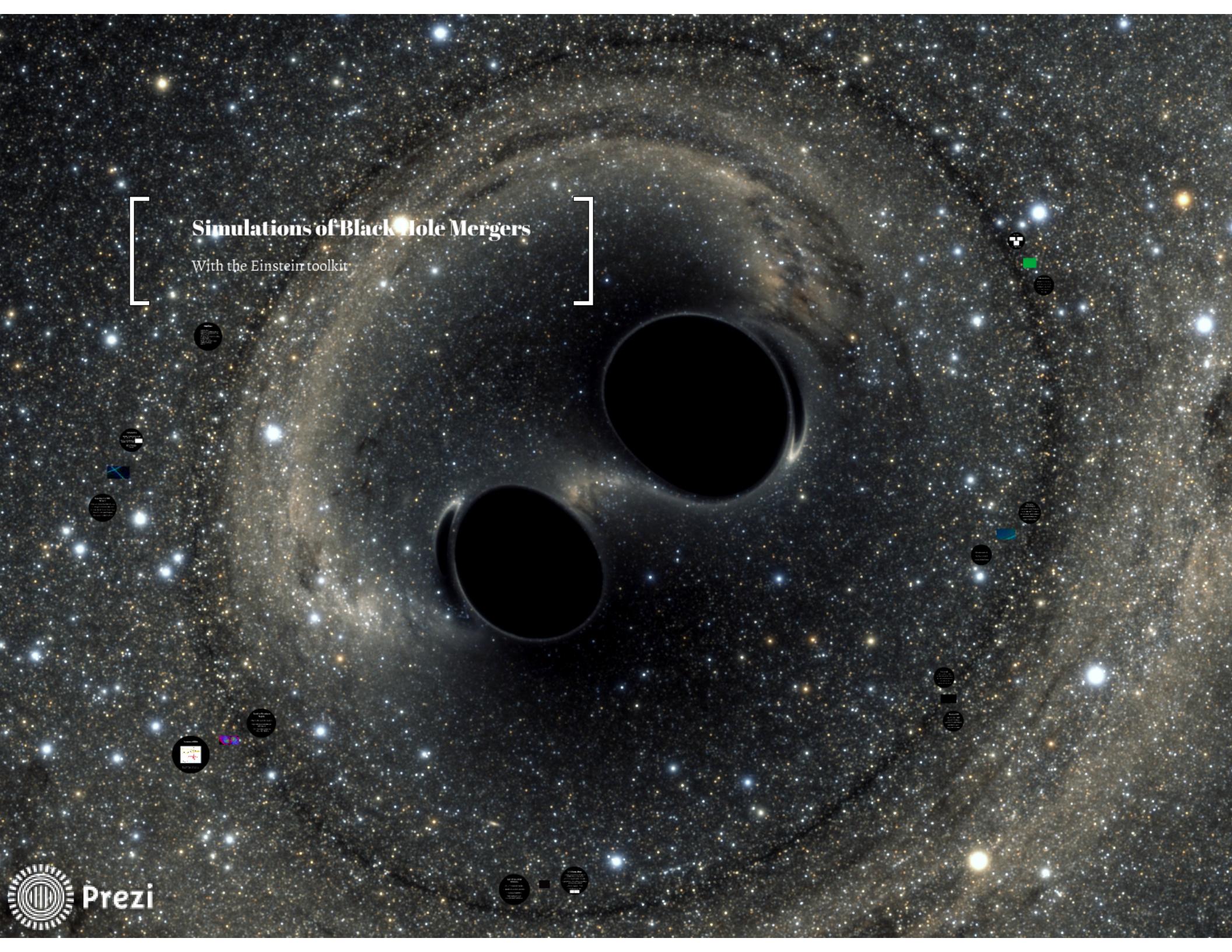


Simulations of Black Hole Mergers

With the Einstein toolkit



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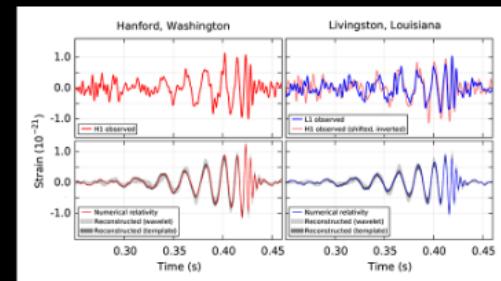


Outline

- Introduction
- Importance of BBH systems
- Role of Numerical Relativity
- 3+1 Formalism
- History of Numerical Relativity
- Cactus Code
- Einstein Toolkit
- Motivation for the Project
- Intitial Conditions
- Results

Introduction

- Simulation of binary black holes with zero spins on quasi-circular orbits
- The era of gravitational wave astronomy has just begun.
- LIGO, VIRGO, KAGRA, LIGO India, NANOGrav, LISA, etc.



