus that attributable to other factors. The results of these analyses are presented in the following section.

2.6 Volatility Drag

After performing the variance decomposition, we aim to understand how much of the ETFs' return loss can be explained by the phenomenon described by Messmore, 1995, known as 'Variance Drain'. Messmore explains that the compounding effect in stocks causes them to 'lose' a portion of their returns due to volatility. The phenomenon is described as follows:

$$\mu_g = \mu_a - \frac{\sigma^2}{2} \tag{11}$$

Then, Leung and Sircar, 2015 expands on Messmore's findings by applying them to leveraged stocks. By introducing a leverage ratio L, the volatility drag becomes amplified by this factor. Consequently, the observed (or realized) returns of a leveraged ETF follow the following process:

$$\mu_L \approx L\mu_M - L(L-1)\frac{\sigma_M^2}{2} \tag{12}$$

Here, μ_L represents the observed returns of a leveraged ETF. In an ideal world, the returns of these ETFs would equal the benchmark returns multiplied by the leverage ratio ($L\mu_M$). However, we aim to understand the sources of the differences between the observed and the ideal returns. The second term on the left-hand side of the expression corresponds to what we define as the volatility "tax."

2.7 Coding

The methodologies described here were developed using Python. Several libraries were employed in our analysis: the Yahoo Finance API library (*yfinance*) was used to download the data for both the stocks and the risk-free rate. For data wrangling and numerical computations, we relied on *Pandas* and *NumPy*, as they offer convenient and efficient tools for data management. For visualization, we used *Matplotlib* (for time-series graphs such as Figure 2) and *Seaborn* (for other visualizations, such as Figure 1). Finally, the econometric analysis and the estimation of the beta coefficients were performed using the *Statsmodels* library. The code used in this study is available in our GitHub repository, allowing readers to replicate the methodology, extend the analysis, or conduct their own experiments. (https://github.com/dan-the-quant/DanTheQuant-Evidence_Lab_Research).

3 Results

Table 2 summarizes the results of the regressions. We can observe that the beta coefficients are practically the values of the leveraged ratio of the ETFs: 2.017 for the SSO and 3.028 and 3.039 for the SPXL and the UPRO, respectively. The conclusion here is that the systematic risk captured by the beta coefficient is practically explaining all the variations of the extra risk captured by the leveraged position. As we saw in the previous descriptive analysis, the standard deviation of the market (the SPY) is multiplied by the leveraged ratio of each asset to obtain the standard deviation of the respective ETF.

Table 2: WLS Regression Estimations

ETF	$\boldsymbol{\alpha}$	**t-stat** (α)	$\boldsymbol{\beta}$	**t-stat** (β)	R^2
SSO	-0.0185	-11.372	2.0171	1362.500	0.998
SPXL	-0.0467	-14.419	3.0279	1025.861	0.996
UPRO	-0.0469	-12.788	3.0394	909.427	0.995

Source: Yahoo Finance Data.

The values of the beta coefficients provide significant insight into the behavior of the ETFs under study. For example, we can confirm that leverage proportionally multiplies the risk of an ETF by the applied leverage ratio; however, this is not the case for returns. A beta of 2 (as in the case of the SSO ETF) implies that the risk experienced by investors is roughly doubled, confirming a 2:1 leverage ratio. Nevertheless, we do not observe that the returns are also multiplied by two. After calculating the cumulative returns of the three ETFs under study, we find that the SSO offers only 1.601 times higher returns than the SPY, while the SPXL and UPRO show ratios of 2.026 and 2.035, respectively.

Naturally, we cannot conclude that the beta coefficient represents the leverage ratio. By construction, the leverage ratios of the ETFs are constant over time, for example, the SSO maintains a 2:1 ratio at all times. However, after examining the time series of rolling betas (Figure 2), we can conclude that the beta coefficient serves as a proxy for the leverage ratio, but its most helpful interpretation is as a risk coefficient. Beta reflects the degree of an asset's exposure to market risk; for example, a beta of 3 indicates that an ETF is three times more sensitive to market movements than the market itself.

