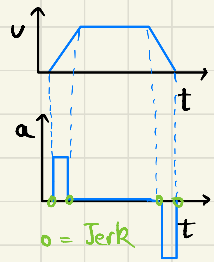
1. DC Motor Model
(exclude phase inductance)

2. Gearbox Model

3. Timing belt drive

4. Power Screw

5. Motion Control
A) Trapezoidal Velocity Profile
B) S Curve velocity Profile

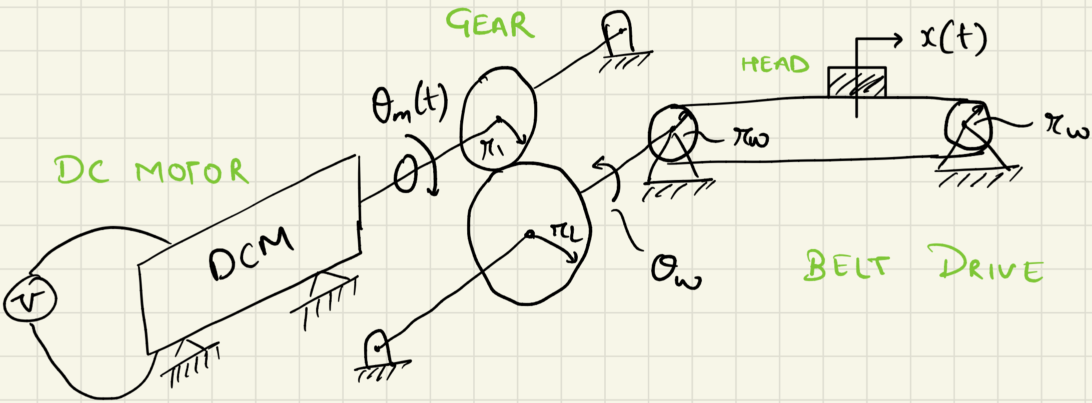


6. PID Controller
A) Linear
- Mobile Robot
- H - Frame 3D Printer
B) Non Linear
- Inverted Pendulum
- Polar Robot

After smoothening corners
⇓

Extra : AC Servo
Stepper
BLDC

Conveyer System



Output = motion of box

Input = voltage to DCM

$\theta_m r_1 = \theta_w r_2$ (no slip)

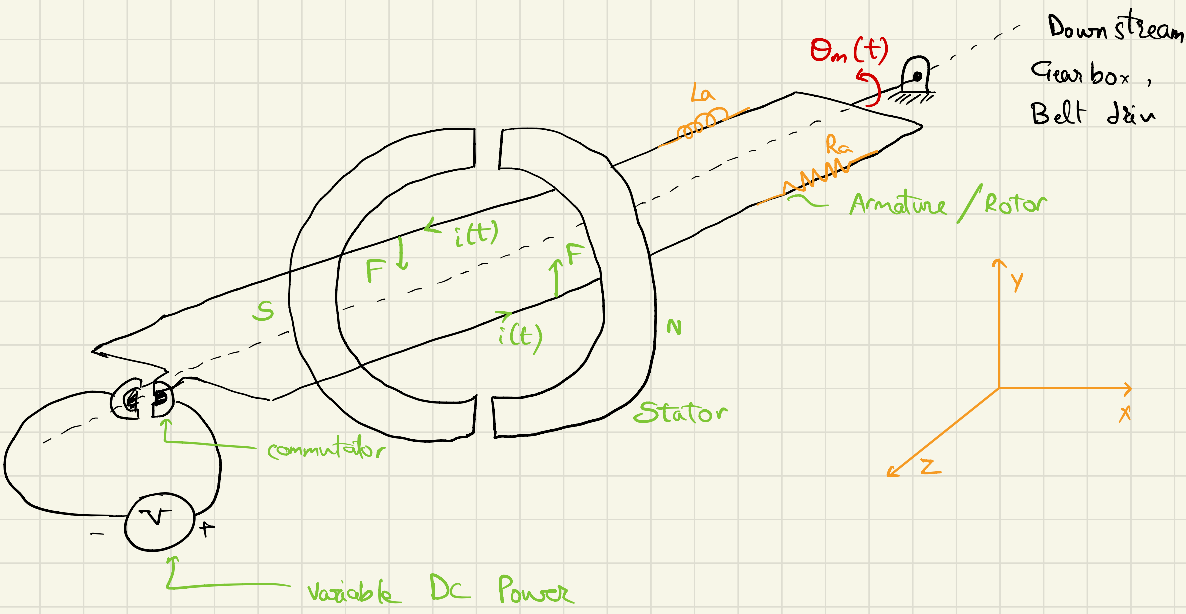
$x(t) = \theta_w r_w$

↳ radians

$$x(t) = \theta_m \frac{r_1}{r_2} r_w$$

Objective :- Relation b/w v (voltage) and $x(t)$ (displacement)

DC MOTOR



Force given by Fleming's Left Hand Rule (FBI)

EOM :-

$e_b = k_b \dot{\theta}_m$
↓
back Emf constant

(back emf)

(electrical eqn) $\quad v(t) = L_a \dfrac{di_a}{dt} + R_a i_a + e_b(t)$

J_m = Rotor Mass MOI about the axis of rotation

D_m = Torsional Damping Nms / rad (Loss of energy in bearings)

(mechanical eq^n)

$$J_m \ddot{\theta}_m + D_m \dot{\theta}_m = t_m(t)$$ (torque created by motor)

$t_m(t) = K_T i_a$

K_T ↓ Torque constant

$L_a << 1$, neglect L_a

from electrical eq^n,

$$R_a i_a + K_b \dot{\theta}_m = v(t)$$

$$i_a(t) = \frac{t_m(t)}{K_T}$$

$$\frac{R_a t_m}{K_T} + K_b \dot{\theta}_m = v(t)$$

$$\frac{R_a}{K_T}\left[J_m \ddot{\theta}_m + D_m \dot{\theta}_m\right] + K_b \dot{\theta}_m = v(t)$$

$$J_m \ddot{\theta}_m + D_m \dot{\theta}_m + \frac{K_b K_T}{R_a} \dot{\theta}_m = \frac{K_b K_T}{R_a} v(t)$$

Take Laplace Transform,

$$s^2 \frac{R_a}{K_T} J_m \Theta_m(s) + s D_m \frac{R_a}{K_T} \Theta_m(s) + s K_b \Theta_m(s) = V(s)$$

↑ uppercase θ_m

$$\Theta_m(s) = \frac{1}{\frac{R_a}{K_T} J_m s^2 + \left(D_m \frac{R_a}{K_T} + K_b\right)s} V(s)$$

$$V(s) = L\left[v(t)\right]$$

$$L\left[V_0\right] = \frac{V_0}{s}$$

$$\Theta_m(s) = \frac{1}{s^2 + \alpha s} \; \frac{V_1}{s}$$

where $$\alpha = \frac{\frac{D_m R_a}{K_T} + k_b}{\frac{J_m R_a}{K_T}}$$

$$V_1 = \frac{K_T}{J_m R_a} V_0$$

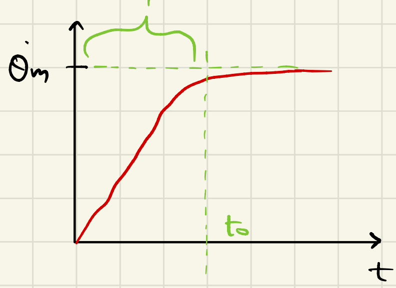
$$\Theta_m(s) = \frac{V_1}{s^3 + \alpha s^2} = \frac{A_1}{s + \alpha} + \frac{B_1}{s^2} + \frac{C_1}{s}$$

$$\theta_m(t) = A_1 e^{-\alpha t} + B_1 r(t) + C_1 u(t)$$

$$\dot{\theta}_m = -\alpha A_1 e^{-\alpha t} + B_1 u(t)$$

so when $t \to \infty$,

$$-\alpha A_1 e^{-\alpha t} = 0$$



Characteristic Curve of DC Motor

$$J_m \ddot{\theta}_m + D_m \dot{\theta}_m + \frac{K_b K_T}{R_a} \dot{\theta}_m = \frac{K_b K_T}{R_a} v(t)$$

$$\frac{R_a}{K_T} t_m(t) + K_b \dot{\theta}_m = v(t)$$

$$t_m(t)\, \dot{\theta}_m(t) = \text{Power} = \text{Constant}$$

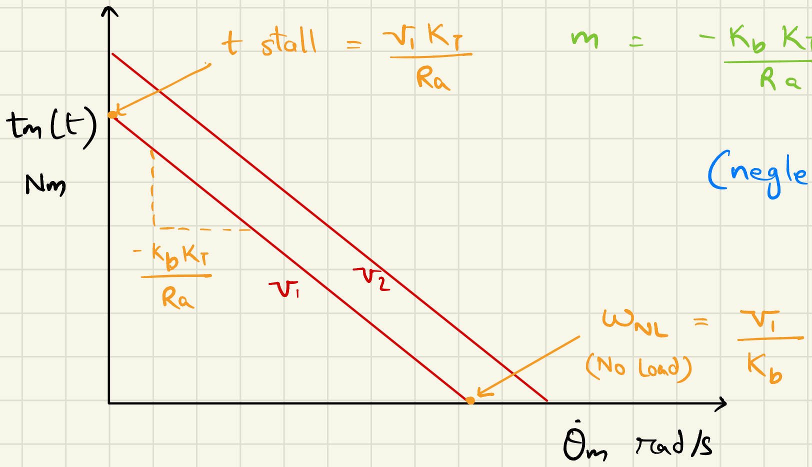
$$y = mx + c$$

$y \downarrow t_m$, $x \downarrow \theta_m$

$$t_m = \underbrace{\left(-\frac{K_b K_T}{R_a}\right)}_{\text{slope}} \dot{\theta}_m + \underbrace{\left(v(t)\frac{K_T}{R_a}\right)}_{\text{y-intercept}}$$

$$m = -\frac{K_b K_T}{R_a}$$

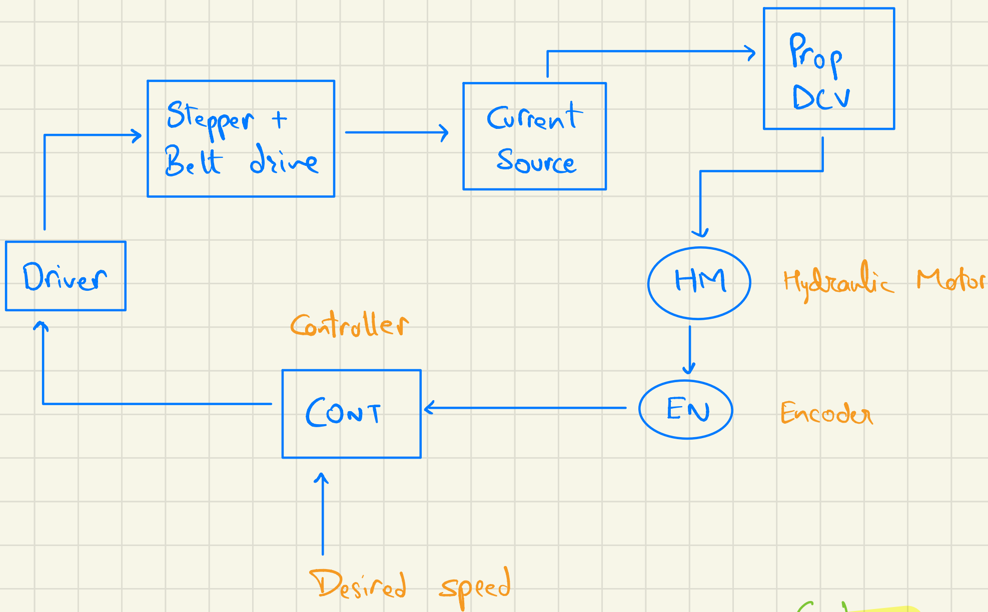
(neglecting La)



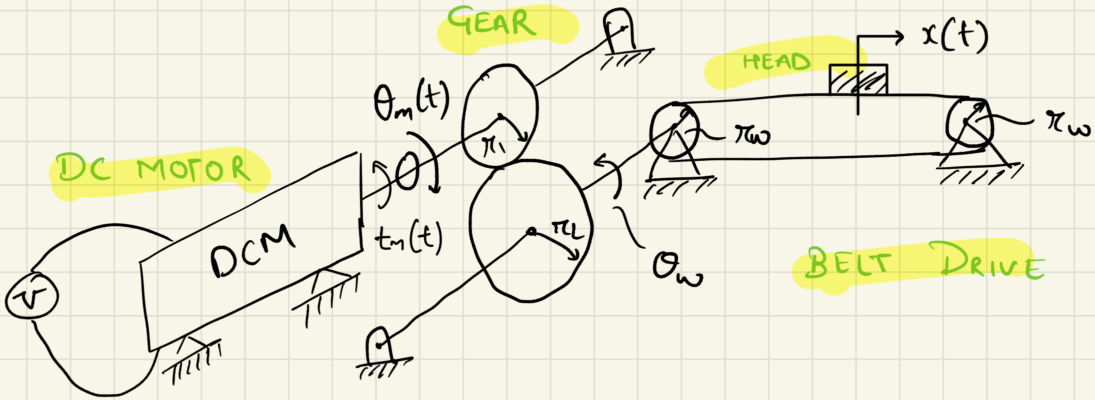
LAB – II

Speed control of Hydraulic motor via positional control of stepper motor timing belt drive assembly.

Schematic :-



Codesys



From DC Motor :-

$$\frac{R_a \, t_m(t)}{K_T} + K_b \dot{\theta}_m = v(t)$$

$$\overset{0}{\cancel{L_a}} \frac{di_a}{dt} \qquad t_m(t) = K_T \, i_a$$

$$\theta_2 r_2 = \theta_1 r_1 \qquad (\because \text{Kinematics})$$

$$\theta_1 = \theta_m$$

$$x(t) = r_m \theta_2(t)$$

Forward Kinematics :-
relation b/w workspace and joint variable

$$\boxed{x(t) = r_w \frac{r_1}{r_2} \theta_1}$$

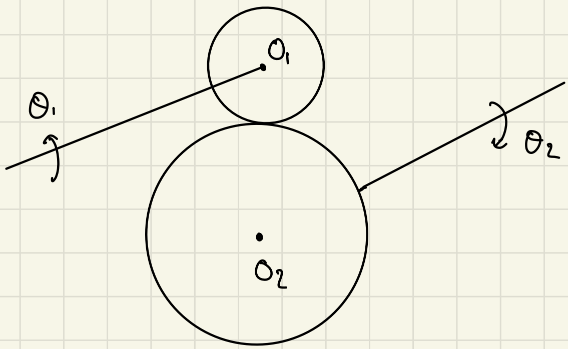
FK
(Forward Kinematic)

$$\boxed{\theta_1(t) = \frac{r_2}{r_1 r_w} \, x(t)}$$

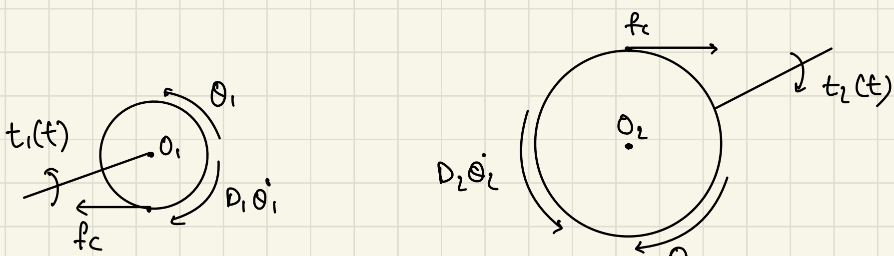
IK

(Inverse Kinematics)

Gear box



FBD



Contact force

$$\sum M_{O_1} = I_{O_1}\ddot{\theta}_1$$

$$\sum M_{O_2} = I_{O_2}\ddot{\theta}_2$$

$$I_{O_1}\ddot{\theta}_1 = t_1(t) - D_1\dot{\theta}_1 - f_c r_1$$

$$-I_{O_2}\ddot{\theta}_2 = -t_2(t) + D_2\dot{\theta}_2 - f_c r_2$$

$$r_1\theta_1 = r_2\theta_2$$ (No Slip)

(write in terms of fc)

$$f_c = \frac{1}{r_2}\left[I_{O_2}\ddot{\theta}_2 + D_2\dot{\theta}_2 - t_2(t)\right]$$

Substituting in θ_1

$$I_{O_1}\ddot{\theta}_1 + D_1\dot{\theta}_1 + \frac{r_1}{r_2}\left[I_{O_2}\ddot{\theta}_2 + D_2\dot{\theta}_2 - t_2(t)\right] = t_1(t)$$

$$\left[I_{O_1} + \left(\frac{r_1}{r_2}\right)^2 I_{O_2}\right]\ddot{\theta}_1 + \left[D_1 + \left(\frac{r_1}{r_2}\right)^2 D_2\right]\dot{\theta}_1 = t_1(t) + \frac{r_1}{r_2}t_2(t)$$

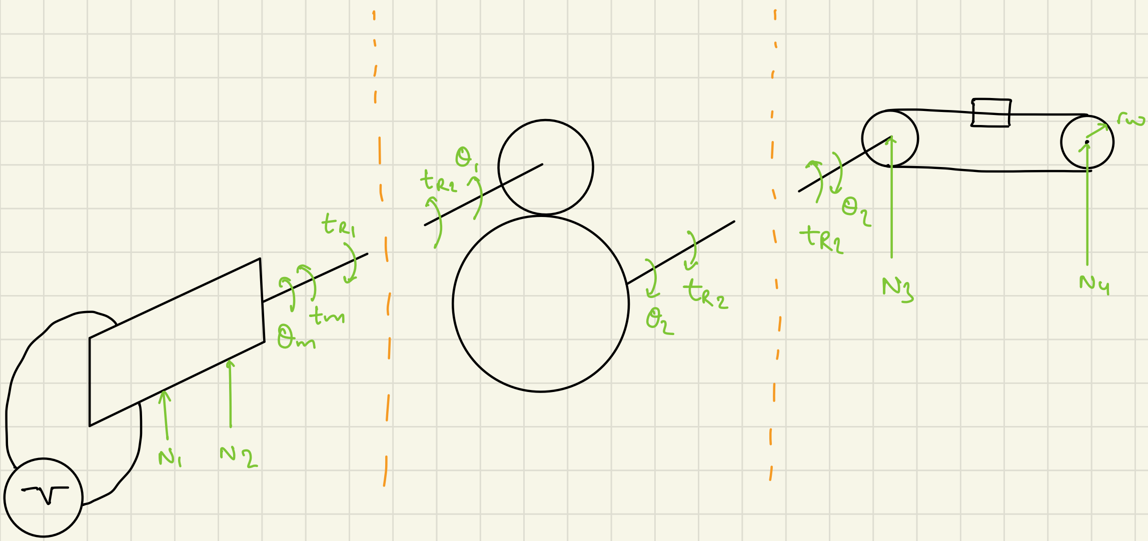
EOM

Replace I_{O_1} as J_{O_1} &

I_{O_2} as J_{O_2}

$$\left[J_{O_1} + \left(\frac{r_1}{r_2}\right)^2 J_{O_2} \right] \ddot{\theta}_1 + \left[D_1 + \left(\frac{r_1}{r_2}\right)^2 D_2 \right] \dot{\theta}_1 = t_1(t) + \frac{r_1}{r_2} t_2(t)$$

