Two types of Control

1. Open Loop control
2. Closed loop control

$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = u(t)$$ ← controlled input

$$x(0) = x_0 \qquad \dot{x}(0) = \dot{x}_0$$

$u(t)$ such that $x(t) \rightarrow x_d(t)$

$x(t)$: Controlled trajectory

$x_d(t)$: desired trajectory

$u(t)$ depends only on time or $x_d(t)$ and its derivative

↳ Open Loop Control

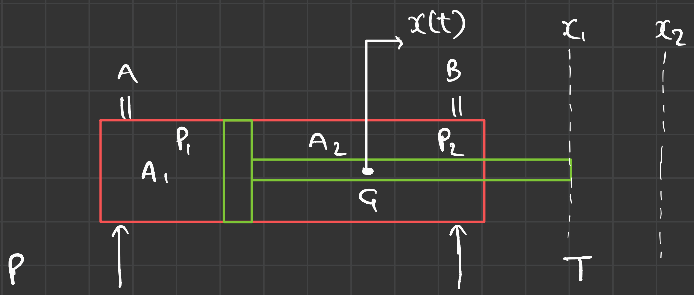
$u(t)$ depends on $x(t)$, $\dot{x}(t)$, $\ddot{x}(t)$, $\int_0^t x(\tau)\,d\tau$

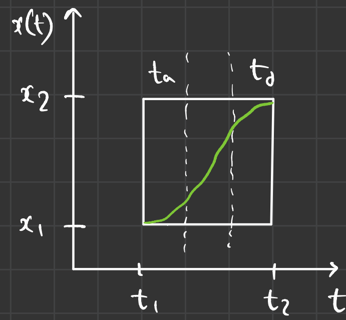
↳ Closed loop control

when, $x_d(t) = 0 \rightarrow$ Regulation

$x_d(t) \neq 0 \rightarrow$ Tracking

Example :-





$$m\ddot{x} + c\dot{x} = P_1 A_1 - P_2 A_2 = u(t)$$

$$u(t) = m\ddot{x}_d + c\dot{x}_d$$

$$m(\ddot{x} - \ddot{x}_d) + c(\dot{x} - \dot{x}_d) = 0$$

$$e(t) = x(t) - x_d(t)$$

$$m\ddot{e} + c\dot{e} = 0 \qquad e(t) \longrightarrow 0$$
$$t \longrightarrow \infty$$

$$\lambda^2 + \alpha\lambda = 0$$
$$\lambda_1 = 0$$
$$\lambda_2 = \alpha$$

$$\dot{e}(0) = 0 \qquad e(0) = 0$$

$$e(t) = A_1 + A_2 e^{-\alpha t}$$
$$e(0) = 0 \Rightarrow A_1 + A_2 = 0$$
$$\dot{e}(0) = 0 \Rightarrow A_2(-\alpha) = 0$$

$$P_1 A_1 - P_2 A_2 = m\ddot{x}_d + c\dot{x}_d$$

$$P_1 = \left(\frac{P_2 A_2 + m\ddot{x}_d + c\dot{x}_d}{A_1}\right)$$

Q. $m\ddot{x} + c\dot{x} + kx = u(t) \longrightarrow (1)$

$x(0) = x_0$

$\dot{x}(0) = \dot{x}_0$

$$u(t) = m\ddot{x}_d + c\dot{x}_d + kx_d$$

Substitute in (1)

then,

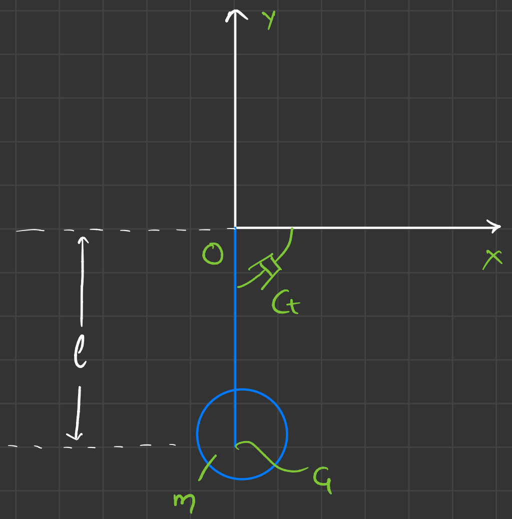
$$m\ddot{e} + c\dot{e} + ke = 0 \longrightarrow \quad \xi < 1 \quad \xi > 1 \quad \xi = 1$$

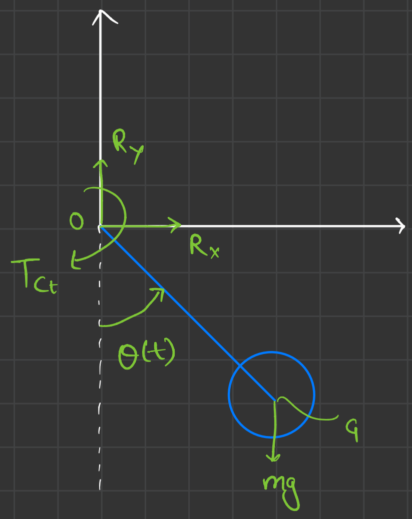
$e(0) = e_0$

$\dot{e}(0) = \dot{e}_0$

$e(t) \to 0$ as $t \to \infty$

LAB





$$T_{Ct} = C_t\,\dot\theta$$

$$\sum M_O = I_O\,\ddot\theta$$

$$m\ell^2\ddot\theta + C_t\dot\theta + mg\ell\sin\theta = 0$$

$$\ddot\theta + \frac{C_t}{m\ell^2}\dot\theta + \frac{g}{\ell}\sin\theta = 0$$

$$\theta(0) = \theta_0 \qquad \dot\theta(0) = \dot\theta_0$$

State Space Form

$$\theta_1 = \theta$$

$$\theta_2 = \dot\theta_1$$

$$\dot\theta_2 = \ddot\theta_1 = -\underbrace{C_{eq}}_{\frac{C_t}{ml^2}}\theta_2 - \frac{g}{\ell}\sin\theta_1$$

$$\frac{d}{dt}\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = \begin{bmatrix}\theta_2\\ -C_{eq}\theta_2 - g/_\ell \sin\theta_1\end{bmatrix}$$

$$\begin{bmatrix}\theta_1(0)\\ \theta_2(0)\end{bmatrix} = \begin{bmatrix}\theta_{10}\\ \theta_{20}\end{bmatrix} = \begin{bmatrix}\theta_0\\ \dot{\theta}_0\end{bmatrix}$$

Q. $\ddot{x} + 4\dot{x} = u(t)$

$u(t) = \ddot{x}_d + 4\dot{x}_d$

$x(0) = 2$ $\quad x_d(t) = 2\cos(\pi t)$

$\dot{x}(0) = 0$

$\ddot{x} + 4\dot{x} = \ddot{x}_d + 4\dot{x}_d$

$(\ddot{x} - \ddot{x}_d) + 4(\dot{x} - \dot{x}_d) = 0 \rightarrow (1)$

$e(t) = x(t) - x_d(t)$

From (1)

$\ddot{e} + 4\dot{e} = 0$

$e(0) = x(0) - x_d(t) = 0$

$\dot{e}(0) = \dot{x}(0) - \dot{x}_d(0) = 0$

$\ddot{x} + 4\dot{x} = -8\pi\sin(\pi t) - 2\pi^2\cos(\pi t)$

$x(0) = 2 \quad \dot{x}(0) = 0$

$$\frac{d}{dt}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}x_2\\ -4x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u(t)$$