Figure 2 illustrates the behavior of the rolling betas adjusted by the respective leverage ratio (2:1 or 3:1) using the ratio β/L where L is the leverage ratio. As observed, the adjusted beta coefficient is not constant over time but fluctuates between approximately 0.97 and 1.09. As mentioned earlier, the leverage ratio of these ETFs remains constant by design. Therefore, the variations in rolling betas suggest that there are periods (such as during the COVID-19 pandemic) when the ETFs become more sensitive to market movements. This may indicate

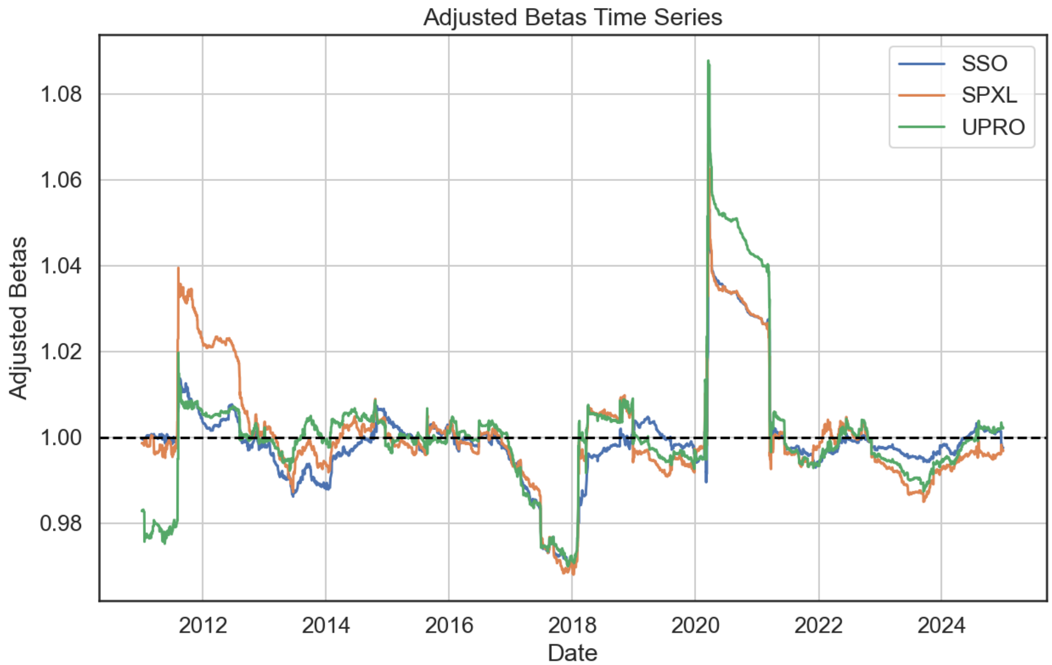

Figure 2: Adjusted Rolling Betas for ETFs.

Source: Own Creation based on Yahoo Finance Data.

the presence of a hidden behavioral factor that amplifies risk in times of market stress, as these highly leveraged assets are often the first to be sold when investors rebalance their portfolios.

Figure 3 shows the decomposition of the cumulative returns for one of the ETFs, in this case, the UPRO ETF, which appears to be the riskiest of the three. The green line represents the CAPM-implied returns of the ETF, calculated by multiplying the estimated beta coefficient by the market (SPY) returns. The blue line, by contrast, corresponds to the actual observed returns of UPRO, which are lower than the factor-implied returns. Finally, the red line represents the inefficiency drift, that is, the portion of returns associated with factors not captured by the market. The reader can analyze the attribution of the other two ETFs in the Appendix section.