Importance of BBH Mergers

- Brightest source of gravitational waves
 - All energy is radiated as GWs (10%)
- Estimate the distribution of black holes
 - Probe GR in strong field regime
 - Discover new insights in physics

Evolution of BBHs

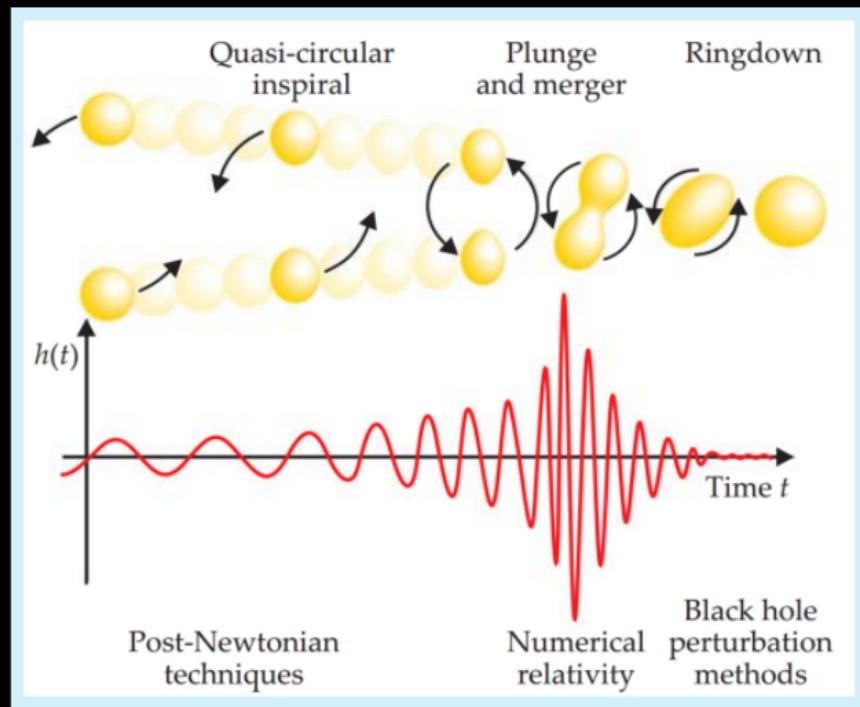
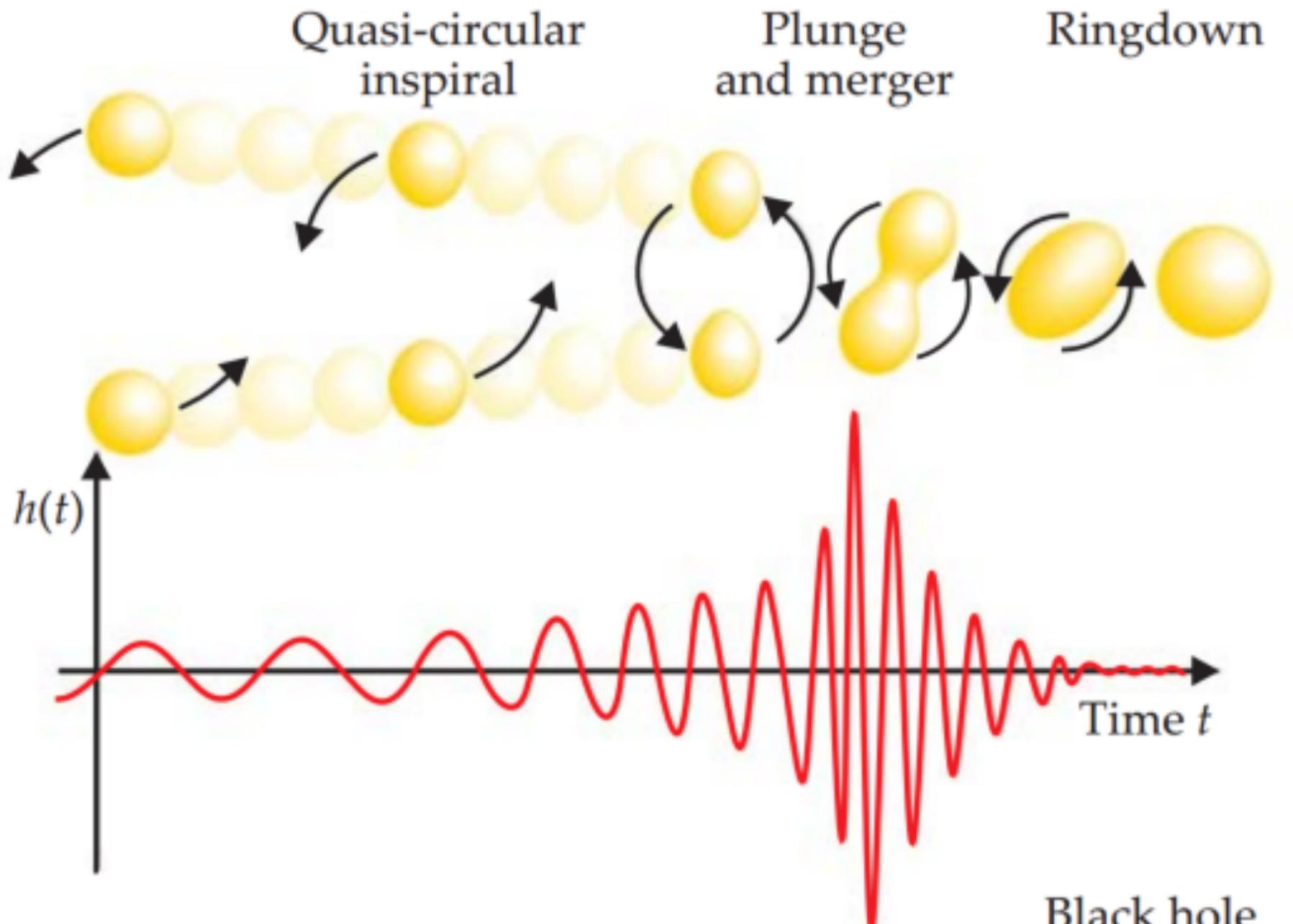