HW

Q. $\ddot{x} + 4\dot{x} + 20x = u(t)$

$x(0) = 3 \qquad \dot{x}(0) = 0$

$x_d(t) = 2\cos(\pi t)$

$$\ddot{x}_d + 4\dot{x}_d + 20x_d = -2\pi^2\cos(\pi t) - 8\pi\sin(\pi t) + 40\cos(\pi t)$$

$x(0) = 3 \qquad \dot{x}(0) = 0$

$$y_1 = x$$
$$\dot{y}_1 = \dot{x} = y_2$$
$$\ddot{y}_1 = \ddot{x} = \dot{y}_2$$

$$\dot{y}_2 + 4y_2 + 20y_1 = u(t)$$

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_2 \\ -4y_2 - 20y_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

LECTURE

State Feedback Control

Q. $\ddot{x} + 3\dot{x} + 16x = u(t)$

$x(0) = x_0$

$\dot{x}(0) = \dot{x}_0$

$u_d(t) \qquad e(t) = x(t) - x_d(t)$

$$u(t) = -k_p\, x(t) - k_D\, \dot{x}(t)$$

$$\ddot{x} + 3\dot{x} + 16x = -k_p x - k_D \dot{x}$$

Proportional gain

Derivative gain

$$\ddot{x} + (3 + k_D)\dot{x} + (16 + k_p)x = 0$$

$$\ddot{x} + \underset{3}{2\zeta\omega_n}\dot{x} + \underset{16}{\omega_n^2}x = 0$$

$\omega_n = 4 \qquad 2\zeta\omega_n = 3$

$\zeta = \frac{3}{8} < 1 \rightarrow$ underdamped

Uncontrolled System Trajectory (Stepper motor)

$$x(t) = e^{-1.5t}\left[A\cos\left(\underbrace{4\sqrt{1-(3/8)^2}}_{\omega_d}\, t\right) + B\sin(\omega_D t)\right]$$

$$\omega_d = 4\sqrt{1-(3/8)^2}$$

Controlled System trajectory (AC Servo motor)

$K_P = 4$ & $K_D = 6$

$$x(t) = Ae^{-4t} + Be^{-5t}$$

$$\ddot{x} + 5\dot{x} + \sin x = u(t)$$

Feedback Linearization

$$u(t) = \sin(x) - K_P x - K_D \dot{x}$$

$K_P = 30$ $K_D = 6$

$x(t) \to 0$ Regulation

$x(t) \to x_d(t)$ Tracking

State Feedback Control

Regulation

$$x_d(t) = 0$$

$$\ddot{x} + 3\dot{x} + 16x = u(t)$$
$$x(0) = 2 \qquad \dot{x}(0) = -2$$

$$u(t) = -K_P x(t) - K_D \dot{x}(t)$$

$$\ddot{x} + (3 + K_D)\dot{x} + (16 + K_P)x = 0$$

Eigenstructure (change of poles)

Tracking

$$x(t) \to x_d(t)$$

$$e(t) = x(t) - x_d(t)$$

$$\ddot{e} + (3 + k_d)\dot{e} + (16 + K_p)e = 0$$

$$\ddot{x} + 3\dot{x} + 16x = u(t)$$
$$x(0) = 2 \qquad \dot{x}(0) = -2$$

$$u(t) = -K_p(x - x_d) + 16x_d - K_d(\dot{x} - \dot{x}_d) + 3\dot{x}_d$$

If $x_d(t)$ is smooth and differentiable,

$$u(t) = -k_p(x - x_d) + 16x_d - K_D(\dot{x} - \dot{x}_d) + 3\dot{x}_d + \ddot{x}_d$$

Feedback + Feedforward

rest terms $\qquad$ x_d, $\dot{x}_d$, $\ddot{x}_d$ terms

Control

Classical | Modern

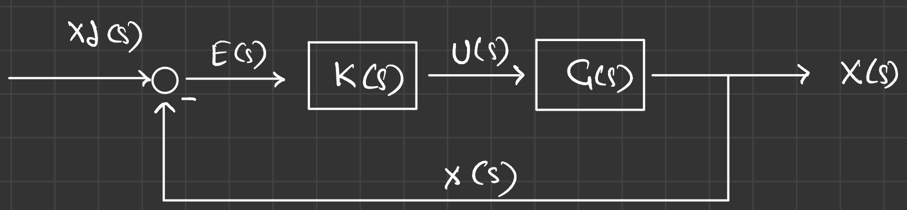
Laplace domain, Bode, R Locus, Nyquist, Compensators, PID, State feedback

Time domain

Feedforward | Feedback

Regulation | Tracking

In Laplace Transform :-



$$E(s) = -x(s) + x_d(s) \qquad U(s) = K(s)\,E(s)$$

$$X(s) = \frac{GK}{1+GK}\,x_d$$

$$X(s) = G(s)\,U(s)$$

$$\xrightarrow{U(s)} \boxed{G(s)} \longrightarrow X(s)$$

$$U(s) = -K(s)\left[X(s) - X_d(s)\right]$$

$$K(s) = K_P + sK_D$$

$$X(s) = -GK\left[X - X_d\right]$$

$$X(1+GK) = GKX_d$$

$$X = \left(\frac{GK}{1+GK}\right)X_d$$

Example

$$G(s) = \frac{1}{s^2+3s+16}$$

$$K(s) = K_P + sK_D$$

$$\frac{GK}{1+GK} = \frac{\dfrac{K_P + sK_D}{\cancel{s^2+3s+16}}}{\dfrac{s^2+3s+16+K_P+K_D s}{\cancel{s^2+3s+16}}}$$

$$\frac{GK}{1+GK} = \frac{K_P + sK_D}{s^2+(3+K_D)s+(16+K_P)}$$

why

$$X(s) = \frac{K_P + sK_D}{s^2 + (3+K_D)s + (16+K_P)} X_d(s)$$

$$E(s) = \frac{1}{1+GK}$$

$$E(s) = \frac{s^2 + 3s + 16}{s^2 + (3+K_D)s + (16+K_P)} X_d$$

LAB

Q. $\ddot{x} + 3\dot{x} + 16x = u(t)$

$x(0) = 1 \qquad \dot{x}(0) = 2$

$x_d(t) = 3\sin(6t)$

closed loop poles @ -10 & -12

$\ddot{x}_d + 3\dot{x}_d + 16x_d = -108\sin 6t + 54\cos 6t + 48\sin 6t$

$= -60\sin 6t + 54\cos 6t$

$\ddot{x} + 3\dot{x} + 16x = -k_P x - k_D\dot{x}$

$\ddot{x} + (3+K_D)\dot{x} + (16+k_P)x = 0$

$\lambda^2 + (3+k_D)\lambda + (16+k_P) = 0$

$\hookrightarrow (\lambda+10)(\lambda+12) = 0$

$\lambda^2 + 12\lambda + 10\lambda + 120$

$\lambda^2 + 22\lambda + 120 = 0$

$22\cancel{\lambda} = (3+K_D)\cancel{\lambda}$

$\boxed{K_D = 19}$

$16 + K_P = 120$

$\boxed{K_P = 104}$

$\ddot{x}_d + 3\dot{x}_d + 16x_d = -60\sin 6t + 54\cos 6t$

$x(0) = 1 \qquad \dot{x}(0) = 2$

$y_1 = x$

$$\dot{y}_1 = \dot{x} = y_2 \qquad x_d = 3\sin 6t$$

$$\dot{y}_1 = \ddot{x} = \dot{y}_2 \qquad \dot{x}_d = 18\cos 6t$$

$$\dot{y}_2 + 3y_2 + 16y_1 = u(t)$$

$$\dot{y}_2 = u(t) \boxed{- 3y_2 - 16y_1}$$

$$u(t) = -K_P(x - x_d) + 16x_d - K_D(\dot{x} - \dot{x}_d) + 3\dot{x}_d + \ddot{x}_d$$

$$u(t) = -104(y_1 - 3\sin 6t) + 18\sin 6t - 19(y_2 - 18\cos 6t) + 54\cos 6t - 108\sin 6t$$