Separating the system into III components



Motor :-

$$\frac{R_a t_m}{K_T} + K_b \dot{\theta}_m = v(t)$$

$$J_m \ddot{\theta}_m + D_m \dot{\theta}_m = t_m(t) - t_{R_1}(t)$$

Gear box :-

$$\left[\overbrace{J_{o_1} + \left(\frac{r_1}{r_2}\right)^2 J_{o_2}}^{\overline{J_{12}}}\right]\ddot{\theta}_1 + \left[\underbrace{D_1 + \left(\frac{r_1}{r_2}\right)^2 D_2}_{\overline{D_{12}}}\right]\dot{\theta}_1 = t_{R_1}(t) + \frac{r_1}{r_2} t_{R_2}(t)$$

Belt drive :-

$$-\left[\underbrace{J_{O_3} + J_{O_4} + m r_w^2}_{\bar{J}_{34}}\right]\ddot{\theta}_2 - \left[\underbrace{D_3 + D_4}_{\bar{D}_{34}}\right]\dot{\theta}_2 = t_{R_2}(t)$$

$$-\bar{J}_{34}\,\ddot{\theta}_2 - \bar{D}_{34}\,\dot{\theta}_2 = t_{R_2}(t)$$

Putting in Gearbox,

$$\left[\bar{J}_{12} + \bar{J}_{34}\left(\frac{r_1}{r_2}\right)^2\right]\ddot{\theta}_1 + \left[\bar{D}_{12} + \bar{D}_{34}\left(\frac{r_1}{r_2}\right)^2\right]\dot{\theta}_1 = t_{R_1}$$

Combine everything

$$R_a \frac{t_m(t)}{K_T} + K_b\,\dot{\theta}_m = v(t)$$

Adding Gearbox + motor, $(\theta_m = \theta_1)$

$$\boxed{\left[J_m + \bar{J}_{12} + \left(\frac{r_1}{r_2}\right)^2 \bar{J}_{34}\right]\ddot{\theta}_m + \left[D_m + \bar{D}_{12} + \bar{D}_{34}\left(\frac{r_1}{r_2}\right)^2\right]\dot{\theta}_m = t_m(t)}$$

Substitute this $t_m(t)$ to motor.

PID controller,

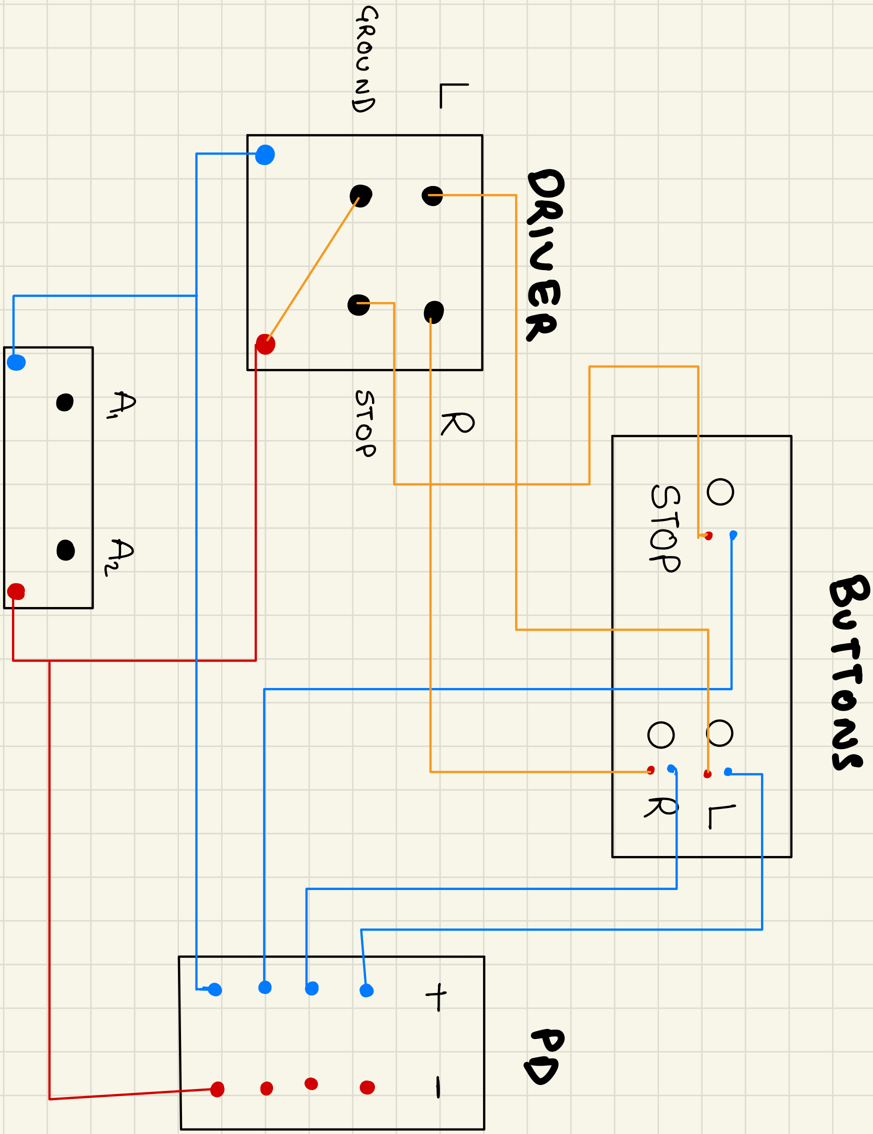
$$v(t) = -K_P(\theta_m - \theta_{md}) - K_D(\dot{\theta}_m - \dot{\theta}_{md}) - K_I \int_0^t (\theta_m(\tau) - \theta_{md}(\tau))\, d\tau$$

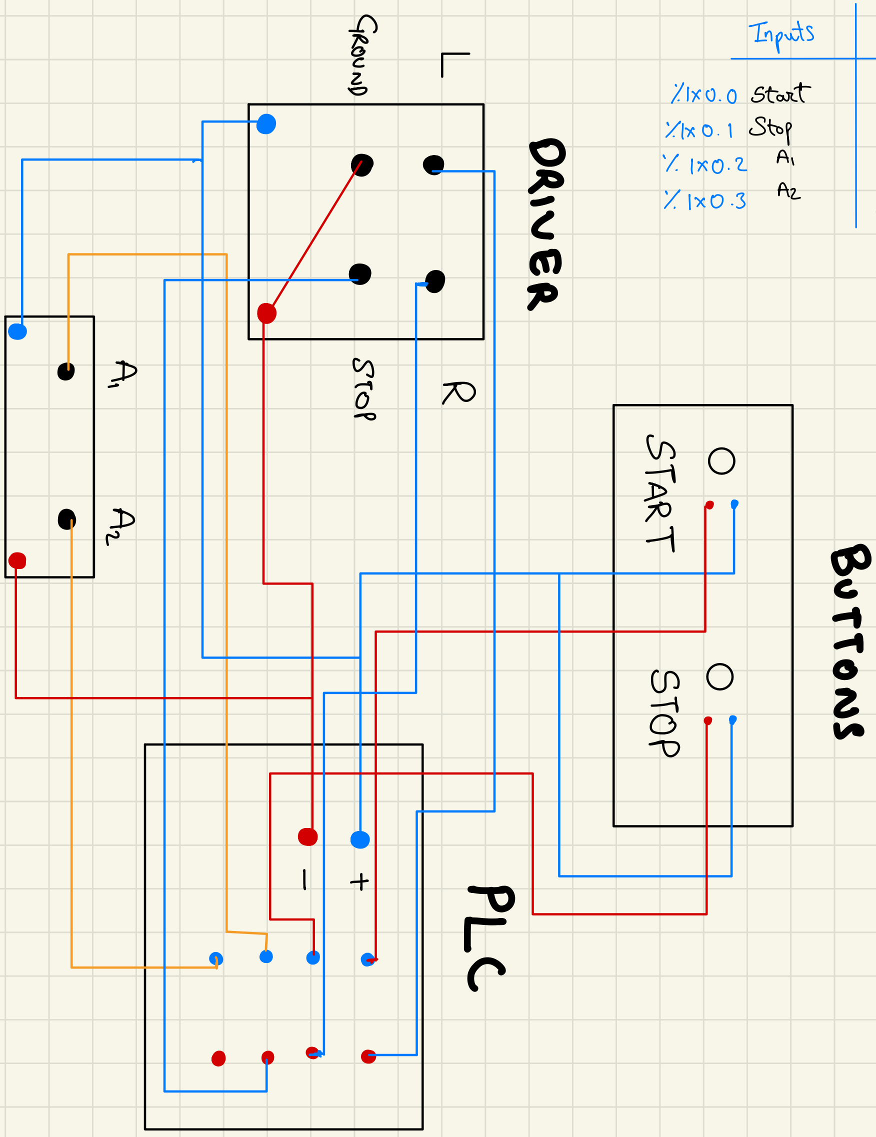
$\theta_{md} \rightarrow$ TVP

(Trapezoidal Velocity Profile)

LAB - III

MANUAL

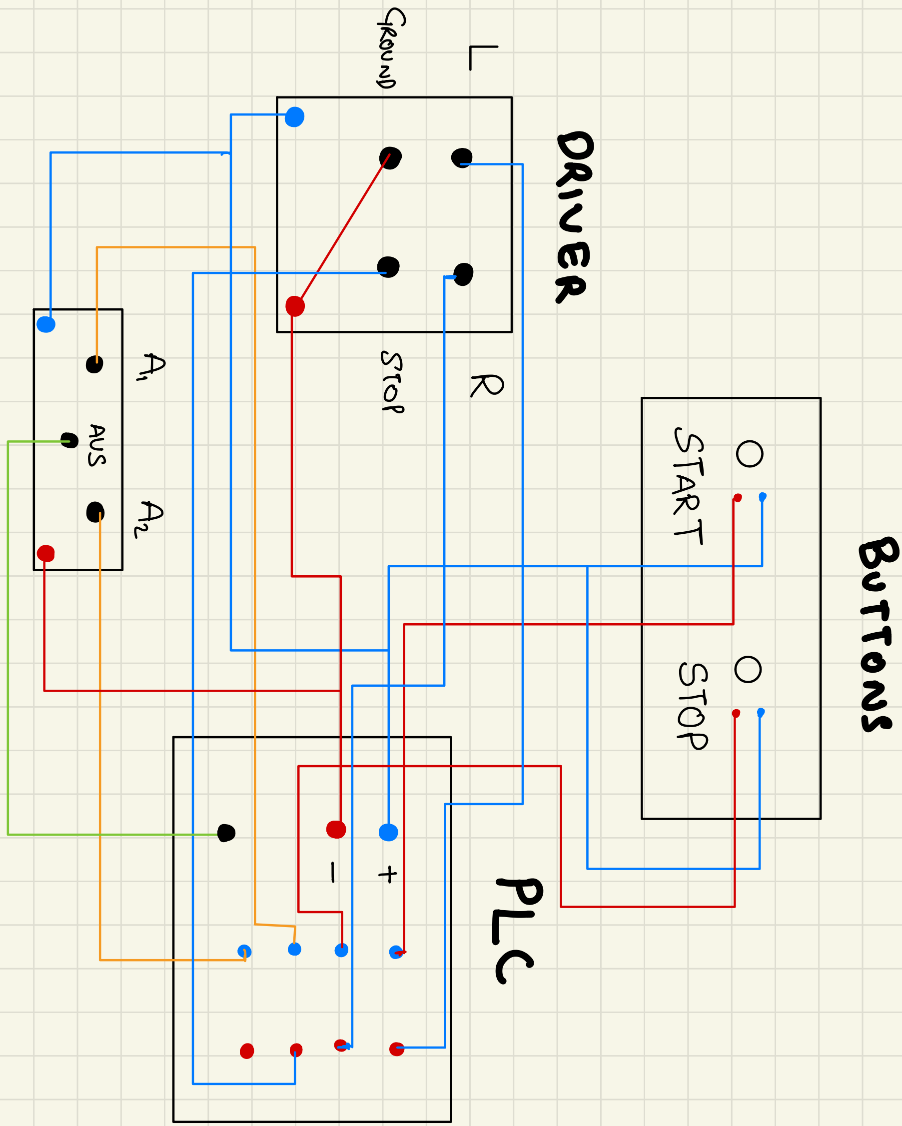




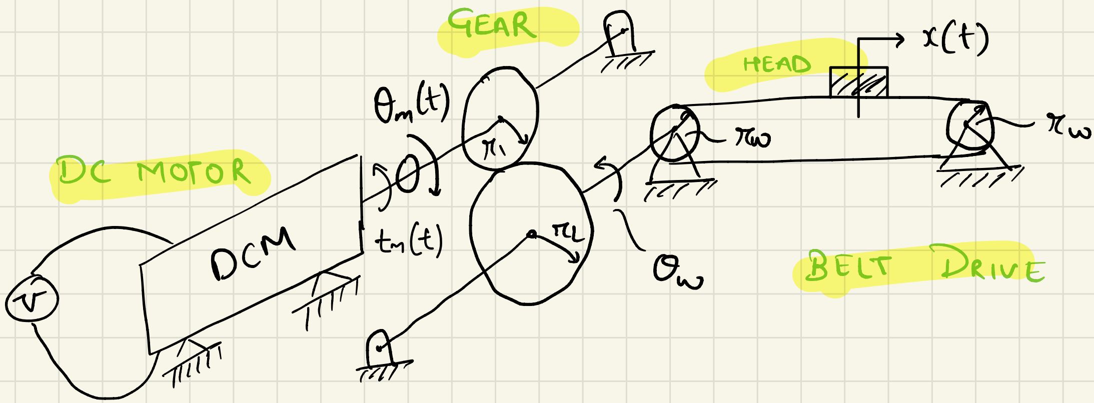
Inputs		Outputs	
%Ix0.0	Start	Stop	%Qx0.0
%Ix0.1	Stop	Left	%Qx0.1
%Ix0.2	A_1	Right	%Qx0.2
%Ix0.3	A_2	Stop-OUT_R	%Qx0.3
		Main Stop	%Qx1.0

PLC

With Analog Input



PLC



General equation :-

$$\bar{J}\ddot{\theta} + \bar{D}\dot{\theta} = v(t)$$

Open loop
(no encoder)

closed loop

$\theta_d(t)$ = Desired Trajectory

Open Loop :-

$$\bar{J}\ddot{\theta} + \bar{D}\dot{\theta} = v(t) \longrightarrow (1)$$

$$v(t) = \bar{J}\ddot{\theta}_d + \bar{D}\dot{\theta}_d \longrightarrow (2)$$

computed torque controller

$$\bar{J}\ddot{\theta} + \bar{D}\dot{\theta} = \bar{J}\ddot{\theta}_d + \bar{D}\dot{\theta}_d$$

$$\bar{J}(\ddot{\theta} - \ddot{\theta}_d) + \bar{D}(\dot{\theta} - \dot{\theta}_d) = 0$$

$$e(t) = \theta(t) - \theta_d(t)$$

$$\bar{J}\ddot{e} + \bar{D}\dot{e} = 0$$

$$e(t) \to 0 \quad , \quad t \to \infty$$
$$e(0) = 0 \quad , \quad \dot{e}(0) = 0$$

Problem of open loop,

→ unable to change the eigenstructure.

→ no poles in the LHP which leads to unstability.