Image from: Binary black hole mergers - Baumgarte, Shapiro - physicstoday.org



Post-Newtonian
techniques

Numerical relativity Black hole perturbation methods

Need for Waveform Models

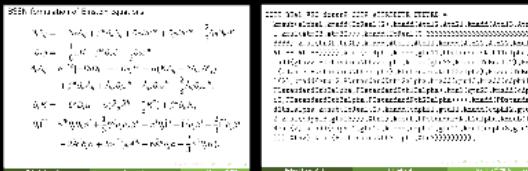
- Guess the correct source
- Estimating parameters of the source
- SNR transient search & matched-filtering

Role of Numerical Relativity

- Full GR close to merger
- Equations are too complex
 - No exact solutions
 - High performance numerical simulations

3+1 Formalism

- Proper choice of coordinates for stability and convergence
- Arnowitt-Deser-Misner formalism
- Spacetime is divided into spacelike 3-dimensional surfaces
- ADM is Ill-posed. BSSN
- C-code using Kranc



BSSN formulation of Einstein equations

$$\partial_t \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij} + \beta^k \partial_k \tilde{\gamma}_{ij} + \tilde{\gamma}_{ik} \partial_j \beta^k + \tilde{\gamma}_{jk} \partial_i \beta^k - \frac{2}{3} \tilde{\gamma}_{ij} \partial_k \beta^k,$$

$$\partial_t \phi = -\frac{1}{6} \alpha K + \beta^k \partial_k \phi + \frac{1}{6} \partial_k \beta^k,$$

$$\partial_t \tilde{A}_{ij} = e^{-4\phi} [-D_i D_j \alpha + \alpha R_{ij}]^{TF} + \alpha (K \tilde{A}_{ij} - 2 \tilde{A}_{ik} \tilde{A}^k{}_j)$$