The behavior of the inefficiency drift (the red line) highlights the reason why the UPRO ETF does not yield the returns predicted by the CAPM model. The leveraged ETFs are inefficiently constructed, which prevents them from achieving the theoretical returns implied by their leverage ratios. We are not receiving three times the SPY's returns, but only slightly more than twice (as discussed above). This may also suggest that, even though these ETFs are leveraged versions of the SPY, they are still subject to independent trading behaviors. Investors may perceive significantly higher risk associated with these funds, leading them to

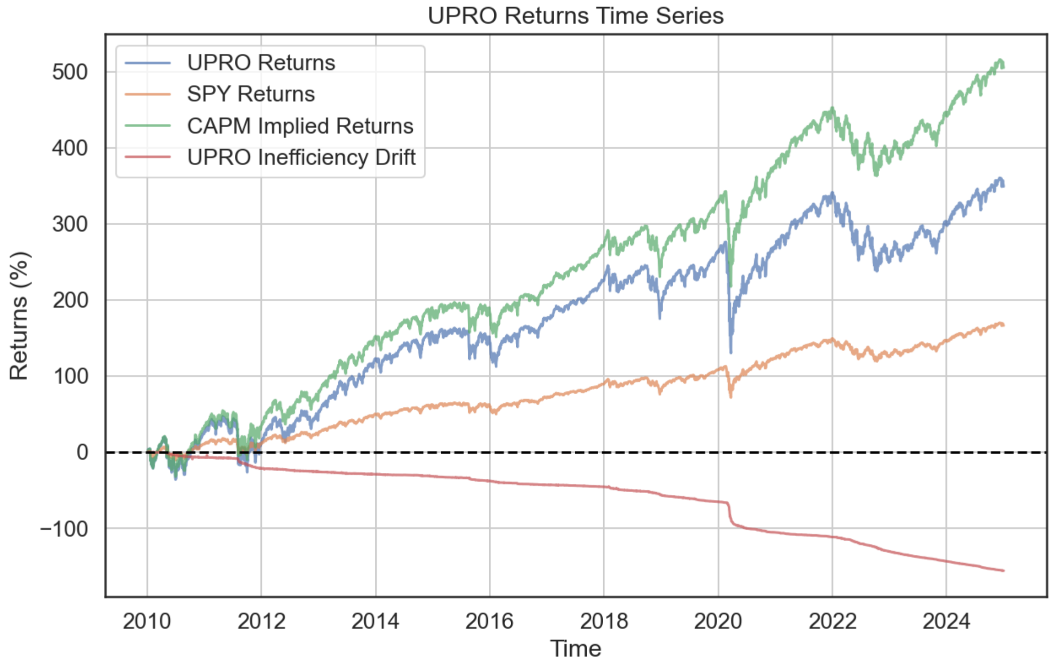

Figure 3: Factor Attribution for the UPRO ETF.

Source: Own Creation based on Yahoo Finance Data.

oversell the ETFs and, consequently, reduce their expected returns.

Table 3 presents the results of the variance decomposition for each ETF. The table reports the ratios of each component's variance relative to the total variance of the asset. The total variance is the sum of the CAPM variance and the inefficiency drift variance, while the market variance is included for additional comparison. For all three ETFs, the CAPM factor accounts for more than 99% of the total variance, with SSO showing the highest CAPM contribution. As expected, the UPRO ETF exhibits the largest inefficiency contribution.

Table 3: Variance Ratios of Leveraged ETFs

ETF	σ^2_{CAPM}	**Attribution**	σ^2_{ε}	**Attribution**	σ^2_i
SSO	4.7138	99.7803%	0.0104	0.2197%	4.7241
SPXL	10.6564	99.6222%	0.0404	0.3778%	10.6969
UPRO	10.6843	99.5021%	0.0535	0.4979%	10.7378

Source: Yahoo Finance Data.

If we recall Table 1, we can see that UPRO is the ETF with the highest standard deviation relative to SPY, and it is also the ETF with the smallest CAPM contribution. This might suggest that the unobserved factors themselves are risky factors. In other words, the additional volatility that these ETFs experience relative to the market (their higher standard deviation) may be attributed to idiosyncratic sources of risk, since SPXL and UPRO have the same leverage ratio (3:1) but the latter has a higher beta. Again, this may serve as further evidence of the inefficiencies involved in these ETFs.

Finally, Table 4 presents the results of the volatility drag analysis. As shown in the fourth column, the ETFs exhibit a loss in daily returns of 0.022%, 0.0541%, and 0.0533%. However, the empirical analysis of the volatility drag (third columns) indicates that only 0.0116% and 0.0348% (for both 3:1 leveraged ETFs) can be explained by the variance of these ETFs. Therefore, the remaining losses of 0.0104%, 0.0193%, and 0.0185% are attributed to unobserved factors.