Closed Loop :-

PID Controller

$$\bar{J}\ddot{\theta} + \bar{D}\dot{\theta} = \tau(t)$$

$$\tau(t) = \bar{J}\Big[-K_p(\theta-\theta_d) - K_D(\dot{\theta}-\dot{\theta}_d) - K_I\int_0^t[\theta(\tau)-\theta_d(\tau)]\,d\tau\Big] + \bar{D}\dot{\theta} + \bar{J}\ddot{\theta}_d$$

Computed torque control

$$\bar{J}\Big[(\ddot{\theta}-\ddot{\theta}_d) + K_D(\dot{\theta}-\dot{\theta}_d) + K_p(\theta-\theta_d) + K_I\int_0^t[\theta(\tau)-\theta_d(\tau)]\,d\tau\Big] = 0$$

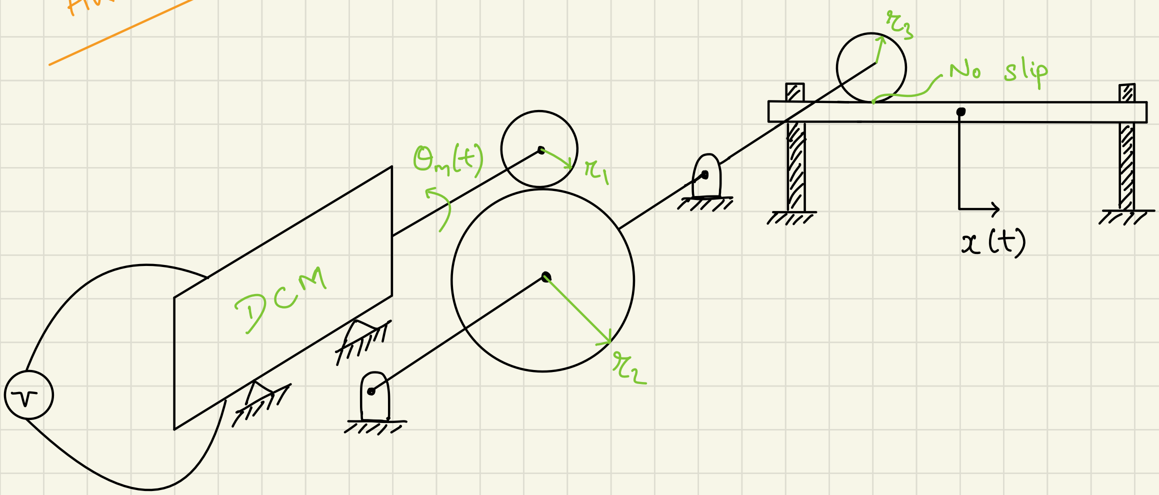
(Differentiating one more time)

$$\dddot{e} + K_D\ddot{e} + K_p\dot{e} + K_I e = 0$$

$$\lambda^3 + K_D\lambda^2 + K_p\lambda + K_I = 0$$

$$\lambda^3 + K_D \lambda^2 + K_p \lambda + K_I = (\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3)$$

HW - 1

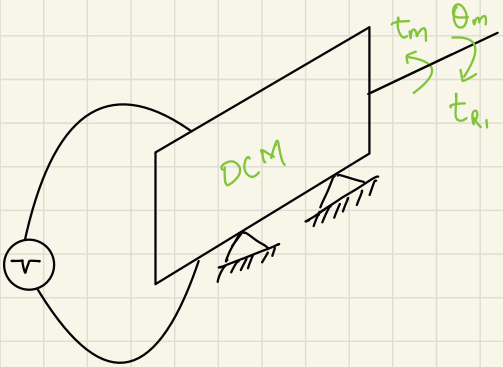


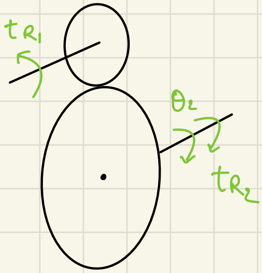
Advantages :-

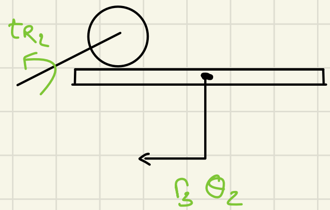
i) Mitigation of downstream inertia and damping constants

ii) Increase in resolution (No slip, $\theta_2 = \frac{r_1}{r_2}\theta_1$)

FBD



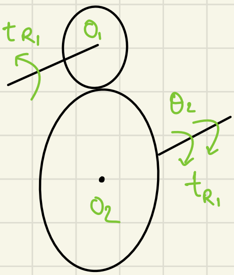


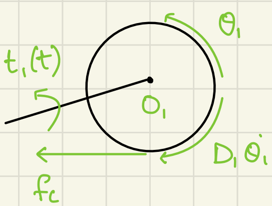


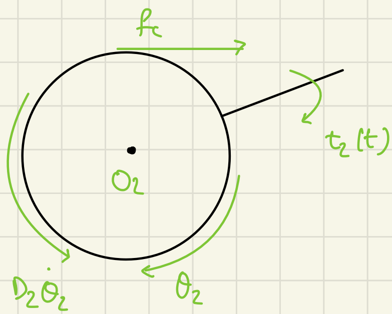
$$x(t) = \frac{r_1}{r_2}\, r_3\, \theta_m(t)$$

$$\theta_2 r_2 = \theta_m r_1$$

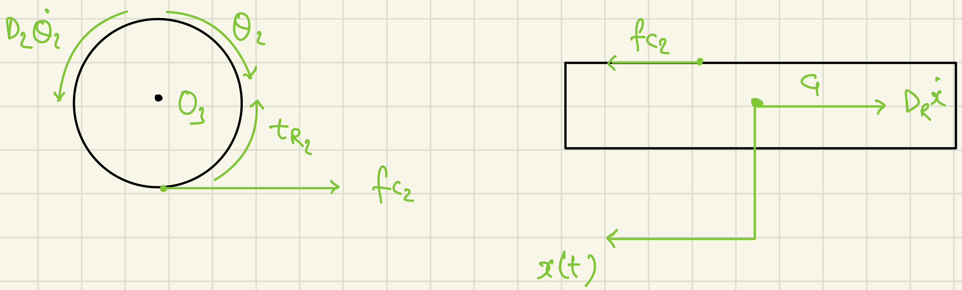
Gearbox







Rack & Pinion



EOM

Motor :-

(electrical eqⁿ)

$$v(t) = \cancel{L_a \frac{di_a}{dt}}^{\,0} + R_a i_a + K_b \dot{\theta}_m$$

(mechanical eqⁿ) $$J_m \ddot{\theta}_m + D_m \dot{\theta}_m = t_m(t)$$

$$R_a i_a + K_b \dot{\theta}_m = v(t)$$

$$\frac{R_a t_m}{K_T} + K_b \dot{\theta}_m = v(t)$$

$$\frac{R_a}{K_T}\left(J_m \ddot{\theta}_m + D_m \dot{\theta}_m \right) + K_b \dot{\theta}_m = v(t)$$

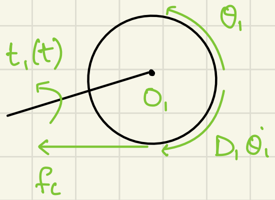
$$J_m \ddot{\theta}_m + D_m \dot{\theta}_m + \frac{K_b K_T}{R_a} \dot{\theta}_m = \frac{K_b K_T}{R_a} v(t)$$

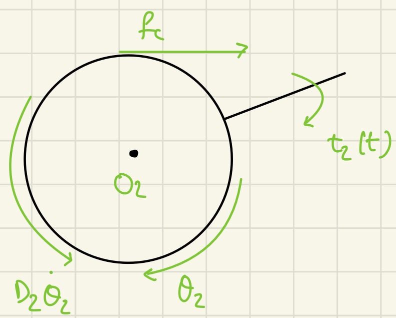
$$\because \; J_m = J_{01}, \quad D_m = D_{01}, \quad \theta_m = \theta_1$$

$$J_{01} \ddot{\theta}_1 + D_{01} \dot{\theta}_1 + \frac{K_b K_T}{R_a} \dot{\theta}_1 = \frac{K_b K_T}{R_a} v(t)$$

$$J_{01} \ddot{\theta}_1 + D_{01} \dot{\theta}_1 = t_m(t) - t_{R_1}(t)$$

Gearbox





$$\sum M_{O_1} = I_{O_1} \ddot{\theta}_1$$

$$\sum M_{O_2} = I_{O_2} \ddot{\theta}_2$$

$$I_{O_1} \ddot{\theta}_1 = t_1(t) - D_1 \dot{\theta}_1 - f_c r_1$$

$$- I_{O_2} \ddot{\theta}_2 = - t_2(t) + D_2 \dot{\theta}_2 - f_c r_2$$

According no slip,

$$r_1 \theta_1 = r_2 \theta_2$$

$$f_2 = \frac{1}{r_2} \left[I_{O_1} \ddot{\theta}_1 + D_2 \dot{\theta}_2 - t_2(t) \right]$$

Substituting in Q_1

$$I_{O_1}\ddot{\theta}_1 + D_1\dot{\theta}_1 + \frac{r_1}{r_2}\left[I_{O_2}\ddot{\theta}_2 + D_2\dot{\theta}_2 - t_2(t)\right] = t_1(t)$$

$$\left[I_{O_1} + \left(\frac{r_1}{r_2}\right)^2 I_{O_2} \right]\ddot{\theta}_1 + \left[D_1 + \left(\frac{r_1}{r_2}\right)^2 D_2 \right]\dot{\theta}_1 = t_1(t) + \frac{r_1}{r_2} t_2(t)$$

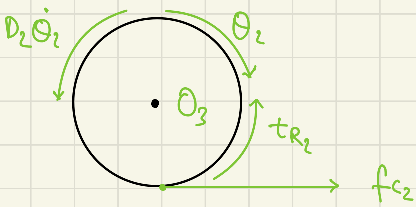
EOM

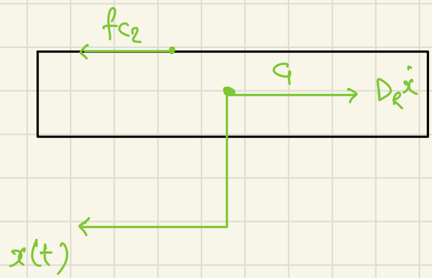
Replace I_{O_1} as J_{O_1} &

I_{O_2} as J_{O_2}

$$\left[J_{O_1} + \left(\frac{r_1}{r_2}\right)^2 J_{O_2} \right]\ddot{\theta}_1 + \left[D_1 + \left(\frac{r_1}{r_2}\right)^2 D_2 \right]\dot{\theta}_1 = t_1(t) + \frac{r_1}{r_2} t_2(t)$$

Rack & Pinion





$$\sum M_{O_3} = -J_{O_3}\ddot{\theta}_2 = r_3 f_{c_2} + D_3\dot{\theta}_2 + t_{R_2}(t)$$

$$\sum F_x = -m_R\ddot{x} = -f_{c_2} + D_R\dot{x}$$

$$f_{c_2} = m_R\ddot{x} + D_R\dot{x}$$

$$t_{R_2}(t) = -\Big[J_{O_3}\ddot{\theta}_2 + D_3\dot{\theta}_2 + m_R r_3^2\ddot{\theta}_2 + D_R r_3^2\dot{\theta}_2 \Big]$$

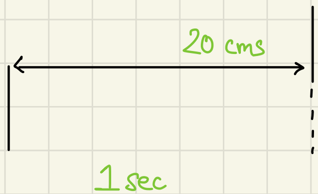
M1

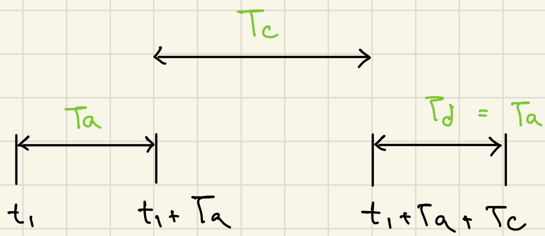
DCMO-S,
MControl 01
Computer Torque Control

Combining all eq's,

$$\bar{J}\ddot{\theta}_m + \bar{D}\dot{\theta}_m = \tau(t)$$

$$\tau(t) = \bar{J}\theta$$





$$t_1 < t \leq t_1 + T_a$$

$$t_1 + T_a < t \leq t_1 + T_a + T_c$$

$$t_1 + T_a + T_c \leq t \leq t_2$$

$$x_d(t) = \text{MC01}$$

$$\theta_d(t) = \frac{r_2}{r_1 r_3} x_d(t)$$

$$T = 2T_a + T_c$$

$$S = 2S_a + S_c$$

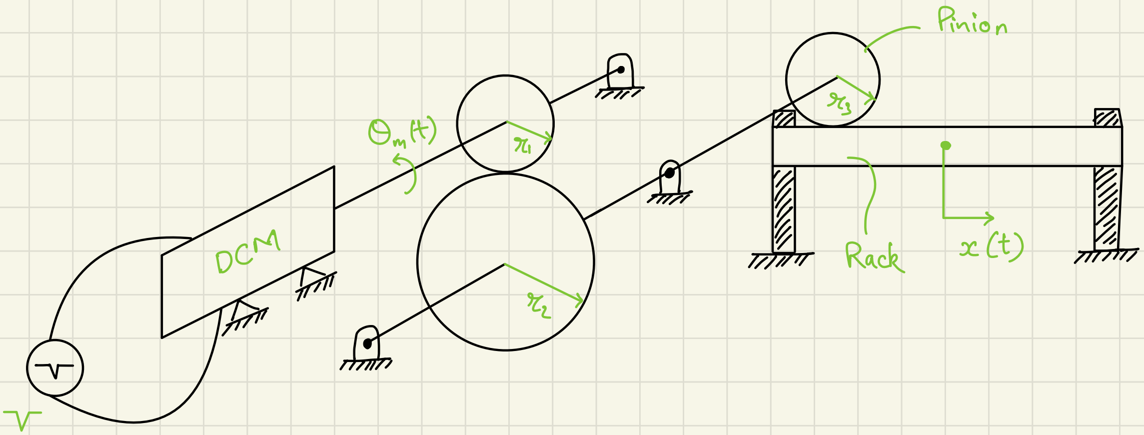
$$\tau(t) = \bar{J}\ddot{\theta}_d + \bar{D}\dot{\theta}_m - \bar{J}\Big[k_p(\theta_m - \theta_d) + K_D(\dot{\theta}_m - \dot{\theta}_d) + K_I \int_0^t (\theta_m(t) - \theta_d(t))\, d\tau \Big]$$

$$e(t) = \theta_m - \theta_d$$

$$\bar{J}\left(\ddot{e} + K_D \dot{e} + k_p e + K_I \int_0^t e(\tau)\, dt \right) = 0$$

$$x(t) = r_3 \frac{r_1}{r_2} \theta_m(t)$$

Q1.



A. Derive the kinematic relationship b/w $x(t)$ and $\theta_m(t)$

B. Draw an FBD of the assembly showing the reactionary forces and torques

C. Derive the EOM relating the voltage $v(t)$ to the rack displacement $x(t)$

D. Assume the EOM is of the form

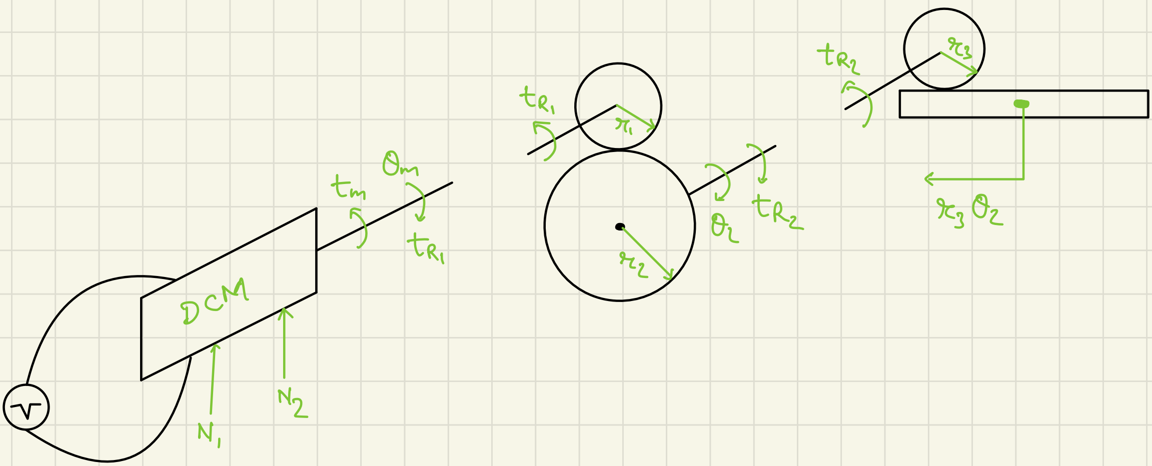
$$\bar{J}\ddot{\theta}_m + \bar{D}\dot{\theta}_m = v(t)$$

Design a computed torque PD controller to place the poles @ -40, -50.

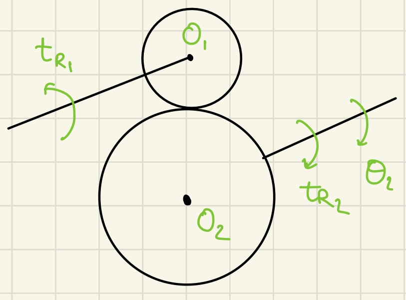
E. The rack has to be moved 50 cms in 3 seconds. Design the TVP for the motion

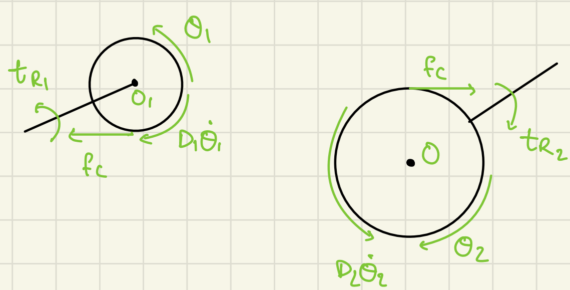
B.

Free Body Diagrams :-

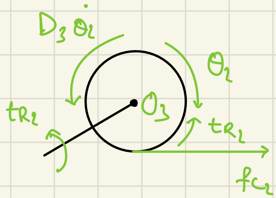


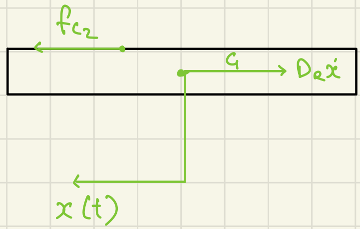
Gearbox :-





Rack and Pinion





C.