$$+ \beta^k \partial_k \tilde{A}_{ij} + \tilde{A}_{ik} \partial_j \beta^k + \tilde{A}_{jk} \partial_i \beta^k - \frac{2}{3} \tilde{A}_{ij} \partial_k \beta^k,$$

$$\partial_t K = -D^i D_i \alpha + \alpha (\tilde{A}_{ij} \tilde{A}^{ij} + \frac{1}{3} K^2) + \beta^k \partial_k K,$$

$$\begin{aligned} \partial_t \tilde{\Gamma}^i = & \tilde{\gamma}^{jk} \partial_j \partial_k \beta^i + \frac{1}{3} \tilde{\gamma}^{ij} \partial_j \partial_k \beta^k + \beta^j \partial_j \tilde{\Gamma}^i - \tilde{\Gamma}^j \partial_j \beta^i + \frac{2}{3} \tilde{\Gamma}^i \partial_j \beta^j \\ & - 2 \tilde{A}^{ij} \partial_j \alpha + 2\alpha (\tilde{\Gamma}^i{}_{jk} \tilde{A}^{jk} + 6 \tilde{A}^{ij} \partial_j \phi - \frac{2}{3} \tilde{\gamma}^{ij} \partial_j K). \end{aligned}$$

```
CCTK_REAL_VEC dottrK CCTK_ATTRIBUTE_UNUSED =
kmsub(alphaL,kmadd(ToReal(2),kmadd(Atm12,Atm21,kmadd(Atm13,Atm3
1,kmul(Atm23,Atm32))),kmadd(ToReal(0.33333333333333333333333333333
3333),kmul(trKL,trKL),kmadd(Atm11,Atm11,kmadd(Atm22,Atm22,kmul(
Atm33,Atm33)))),kmul(em4phi,kmadd(gtu11,PDstandardNth11alpha,km
add(gtu22,PDstandardNth22alpha,kmadd(gtu33,kmadd(ToReal(2),kmu
1(cdphi3,PDstandardNth3alpha),PDstandardNth33alpha),kmadd(ToRea
1(2),kmadd(gtu12,PDstandardNth12alpha,kmadd(gtu13,kmadd(cdphi1,
PDstandardNth3alpha,PDstandardNth13alpha),kmul(gtu23,kmadd(cdph
i2,PDstandardNth3alpha,PDstandardNth23alpha)))),kmadd(PDstandar
dNth1alpha,kmsub(ToReal(2),kmadd(cdphi1,gtu11,kmadd(cdphi2,gtu1
2,kmul(cdphi3,gtu13))),Xtn1),kmsub(PDstandardNth2alpha,kmsub(To
Real(2),kmadd(cdphi1,gtu12,kmadd(cdphi2,gtu22,kmul(cdphi3,gtu23
))),Xtn2),kmul(PDstandardNth3alpha,Xtn3))))))));
```

History of NR

- 1964 - Hahn and Lindquist
- 1979 - Smarr and Eppley
- 1998 - Black Hole Grand Challenge
- 1999- BSSN Formulation
- 2005 - Pretorius & others



Cactus Code

- Open source modular framework
- NR , CFD, Cosmology, Quantum Gravity).
- Descendant of code developed at NCSA
 - Later developed at AEI
- Flesh and Thorns (Carpet, Driver)