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_2 \\ -3y_2 - 16y_1 \end{bmatrix} + \begin{bmatrix} 0 \\ u(t) \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

Regulation

Q. $\ddot{x} + a\dot{x} + bx = u(t)$

Regulation $x(t) \longrightarrow 0$

$$u(t) = -k_P x(t) - k_D \dot{x}(t)$$

$$\ddot{x} + (a+k_D)\dot{x} + (b+k_P)x = 0$$

$$\lambda^2 + (a+K_D)\lambda + (b+K_P)\lambda = 0$$

$-\lambda_1$, $-\lambda_2$ are closed loop poles

$$(\lambda+\lambda_1)(\lambda+\lambda_2) = \lambda^2 + (a+K_D)\lambda + (b+K_P)$$

$$x(t) = Ae^{\lambda t}$$

Q. $U(s) \longrightarrow \boxed{G(s)} \longrightarrow X(s)$

$$X(s) = G(s)\,U(s)$$

$$U(s) = G^{-1}\,X_d(s)$$

↳ Desired output

In Open loop,

$$\ddot{x} + a\dot{x} + bx = u(t)$$

$$x(t) \longrightarrow x_d(t)$$

$$u(t) = \ddot{x}_d + a\dot{x}_d + bx_d$$

So,

$$\left[s^2 + as + b\right] X(s) = U(s)$$

$$X(s) = \frac{1}{s^2 + as + b}\,U(s)$$

$$U(s) = G^{-1}\,X_d(s)$$

$$U(s) = s^2 X_d(s) + as\,X_d(s) + b\,X_d(s)$$

Q. $G(s) = \dfrac{8s + 2}{s^2 + as + b}$ $\quad U(s) \longrightarrow \boxed{G(s)} \longrightarrow X(s)$

$$X(s) = G(s)\,U(s)$$

A:-

$$U(s) = G^{-1}(s)\, X_d(s) = \frac{s^2 + as + b}{3s + 2}\, X_d(s)$$

$$U(s) = \frac{s^2 + as + b}{3s + 2}\, A \frac{\omega}{s^2 + \omega^2}$$

$$u(t) = L^{-1}[U(s)] = L^{-1}\left[\frac{A\omega\,(s^2 + as + b)}{(3s+2)(s^2+\omega^2)}\right]$$

$$= A_1 e^{2/3\, t} + B_1 \sin(\dots) + B_2 \cos(\dots)$$

Objectives of MCS

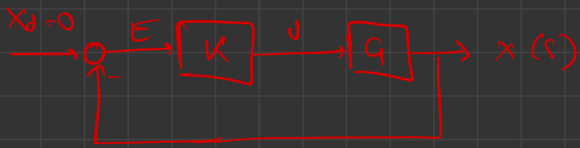
Time domain, Laplace domain, PID, PI
Closed loop, Open loop, SISO

Bode, Root locus, Routh ..., Nyquist

→ underdamped, overdamped
→ state space

Closed loop

a.



$X_d \to 0$ (Regulation)

$$X(s) = G(s)\,U(s)$$

$$X_d \longrightarrow \boxed{G_c} \longrightarrow X$$

$$G(s) = \frac{1}{s^2 + as + b}$$

$$K(s) = K_P + sK_D$$

$$G_c(s) = \frac{GK}{1+GK} = \frac{\dfrac{K_P + sK_D}{s^2 + as + b}}{1 + \dfrac{K_P + sK_D}{s^2 + as + b}}$$

$$G_c(s) = \frac{K_P + sK_D}{s^2 + (a + K_D)s + (b + K_P)}$$

Tracking

Q: $\ddot{x} + a\dot{x} + b = u(t)$

$$u(t) = -K_P(x - x_d) - K_D(\dot{x} - \dot{x}_d) + \ddot{x}_d + a\dot{x}_d + b x_d$$

$$\ddot{e} + (a + K_D)\dot{e} + (b + K_P)e = 0$$

Open loop

Closed loop

Regulation
Tracking } Time Domain

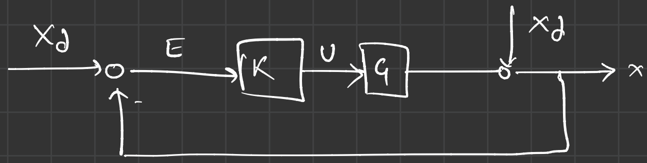
Regulation
Tracking } Laplace Domain

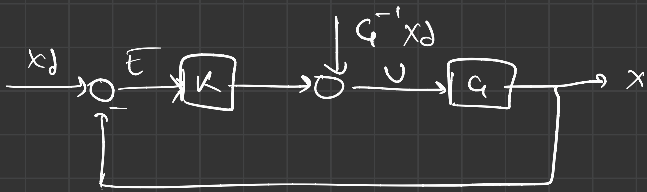
Tracking

$$X(s) = GU \qquad U = -K(s)(x - x_d)$$

$$X = -GK(x - x_d) + x_d$$

$$(1 + GK)x = (1 + GK)x_d$$





$$E = x_d - x \qquad U = -K(x - x_d) + G^{-1}x_d$$

$$X = GU = -GK(x - x_d) + x_d$$

$$(1 + GK)x = (1 + GK)x_d$$

$$\ddot{x} + a\dot{x} + bx = u(t) = -K_p(x - x_d) - K_D(\dot{x} - \dot{x}_d) + \ddot{x}_d + a\dot{x}_d + bx_d$$

$$L\left[\ddot{x}_d + a\dot{x}_d + bx_d\right] = G^{-1}x_d$$

$$G^{-1} = s^2 + as + b$$

PID

Time Domain :-

Regulation :-

$$\ddot{x} + a\dot{x} + bx = u(t) = -k_P x - k_D \dot{x} - k_I \int_0^t x(\tau)\, d\tau$$

differentiate

$$\dddot{x} + a\ddot{x} + b\dot{x} = -k_P \dot{x} - k_D \ddot{x} - k_I x$$

$$\dddot{x} + (a + k_D)\ddot{x} + [b + k_P]\dot{x} + k_I x = 0$$

$$\lambda^3 + (a + k_D)\lambda^2 + (b + k_P)\lambda + k_I = 0$$

Original system, $\lambda^2 + a\lambda + b = 0$

Closed loop poles @ $-\lambda_1, -\lambda_2, -\lambda_3$

$$\lambda^3 + (a + k_D)\lambda^2 + (b + k_P)\lambda + k_I$$

$$= (\lambda + \lambda_1)(\lambda + \lambda_2)(\lambda + \lambda_3)$$

Tracking :-

$$\ddot{x} + a\dot{x} + bx = u(t) \qquad x(t) \rightarrow x_d t$$

in finite time

$$u(t) = -k_p(x - x_d) - k_D(\dot{x} - \dot{x}_d) - K_I \int_0^t [x(\tau) - x_d(\tau)]\, d\tau$$
$$+ \ddot{x}_d + a\dot{x}_d + bx_d$$

Error variable :- $e(t) = x(t) - x_d(t)$

$$(\ddot{x} - \ddot{x}_d) + (a + K_D)(\dot{x} - \dot{x}_d) + (b + K_P)(x - x_d)$$
$$+ K_I \int_0^t [x(t) - x_d(t)]\, d\tau = 0$$