Motor :-

(electrical eq^n) $$v(t) = \underbrace{\cancel{L_a \frac{di_a}{dt}}}_{0} + R_a i_a + K_b \dot{\theta}_m$$

$$R_a i_a + K_b \dot{\theta}_m = v(t)$$

$$\frac{R_a t_m}{K_T} + K_b \dot{\theta}_m = v(t) \quad (\because t_m = K_T i_a)$$

$\longrightarrow$ 1

(mechanical eq^n) $$J_{O_1}\ddot{\theta}_1 + D_{O_1}\dot{\theta}_1 = t_m$$

Substitute in 1

$$\frac{R_a}{K_T}\left(J_{O_1}\ddot{\theta}_1 + D_{O_1}\dot{\theta}_1\right) + K_b \dot{\theta}_m = v(t)$$

$$J_{O_1}\dot{\theta}_1 + D_{O_1}\hat{\theta}_1 + \frac{K_b K_T}{R_a}\dot{\theta}_1 = K_b K_T v(t)$$

$$J_{o_1}\ddot{\theta}_1 + D_{o_1}\dot{\theta}_1 = \frac{K_b K_T}{} v(t) - \frac{K_b K_T}{R_a}\dot{\theta}_1$$

$$J_{o_1}\ddot{\theta}_1 + D_{o_1}\dot{\theta}_1 = t_m(t) - t_{R_1}(t) \longrightarrow (2)$$

Gearbox :-

$$\Sigma M_{o_1} = J_{o_1}\ddot{\theta}_1$$

$$J_{o_1}\ddot{\theta}_1 = t_{R_1} - f_{c_1} r_1 - D_1\dot{\theta}_1$$

$$t_{R_1} = J_{o_1}\ddot{\theta}_1 + f_{c_1} r_1 + D_1\dot{\theta}_1 \longrightarrow (3)$$

$$\Sigma M_{o_2} = -J_{o_2}\ddot{\theta}_2$$

$$-J_{o_2}\ddot{\theta}_2 = D_2\dot{\theta}_2 - f_{c_1} r_2 - t_{R_2}$$

$$f_{c_1} r_2 = J_{o_2}\ddot{\theta}_2 + D_2\dot{\theta}_2 - t_{R_2}$$

$$f_{c_1} = \left(J_{o_2}\ddot{\theta}_2 + D_2\dot{\theta}_2 - t_{R_2}\right)\frac{1}{r_2} \longrightarrow (4)$$

Rack and Pinion :-

$$\Sigma M_{o_3} = -J_{o_3}\ddot{\theta}_2$$

$$-J_{o_3}\ddot{\theta}_2 = r_3 f_{c_2} + D_3\dot{\theta}_2 + t_{R_2}$$

$$t_{R_2} = -J_{o_3}\ddot{\theta}_2 - r_3 f_{c_2} - D_3\dot{\theta}_2 \longrightarrow (5)$$

$$\sum F_x = -m_R \ddot{x}$$

$$-m_R \ddot{x} = -f_{c_2} + D_R \dot{x}$$

$$f_{c_2} = m_R \ddot{x} + D_R \dot{x}$$

Substituting in 5

$$t_{R_2} = -J_{O_3} \ddot{\theta}_2 - r_3 m_R \ddot{x} - r_3 D_R \dot{x} - D_3 \dot{\theta}_2 \longrightarrow 6$$

Combine everything,

Put 6 $\rightarrow$ 4,

$$f_{c_1} = \left(J_{O_2} \ddot{\theta}_2 + D_2 \dot{\theta}_2 + J_{O_3} \ddot{\theta}_2 + r_3 m_R \ddot{x} + r_3 D_R \dot{x} + D_3 \dot{\theta}_2 \right) \frac{1}{r_2} \longrightarrow 7$$

Put 7 $\rightarrow$ 3,

$$t_{R_1} = J_{O_1} \ddot{\theta}_1 + D_1 \dot{\theta}_1 + \frac{r_1}{r_2} \left(J_{O_2} \ddot{\theta}_2 + D_2 \dot{\theta}_2 + J_{O_3} \ddot{\theta}_2 + r_3 m_R \ddot{x} + r_3 D_R \dot{x} + D_3 \dot{\theta}_2 \right) \longrightarrow 8$$

Put 8 $\rightarrow$ 2 ,

$$J_{O_1}\ddot{\theta}_1 + D_1\dot{\theta}_1 = t_m(t) - J_{O_1}\ddot{\theta}_1 - D_1\dot{\theta}_1 - \frac{r_1}{r_2}\Big(J_{O_2}\ddot{\theta}_2 + D_2\dot{\theta}_2 + J_{O_3}\ddot{\theta}_2 + r_3 m_R \ddot{x} + r_3 D_R \dot{x} + D_3\dot{\theta}_2 \Big)$$

$$t_m(t) = 2J_{O_1}\ddot{\theta}_1 + 2D_1\dot{\theta}_1 + \frac{r_1}{r_2}\Big(J_{O_2}\ddot{\theta}_2 + D_2\dot{\theta}_2 + J_{O_3}\ddot{\theta}_2 + r_3 m_R \ddot{x} + r_3 D_R \dot{x} + D_3\dot{\theta}_2 \Big) \longrightarrow 9$$

Put 9 $\rightarrow$ 1

$$\frac{R_a}{K_T}\Big[2J_{O_1}\ddot{\theta}_1 + 2D_1\dot{\theta}_1 + \frac{r_1}{r_2}\Big(J_{O_2}\ddot{\theta}_2 + D_2\dot{\theta}_2 + J_{O_3}\ddot{\theta}_2 + r_3 m_R \ddot{x} + r_3 D_R \dot{x} + D_3\dot{\theta}_2 \Big)\Big] + K_b\dot{\theta}_m = v(t)$$

A.

$$\theta_2 \, r_2 = \theta_m \, r_1$$

$$x(t) = r_3 \, \theta_2(t)$$

$$\boxed{x(t) = \frac{r_1}{r_2} \cdot r_3 \cdot \theta_m(t)}$$

D.

$$\bar{J}\ddot{\theta} + \bar{D}\dot{\theta} = \tau(t)$$

PD Controller,

$$\tau(t) = \bar{J}\left[-K_P(\theta - \theta_d) - K_D(\dot{\theta} - \dot{\theta}_d) \right] + \bar{D}\dot{\theta} + \bar{J}\ddot{\theta}_d$$

Computed Torque control

$$\bar{J}\left[(\ddot{\theta} - \ddot{\theta}_d) + K_D(\dot{\theta} - \dot{\theta}_d) + K_P(\theta - \theta_d) \right] = 0$$

$$\ddot{e} + K_D \dot{e} + K_P e = 0$$

$$\lambda^2 + K_D \lambda + K_P = (\lambda + 40)(\lambda + 50)$$

$$(\lambda + 40)(\lambda + 50) = \lambda^2 + 50\lambda + 40\lambda + 2000$$

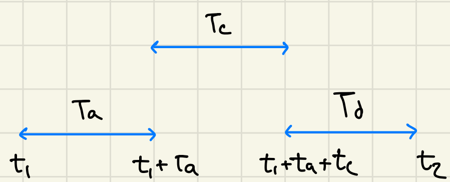
$$(\lambda + 40)(\lambda + 50) = \lambda^2 + 90\lambda + 2000$$

$$\boxed{\therefore \quad K_D = 90 \qquad K_P = 2000}$$

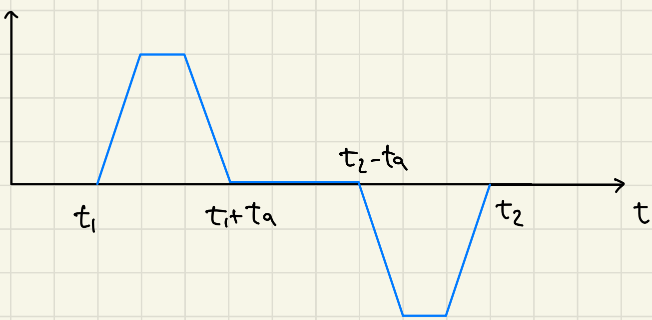
E.

50 cms $\longrightarrow$ 3 seconds

For TVP,



where,
T_a = acceleration
T_c = constant speed
T_d = deceleration



Assume S_a = 5 cms

$\therefore$ $S_d = S_a$ = 5 cms

$$S = S_a + S_c + S_d$$

$$S_c = 50 - 10 = 40 \text{ cms}$$

$$T_a = \frac{S_a T}{2 S_a + \frac{S_c}{2}}$$

$$= \frac{5 \times 3}{2(5) + \frac{40}{2}} = \frac{15}{30} = \frac{1}{2}$$

$$T_a = 0.5 \text{ s} = T_d$$

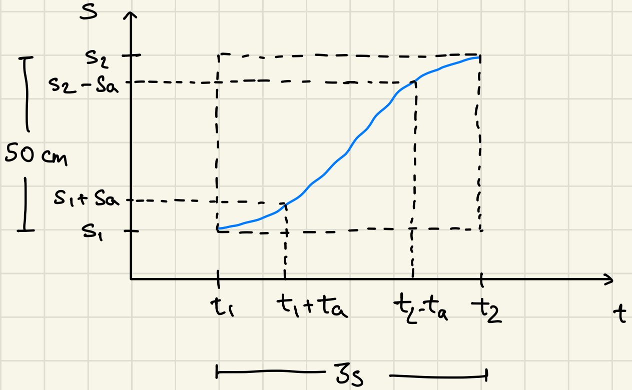
$$T = T_a + T_c + T_d$$

$$T_c = 3 - 1$$

$$T_c = 2 \text{ s}$$

$\therefore$ $S_a = 5\text{cms}$ $\quad$ $S_c = 40 \text{ cm}$ $\quad$ $S_d = 5 \text{ cms}$

$T_a = 0.5\text{s}$ $\quad$ $T_c = 2\text{s}$ $\quad$ $T_d = 0.5\text{s}$



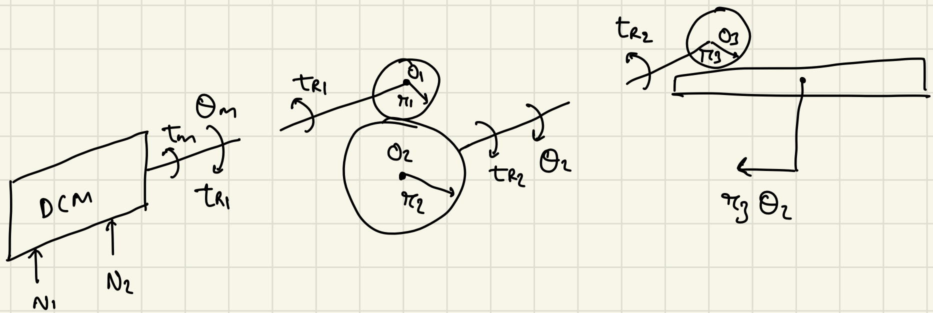
s
s_2
$s_2 - s_a$
50 cm
$s_1 + s_a$
s_1
t_1
$t_1 + t_a$
$t_2 - t_a$
t_2
t
3s

Trapezoidal Velocity Profile

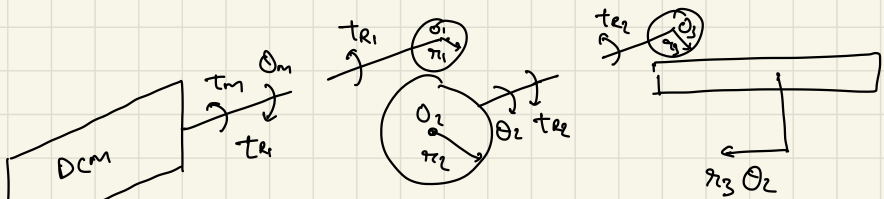
$$t_1 \leq t < t_1 + T_a$$
$$t_1 + T_a \leq t < t_1 + T_a + T_c$$
$$t_2 - T_a \leq t \leq t_2$$

write with values of t_1, t_2, t_3

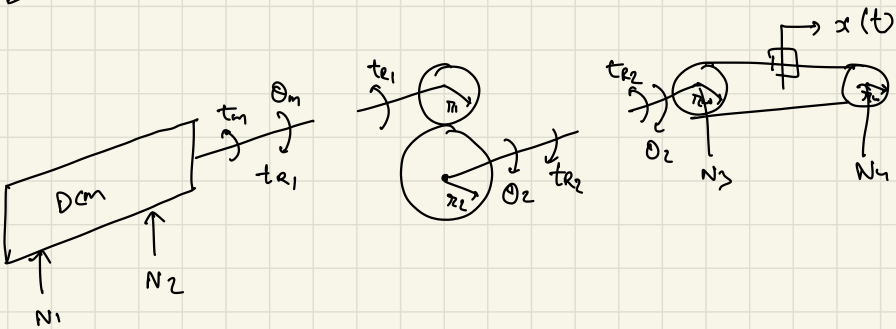
M-1



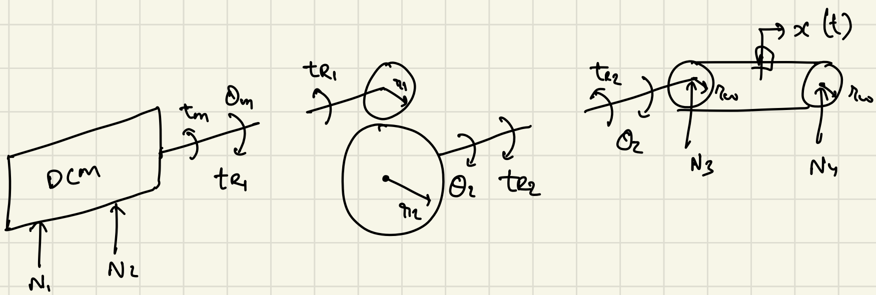
DCM
N1
N2
tm
θm
tR1
r1
θ1
θ2
r2
tR2
θ2
tR2
θ3
r3
r3 θ2



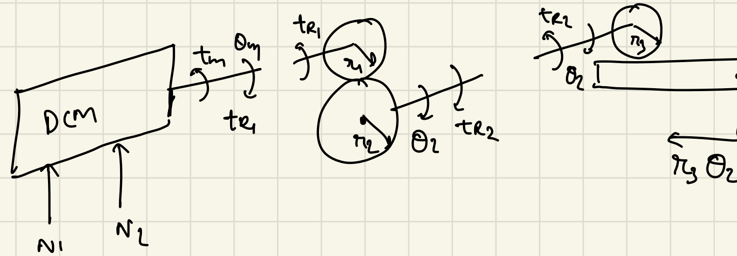
DCM
tm
θm
tR1
tR1
θ1
r1
θ2
r2
θ2
tR2
tR2
θ3
r3
r3 θ2

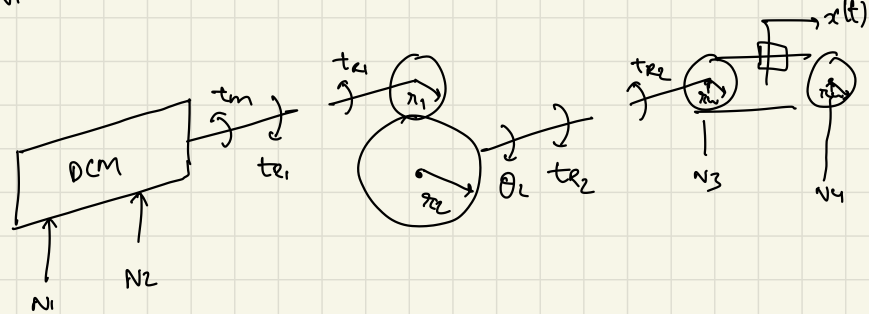


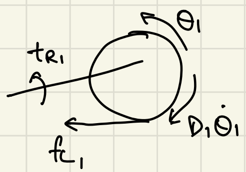
DCM
N1
N2
tm
θm
tR1
tR1
r1
r2
θ2
tR2
tR2
θ2
N3
N4
x (t)

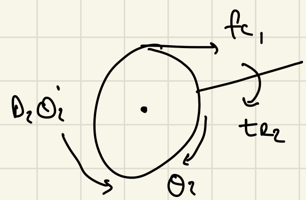


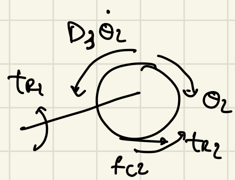
DCM
N1
N2
tm
θm
tR1
tR1
r1
r2
θ2
tR2
tR2
θ2
N3
N4
rw
rw
x (t)

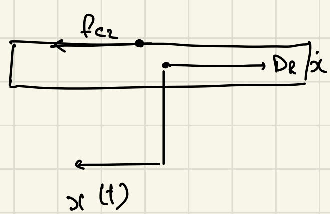

DCM
tm
θm
tR1
r1
r2
θ2
tR2
r3
r3 θ2
N1
N2

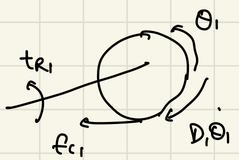

DCM
tm
tR1
r1
r2
θ2
tR2
x(t)
N3
N4
N1
N2

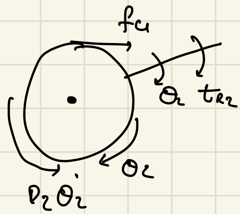

tR1
θ1
D1 θ̇1
fc1

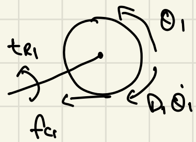

fc1
D2 θ̇2
tR2
θ2

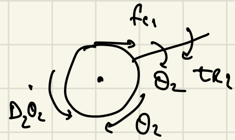

D3 θ̇2
tR1
θ2
tR2
fc2

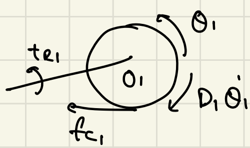

fc2
DR ẋ
x(t)

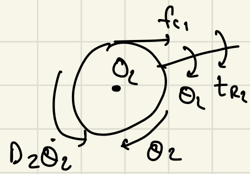

t_{R1}
θ_1
f_{c1}
$D_1\dot{\theta}_1$

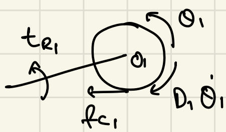

f_{c1}
θ_2
t_{R2}
θ_2
$D_2\dot{\theta}_2$

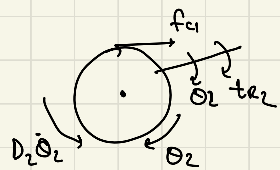

t_{R1}
θ_1
$D_1\dot{\theta}_1$
f_{c1}

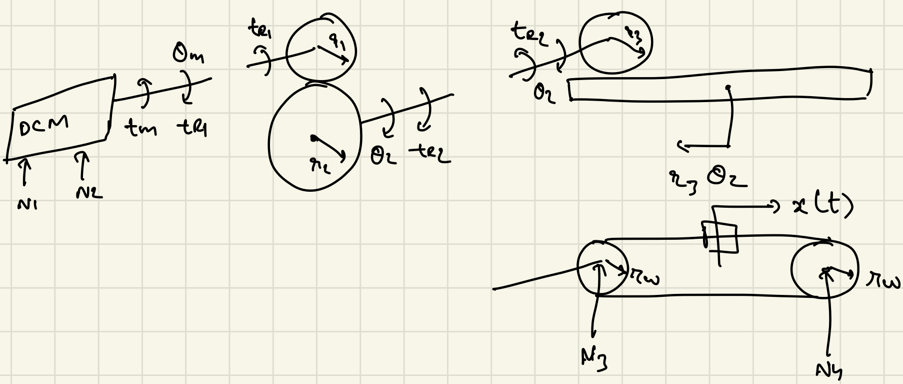

f_{c1}
θ_2
t_{R2}
$D_2\dot{\theta}_2$
θ_2

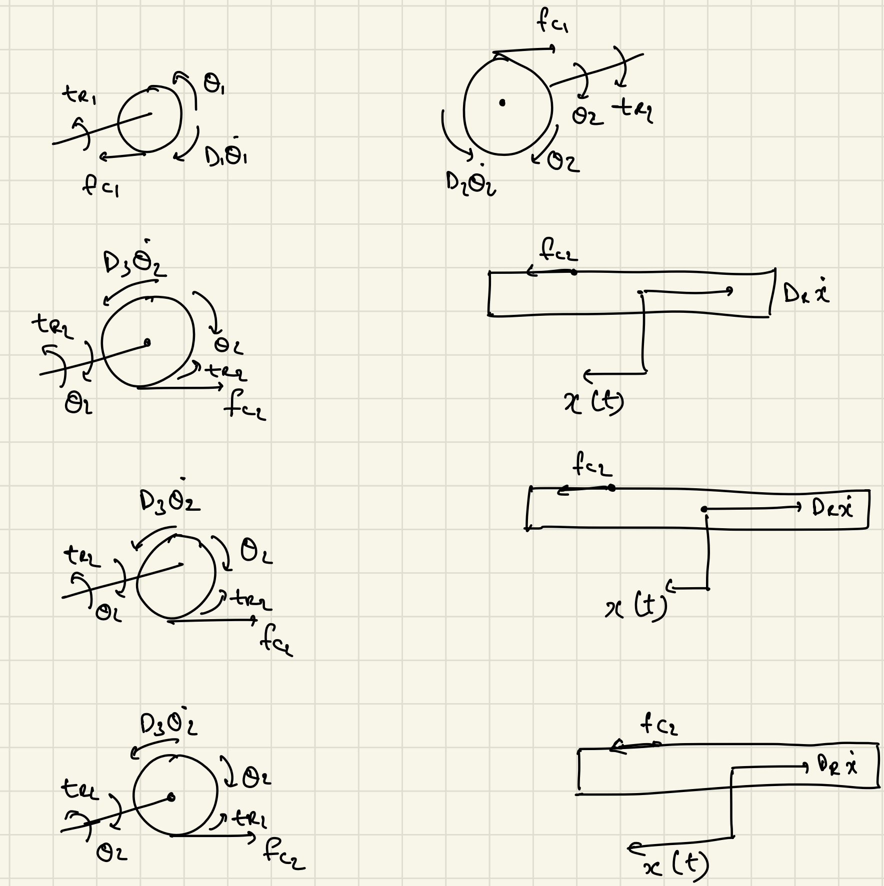

t_{R1}
θ_1
O_1
$D_1\dot{\theta}_1$
f_{c1}


f_{c1}
O_2
θ_2
t_{R2}
$D_2\dot{\theta}_2$
θ_2


t_{R1}
θ_1
O_1
$D_1\dot{\theta}_1$
f_{c1}


f_{c1}
θ_2
t_{R2}
$D_2\dot{\theta}_2$
θ_2


DCM
N_1
N_2
t_m
t_{R1}
θ_m
t_{R1}
r_1
r_2
θ_2
t_{R2}
t_{R2}
r_3
θ_2
$r_3\,\theta_2$
$x(t)$
r_w
r_w
N_3
N_4