Table 4: Volatility Drag Decomposition for Leveraged ETFs (in Percentage Terms)

ETF	**Mean Daily Returns**	**Ideal Daily Returns**	**Theoretical Drag**	**Observed Drag**	**Unexplained Drag**
SSO	0.0797	0.1017	0.0116	0.0220	0.0104
SPXL	0.0984	0.1526	0.0348	0.0541	0.0193
UPRO	0.0993	0.1526	0.0348	0.0533	0.0185

Source: Author's calculations using Yahoo Finance data.

This helps us understand that the ETFs exhibit inefficiencies not only attributable to the additional volatility they experience, but also to other factors that may be related to their structural design. Hence, in the following section, we discuss the conclusions and implications of these results, as well as the potential directions for future research.

4 Conclusions

The analysis confirms that leverage effectively magnifies the systematic risk of an ETF almost in direct proportion to its nominal leverage ratio, as evidenced by the estimated beta coefficients near 2 and 3 for the SSO and the two triple-leveraged funds (SPXL and UPRO), respectively. However, this relationship does not hold for returns, which increase at a slower pace than risk. The descriptive and regression results consistently show that the excess risk introduced by leverage is not fully compensated by proportional increases in expected returns, implying that the use of leverage introduces inefficiencies in the risk–return trade-off.

The time-varying behavior of the rolling betas reveals that market sensitivity is not constant, particularly during episodes of market stress such as the 2020 crisis. These fluctuations suggest that leverage amplifies not only exposure to the market but also to dynamic behavioral and liquidity factors. The variance decomposition further reinforces this view, showing that while systematic risk explains nearly all variance, the small residual (representing the inefficiency drift) remains nontrivial and may carry significant implications for performance during turbulent periods.

Overall, the findings highlight that leveraged ETFs, though designed to mechanically scale market exposure, behave as imperfect amplifiers of the benchmark. Their structural design leads to deviations from theoretical leverage ratios and exposes investors to compounding effects that erode performance over time. These results indicate that behavioral and mechanical inefficiencies jointly prevent leveraged ETFs from achieving their nominal leverage targets in the long run.

We can also conclude that the Capital Asset Pricing Model is not only a useful tool for predicting returns, but also, it is a framework that can be used for estimating risk exposures to certain factors. This educational paper helped us understand, through a very specific example, that the returns offered by a riskier investment often do not fully compensate for the additional risk. Therefore, interpreting CAPM Beta as an indicator of systematic risk exposure is very helpful. In practice, among institutional investors, this analysis is expanded and made more precise by using commercial factor models that include hundreds of factors as potential sources of systematic risk, such as countries, industries, and investing styles.

The door is open for future research papers that investigate the sources of these inefficiencies, which might be related to transaction costs, asset allocation, and sources of leverage. Additionally, there remains work to be done in identifying potential non-linear relationships between systematic risk and the excess returns of assets. Future studies could also explore the role of institutions, such as market microstructure, investor behavior, and regulatory frameworks in shaping these dynamics, providing a more comprehensive understanding of the factors that drive deviations from investor expectations.

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6 Appendix

In this section, we present the figures that were not included in the previous analysis so that readers can compare certain results with those shown above. The purpose of this section is to complement the previous analysis and ensure that no relevant information is overlooked. Table 5 shows the descriptive statistics of the raw returns, without subtracting the risk free rate. Finally, Figure 4 shows the attribution for the SSO ETF and the SPXL ETF.

Table 5: Descriptive Statistics for ETFs' Returns

Statistics	**SSO**	**SPXL**	**UPRO**	**SPY**
Count	3773	3773	3773	3773
Mean	0.0797	0.0984	0.0993	0.0509
Std. Dev.	2.1735	3.2706	3.2769	1.0772
Min	-26.6028	-41.3577	-42.9954	-11.5887
25%	-0.7461	-1.1256	-1.1149	-0.3716
50% (Median)	0.1321	0.2020	0.2011	0.0682
75%	1.1199	1.6928	1.6881	0.5773
Max	17.3052	24.5346	24.6592	8.6731

Source: Yahoo Finance.