→ Integro - differential equation

differentiating,

$$\dddot{e} + (a + k_D)\ddot{e} + (b + k_P)\dot{e} + K_I e = 0$$

$$\lambda^3 + (a + K_D)\lambda^2 + (b + K_P)\lambda + K_I = 0$$

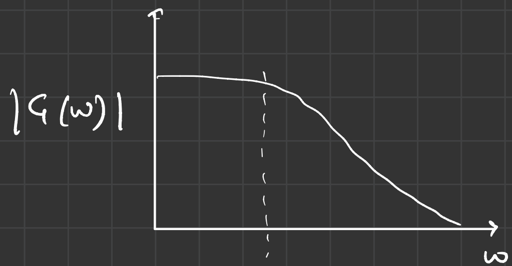
Original system, $\lambda^2 + a\lambda + b = 0$

→ Closed loop poles @ $-\lambda_1, -\lambda_2, -\lambda_3$

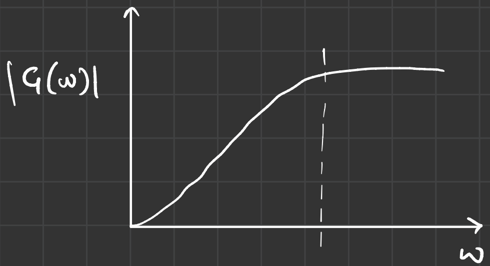
$$\lambda^3 + (a + K_D)\lambda^2 + (b + K_P)\lambda + K_I$$
$$= (\lambda + \lambda_1)(\lambda + \lambda_2)(\lambda + \lambda_3)$$

LAB

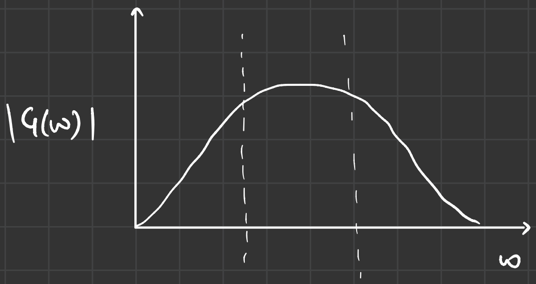
Low Pass



High Pass



Band Pass

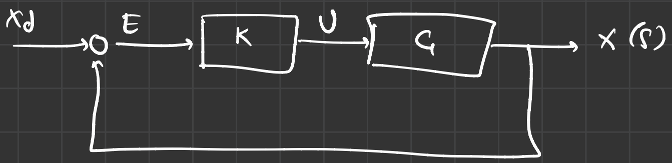


$K(s) = K_p$ $\quad G(s) = \dfrac{1}{s^2 + 0.02s + 4}$

1. Steady state error to a unit step should be less than 0.3 %

2. % OV less than 15 %

A ⇒ $\quad GK = \dfrac{K_p}{s^2 + 0.02s + 4}$



1) $\quad E(s) = \dfrac{1}{1 + GK} \, X_d(s)$

Using FVT,

$$e_{ss}(t) = \lim_{s \to 0} s \, \frac{1}{1 + GK} \, X_d(s)$$

$$e_{ss}(t) = \lim_{s \to 0} \frac{1}{1 + \dfrac{K_p}{s^2 + 0.02s + 4}} = \frac{4}{4 + K_p} < 0.003$$

$K_p \geq 1330$

Threshold value

2. Bode $\left(\dfrac{1330}{s^2 + 0.02s + 4}\right)$

23/2/22

Book :– Robust & Optimal Control
Kemin Zhou
LSU

General linear system in state space :–

$$\dot{x} = Ax + B_1 w + B_2 u$$
$$z = C_1 x + D_{11} w + D_{12} u$$
$$y = C_2 x + D_{21} w + D_{22} u$$

$x(t)$ $n \times 1$ state vector
$u(t)$ $m \times 1$ control input
$w(t)$ $l \times 1$ disturbance input

$|w(t)| \leq \bar{w}$ $w(t) \to 0$ as $t \to \infty$

$z(t)$ – $q \times 1$ controlled output

$y(t)$ – $p \times 1$ measurement output

A $n \times n$ system matrix

Bs, Cs, Ds – Gain matrices

Mitigate the effect of $w(t)$ on $z(t)$.
Designing $u(t)$ using $y(t)$

$$\dot{x} = Ax + Bu$$

$$\cancel{y = Cx + Du}$$

$$y \qquad z(t) = x(t)$$

$$C = I_{n\times n} \qquad D = 0_{m\times m}$$

Design $u(t) = -Kx(t) \rightarrow$ Feedback control

Example :- $m\ddot{x} + c\dot{x} + kx = u(t)$

$$x(t) \rightarrow x_d(t)$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u(t)$$

$$u(t) = -k_p x_1 - k_D x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix}\begin{bmatrix} -k_p & -k_D \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

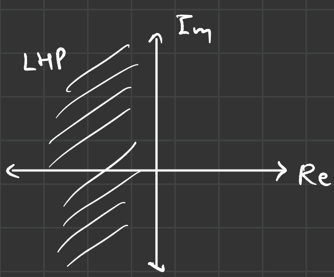
$$\dot{x} = (A - BK)x \qquad \dot{x} = (A + BK)x$$

$$K = \begin{bmatrix} K_p & K_D \end{bmatrix} \qquad K = -\begin{bmatrix} K_p & K_D \end{bmatrix}$$

closed loop system matrix

$$\sigma(A + Bk) < 0 \quad \Rightarrow$$

↳ eigen values



To have eigen values < 0, we use an algorithm called pole placement.

pole placement

Design k such that $\sigma(A+Bk) < 0$

$$A = \begin{bmatrix} 0 & 1 \\ x_1 & x_2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ x_3 \end{bmatrix}$$

$$K = \begin{bmatrix} y_1 & y_2 \end{bmatrix}$$

Ex :- $A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad K = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$

$$A + Bk = \begin{bmatrix} 0 & 1 \\ 2+k_1 & -3+k_2 \end{bmatrix}$$

This system is an Companion Canonical form.

$$A = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} \qquad B = \begin{bmatrix} x_5 \\ x_6 \end{bmatrix} \qquad k = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$$

using Similarity Transformation to transform above to canonical form.

$$\dot{x} = Ax + Bx \longrightarrow (1)$$

$$x = Ty \Rightarrow \dot{x} = T\dot{y}$$

Substitute in (1)

$$T\dot{y} = ATy + Bu$$

T is $n \times n$ non-singular

$$\dot{y} = T^{-1}ATy + T^{-1}Bu$$

$$T^{-1}AT = \begin{bmatrix} 0 & I \\ x_1 & x_2 \end{bmatrix} \qquad T^{-1}B = \begin{bmatrix} 0 \\ x_3 \end{bmatrix}$$

Example :-

$$A = \begin{bmatrix} 2 & -3 & 9 \\ 10 & 12 & 13 \\ 5 & -7 & 14 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

Convert to canonical form,

$$A_c = T^{-1}AT = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ x_1 & x_2 & x_3 \end{bmatrix}$$

$$B_c = T^{-1}BT = \begin{bmatrix} 0 \\ 0 \\ x_4 \end{bmatrix}$$

This system is single input system.