Kinematics :- i) Rack & Pinion

$$\theta_1 r_1 = \theta_2 r_2$$

$$\theta_1 = \theta_m$$

$$\theta_m r_1 = \theta_2 r_2$$

$$x(t) = r_3 \theta_2(t)$$

$$\boxed{x(t) = \frac{r_1}{r_2} r_3 \theta_m(t)}$$

$$\theta_1 r_1 = \theta_2 r_2$$
$$\theta_1 = \theta_m$$

$$\theta_m r_1 = \theta_2 r_2$$

$$x(t) = r_3 \theta_2(t)$$

$$\boxed{x(t) = r_3 \frac{r_1}{r_2} \theta_m(t)}$$

$$\theta_1 r_1 = \theta_2 r_2$$
$$\theta_1 = \theta_m$$

$$\theta_m r_1 = \theta_2 r_2$$

$$x(t) = r_3 \theta_2(t)$$

$$\boxed{x(t) = r_3 \frac{r_1}{r_2} \theta_m(t)}$$

ii) Belt drive

$$\theta_1 r_1 = \theta_2 r_2$$
$$\theta_1 = \theta_m$$
$$\theta_2 = \theta_w$$

$$\theta_m r_1 = \theta_w r_2$$

$$x(t) = \theta_w r_w$$

$$\boxed{x(t) = \frac{r_1}{r_2} \cdot r_w \theta_m}$$

$$\theta_1 r_1 = \theta_2 r_2$$
$$\theta_1 = \theta_m$$
$$\theta_2 = \theta_w$$

$$\theta_m r_1 = \theta_w r_2$$

$$x(t) = r_w \theta_w$$

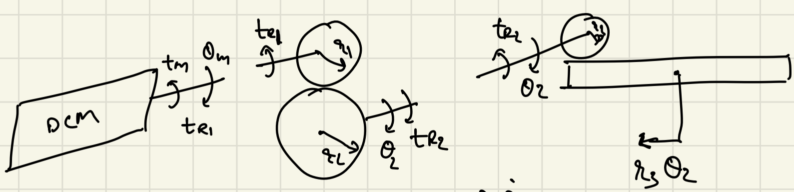
$$\boxed{x(t) = r_w \frac{r_1}{r_2} \theta_m}$$

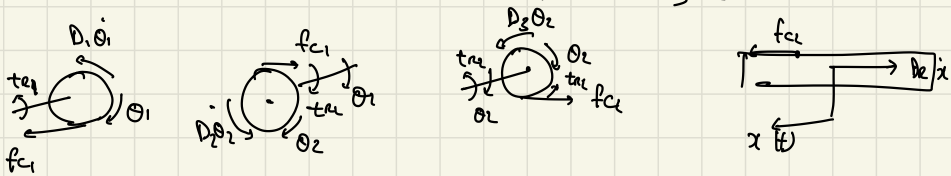
$$\theta_1 r_1 = \theta_2 r_2$$
$$\theta_1 = \theta_m$$
$$\theta_2 = \theta_w$$

$$\theta_m r_1 = \theta_w r_2$$

$$x(t) = r_w \theta_w$$

$$\boxed{x(t) = r_w \frac{r_1}{r_2} \theta_m}$$





EOM :-

$$v(t) = L_a \cancel{\frac{di_a}{dt}}^{0} + R_a i_a + K_b \dot{\theta}_m$$

$$v(t) = L_a \frac{di_a}{dt} + R_a i_a + K_b \dot{\theta}_m$$

$$v(t) = L_a \cancel{\frac{di_a}{dt}}^{0} + R_a i_a + K_b \dot{\theta}_m$$

$$t_m = K_T i_a$$

$$J_{01} \ddot{\theta}_1 + D_{01} \dot{\theta}_1 = t_m$$

$$J_{01} \ddot{\theta}_1 + D_{01} \dot{\theta}_1 = t_m$$

$$v(t) = L_a \frac{di_a}{dt} + R_a i_a + K_b \dot{\theta}_m$$

$$t_m = K_T i_a$$

$$J_{01} \ddot{\theta}_1 + D_{01} \dot{\theta}_1 = t_m$$

$$\frac{R_a}{K_T} (J_{01} \ddot{\theta}_1 + D_{01} \dot{\theta}_1) + K_b \dot{\theta}_m = v(t)$$

$$J_{01} \ddot{\theta}_1 + D_{01} \dot{\theta}_1 + \frac{K_b K_T}{R_a} \dot{\theta}_m = K_b K_T v(t)$$

$$= t_m(t) - t_{R1}(t)$$

$$t_m(t) - t_{R1}(t)$$

$$t_m(t) - t_{R1}(t)$$

$$t_m(t) - t_{R1}(t)$$

Gearbox :-

$$\Sigma M_{o1} = J_{o1} \ddot{\theta}_1$$

$$J_{o1} \ddot{\theta}_1 = t_{R1} - D_1 \dot{\theta}_1 - f_c r_1$$

$$t_{R1} = \longrightarrow (1)$$

$$\Sigma M_{o1} = J_{o1} \ddot{\theta}_1$$

$$J_{o1} \ddot{\theta}_1 = t_{R1} - f_{c1} r_1 - D_1 \dot{\theta}_1$$

$$t_{R1} = \longrightarrow (1)$$

$$\Sigma M_{o2} = - J_{o2} \ddot{\theta}_2$$

$$- J_{o2} \ddot{\theta}_2 = D_2 \dot{\theta}_2 - f_{c1} r_2 - t_{R2}$$

$$f_{c1} = \left(J_{o2} \ddot{\theta}_2 + D_2 \dot{\theta}_2 - t_{R2} \right) \frac{1}{r_2} \longrightarrow (2)$$

Rack & Pinion :-

$$\Sigma M O_3 = - J_{o3} \ddot{\theta}_2$$

$$- J_{o3} \ddot{\theta}_2 = t_{R2} + D_3 \dot{\theta}_2 + f_{c2} r_3$$

$$t_{R2} = \longrightarrow (3)$$

$$\sum M_{O_3} = -J_{O_3}\ddot{\theta}_2$$

$$-J_{O_3}\ddot{\theta}_2 = t_{R_2} + D_3\dot{\theta}_2 + f_{c_2} r_3$$

$$t_{R_2} = \longrightarrow (4)$$

$$\sum F_x = -m_R\ddot{x}$$

$$-m_R\ddot{x} = D_R\dot{x} - f_{c_2}$$

$$f_{c_2} = m_R\ddot{x} + D_R\dot{x}$$

$$\sum F_x = -m_R\ddot{x}$$

$$-m_R\ddot{x} = D_R\dot{x} - f_{c_2}$$

PD controller,

$$\bar{J}\ddot{\theta} + \bar{D}\dot{\theta} = v(t)$$

$$v(t) = \bar{J}\left[-K_P(\theta_m - \theta_d) - K_D(\dot{\theta}_m - \dot{\theta}_d) - K_I\int_0^t(\theta_m - \theta_{md}(\tau))d\tau\right] + \bar{D}\dot{\theta} - \bar{J}\ddot{\theta}_d$$

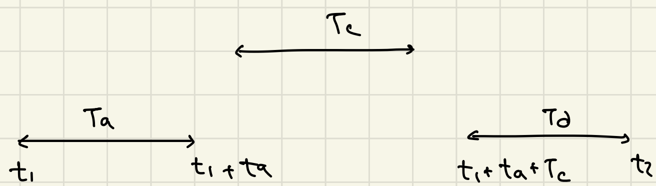
$$\bar{J}\ddot{\theta} + \bar{D}\dot{\theta} = \tau(t)$$

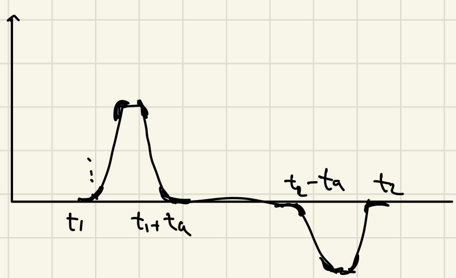
$$\tau(t) = \bar{J}\Big[-K_P(\theta_m - \theta_{md}) - K_D(\dot{\theta}_m - \dot{\theta}_{md}) - K_I\int_0^{\tau}(\theta_m - \theta_{md})\,d\tau\Big] + \bar{D}\dot{\theta} + \bar{J}\ddot{\theta}_d$$

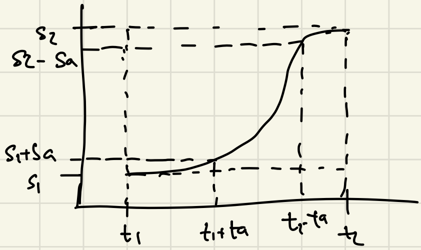
$$\bar{J}\Big[(\ddot{\theta} - \ddot{\theta}_d) + K_D(\dot{\theta}_m - \dot{\theta}_{md}) + K_P(\theta - \theta_d)\Big] = 0$$

$$\dddot{e} + K_D\ddot{e} + K_P\dot{e} + K_I e = 0$$

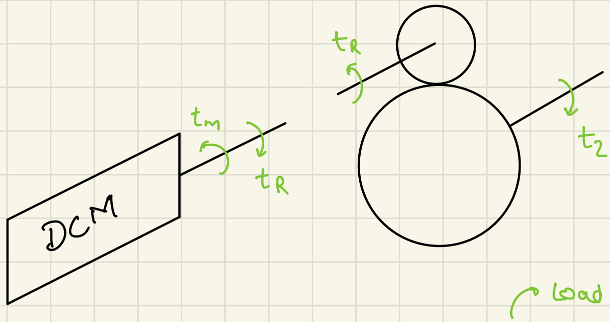
$$\lambda^3 + K_D\lambda^2 + K_P\lambda + K_I = (\lambda-\lambda_1)(\lambda-\lambda_2)(\lambda-\lambda_3)$$







s2
s2 - sa
s1+sa
s1
t1
t1+ta
t2-ta
t2



load

$$\frac{t_R}{r_1} = \frac{t_L}{r_2}$$

$$t_R = \frac{r_1}{r_2} t_L$$

$$\{\text{Motor Dynamics}\} = t_m(t) - t_R(t)$$

$$\ddot{\theta}_{m_1}, \dot{\theta}_m = t_m(t) - \frac{r_1}{r_2} t_2(t)$$

ACD = TVP

Acceleration Cruise Deceleration

TVP :-

$$\underbrace{[s_1, s_2]}_{S} \quad \text{in} \quad \underbrace{[t_1, t_2]}_{T}$$

$$v_{max} = \frac{S_c}{T_c}$$

$$S_a = \frac{1}{2} a T_a^2 \quad \therefore \quad a_{max} = \frac{2 S_a}{T_a^2}$$

$$\frac{2 S_a}{T_a} = \frac{S_c}{T_c}$$

In exam, assume S_a, if Q asks a_{max} below certain level, then reiterate.

$A_{max} \longrightarrow$ low

$S_a \longrightarrow$ higher

max acceleration

$\longrightarrow a = \dfrac{2 S_a}{T_a^2}$

$v_c = a T_a$

constant velocity

LAB

Right = 1280 / 1536

Left = 2560 / 2816

fluctuation = 256 / 256

mid - point = 1920 / 2176

MR - Move Right

ML - Move left

if(setp > Right val)

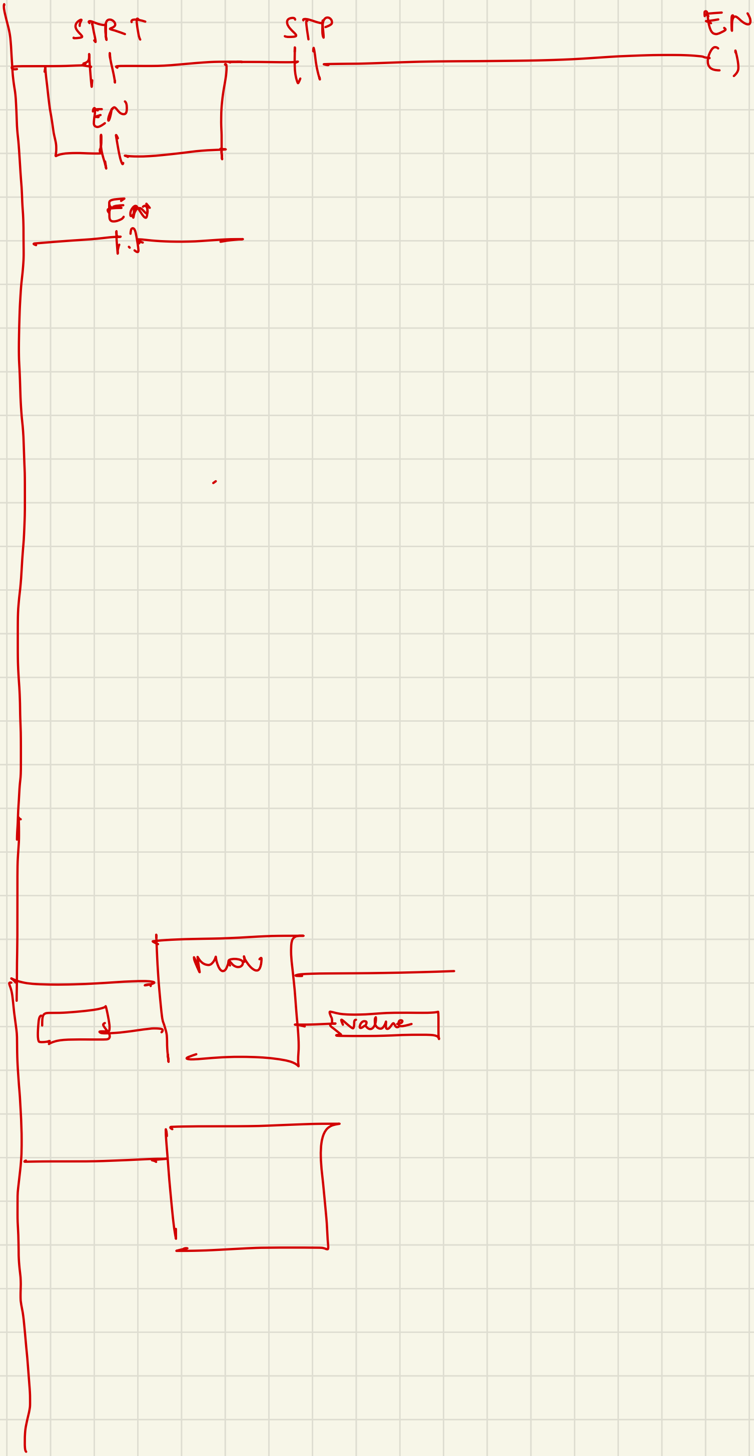
Move left

else if (setp < left val)

Move Right

setp > $\boxed{1536}$

Mov -


STRT
STP
EN
EN
EN
MOV
Value

When not neglecting $L_a \frac{d}{dt} i_a$

$$L_a \frac{d}{dt} i_a + R_a i_a + K_b \dot{\theta}_m = v(t) \longrightarrow (1)$$

Differentiate

$$J \ddot{\theta}_m + D \dot{\theta}_m = t_m(t)$$

$$J \dddot{\theta}_m + D \ddot{\theta}_m = \dot{t}_m(t)$$

$$t_m = K_T i_a$$

$$\frac{t_m}{K_T} = i_a$$

Substitute in (1),

$$L_a \frac{d}{dt}(i_a) + R_a i_a + K_b \dot{\theta}_m = v(t)$$

$$L_a \frac{\dot{t}_m}{K_T} + R_a i_a + K_b \dot{\theta}_m = v(t)$$

Convert to State Space Form for Simulation

Method I :- Differentiation

$$\bar{J}\ddot{\theta}_m + \bar{D}\dot{\theta}_m = \tau(t)$$

$$\bar{J}\dddot{\theta}_m + \bar{D}\ddot{\theta}_m = \dot{\tau}(t) \longrightarrow (1)$$

$$\tau(t) = \bar{J}\Big[-K_p(\theta_m - \theta_d) - K_D(\dot{\theta}_m - \dot{\theta}_d) - K_I\int_0^t(\theta_m(\tau) - \theta_d(\tau))\,d\tau\Big] + \bar{J}\ddot{\theta}_d + \bar{D}\dot{\theta}_m$$

Differentiating $\tau(t)$ & putting in (1)

$$\bar{J}\dddot{\theta}_m + \bar{D}\ddot{\theta}_m = \bar{J}\Big(-K_p(\dot{\theta}_m - \dot{\theta}_d) - K_D(\ddot{\theta}_m - \ddot{\theta}_d) - K_I(\theta_m - \theta_d)\Big) + \bar{J}\dddot{\theta}_d + \bar{D}\ddot{\theta}_m$$

error equation :-

$$\bar{J}(\dddot{e} + K_D\ddot{e} + K_p\dot{e} + K_I e) = 0$$

For state space form :-

$$\theta_m = \theta_1 \qquad \dot{\theta}_1 = \theta_2$$
$$\dot{\theta}_m = \theta_2 \qquad \dot{\theta}_2 = \theta_3$$
$$\ddot{\theta}_m = \theta_3$$

$$\bar{J}\dot{\theta}_3 = \bar{J}\dddot{\theta}_m = -\bar{D}\ddot{\theta}_1 + \bar{D}\ddot{\theta}_1 + \bar{J}\Big(-K_P(\dot{\theta}_1 - \dot{\theta}_d) - K_D(\ddot{\theta}_1 - \ddot{\theta}_d) - K_I(\theta_1 - \theta_d)\Big) + \bar{J}\ddot{\theta}_d$$