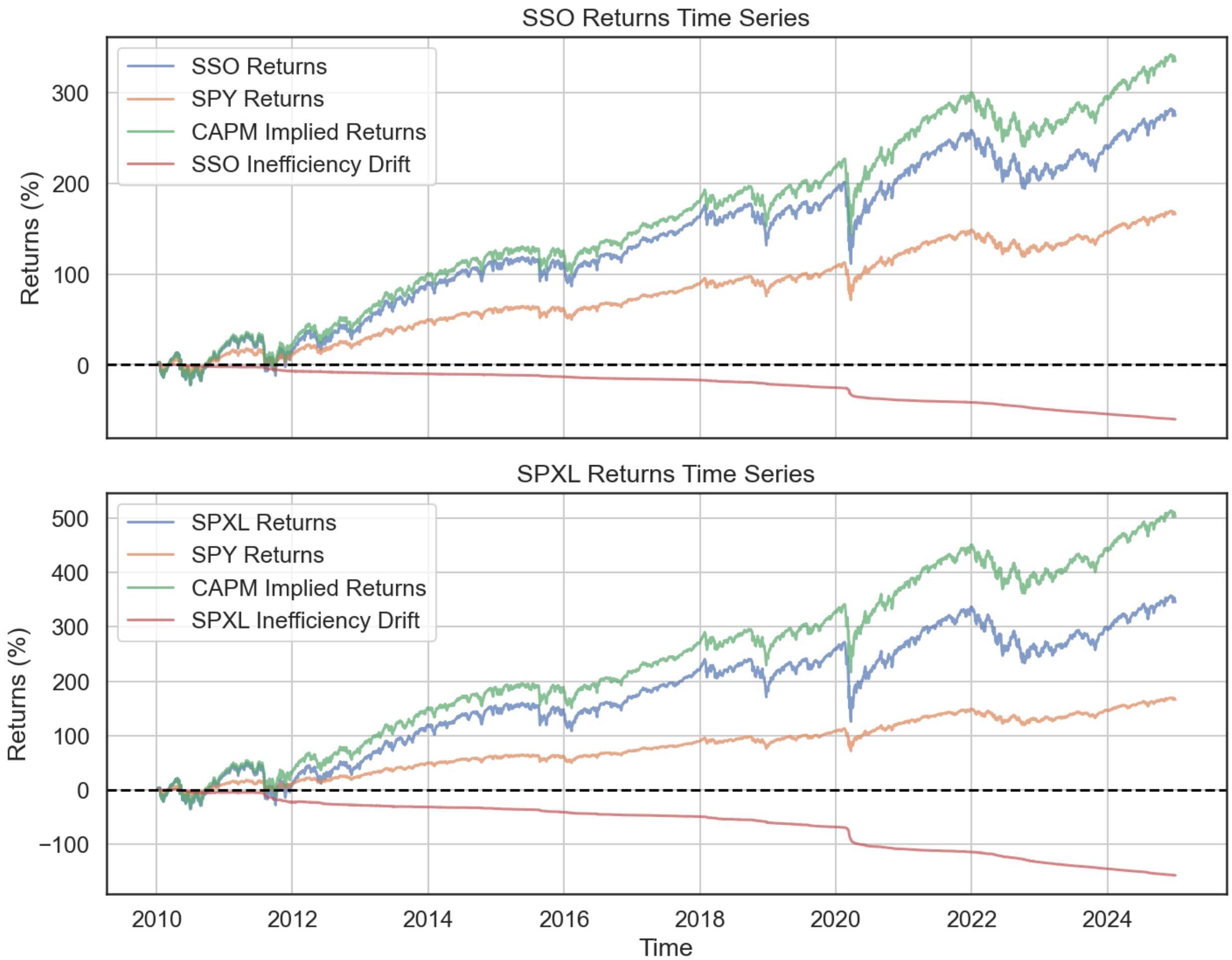

Figure 4: Factor Attribution for the SSO and SPXL ETFs.

Source: Own Creation based on Yahoo Finance Data.