Example :- 2 DOF

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & -9 & 10 & 23 \\ -7 & 8 & 21 & 0 \\ -1 & 2 & 10 & 13 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 4 \\ -1 & 8 \\ 2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Controllability Index $\sum_{k=1}^{m} n_k = n$

Rank of Controllability matrix

$$\begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix} \qquad \begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}B \end{bmatrix}$$

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MCS HW 1

1. $\dot{x} = 5x + u(t)$

$u(t) = -k_p(x - x_d) + \dot{x}_d - 5x_d$

$(\dot{x} - \dot{x}_d) - 5x + k_p(x - x_d) + 5x_d = 0$

$\dot{e} + (k_p - 5)e = 0$

$e(0) = 1 - 0 = 1$

$e_n(t) = Ae^{\lambda t}$

$\lambda + (k_p - 5) = 0$

$\lambda = 5 - k_p$

For a stable system, $\lambda < 0$

for $e(t) \to 0$, $\lambda < 0$

$\underline{\underline{k_p > 5}}$

2. $\dot{x} + 2x = u(t)$

$x_d(t) = \cos(10t)$

$u(t) = \dot{x}_d + 2x_d$

$= -10\sin(10t) + 2\cos(10t)$

$\dot{x} + 2x = 2\cos(10t) - 10\sin(10t)$

$x(t) = x_n(t) + x_p(t)$

$x_n(t) = Ae^{\lambda t}$

$\lambda + 2 = 0 \qquad \lambda = -2 \qquad x_n(t) = Ae^{-2t}$

$x_p(t) = B\cos(10t) + D\sin(10t)$

$\dot{x}_p(t) = -10B\sin(10t) + 10D\cos(10t)$

$x_p(t) : -10B\sin(10t) + 10D\cos(10t) + 2B\cos(10t) + 2D\sin(10t) = -10\sin(10t) + 2\cos(10t)$

$-10B + 2D = -10 \qquad D = 0$

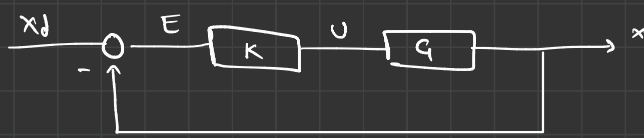
$10D + 2B = 2 \qquad B = 1$

$x_k(t) = Ae^{-2t} + \cos(10t)$

$A = 0$ for $x(0) = 1$

$x(t) = \cos(10t)$

3. $X(s) = \dfrac{3s+2}{s^2+3s+16} U(s)$



$$x = \frac{GK}{1+GK} x_d$$

$$= \frac{\frac{3s+2}{s^2+3s+16}(K)}{1 + \frac{3s+2}{s^2+3s+16}(K)} x_d$$

$$x = \frac{(3s+2)\,K_p}{s^2 + 3s + 16 + 3sK_p + 2K_p} \qquad K = K_p$$

$$= \frac{(3s+2)\,K_p}{s^2 + 3s(1+K_p) + (2K_p + 16)}$$

zero at $-2/3$

For stable system,

$$\omega_n = \sqrt{2K_p + 16}$$

$$\xi_g = \frac{3(1+K_p)}{2 \cdot \sqrt{2K_p + 16}} > 1$$

at $K_p = 0$ $\quad \xi_g = 3/8 < 1$

$K_p = 1$ $\quad \xi_g = \frac{6}{2\sqrt{18}}$

$K_p = 2$ $\quad \xi_g = \frac{9}{2\sqrt{20}}$

$K_p = 3$ $\quad \xi_g = \frac{12}{2\sqrt{22}}$

$$3(1+K_p) < 2 \cdot \sqrt{K_p + 16}$$

$$9(1 + K_p^2 + 2K_p) > 2K_p + 32$$

$$9 + 9K_p^2 + 18K_p > 2K_p + 32$$

$$9K_p^2 + 20K_p - 23 > 0$$

$K_p < 0.835$

$K_p < -3.057$

$0 < K_p < 0.835$

4. If numerator is $3s-2$, the zero will be at $2/3$ which would make the system a non-minimum phase system.

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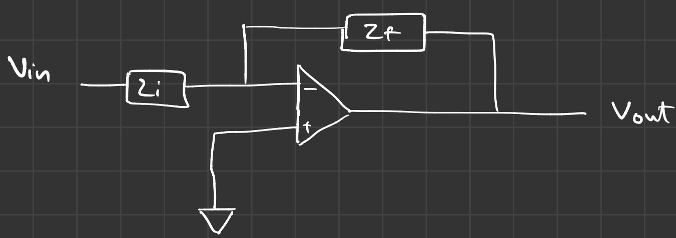
MCS HW II

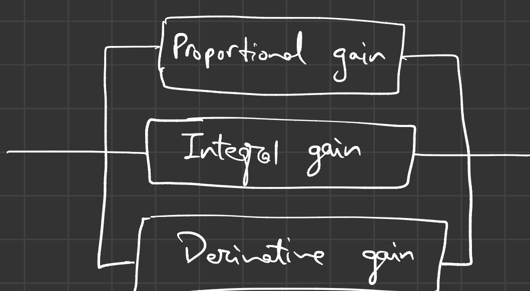
1. PID controller using op-Amps

$$K_s = K_p + sK_p + \frac{K_i}{s} = \frac{s^2 K_D + sK_p + K_i}{s}$$

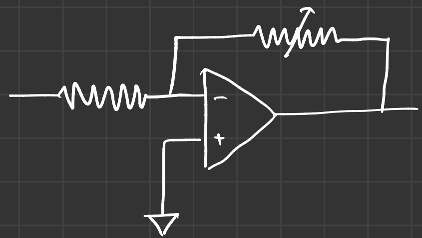


$$K = \frac{Z_f(s)}{Z_i(s)}$$

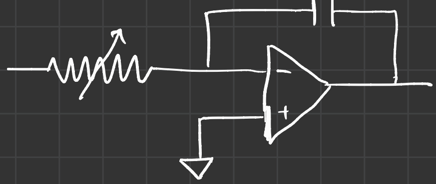
A PID controller :



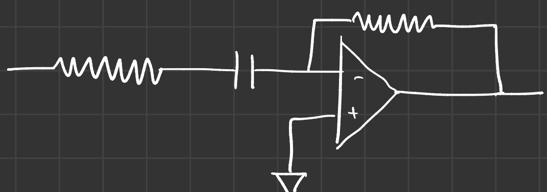
Proportional gain



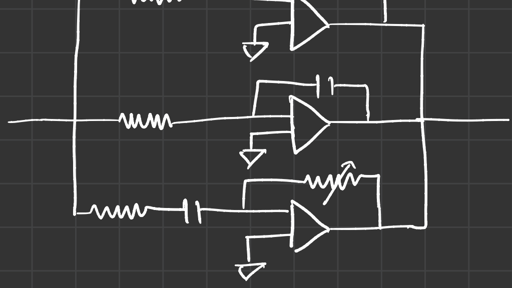
Integral gain



Derivative gain



Putting all together



The potentiometer can be controlled to tune the controller.

2. $G(s) = \dfrac{1}{s^2 + 9s + 20}$ $K(s) = K_P + sK_D + \dfrac{K_i}{s}$

$$\zeta = 0.4$$

$e_{ss} < 0.2$ of $x_d(t) = r(t)$

$$X_d(s) = \frac{1}{s^2}$$

$$X(s) = \frac{\dfrac{K_P + sK_D + \dfrac{K_i}{s}}{s^2 + 9s + 20}}{1 + \dfrac{K_P + sK_D + \dfrac{K_i}{s}}{s^2 + 9s + 20}} \cdot \frac{1}{s^2}$$

$$X(s) = \frac{s^2 K_D + sK_P + K_i}{s^3 + s^2(9 + K_D) + (20 + K_P)s + K_i}$$

To design :-

1. Open loop step response < MATLAB

Assume $K(s) = K_P$

$$T_f = \frac{K_P}{s^2 + 9s + (20 + K_P)}$$ $K_P = 300$

w/ $K_P = 0.88$, $\zeta = 0.4$

$K_i = 1 \qquad e_{ss} < 0.2\%$

$K_D = 0.002$

$$e_{ss} = \lim_{s \to 0} \frac{s(s^2 K_D + s K_p + K_i)}{s^3 + s^2(9 + K_D) + (20 + K_p)s + K_i} \cdot \frac{1}{s^2} = 0.2\%$$