Method II :— Without Differentiation

$$\bar{J}\ddot{\theta}_m + \bar{D}\dot{\theta}_m = \tau(t)$$

$$\tau(t) = \bar{J}\Big[-K_P(\theta_m - \theta_d) - K_D(\dot{\theta}_m - \dot{\theta}_d) - K_I\int_0^{\tau}(\theta_m(\tau) - \theta_d(\tau))\,d\tau\Big] + \bar{J}\ddot{\theta}_d + \bar{D}\dot{\theta}_m$$

$$\theta_1 = \int_0^{t}\theta_m(\tau)\,d\tau \qquad \dot{\theta}_1 = \theta_2$$

$$\theta_2 = \theta_m(t) \qquad \dot{\theta}_2 = \theta_3$$

$$\theta_3 = \dot{\theta}_m$$

$$\bar{J}\dot{\theta}_3 = \bar{J}\ddot{\theta}_m = \bar{J}\Big[-K_P(\theta_2 - \theta_d) - K_D(\theta_3 - \dot{\theta}_d) - K_I\Big(\theta_1 - \int_0^{t}\theta_d(\tau)\,d\tau\Big)\Big] + \bar{J}\ddot{\theta}_d + \bar{D}\theta_3$$

$\iiiint$ abserk

$\iiint$ abseleration

$\iint$ absecity

$\int$ absement

displacement

$\cdot$ velocity

$\cdot\cdot$ acceleration

$\cdot\cdot\cdot$ jerk

$\cdot\cdot\cdot\cdot$ jounce

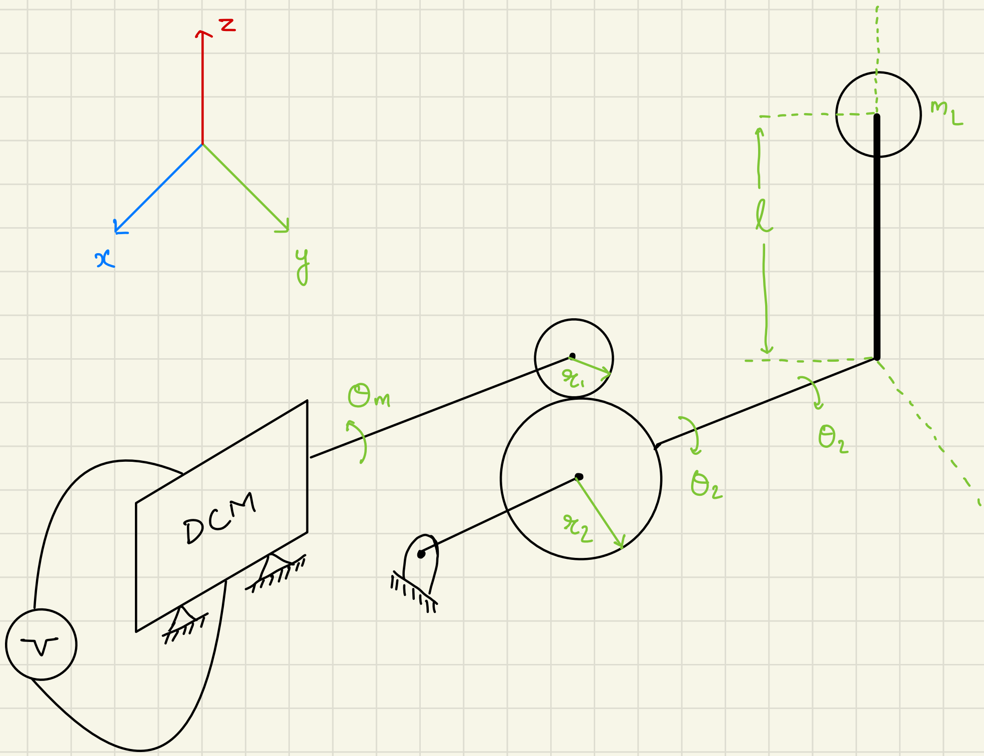
$\cdot\cdot\cdot\cdot\cdot$ founce

$\cdot\cdot\cdot\cdot\cdot\cdot$ bounce

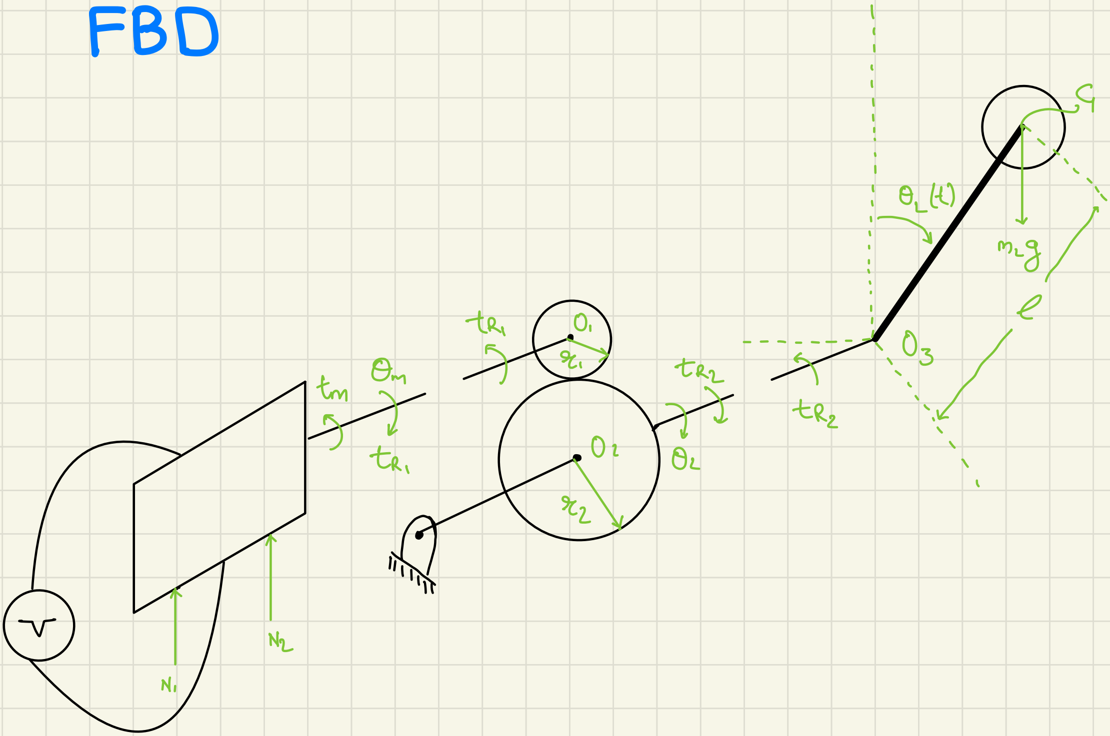
$\uparrow$ Integration

$\downarrow$ Differentiation

Position Control of Inverted Pendulum

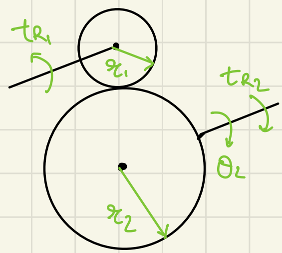


FBD

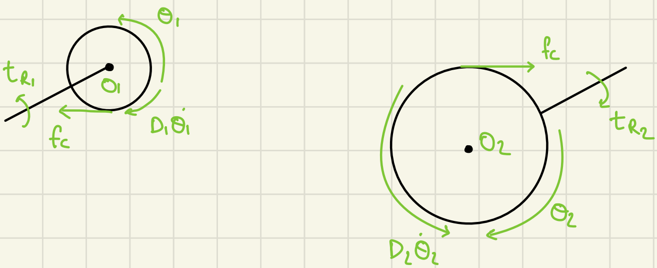


t_m
θ_m
t_{R_1}
N_1
N_2
O_1
r_1
O_2
r_2
t_{R_2}
θ_2
O_3
$\theta_L(t)$
$m_L g$
ℓ

Gearbox :-



t_{R_1}
r_1
t_{R_2}
θ_2
r_2



θ_1
t_{R_1}
O_1
f_c
$D_1\dot{\theta}_1$
f_c
t_{R_2}
O_2
θ_2
$D_2\dot{\theta}_2$

Motor :-

(Electrical eq^n)

$$v(t) = \cancelto{0}{L_a \frac{di_a}{dt}} + R_a i_a + K_b \dot{\theta}_m$$

$$t_m = K_T i_a$$

$$\frac{R_a t_m}{K_T} + K_b \dot{\theta}_m = v(t) \longrightarrow ①$$

(Mechanical eq^n)

$$J_m \ddot{\theta}_1 + D_m \dot{\theta}_1 = t_m$$

$$\frac{R_a}{K_T} \left(J_m \ddot{\theta}_1 + D_m \dot{\theta}_1 \right) + K_b \dot{\theta}_m = v(t)$$

$$J_m \ddot{\theta}_1 + D_m \dot{\theta}_1 + \frac{K_b K_T}{R_a} \dot{\theta}_1 = K_b K_T v(t)$$

$$J_m \ddot{\theta}_1 + D_m \dot{\theta}_1 = t_m(t) - t_{R_1}(t) \longrightarrow ②$$

Gear box :-

$$\Sigma M_{O_1} = J_{O_1} \ddot{\theta}_1$$

$$J_{O_1} \ddot{\theta}_1 = t_{R_1} - f_{c_1} \pi_1 - D_1 \dot{\theta}_1$$

$$t_{R_1} = J_{O_1} \ddot{\theta}_1 + f_{c_1} \pi_1 + D_1 \dot{\theta}_1 \longrightarrow ③$$

$$\Sigma M_{O_2} = -J_{O_2}\ddot{\theta}_L$$

$$-J_{O_2}\ddot{\theta}_L = D_2\dot{\theta}_L - f_{c_1} r_2 - t_{R_2}$$

$$f_{c_1} r_2 = J_{O_2}\ddot{\theta}_L + D_2\dot{\theta}_L - t_{R_2}$$

$$f_{c_1} = \left(J_{O_2}\ddot{\theta}_L + D_2\dot{\theta}_L - t_{R_2}\right)\frac{1}{r_2} \longrightarrow (4)$$

Inverted Pendulum :-

$$\Sigma M_{O_3} = -J_{O_3}\ddot{\theta}_L$$

$$-J_{O_3}\ddot{\theta}_L = D_2\dot{\theta}_L + t_{R_2} - m_2 g \sin\theta_L$$

$$-t_{R_2} = D_2\dot{\theta}_L + J_{O_3}\ddot{\theta}_L - m_2 g \sin\theta_L \longrightarrow (5)$$

EOM :-

(5) in (4),

$$f_{c_1} = \left(J_{O_2}\ddot{\theta}_L + D_2\dot{\theta}_L + D_2\theta_L + J_{O_3}\ddot{\theta}_L - m_L g \sin\theta_L\right)\frac{1}{r_2} \longrightarrow (6)$$

⑥ in ③

$$t_{R_1} = J_{o_1}\ddot{\theta}_1 + \frac{r_1}{r_2}\left[J_{o_2}\ddot{\theta}_L + 2D_2\dot{\theta}_L + J_{o_3}\ddot{\theta}_L - m_L g \sin\theta_L \right] + D_1\dot{\theta}_1 \longrightarrow (7)$$

Put ⑦ in ②

$$J_m\ddot{\theta}_1 + D_m\dot{\theta}_1 = t_m(t) - J_{o_1}\ddot{\theta}_1 - \frac{r_1}{r_2}\left[J_{o_2}\ddot{\theta}_L + 2D_2\dot{\theta}_L + J_{o_3}\ddot{\theta}_L - m_L g \sin\theta_L \right] - D_1\dot{\theta}_1$$

$$t_m(t) = -\ddot{\theta}_1\left[J_m + J_{o_1}\right] - \dot{\theta}_1\left[D_m + D_1\right] - \frac{r_1}{r_2}\left[J_{o_2}\ddot{\theta}_L + 2D_2\dot{\theta}_L + J_{o_3}\ddot{\theta}_L - m_L g \sin\theta_L \right] \longrightarrow (8)$$

check the signs

Put 8 in 1

$$\frac{R_a}{K_T}\left[-\ddot{\theta}_1\left[J_m+J_{o1}\right]-\dot{\theta}_1\left[D_m+D_1\right]-\frac{r_1}{r_2}\left[J_{o2}\ddot{\theta}_L + 2D_2\dot{\theta}_L + J_{o3}\ddot{\theta}_L - m_L g\sin\theta_L\right]\right] + K_b\dot{\theta}_m = v(t)$$

$$\boxed{\begin{aligned}\theta_1 &= \theta_m\\ \theta_2 &= \theta_L\\ \theta_L &= \frac{r_1}{r_2}\theta_m\end{aligned}}$$

use this

for → 8

EOM for inverted pendulum

$$\bar{J}\ddot{\theta}_m + \bar{D}\dot{\theta}_m + \left(\frac{r_1}{r_2}\right)^2 \left[J_{02}\ddot{\theta}_m + 2D_2\dot{\theta}_m + J_{0s}\ddot{\theta}_m \right] - \frac{r_1}{r_2} m_2 g l \sin\left(\frac{r_1}{r_2}\theta_m\right) = t_m(t) = K_T i_a$$

Electrical eq^n of DCM :

$$L_a \frac{di_a}{dt} + R_a i_a + K_b\dot{\theta}_m = v(t)$$

$$t_m(t) = K_T i_a \qquad i_a = \frac{t_m}{K_T}$$

$$\frac{R_a}{K_T}\left[\bar{J}\ddot{\theta}_m + \bar{D}\dot{\theta}_m + \left(\frac{r_1}{r_2}\right)^2 \left[\quad \right] - \frac{r_1}{r_2} m_2 g l \sin\left(\frac{r_1}{r_2}\theta_m\right) \right] + K_b\dot{\theta}_m = v(t)$$

Just combine J terms

$$J_T = \bar{J} + \left(\frac{r_1}{r_2}\right)^2 (J_{o2} + J_{o3})$$

$$D_T = \bar{D} + \left(\frac{r_1}{r_2}\right)^2 (2D_2) + K_b$$

$$J_T \ddot{\theta}_m + D_T \dot{\theta}_m - \frac{r_1}{r_2} m_2 g l \sin\left(\frac{r_1}{r_2}\theta_m\right) = v(t)$$

Computed Torque Control :-

PID controller

Control law

$$v(t) = -\frac{r_1}{r_2} m_2 g l \sin\left(\frac{r_1}{r_2}\theta_m\right) + D_T \dot{\theta}_m$$

$J\ddot{\theta}_d$ (take common)

$$+ J_T \Big[\ddot{\theta}_d - K_p(\theta_m - \theta_d) - K_D(\dot{\theta}_m - \dot{\theta}_d) - K_I \int_0^t [\theta_m(\tau) - \theta_d(\tau)]\, d\tau \Big]$$

(J_T is factored out & equal to 0)

$$J_T(\ddot{\theta}_m - \ddot{\theta}_d) + J_T(K_D(\dot{\theta}_m - \dot{\theta}_d)) + J_T(K_p(\theta_m - \theta_d)) + J_T\left(K_I\left(\int_0^t (\theta_m(\tau) - \theta_d(\tau))\, d\tau\right)\right) = 0$$

$$\ddot{e} + K_D \dot{e} + K_p e + K_I \int e(\tau)\, d\tau = 0$$

differentiating again,

$$\lambda^3 + K_D \lambda^2 + K_p \lambda + K_I = 0$$

LAB A+B+B−A−

Qx0.0 = A
Qx0.3 = B

% Ix 0.1 = A(ext)

% Ix 0.3 = A (retracted)

% Ix 0.2 = B(ext)

% Ix 0.4 = B (retracted)

• % Ix 0.0 = Start

• % Ix 0.0 = Stop

Script :-

Simulation of closed loop system of inverted pendulum

$$J\ddot{\theta} + c\dot{\theta} + k\theta - 0.5\sin\theta = \tau(t)$$

$$\tau(t) = -0.5\sin\theta + c\dot{\theta} + k\theta$$
$$+ \Big[\ddot{\theta}_d - K_P(\theta - \theta_d)$$
$$- K_D(\dot{\theta} - \dot{\theta}_d)$$
$$- K_I\Big(\int_0^t (\theta(\tau) - \theta_d(\tau))\, d\tau\Big)\Big] J$$

$\theta_1 = y(1,1)$ $\quad\int_0^t \theta(t)\, d\tau = \theta_1$

$\theta_2 = y(2,1)$ $\quad\dot{\theta}_1 = \theta(t) = \theta_2$

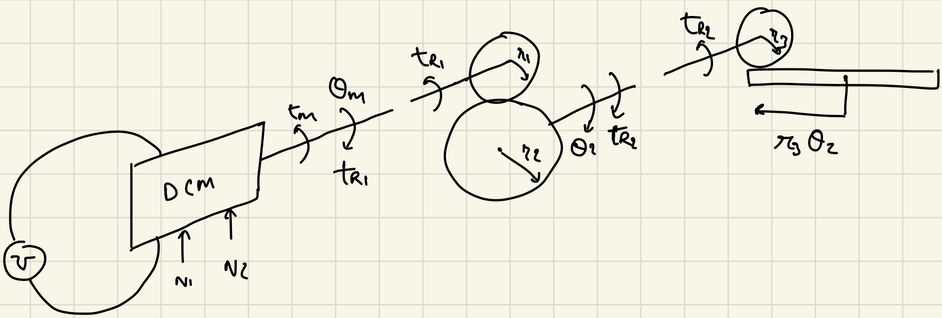
$\theta_3 = y(3,1)$ $\quad\dot{\theta}_2 = \dot{\theta} = \theta_3$

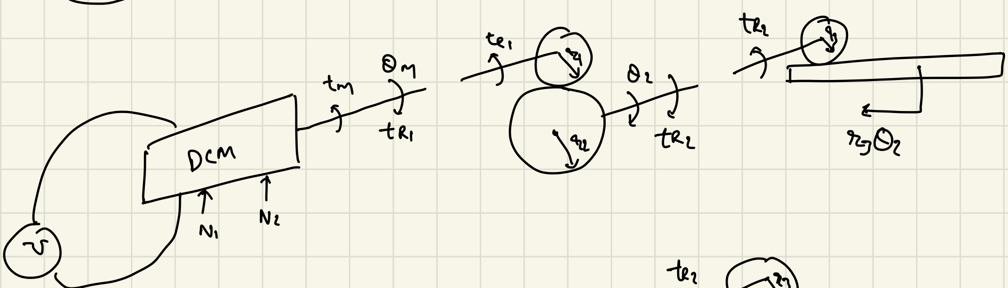
$$\dot{\theta}_3 = 0.5\sin(y(2,1)) + 0.5\sin(y(2,1))$$
$$+ c\theta_3 - c\theta_3 + k\theta_1 - k\theta_1 + \Big[a$$
$$- K_P(\theta_2 - s) - K_D(\theta_3 - v) - K_I(\theta_1 - si)\Big] J$$

