HW 8 - ASTR501

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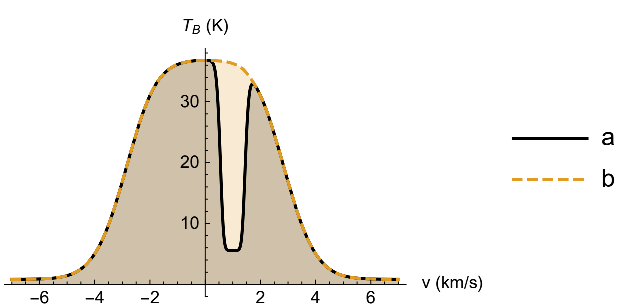
```
Unprotect@Quantity; Quantity[0. | 0, _] = 0; Protect@Quantity;
SetOptions[Plot, {Filling → Bottom, ImageSize → 250, AxesLabel → {"v (km/s)", "T_B (K)"}}];
CO[nH_] := 2 nH 6.6 × 10^-5 /cm^3 ; ν0 = 115.272 GHz ;
B0 = Solve[h ν0 == k B J (J + 1) / 2 - 0 /. J → 1, B][[1, 1, 2]];
g[j_] := 2 j + 1;
f[T_, j_] := g[j] Exp[- B0 / T j (j + 1) / 2] / Sum[g[i] Exp[- B0 i (i + 1) / 2 / T], {i, 0, 8}];
n@i_ := f[T, i - 1] CO[nH];   σν[v_] := v / c ν0;
ϕ = Exp[- (ν - (1 + vz / c) ν0) ^2 / (2 σ^2)] / (Sqrt[2 π] σ);
A21 = 7.166 × 10^-8 Hz ;
{B21, B12} =
  NSolve[{A21 == 2 h ν0^3 / c ^2 b21, g@0 / g@1 == b21 / b12}, {b12, b21}][[1, ;; , 2]];
jν[nH_, vz_, T_, σ_] = n@2 A21 h ν / (4 π) ϕ;
αν[nH_, vz_, T_, σ_] = h ν / (4 π) (n@1 B12 - n@2 B21) ϕ;
TB[d_, nH_, vz_, Δv_, T_, nH2_: 0, vz2_: 1, Δv2_: 1, T2_: 1] := ParametricNDSolveValue[
    UnitConvert@{Iν'@z / 1.0 pc == - (αν[nH, vz, T, σν@Δv] + αν[nH2, vz2, T2, σν@Δv2]) Iν@z +
        (jν[nH, vz, T, σν@Δv] + jν[nH2, vz2, T2, σν@Δv2]),
      Iν[-d] == 2 h ν^3 / c ^2 / (Exp[h ν / (k 2.725 K)] - 1)} /. Quantity[x_, _] ⧴ x,
    QuantityMagnitude@UnitConvert[c ^2 / (2 k) / ν^2] Iν@0, {z, -d, 0}, ν,
    MaxStepFraction → 0.0001]@QuantityMagnitude[(1 + # 1 km/s / c) ν0, 1 Hz] &
```

a) and b)

In[27]:=
```
Plot[Evaluate@
  {TB[8, 500 UnitBox[(z + 5) / 6], 0, 1.5 km/s , 40 K , 50 UnitBox[z + 1], 1 km/s , .2 km/s , 8 K ]@x,
   TB[8, 500 UnitBox[(z + 3) / 6], 0, 1.5 km/s , 40 K , 50 UnitBox[z + 7], 1 km/s , .2 km/s , 8 K ]@x},
 {x, -7, 7}, PlotStyle → {Black, Dashed}, PlotLegends → {"a", "b"}]
```

Out[27]=



c)

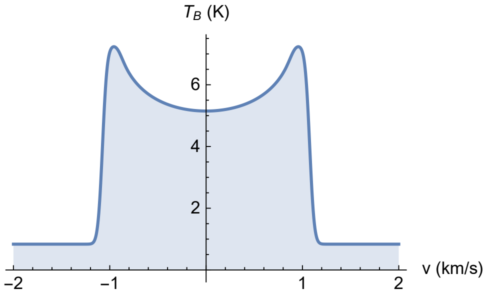
In[28]:=
```
σv = Solve[3 × 1 / 2 mass 1 CO  vx^2 == 3 / 2 k 10 K , vx][[2, 1, 2]]
```

Out[28]= 54.483 m/s

d)

```
In[29]:= Plot[Evaluate@TB[1, 50, Sin[2 π z] 1. km/s, σv, 10 K]@x, {x, -2, 2}]
```

Out[29]=



e)

No. There can also be dips due to absorption of cool gas or peculiar motion of low density gas.

f)

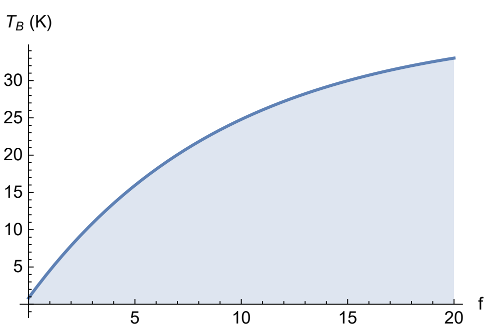
```
In[30]:= c^2/(2 k)/ν^2 2 h ν^3/c^2/(Exp[h ν/(k 2.725 K)] - 1) /. ν -> 115 GHz // UnitConvert

Out[30]= 0.838913 K
```

g)

```
In[31]:= Plot[TB[1, x 50, 0, 1 km/s, 40 K]@0, {x, 0, 20}, AxesLabel → {"f", "T_B (K)"}]
```

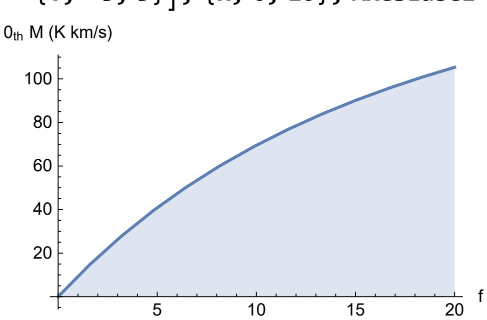
Out[31]=



h)

```
In[32]:= Plot[NIntegrate[Evaluate@(TB[1, x 50, 0, 1 km/s, 40 K]@v - TB[1, 50, 0, 1 km/s, 40 K]@5),
           {v, -5, 5}], {x, 0, 20}, AxesLabel → {"f", "0_th M (K km/s)"}, PlotPoints → 4, MaxRecursion → 2]
```

Out[32]=



The curve of growth is initially linearly proportional to the column density but then saturates due to high optical depth.