if $K_i = 0$

$$\lim_{s \to 0} \frac{s K_D + K_p}{s^3 + (9 + K_D)s^2 + (20 + K_p)s} = 0.002$$

if $K_i = 0$

$$\frac{s K_D + K_p}{s^2 + (9 + K_D)s + (20 + K_p)}$$

$$9 + K_D = 0.8\sqrt{20 + K_p}$$

$$\lim_{s \to 0} \frac{s K_D + K_p}{s^2 + (K_D + 9)s + (20 + K_p)} = 0.002$$

$$\frac{K_p}{20 + K_p} = 0.002$$

$$K_p = 20.040$$

$$9 + K_D = 0.8\sqrt{40.040}$$

$$K_D = -3.9378$$

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MCS HW - III

$$G(s) = \frac{1}{s^2+10s} \qquad K(s) = K_p$$

$$e_{ss}(t) = \lim_{s\to 0} \frac{s \cdot 1}{1+GK} \cdot \frac{1}{s^2}$$

$$= \lim_{s\to 0} \frac{1}{1+\frac{K_p}{s(s+10)}} \cdot \frac{1}{s} = \lim_{s\to 0} \frac{s+10}{s^2+10s+K_p}$$

$$\frac{10}{K_p} \leq 0.003$$

$$K_p = 3333.33$$

$$e^{-\xi\pi/\sqrt{1-\xi^2}} \leq 0.15 \Rightarrow \xi = 0.517$$

$$10 = 2\xi\sqrt{K_p} \Rightarrow \frac{5}{0.517} = \sqrt{K_p}$$

$$K_p = 93.5321$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Let $r = \frac{\omega}{\omega_n}$

$$|GK| = \frac{\omega_n^2}{\sqrt{\omega^4 + (2\zeta\omega_n)^2}} = \frac{1}{\sqrt{r^4 + (2\zeta r)^2}}$$

$$\angle GK = 0 - \tan^{-1}\left[\frac{2\zeta\omega_n\omega}{\omega^2}\right] = \tan^{-1}\left(\frac{2\zeta}{r}\right)$$

For frequency gain crossover,

$$r^4 + 4r^2\zeta^2 - 1 = 0$$

$$r^2 = \sqrt{4\zeta^4 + 1} - 2\zeta^2$$

$$\phi_{GK} = -\tan^{-1}\left[\frac{2\zeta}{(\sqrt{4\zeta^4+1} - 2\zeta^2)^{1/2}}\right]$$

$$PM = 180 + \phi$$

$$\phi = \tan^{-1}\left[\frac{2 \cdot 0.517}{(\sqrt{4\zeta^2+1} - 2\zeta)^{1/2}}\right]$$

$$PM = 51.7^\circ$$

Bode of $\frac{K_p}{s^2+10s} \Rightarrow \frac{3333.33}{s^2+10s}$

e_{ss} req. needs $K_p = 3333.33$

$\%OV$ needs $K_p = 93.534$ for $\zeta = 0.517$

ζ needs $PM = 51.7^\circ$

$\omega_{gc} = 57.5$ rad/s

$PM = 10^\circ$

$$G_p(s) = \frac{1 + \lambda G s}{\lambda (1 + G s}$$

$$\omega_{gc} = \frac{1}{G\sqrt{\lambda}}$$

$$\hat{\phi} = 51.7 - 10 = 45.7^\circ$$

$$\hat{\phi} = \sin^{-1}\left(\frac{\lambda - 1}{\lambda + 1}\right)$$

$$0.989 = \frac{\lambda - 1}{\lambda + 1}$$

$$\lambda = 184.65$$

$$57.5 = \frac{1}{G\sqrt{\lambda}} \qquad G = 1.279 \times 10^{-3}$$

$$G_p K_p G = \frac{(1 + 184.65\,(1.279\times10^{-3})s)}{184.65\,(1 + (1.279\times10^{-3})s)} \cdot \frac{3333.33}{(s^2+10s}$$

$$K_p = \frac{3333.3}{184.65}$$

$$-10 \log \lambda = -22.663$$

$$\omega_m = 211 \text{ rad/s}$$

$$211 = \frac{1}{G\sqrt{184.65}}$$

$$G = 3.48 \times 10^{-4}$$

Final values :-

$K_p = 3333.3 \ (184.65)$

$q = 3.48 \times 10^{-4}$

$\lambda = 184.65$

LAG :-

$$G_p(s) = \frac{1 + \tau_2 s}{1 + \beta \tau_2 s}$$

at $51.7'$

$\omega_{gc} = 2.08 \text{ rad/s}$

$\text{gain} = 72.2 \text{ dB}$

$$\beta = 20 \log_{10} \beta = 72.2 \text{ dB}$$

$$\beta = 4073.80$$

$$\frac{1}{\tau_2} = \frac{2.08}{10} = 0.208 \Rightarrow 4.807 = \tau_2$$

$$G_p = \frac{1 + 4.807 s}{1 + (4073.80 \cdot 4.807) s}$$

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MCS LAB 7

$$G(s) = \frac{1}{s^2 + 100s + 4}$$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot 1}{1 + GK} \cdot \frac{1}{s^2}$$

$$= \lim_{s \to 0} \frac{1}{s^3 + 100s^2 + s(4 + K_p)} \quad (s^2 + 0.02s + 4)$$

$$\frac{4}{4 + K_p} \leq 0.003$$

$$\Rightarrow 4 = 0.012 + 0.003 K_p$$

$$K_p = 1.329$$

for $\% OV < 15\%$ $\quad \zeta = 0.517$

$$e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.15$$

$$s^2 - 1(0.025) + (4 + K_p)$$

$$\omega_n = \sqrt{4 + K_p}$$

$$2\zeta\omega_n = 0.02 \quad \Rightarrow \quad \sqrt{4 + K_p} = \frac{0.02}{2 \cdot 0.517}$$

$K_p = -3.999$

$\phi = \tan^{-1}[2(0.317)]$

$PM = 51.7°$

Bode of $G_R = \dfrac{K_p}{s^2 + 0.02s + 4}$

$\omega_{gc} = 36.5°$
$PM = 0°$

$\phi = 51.7°$

$\sin\phi = \left(\dfrac{\lambda - 1}{\lambda + 1}\right) = 0.9907\lambda + 0.9907 = \lambda - 1$

$\lambda = 214.709$

$\omega_{gc} = \dfrac{1}{a\sqrt{\lambda}} = 36.5\left(\sqrt{214.709}\right) = \dfrac{1}{a}$

$a = 1.27601$

$-10\log_{10}\lambda = -23.318$

$\omega_m = 140$ rad/s

$\omega_m = \dfrac{1}{z_o\sqrt{\lambda}}$

$z_o = 4.87 \times 10^{-4}$

$K_p = -3.999 \times (214.709)$
$z_o = 4.87 \times 10^{-4}$

$\lambda = 214.705$

LAG :-

$$G_p(s) = \frac{1 + \tau_2 s}{1 + \beta \tau_2 s}$$

at 51.7°

$\omega_{gc} = 2.08$ rad/s
gain $= 72.2$ dB

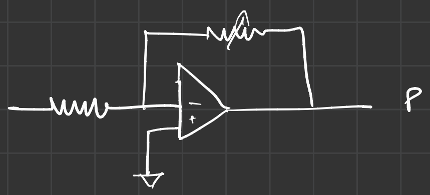
$$\omega_{gc} = \frac{1}{\tau_2 \sqrt{\beta}}$$