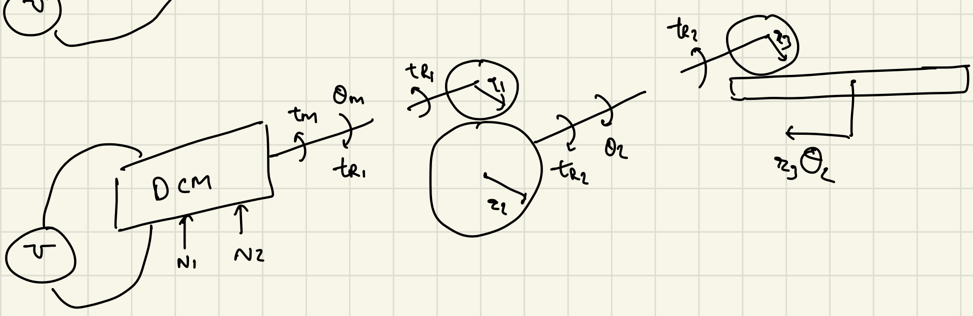
M-2

FBD

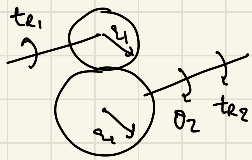
Rack & Pinion



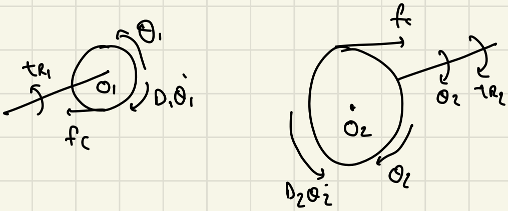




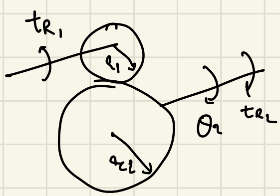
Gearbox



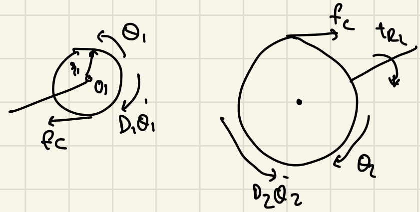
t_{R_1}
r_1
r_2
θ_2
t_{R_2}



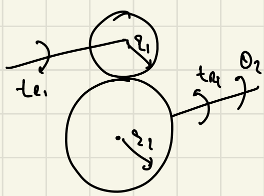
t_{R_1}
θ_1
O_1
$D_1\dot{\theta}_1$
f_c
f_c
O_2
θ_2
t_{R_2}
$D_2\dot{\theta}_2$
θ_2



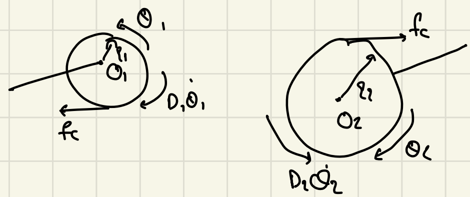
t_{R_1}
r_1
r_2
θ_2
t_{R_2}



θ_1
r_1
O_1
$D_1\dot{\theta}_1$
f_c
f_c
t_{R_2}
θ_2
$D_2\dot{\theta}_2$

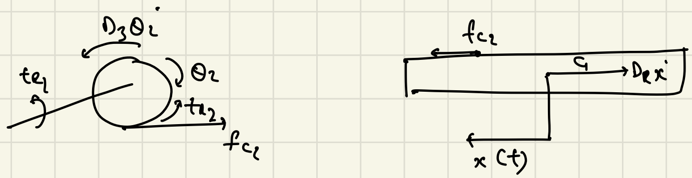


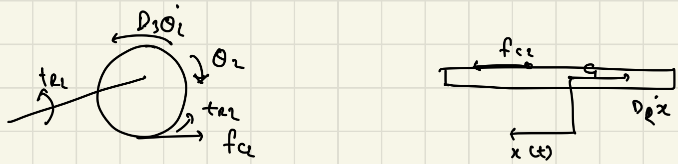
r_1
t_{R_1}
t_{R_2}
θ_2
r_2

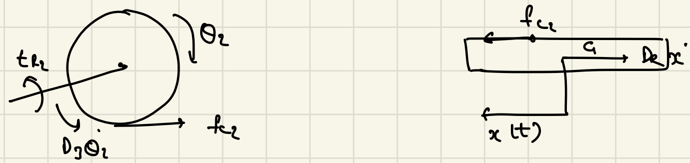


θ_1
r_1
O_1
$D_1\dot{\theta}_1$
f_c
f_c
r_2
O_2
θ_2
$D_2\dot{\theta}_2$

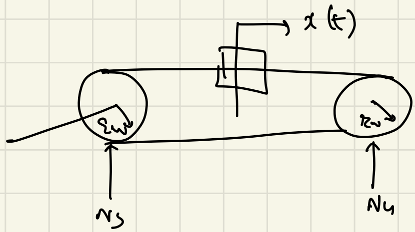
Rack & Pinion

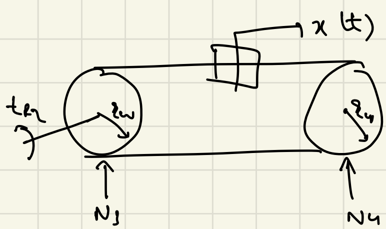






Belt drive





KINEMATICS

Rack & Pinion :-

$\rightarrow$ $\theta_1 r_1 = \theta_2 r_2$

$\theta_1 = \theta_m$

$\theta_m r_1 = \theta_2 r_2$

$x(t) = r_3 \, \theta_2(t)$

$$\boxed{x(t) = r_3 \, \theta_m \frac{r_1}{r_2}}$$

$\rightarrow$ $\theta_1 r_1 = \theta_2 r_2$

$\theta_1 = \theta_m$

$\theta_m r_1 = \theta_2 r_2$

$x(t) = r_3 \, \theta_2(t)$

$$\boxed{x(t) = r_3 \frac{r_1}{r_2} \theta_m(t)}$$

$\rightarrow$ $\theta_1 r_1 = \theta_2 r_2$

$\theta_1 = \theta_m$

$\theta_m r_1 = \theta_2 r_2$

$x(t) = r_3 \, \theta_2(t)$

$$\boxed{x(t) = r_3 \, \theta_m \frac{r_1}{r_2}}$$

Belt Drive :- $\rightarrow$ $r_1\theta_1 = r_2\theta_2$

$$\theta_1 = \theta_m$$

$$\theta_2 = \theta_w$$

$$r_1\theta_m = r_2\theta_w$$

$$x(t) = r_w\theta_w$$

$$x(t) = r_w\frac{r_1}{r_2}\theta_m(t)$$

$\rightarrow$ $r_1\theta_1 = r_2\theta_2$

$$\theta_1 = \theta_m$$

$$\theta_2 = \theta_w$$

$$r_1\theta_m = r_2\theta_w$$

$$x(t) = r_w\theta_w$$

$$\boxed{x(t) = r_w\frac{r_1}{r_2}\theta_m}$$

$\rightarrow$ $r_1\theta_1 = r_2\theta_2$

$$\theta_1 = \theta_m$$

$$\theta_2 = \theta_w$$

$$r_1\theta_m = r_2\theta_w$$

$$x(t) = r_w\theta_w$$

$$\boxed{x(t) = r_w\frac{r_1}{r_2}\theta_m}$$

EOM

Motor

$$v(t) = \cancel{L_a \frac{di_a}{dt}}^{0} + R_a i_a + K_b \dot{\theta}_m$$

$$v(t) = R_a i_a + K_b \dot{\theta}_m$$

$$t_m = K_T i_a$$

$$R_a \frac{t_m}{K_T} + K_b \dot{\theta}_m = v(t) \longrightarrow (1)$$

$$J_{o1} \ddot{\theta}_1 + D_{o1} \dot{\theta}_1 = t_m$$

$$\frac{R_a}{K_T} (J_{o1} \ddot{\theta}_1 + D_{o1} \dot{\theta}_1) + K_b \dot{\theta}_m = v(t)$$

$$J_{o1} \ddot{\theta}_1 + D_{o1} \dot{\theta}_1 = t_m(t) - t_{R1}(t) \longrightarrow (2)$$

Motor

$$v(t) = \cancel{L_a \frac{di_a}{dt}}^{0} + R_a i_a + K_b \dot{\theta}_m$$

$$R_a i_a + K_b \dot{\theta}_m = v(t)$$

$$t_m = K_T i_a$$

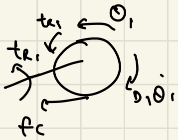
$$R_a \frac{t_m}{K_T} + K_b \dot{\theta}_m = v(t) \longrightarrow (1)$$

$$J_{o1} \ddot{\theta}_1 + D_{o1} \dot{\theta}_1 = t_m$$

$$\frac{R_a}{K_T} (J_{o1} \ddot{\theta}_1 + D_{o1} \dot{\theta}_1) + K_b \dot{\theta}_m = v(t)$$

$$J_{o1} \ddot{\theta}_1 + D_{o1} \dot{\theta}_1 = t_m(t) - T_{R1} \longrightarrow (2)$$

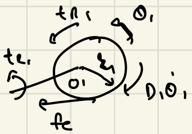
Gearbox



$$\sum M_{O1} = J_{O1}\ddot{\theta}_1$$

$$J_{O1}\ddot{\theta}_1 = t_{R1} - f_{c1} r_1 - D_1\dot{\theta}_1$$

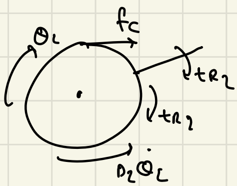
$$t_{R1} = \longrightarrow \quad 3$$



$$\sum M_{O1} = J_{O1}\ddot{\theta}_1$$

$$J_{O1}\ddot{\theta}_1 = t_{e1} - f_{c1} r_1 - D_1\dot{\theta}_1$$

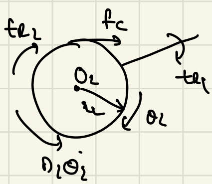
$$t_{e1} = \longrightarrow \quad 3$$



$$\sum M_{O2} = -J_{O2}\ddot{\theta}_2$$

$$-J_{O2}\ddot{\theta}_2 = D_2\dot{\theta}_2 - f_{c1} r_2 - t_{R1}$$

$$f_{c1} = \longrightarrow \quad 4$$

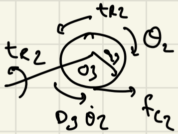


$$\sum M_{O2} = -J_{O2}\ddot{\theta}_2$$

$$-J_{O2}\ddot{\theta}_2 = D_2\dot{\theta}_2 - f_{c1} r_2 - t_{R2}$$

$$f_{c1} = \longrightarrow \quad 4$$

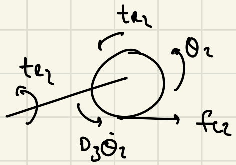
Rack & Pinion



$$\sum M_{O3} = -J_{O3}\ddot{\theta}_2$$

$$-J_{O3}\ddot{\theta}_2 = r_3 f_{c2} + t_{R2} + D_3\dot{\theta}_2$$

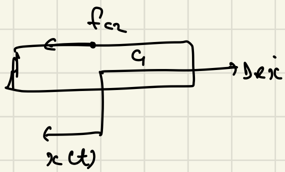
$$t_{e2} = \longrightarrow \quad 5$$



$$\Sigma M_{O3} = -J_{O3}\ddot{\theta}_2$$

$$-J_{O3}\ddot{\theta}_2 = D_3\dot{\theta}_2 + f_{c2}x_3 + t_{R2}$$

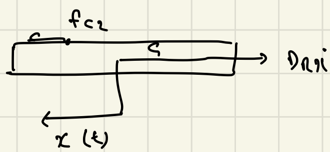
$$t_{R2} = \longrightarrow \; 5$$



$$\Sigma f_x = -m_R\ddot{x}$$

$$-m_R\ddot{x} = -f_{c2} + D_R\dot{x}$$

$$f_{c2} = \longrightarrow \; 6$$



$$\Sigma F_x = -m_R\ddot{x}$$

$$-m_R\ddot{x} = -f_{c_2} + D_R\dot{x}$$

$$f_{c2} = \longrightarrow \; 6$$

COMPUTED TORQUE CONTROL

Belt Drive

$$\bar{J}\ddot{\theta} + \bar{D}\dot{\theta} = \tau(t)$$

$$\tau(t) = \bar{J}\Big[-K_p(\theta - \theta_d) - K_D(\dot{\theta} - \dot{\theta}_d) - K_I\int_0^t [\theta(\tau) - \theta_d(\tau)]\,d\tau\Big] + \bar{D}\dot{\theta} + \bar{J}\ddot{\theta}_d$$

$$\bar{J}\ddot{\theta} + \bar{D}\dot{\theta} = \tau(t)$$

$$\tau(t) = \bar{J}\Big[-K_p(\theta - \theta_d) - K_D(\dot{\theta} - \dot{\theta}_d) - K_I\int_0^t [\theta(\tau) - \theta_d(\tau)]\,d\tau\Big] + \bar{D}\dot{\theta} + \bar{J}\ddot{\theta}_d$$

$$\bar{J}\left[(\ddot{\theta}-\ddot{\theta}_d) + K_D(\dot{\theta}-\dot{\theta}_d) + K_P(\theta-\theta_d) + K_I\int_0^t[\theta(\tau)-\theta_d(\tau)]\,d\tau\right] = 0$$

differentiate

$$\dddot{e} + K_D\ddot{e} + K_P\dot{e} + K_I e = 0$$

$$\lambda^3 + K_D\lambda^2 + K_P\lambda + K_I = (\lambda-\lambda_1)(\lambda-\lambda_2)(\lambda-\lambda_3)$$

For Tvp

T_c

T_a T_d

t_1 t_1+T_a $t_1+T_a+T_c$ t_2

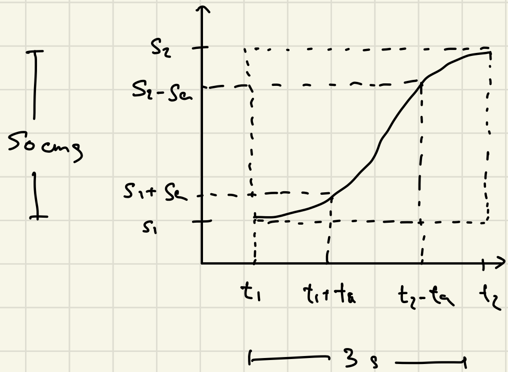
Assume $S_k = S_{cm}$

$S_a = S_d = S_{cm}$

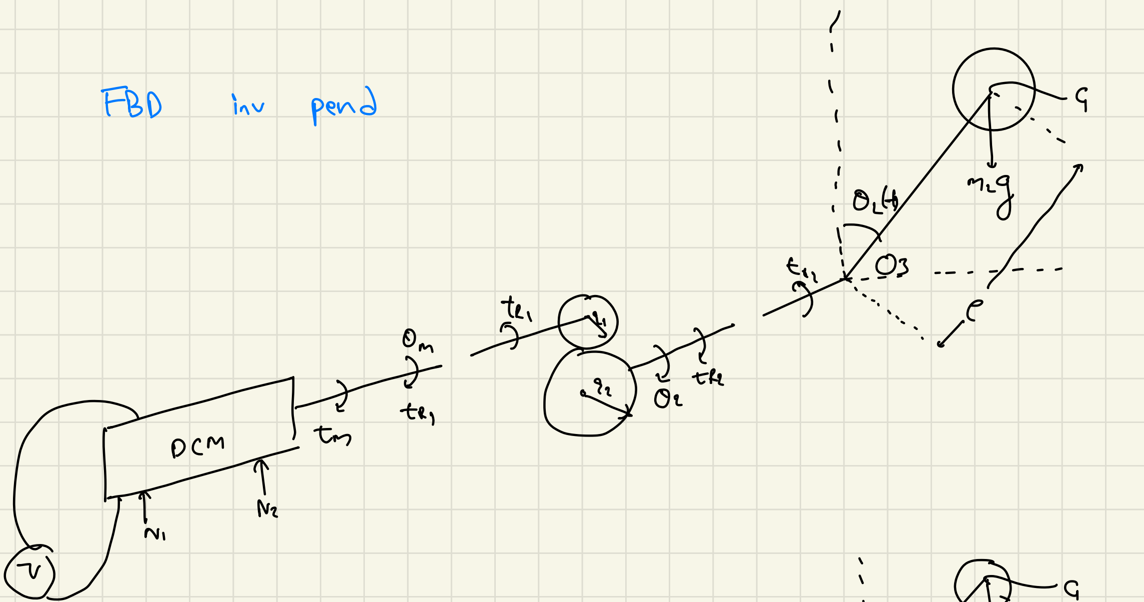
$$S_c = S - S_a - S_d$$

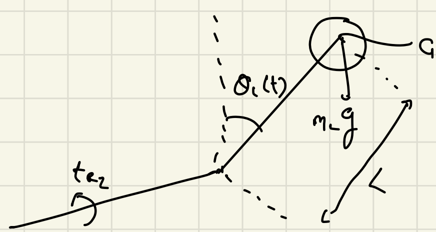
$$T_a = \frac{S_a T}{2S_a + \frac{S_c}{2}}$$

$$T_a = \frac{S_a T}{2S_a + \frac{S_c}{2}}$$



FBD inv pend





EOM for inv pend

$$\Sigma M_{O_3} = -J_{O_3}\ddot{\theta}_L$$

$$-J_{O_3}\ddot{\theta}_L = t_{e_2} + D_2\dot{\theta}_L - m_L g \sin\theta_L$$

$$t_{e_2} = \longrightarrow 5$$

$$\Sigma M_{O_3} = -J_{O_3}\ddot{\theta}_L$$
$$-J_{O_3}\ddot{\theta}_L = t_{R_2} + D_2\dot{\theta}_L - m_R g \sin\theta$$
$$t_{R_2} = \longrightarrow (S)$$

$$\Sigma M_{O_3} = -J_{O_3}\ddot{\theta}_L$$
$$-J_{O_3}\ddot{\theta}_L = t_{R_2} + D_2\dot{\theta}_L - m_R g \sin\theta_L$$
$$t_{R_2} = \longrightarrow (S)$$