einstein toolkit



- Fork of Cactus for NR
- Actively developed
- Comes with SimFactory

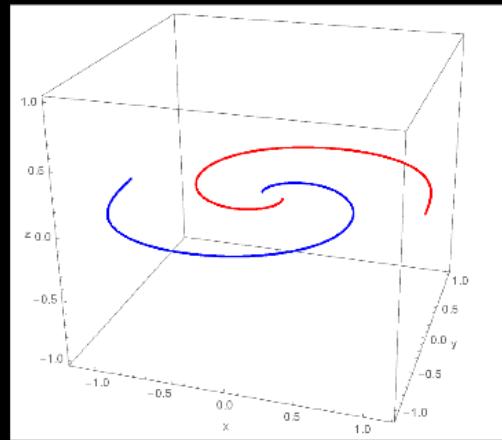
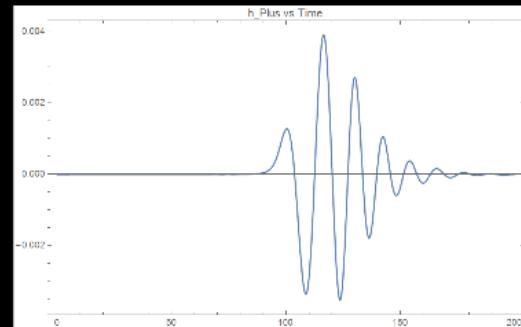
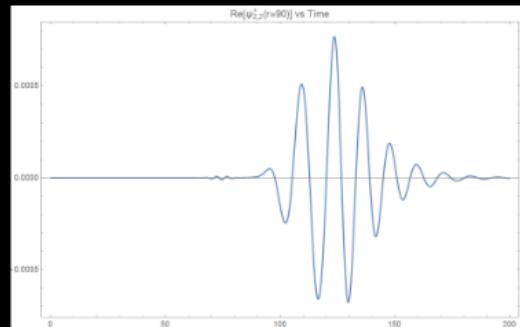
Motivation for this Project

- Open source simulation of GW150914
 - Gain familiarity with ET for research
 - Extend to eccentric systems with spins
 - Validate semi-analytic model
 - Catalogs using Blue Waters

Initial Conditions

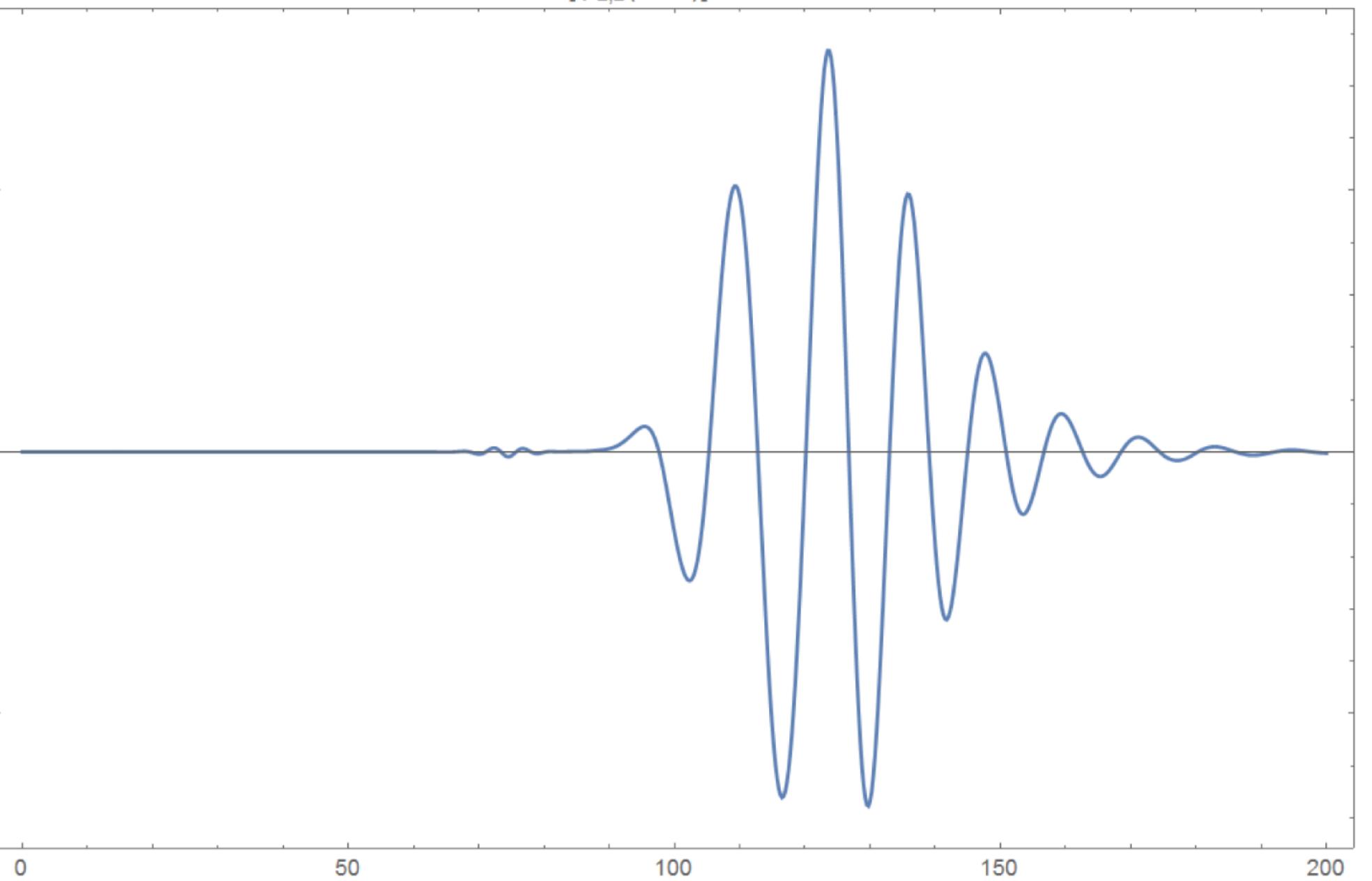
- Modified qco-mclachlan
- BHs located at $x = +/- 1.168$
- Initial mass ratio = $36/29$
- $m_1 = .45, r_1 = .25, r_2 = .20$
- $P = ? (.33)$