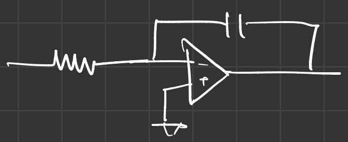
$$\beta = 20\log_{10}\beta = 72.2 \text{ dB}$$
$$\beta = 4073.80$$

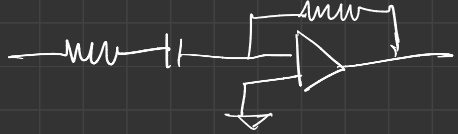
$$\tau_2 = \frac{1}{\omega_{gc}\sqrt{\beta}}$$

$$\frac{1}{\tau_2} = \frac{2.08}{10} = 0.208 \Rightarrow 4.807 = \tau_2$$

$$G_p = \frac{1 + 4.807s}{1 + (4073.80 \cdot 4.807)s}$$







Q. $G(s) = \dfrac{3}{s^2 + 3s + 16}$

Assume $k_s = k_p$

$$e_{ss}(t) = \lim_{s \to 0} s \cdot \frac{1}{1 + GK} \cdot X(s)s$$

$$= \lim_{s \to 0} s \cdot \frac{1}{1 + \dfrac{3K_p}{s^2 + 3s + 16}} \cdot \frac{1}{s}$$

$$= \frac{s^2 + 3s + 16}{(s^2 + 3s + 16 + 3K_p)}$$

$$\frac{16}{16 + 3K_p} = \frac{0.03}{100}$$

$$1600 = 0.48 + 0.09 K_p$$

$$K_p = 17772.44$$

for % OV < 15 %

$$e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.15$$

$$\zeta = 0.517$$

$$\omega_n = \sqrt{16 + 3K_p}$$

$$\sqrt{16+3K_p} = \frac{3}{2(0.517)}$$

2ζωn = 3

$$16+3K_p = 8.417$$

$$K_p = -2.527$$

$$\phi = \tan^{-1}(2(0.517))$$

$$\phi = 45.9 \approx 46$$

Bode of $GK = \frac{3K_p}{s^2+3s+16}$

$$\omega^* = 3.391$$

$$PM = 114$$

$$\phi = 180 + (-66) = 114$$

Required $\phi = 51.7$

$$\phi' = 51.7 - 114$$

$$\phi' = -62.3$$

Absolute value :- $\phi' = 62.3$

$$\sin\theta' = \frac{\lambda - 1}{\lambda + 1}$$

$$0.8853 = \frac{\lambda - 1}{\lambda + 1}$$

$$0.8853 \lambda + 0.8853 = \lambda - 1$$

$$0.1147 \lambda = 1.8853$$

$$\lambda = 16.436$$

$$\omega^* = \frac{1}{\tau_1 \sqrt{\lambda}}$$

$$3.24 = \frac{1}{\tau_1 \sqrt{16.436}}$$

$$\tau_1 = 0.0761$$

$$-10 \log_{10} \lambda = -12.1579$$

$$\omega_m = 464 \text{ rad/s}$$

$$464 = \frac{1}{\tau_1 \sqrt{16.426}}$$

$$\text{actual } \tau_1 = 0.0005315$$

$$\text{actual } K_p = \frac{K_p \lambda}{3}$$

$$= \frac{17772.44 \, (16.436)}{3}$$

$$\text{actual } K_p = 97369.27$$

Controller, Observer & Observer Based Controller

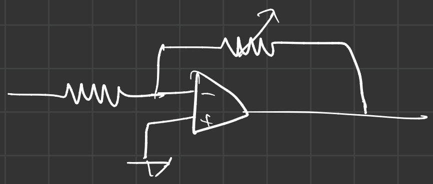
Achieve stability

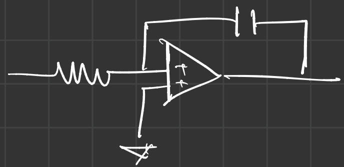
Pole placement

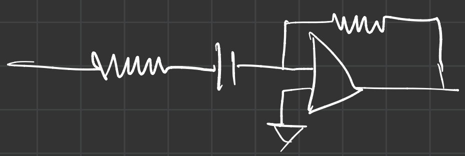
→ Matlab Command : place

→ Canonical form Transformations → SISO (For TEE)
MIMO

```
D = [-20 ; -25 ; -30 ; -35] ;
K = place (A, B, D) ;
```







Q. State feedback

$$\dot{x} = 5x + u(t)$$

$$x(0) = 1 \qquad \dot{x}(0) = 0$$

$$u(t) = -k_p(x - x_d) + \dot{x}_d - 5\dot{x}_d$$

$$(\dot{x} - \dot{x}_d) + (k_p - 5)(x - x_d) = 0$$

$$\dot{e} + (k_p - 5)e = 0$$

$$e(0) = x(0) - x_d(0) = 1$$

$$e_n(t) = Ae^{\lambda t}$$

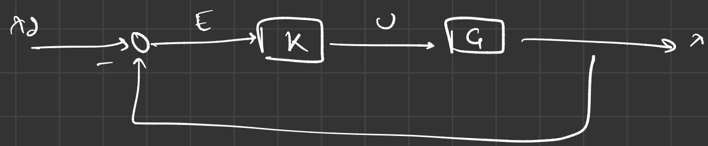
$$\lambda + (k_p - 5) = 0$$

$$\lambda = 5 - k_p$$

for $e(t) \to 0$, $\lambda < 0$

$$\boxed{k_p > 5}$$

Q. $x(s) = \dfrac{3s+2}{s^2+3s+16} \quad U(s)$



$$x = \frac{GK}{1+GK} \, x_d$$

$$x = \frac{\frac{3s+2}{s^2+3s+16} K_p}{1 + \frac{3s+2}{s^2+3s+16} K_p}$$

$$= \frac{(3s+2) K_p}{s^2 + 3s(1+K_p) + (2K_p+16)}$$

For stable system,

$$\omega_n^2 = 2K_p + 16$$
$$\omega_n = \sqrt{2K_p + 16}$$

$$2\xi\omega_n = 3s(1+K_p)$$

$$\xi = \frac{3s(1+K_p)}{2\left(\sqrt{2K_p+16}\right)} > 1$$

at $K_p = 0$, $\xi = 3/8 < 1$

$K_p = 1$

$K_p = 2$

$K_p = 3$

$$3(1 + K_p) < 2\left(\sqrt{K_p + 16}\right)$$

$$9(1 + K_p^2 + 2K_p) < 2K_p + 32$$

$$9K_p^2 + 20K_p - 23 > 0$$

$k_p < 0.835$
$k_p < -3.057$

$$0 < K_p < 0.85$$

Q. If num= $3s - 2$, zero at $\frac{2}{3}$ which would make the system a non - min phase system

Q. $\ddot{x} + 3\dot{x} + 16x = u(t)$ $\quad x(0) = 1 \quad \dot{x}(1) = 2$

$x(t) \longrightarrow 2\sin(\pi t)$

$$u(t) = \ddot{x} + 3\dot{x} + 16x$$
$$= 32\sin(\pi t) + 6\pi\cos(\pi t) - 2\pi^2\sin(\pi t)$$

$$\ddot{e} + 3\dot{e} + 16e = 0$$

$$\omega_n^2 = \sqrt{16} \quad , \quad \omega_n = 4$$

$$2\xi\omega_n = 3$$

$$\xi = \frac{3}{8}$$

$$e(0) = x(0) - x_d(0) = -1$$
$$\dot{e}(0) = \dot{x}(0) - \dot{x}_d(0) = 2\pi - 2 = 4.28$$

$$e(t) = e^{-\xi\omega_n t}\left[A\cos(\omega_d t) + B\sin(\omega_d t)\right]$$

$$\omega_D = \omega_n \times \sqrt{1-\xi^2} = \frac{\sqrt{55}}{2}$$

$$e(0) \Rightarrow A = -1$$

$$\dot{e}(0) =$$

Q. PID

$$u(t) = -k_P(x - x_d) - k_P(\dot{x} - \dot{x}_d) - K_I \int_0^t (x(t) - x_d(t))\, d\tau + \ddot{x}_d + a\dot{x}_d + b x_d$$

$$u(t) = -k_P(x - x_d) - k_D(\dot{x} - \dot{x}_d) - K_I \int_0^t [x(t) - x_d(t)]\, dt + \ddot{x} + a\dot{x}_d + b x_d$$

