HW 9 - ASTR501

Daniel George - dgeorge5@illinois.edu

Q1) Reference: http://iopscience.iop.org/article/10.1086/648121/pdf

In[1]:=
```
Q=Quiet@ToExpression@StringReplace[TextRecognize[ <image> ,"Line"],{"X"→"*","10-"→"10^-","~"→""}];

Te = 10000K ;   Evaluate@Array[w,5,0] = {4,6,4,4,2};   ij = Flatten[Table[{i,j},{i,0,4},{j,i+1,4}],1];
Evaluate[λ@@@ij] = Q[[;;10]];   Evaluate[A@@@ij] = Q[[-10;;]]; A[0,2] = 1.6×10^-4;
Evaluate[ϒ@@@ij] = Q[[13;;22]]*{6/10,4/10,4/6,2/6,1,1,1,1,1,1};
ϒ[j_,i_]/;j>i = ϒ[i,j] Exp[-h c/(λ[i,j] 1μm k Te)];   ϒ[i_,i_] = 0;
c[i_,j_]=h^2/(2π 1m_e)^1.5/Sqrt[k Te] x/cm^3 ϒ[j,i]/w@i;  (* b) Collisional (de)excitation rates *)
f@x_ = First@Solve[Append[Total@Array[n,5,0]==1]@UnitConvert@Table[  (* a) Level population equations *)
  Sum[n(j)c(j,i),{j,0,4}]+Sum[n(j)A(i,j),{j,i+1,4}]==Sum[n(i)c(i,j),{j,0,4}]+Sum[n(i)A(j,i),{j,0,i-1}],{i,0,4}]/.Quantity[z_,_]:>z,Array[n,5,0]];
```

c) Stimulated emission can be ignored because there are few atoms in higher states which radiatively decay immediately.

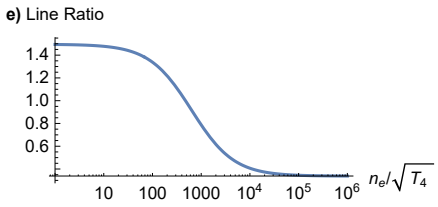
In[7]:=
```
Grid[Prepend[Prepend[f[#][[;;,2]],#]&/@{1,10^4},Prepend["d) n_e"]@Array[n,5,0]],Frame→All]
```

Out[7]=

d) n_e	n[0]	n[1]	n[2]	n[3]	n[4]
1	0.999988	0.0000104704	1.57608×10^{-6}	7.56812×10^{-11}	4.91647×10^{-11}
10 000	0.969929	0.0193661	0.0106992	4.03822×10^{-6}	1.74191×10^{-6}

In[8]:=
```
r@ne_ := Divide@@(n[#2] A@##/(λ@##)&@@@{{0,1},{0,2}}/.f@ne);
{LogLinearPlot[r@ne,{ne,1,10^6},PlotRange→All,ImageSize→220,AspectRatio→.48,
  AxesLabel→{"n_e/√T_4","e) Line Ratio"}],#->r@#&/@{1,10^4}//Column}//GraphicsRow
```

Out[8]=



1 → 1.49362
10 000 → 0.406956

Q2 a)

In[9]:=
```
RS = 9.77×10^18 (10^48.75/10^49)^(1/3) 1 cm // UnitConvert
```

Out[9]= 8.0642×10^{16} m

Q2 b) (*Same order as in the question*)

In[10]:=
```
UnitConvert[{1.22×10^3 1 yr, 5 Myr, 2.39×10^5 1 yr (10^48.75/10^49)^(1/3)},"Years"]
```

Out[10]= {1220. yr, 5 000 000 yr, 197 272. yr}

Q2 c) *Dust is important.* Reference: http://www.scielo.org.mx/pdf/rmaa/v51n2/v51n2a10.pdf

In[11]:=
```
nd = NSolve[{(100/cm^3 1.67×10^-24 g + nd md).01 == nd md, 3 g/cm^3 == md/(4/3 π 0.1 μm^3)},{nd,md}][[1,1,2]];
FindRoot[UnitConvert@((R0/RS)^3 == Exp[-(nd π 0.1 μm^2 R0)]) /. Quantity[x_,_]:>x,{R0,10^18}][[1,2]] m
```

Out[12]= 7.27978×10^{16} m