4. $((\forall R)C) \sqcap (\exists R)C' \sqsubseteq (\exists R)C$

① Form counter example

$\neg((\exists R)C) \rightsquigarrow \forall R(\neg C)$

$\mathcal{A} := \{((\forall R)C \sqcap (\exists R)C')(a), (\forall R(\neg C))(a)\}$

② Negations pushed inside

③ Apply Rules

$\sqcap$-Rule on $(\forall R.C \sqcap \exists R.C')(a)$

$\mathcal{A} := \mathcal{A} \cup \{((\forall R)C)(a), ((\exists R)C')(a)\}$

$\exists$-Rule on $(\exists R.C')(a)$

$\mathcal{A} := \mathcal{A} \cup \{(R(a,b)), C'(b)\}$

$\forall$-Rule on $(\forall R.C)(a)$

$\mathcal{A} := \mathcal{A} \cup \{C(b)\}$

...

$\forall$-Rule on $(\forall R.\neg C)(a)$

$\mathcal{A} := \mathcal{A} \cup \{(\neg C)(b)\}$

... contradiction as $C(b) \sqcap \neg C(b)$

However

no contradiction to $C'(b)$

$\therefore$ subsumption does not hold

5. $\forall R(C \sqcup C') \sqsubseteq (\forall R)C \sqcup (\forall R)C'$

① Form counter Example

$\neg(\forall R(C) \sqcup \forall R(C'))$

$\rightsquigarrow \neg(\forall R(C)) \sqcap \neg(\forall R(C'))$

$\rightsquigarrow \exists R(\neg C) \sqcap \exists R(\neg C')$

$\mathcal{A} := \{ \forall R(C \sqcup C')(a), (\exists R(\neg C) \sqcap \exists R(\neg C'))(a) \}$

② Negations pushed inside

③ Apply Rules

$\sqcap$-Rule on $(\exists R(\neg C) \sqcap \exists R(\neg C'))(a)$

$\mathcal{A} := \mathcal{A} \cup \{ (\exists R(\neg C))(a), (\exists R(\neg C'))(a) \}$

$\sqcup$-Rule on $(\forall R(C \sqcup C'))(a)$

$\mathcal{A}_1 := \mathcal{A} \cup \{ (\forall R(C))(a) \}$

$\mathcal{A}_2 := \mathcal{A} \cup \{ (\forall R(C'))(a) \}$

$\exists$-Rule on $(\exists R(\neg C))(a)$ on $\mathcal{A}_1$

$\mathcal{A}_1 := \mathcal{A}_1 \cup \{ R(a,b), \neg C(b) \}$

$\forall$-Rule on $(\forall R(C))(a)$ on $\mathcal{A}_1$

$\mathcal{A}_1 := \mathcal{A}_1 \cup \{ C(b) \}$

... contradiction as $C(b) \sqcap \neg C(b)$

5. contd.

$\exists$-Rule on $(\exists R(\neg C'))(a)$ on A_2

$A_2 := A_2 \cup \{R(a,c), \neg C'(c)\}$

$\forall$-Rule on $(\forall R(C'))(a)$ on A_2

$A_2 := A_2 \cup \{C'(c)\}$

... contradiction as $C'(c) \sqcap \neg C'(c)$

$\therefore$ subsumption holds.