Q. $-20, -25, -30$

$$(\lambda+20)(\lambda+25)(\lambda+30) = \lambda^3 + 75\lambda^2 + 1850\lambda + 15000$$

3 2 1

M 1

A1. $$G(s) = \frac{3s-5}{s^2+3s+16}$$

$$\ddot{x} + 3\dot{x} + 16x = u(t)$$

$$u(t) = -K_D\dot{x} - K_p x$$

$$\ddot{x} + (3+K_D)\dot{x} + (16+K_p)x = 0$$

$3 + K_D = 10 + 12$ $\qquad\qquad$ $16 + K_p = 10 \times 12$

$K_D = 19$ & $K_p = 104$

B) $$\frac{104 + 19s}{3s-5} \to \frac{104}{3s-5} + \frac{19s}{3s-5}$$

$$u(t) = L.T^{-1}\left[\frac{104}{3(s-5/3)}\right] + LT^{-1}\left[\frac{19s}{3s-5}\right]$$

$$u(t) = \frac{104}{3} e^{5/3 t} + L.T^{-1}\frac{19s}{3s-5}$$

As $t \to \infty$, $u(t) \to \infty$

$u(t)$ becomes unbounded

$1/s^2 \to$ ramp $\qquad$ step $\to 1/s$

Q.

$$g(s) = \frac{1}{s^2+10s} \qquad K(s) = K_p$$

$$e_{ss}(t) = \lim_{s\to 0} \frac{s}{1+GK} \frac{1}{s^2}$$

$$= \lim_{s\to 0} \frac{1}{1+\frac{K_p}{s^2+10s}} \left(\frac{1}{s}\right)$$

$$= \lim_{s\to 0} \frac{s^2+10s}{s^2+10s+K_p} \frac{1}{s}$$

$$= \lim_{s\to 0} \frac{s+10}{s^2+10s+K_p}$$

$$\frac{10}{K_p} \leq \frac{0.4}{100}$$

$$K_p = 3333.33$$

$$e^{-\xi\pi/\sqrt{1-\xi^2}} \leq \frac{15}{100}$$

$$\xi = 0.517$$

$$2\xi\omega_n = 10 \qquad \omega_n = \sqrt{K_p}$$

$$2\xi\sqrt{K_p} = 10$$

$$K_p = 93.5321$$

$$\phi = \tan^{-1}(2\xi)$$

$$PM = 180 + \phi = 51.7$$

$$\omega_{gc} = 52.5 \text{ rad/s}$$

$$PM = 10^{\circ}$$

$$G_P(s) = \frac{1 + \lambda T_1 s}{\lambda(1 + T_1 s)}$$

$$PM = 51.1 - 10 = 45.7$$

$$\phi = \sin^{-1}\left(\frac{\lambda - 1}{\lambda + 1}\right)$$

$$\lambda = 184.65$$

$$\omega_{gc} = \frac{1}{T_1\sqrt{\lambda}} \qquad T_1 = \_\_\_\_\_\_$$

$$K_P = K_P\lambda$$

$$-10\log\lambda = \_\_\_\_\_\_$$

$$\omega_m = \_\_\_\_\_\_$$

$$\omega_m = \frac{1}{T_1\sqrt{\lambda}}$$

$$\boxed{T_1 = \_\_\_\_\_\_} \qquad \boxed{\lambda = }$$

$$\boxed{K_P = K_P\lambda}$$

Q $\quad G(s) = \dfrac{1}{s^2+10s}$

$$e_{ss}(t) = \lim_{s\to 0} \frac{s}{1+GK} \frac{1}{s^2}$$

$$= \frac{s+10}{s^2+10s+Kp}$$

$$\frac{10}{Kp} \leq \frac{0.4}{100}$$

$$Kp = 3333.33$$

$$e^{-\xi\pi/\sqrt{1-\xi^2}} = \frac{15}{100} = 0.15$$

$$\xi = 0.517$$

$$2\xi\omega_n = 10 \qquad \omega_n = \sqrt{Kp}$$

$$2\xi\sqrt{Kp} = 10$$

$$Kp = 93.5321$$

$$\phi = \xi \times 100 = 51.7^{\circ}$$

$$PM = \xi \times 100 = 51.7$$

Bode $\quad \omega_{gc}$

PM

$$\phi = PM - PM' = 57.1 - 10 = 47.1$$

$$\phi = \sin^{-1}\left(\frac{\lambda - 1}{\lambda + 1}\right)$$

$$\boxed{\lambda = \_\_\_\_}$$

$$\omega_{gc} = \frac{1}{G\sqrt{\lambda}}$$

$$G = \_\_\_\_$$

$$-10\log\lambda = \_\_\_\_$$

$$\omega_m = \_\_\_\_$$

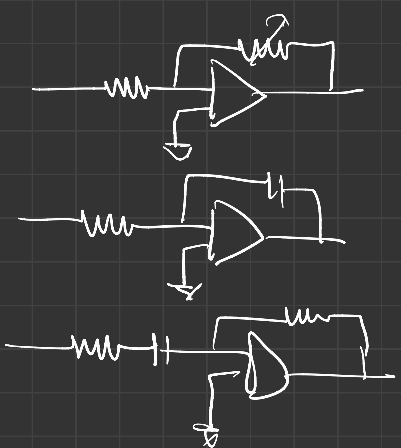
$$\omega_m = \frac{1}{G\sqrt{\lambda}}$$

$$\boxed{G =}$$

$$\boxed{\lambda =}$$

$$\boxed{K_P = K_e\lambda}$$

LAGr



$$G_P(s) = \frac{1 + \tau_2 s}{1 + \beta\tau_2 s}$$

PM = 51.7'

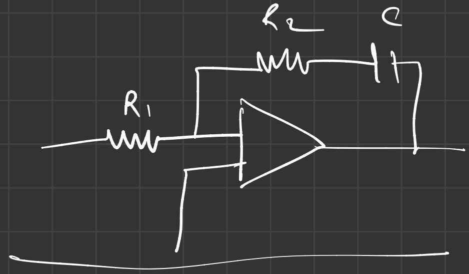
ω_{gc} = 2.08 rad/s

gain = 72.2 dB

$$20\log\beta = 72.2 \text{ dB}$$

$$\beta = 4073.80$$

$$\frac{1}{\tau_2} = \frac{\omega_{gc}}{}$$



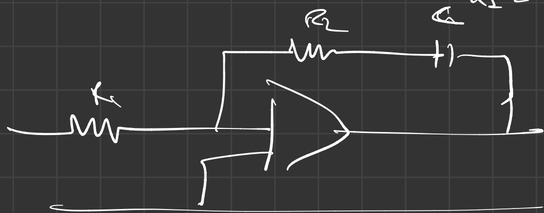
$$K_P = \frac{R_2}{R_1}$$

$$K_I = \frac{1}{R_1 C_1}$$

$$K_P = \frac{R_2}{R_1}$$

$$K_I = \frac{1}{R_1 C_1}$$

MOO



$$K_P = \frac{R_2}{R_1}$$

$$K_I = \frac{1}{R_1 C_1}$$