Computed Torque Control

$$\bar{J}\ddot{\theta} + \bar{D}\dot{\theta} = v(t)$$

$$v(t) = -\frac{r_1}{r_2} m_R g l \sin\left(\frac{r_1}{r_2}\theta_m\right) + D_T\dot{\theta}_m$$
$$+ J_T\Big[\ddot{\theta}_d - K_P(\theta_m - \theta_d)$$
$$-K_D(\dot{\theta}_m - \dot{\theta}_d)$$
$$-K_I\int_0^t [\theta_m(\tau) - \theta_d(\tau)]\, d\tau\Big]$$

$$v(t) = -\frac{r_1}{r_2} m_L g \sin\left(\frac{r_1}{r_2}\theta_m\right) + D_T\dot{\theta}_m$$
$$+ J_T\Big[\ddot{\theta}_d - K_P(\theta_m - \theta_d)$$
$$-K_D(\dot{\theta}_m - \dot{\theta}_d)$$
$$-K_I\int_0^t [\theta_m(\tau) - \theta_d(\tau)]\, d\tau\Big]$$

$$\tau(t) = -\frac{r_1}{r_2} m_c g \sin\left(\frac{r_1}{r_2}\theta_m\right)$$

$$+ J_T\left(\ddot{\theta}_d - K_p \cdots\right.$$

$$J_T(\ddot{\theta}_m - \ddot{\theta}_d) + J_T(K_p(\theta_m - \theta_d))$$

$$+ J_T(K_p(\dot{\theta}_m - \dot{\theta}_d)) \qquad = 0$$

$$+ J_T\left(K_I \int_0^t (\theta_m(\tau) - \theta_d(\tau))\, d\tau\right)$$

$$\dddot{e} + K_D \ddot{e} + K_p \dot{e} + K_I e = 0$$

Elec: $v(t) = L_a \frac{di_a}{dt} + R_a i_a + K_b \dot{\theta}_m \Rightarrow \frac{R_a \tau_m}{K_t} + K_b \dot{\theta}_m$

Mech: $J_m \ddot{\theta}_m + D_m \dot{\theta}_m = \tau_m(t)$

$$\frac{R_a}{K_t}(J_m \ddot{\theta}_m + D_m \dot{\theta}_m) + K_b \dot{\theta}_m$$

$\sum M_{o_1} = J_o \ddot{\theta}_1$ $\qquad \theta_1 r_1 = \theta_2 r_2$

$= f R_1 - f_c R_1 - D_1 \dot{\theta}_1$ $\qquad \theta_1 = \frac{r_2}{r_1}\theta_2$

$\sum M \theta_2 = J_o \ddot{\theta}_2$ $\qquad f_c$:

$= D_2 \dot{\theta}_2 - f_c R_2 - f R_2$ $\qquad f_c$ is ⊘

$$\tau(t) = \bar{J}\left[-K_p(\theta - \theta_d) - K_D(\dot{\theta} - \dot{\theta}_d) - \cancel{K_I \int_0^t (\theta(t) - \theta_d(t))}\right] + \bar{D}\dot{\theta} + \bar{J}\ddot{\theta}_d$$

$40 + 50$

$$\bar{J}\left[(\ddot{\theta} - \ddot{\theta}_d) - K_D(\dot{\theta} - \dot{\theta}_d) - K_p(\theta - \theta_d)\right] = 0$$

$$\ddot{e} + K_D \dot{e} + K_p e = 0$$

$$\lambda^2 + K_D \lambda + K_p = 0 \quad (\lambda - \lambda_1)(\lambda - \lambda_2)$$

$$\lambda^2 + K_D \lambda + K_p = (\lambda + 40)(\lambda + 50) = 0$$

$v(t) = \bar{J}[$

$= \lambda(\lambda + 40) + 50\lambda + (40)$

$\bar{J}[-K_p($

$= \lambda^2 + 90\lambda + 2000$

$\dot{e} = \lambda^3$

$e' =$

$e = ($

$K_D = 90$

$K_p = 2000$

$$\ddot{e} + K_D \dot{e} + K_p e = 0$$

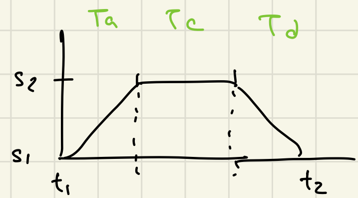
$$\lambda^2 + K_D \lambda + K_p = 0$$

$$(\lambda - \lambda_1)(\lambda - \lambda_2) = 0$$

$$\lambda^2 + K_D \lambda + K_p = (\lambda + 40)(\lambda + 50)$$

Function of Time TVP

$$a = \frac{2 s_a}{T_a^2}$$



when $t \in [t_1, t_1 + T_a]$

$$x(t) = s_1 + \frac{1}{2} a t_3^2$$

$\hookrightarrow t - t_1$

when $t \in [t_1 + T_a, t_1 + T_a + T_c]$

$$x(t) = s_1 + s_c + v_c t_4$$

$\hookrightarrow t_1 - (t_1 + T_a)$

when $t \in [t_1 + T_a + T_c, t_1 + T]$

$$x(t) = s_1 + s_c + s_a + v_c t_6 - \frac{1}{2} a t_6^2$$

$\hookrightarrow t - [t_1 + T_a + T_c]$

$$a = \frac{2 s_a}{T_a^2}$$

when $T \in [t_1, t_1 + T_a]$

$$x(t) = s_1 + \frac{1}{2} a t_3^2$$

$\hookrightarrow t - t_1$

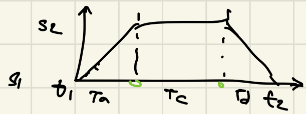
when $t \in (t_1 + T_a, t_1 + T_a + T_c]$

$$x(t) = s_1 + s_a + v t_4$$

$\hookrightarrow t - [t_1 + T_a]$

when $t \in [\, t_1 + T_a + T_c \,,\; t_1 + T \,]$

$$x(t) = s_1 + s_a + s_c + v_c t_6 - \frac{1}{2} a t_6^2$$



when $T \in [\, t_1 \,,\; t_1 + T_a \,]$,

$$x(t) = s_1 + \frac{1}{2} a t_3^2$$

$t - t_1$

when $T \in [\, t_1 + T_a \,,\; t_1 + T_a + T_c \,]$,

$$x(t) = s_1 + s_a + v_c t_4$$

$t - [t_1 + T_a]$

when $T \in [\, t_1 + T_a + T_c \,,\; t_1 + T \,]$

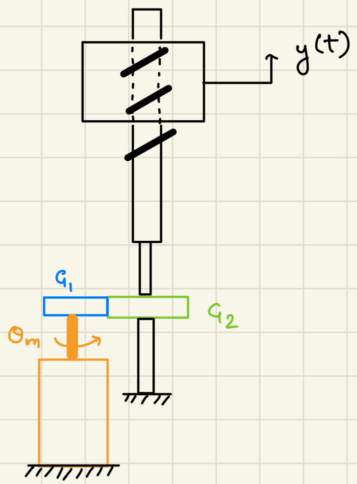
$$x(t) = s_1 + s_a + s_c + v_c t_6 - \frac{1}{2} a t_6^2$$

$t - [\, t_1 + T_a + T_c \,]$

Power Screw

(simp rob 7)

With Gearbox :-



HW 2

$P = (0.3, 0.5)$

$Q = (0.7, -0.1)$

$O = (0, 0)$ origin

$$s_1 = \sqrt{0.3^2 + 0.5^2} = 0.58$$

distant b/w PQ :

$$S = \sqrt{0.4^2 + 0.5^2} = 0.7211$$

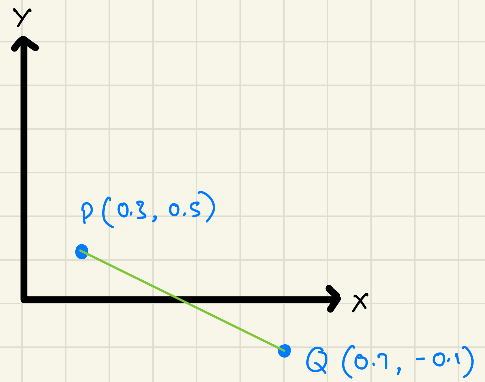
$t_1 = 3s$ $\quad t_0 = 2s$ $\quad T = 5s$

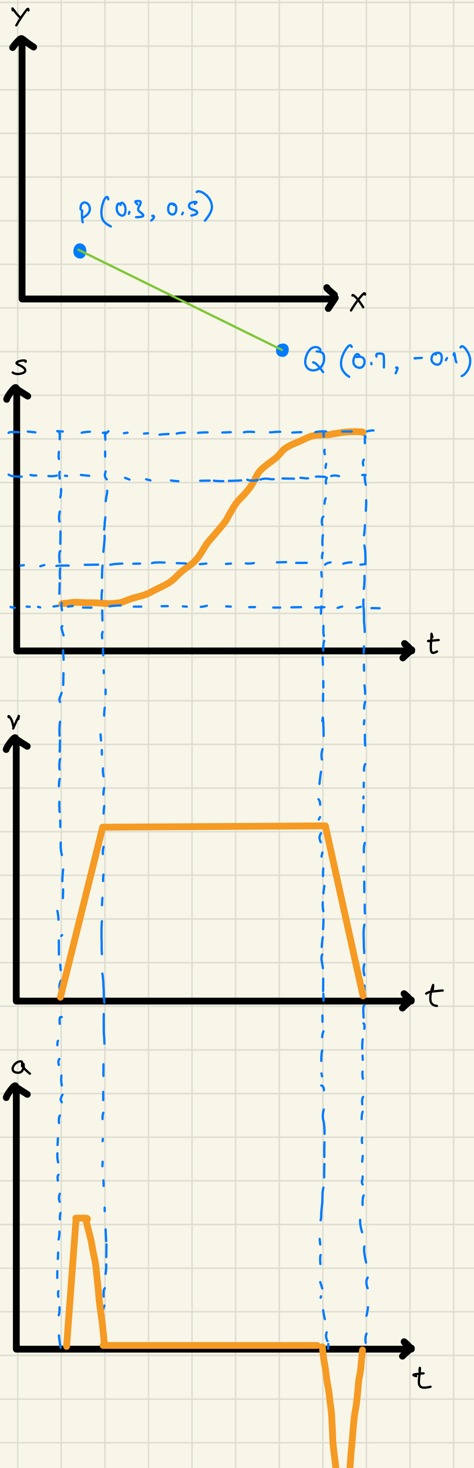
$$a_{max} = \frac{2s_1}{t_1^2} = \frac{2(0.58)}{9}$$

$$a_{max} = 0.128 \ m/s^2$$

which is under the given a_{max} constraint $(1 m/s^2)$

$S_a = 0.025$ $\quad S_{aa} = 0.005$ $\quad t = 0:10$

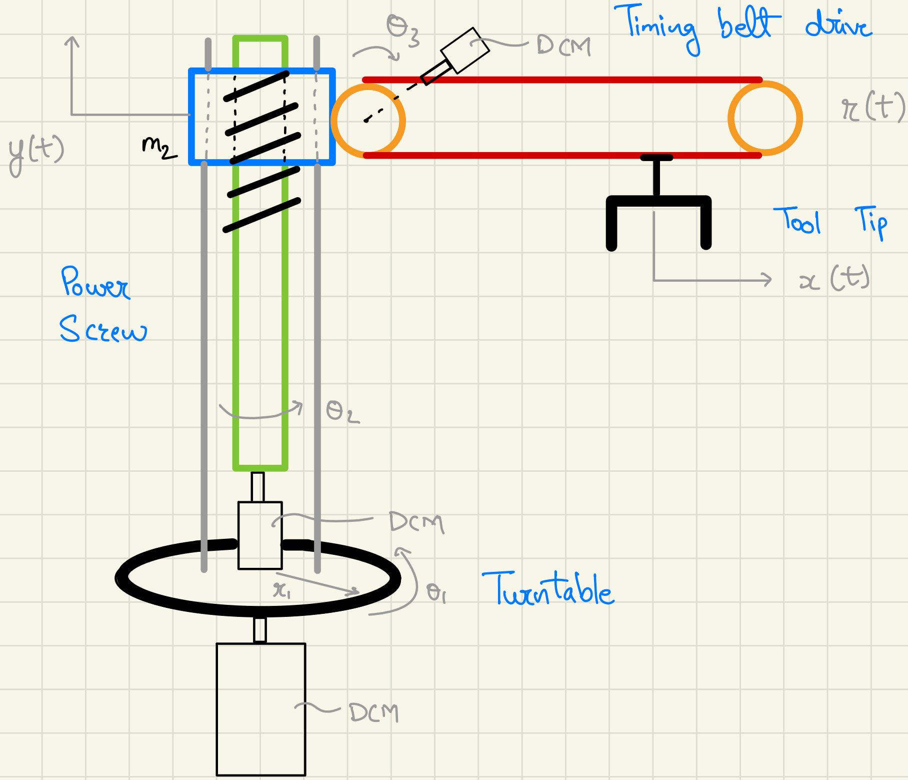




Y
X
P (0.3, 0.5)
Q (0.7, -0.1)
s
t
v
t
a
t

HW 3

Q. Turntable + Power Screw + Timing Belt



Here, $x(t) = r(t) \cdot \theta_3$

$$y(t) = \frac{\theta_2}{2\pi} \lambda \longrightarrow \text{Lead}$$

3 motors to power each component respectively.

No. of EOMs = 3

Tooltip $\equiv (\theta_1, y(t), x(t))$

1. Turntable :- DCM :- $\frac{R_a}{K_T} t_{m_1} + K_b \dot{\theta}_m = v_1(t)$

$$J_m \ddot{\theta}_1 + D_m \dot{\theta}_1 = t_{m_1}(t) - t_{R_1}(t)$$

GB :- $\Sigma M_{o_1} = J_{o_1} \ddot{\theta}_1$

$$J_{o_1} \ddot{\theta}_1 = t_{R_1} - f_{cr_1} - D_1 \dot{\theta}_1$$

$$t_{R_1} = J_{o_1} \ddot{\theta}_1 + f_{cr_1} + D_1 \dot{\theta}_1$$

$$J_m \ddot{\theta}_1 + D_m \dot{\theta}_1 = t_{m_1}(t) - J_{o_1} \ddot{\theta}_1 - f_{cr_1} - D_1 \dot{\theta}_1$$

$$2 J_m \ddot{\theta}_1 + 2 D_m \dot{\theta}_1 + f_{c_1 r_1} = t_{m_1}(t)$$

$\therefore$ EOM$_1$ $\Rightarrow$ $$\boxed{\frac{R_a}{K_T}\left(2 J_m \ddot{\theta}_1 + 2 D_m \dot{\theta}_1 + f_{c_1 r_1}\right) + K_b \dot{\theta}_m = v_1(t)}$$

2. Power Screw :-

DCM :- $\frac{R_a}{K_T} t_{m_2} + K_b \frac{2\pi}{\lambda}\dot{y} = v(t)$

$J_{m_2}\ddot{\theta}_2 + D_{m_2}\dot{\theta}_2 = t_{m_2}(t) - t_{R_2}(t)$

Slide :- $\Sigma F_y = M_2\ddot{y} = f - m_2 g$

$f = m_2\ddot{y} + m_2 g$

$$G(\mu,\alpha) = \frac{\mu\cos\alpha + \sin\alpha}{\cos\alpha - \mu\sin\alpha}$$

Screw :- $J_{sc}\frac{2\pi}{\lambda}\ddot{y} + G(\mu,\alpha)\, r_{sc}\, m\ddot{y}$

$+ G(\mu,\alpha)\, r_{sc}\, mg = t_r(t)$

$\therefore$ Eom 2 $\Rightarrow$

$$\boxed{\left(J_{sc} + J_{m_2}\right)\frac{2\pi}{\lambda}\ddot{y} + G(\mu,\alpha)\, r_{sc}\, m\ddot{y} + D_{m_2}\frac{2\pi}{\lambda}\dot{y} + G(\mu,\alpha)\, r_{sc}\, mg} = t_{m_2}(t)$$

3. Timing belt Drive

Motor :- $\frac{R_a t_{m_3}}{K_T} + K_b\dot{\theta}_{m_3} = v_3(t)$

$J_{m_3}\ddot{\theta}_{m_3} + D_{m_3}\dot{\theta}_{m_3} = t_{m_3}(t) - t_{R_3}(t)$

Belt Drive :-

$$\left.\begin{aligned} &-\left[J_{o_3} + J_{o_4}\, m r_w^2\right] \ddot{\theta}_3 \\ &-\left[D_3 + D_4\right] \dot{\theta}_3 \end{aligned}\right\} = t_{R_3}(t)$$

So,

$$J_{m_3} \ddot{\theta}_{m_3} + D_{m_3} \dot{\theta}_{m_3} = t_{m_3}(t) + \left[J_{o_3} + J_{o_4}\, m r_w^2\right] \ddot{\theta}_3 + \left[D_3 + D_4\right] \dot{\theta}_3$$

$$\begin{aligned} t_{m_3}(t) = \; & J_{m_3} \dot{\theta}_{m_3} + D_{m_3} \dot{\theta}_{m_3} \\ & - \left[J_{o_3} + J_{o_4}\, m r_w^2\right] \ddot{\theta}_3 \\ & - \left[D_3 + D_4\right] \dot{\theta}_3 \end{aligned}$$

$\therefore$ EOM$_3$:-

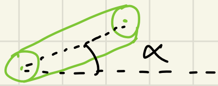
$$\boxed{\frac{R_a}{K_T}\left[J_{m_3} \dot{\theta}_{m_3} + D_{m_3} \dot{\theta}_{m_3} - \left[J_{o_3} + J_{o_4}\, m r_w^2\right] \ddot{\theta}_3 - \left[D_3 + D_4\right] \dot{\theta}_3 \right] + K_b \dot{\theta}_{m_3} = V_3(t)}$$

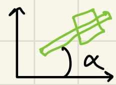
3. Power Screw :-

TEE

Power = Torque x Omega

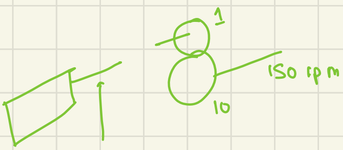
1. Complete FBD / Component FBD
2. TB / PS   DCM + GB

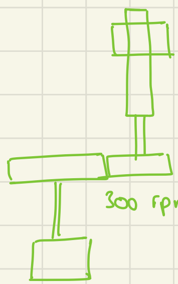




F = Mass (Acceleration) + C (Velocity)

Damping





lead = 5 mm

10 : 1

5mm → 1 rotation