Results

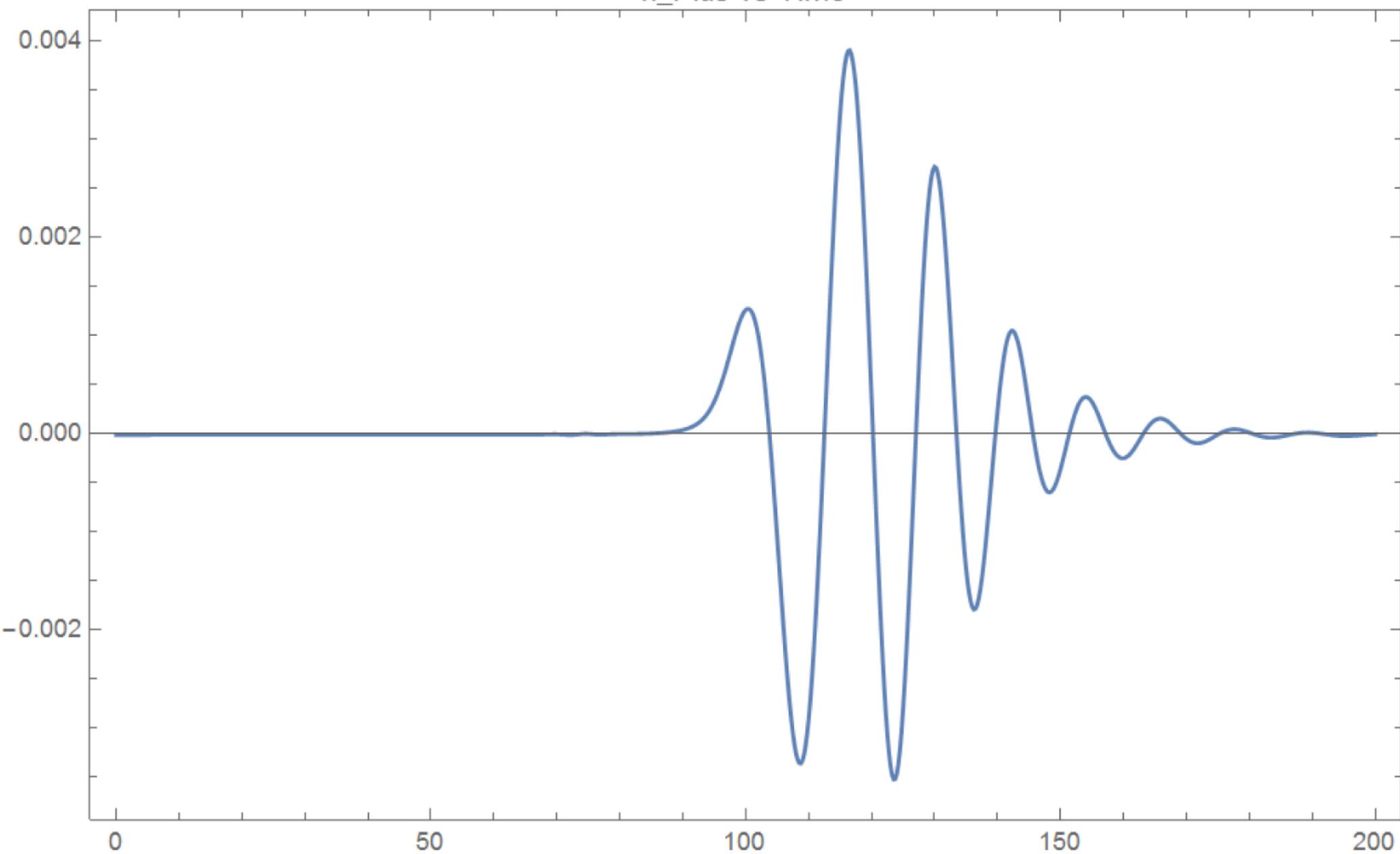


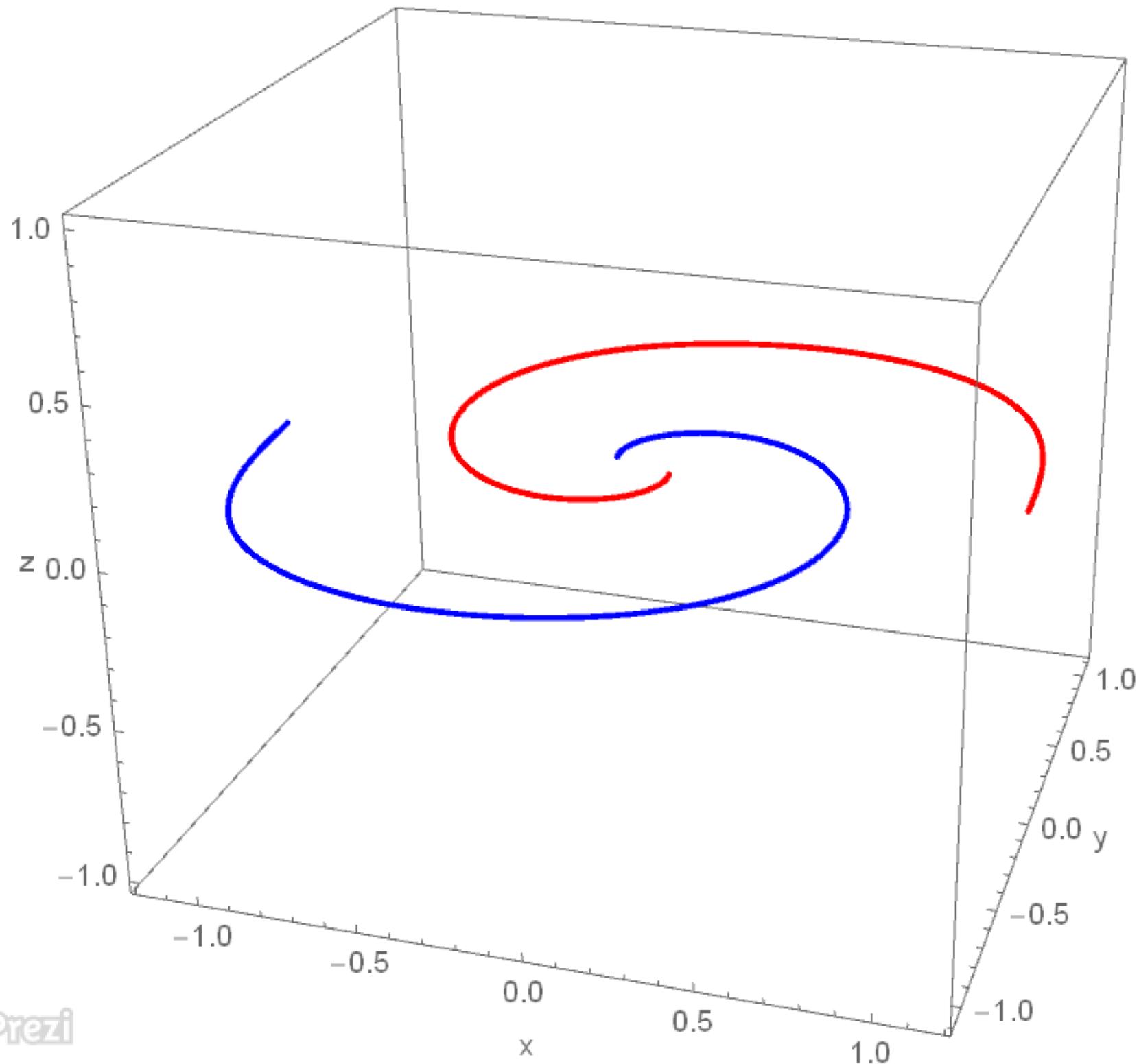
Analysis using SimulationTools by
Ian Hinder and Barry Wardell

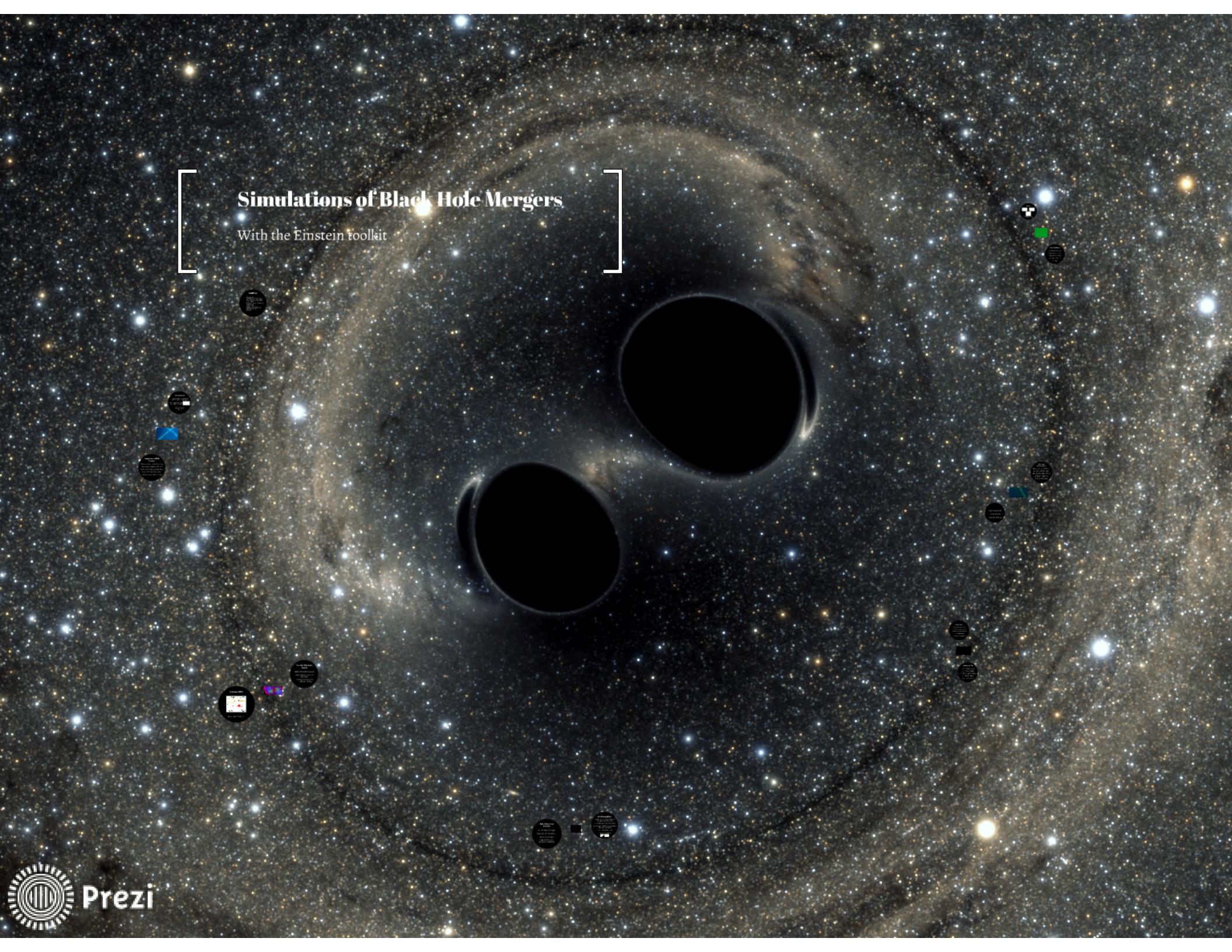
$\text{Re}[\psi_{2,2}^4(r=90)]$ vs Time



h_Plus vs Time







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