HW 6 - ASTR501

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Q1

a)

Using relativistic Doppler shift equation

In[165]:=
```
sν = {νSource1, νSource2} → (νObs Sqrt[1 + β] / Sqrt[1 - β] /. β → v Cos[θ] / c /.
    Solve[γ == 1/Sqrt[1 - (1/c^2) v^2], v]) // Thread
```

Out[165]=
$$\left\{\nu\text{Source1} \to \frac{\nu\text{Obs}\sqrt{1-\frac{\sqrt{-1+\gamma^2}\,\text{Cos}[\theta]}{\gamma}}}{\sqrt{1+\frac{\sqrt{-1+\gamma^2}\,\text{Cos}[\theta]}{\gamma}}},\ \nu\text{Source2} \to \frac{\nu\text{Obs}\sqrt{1+\frac{\sqrt{-1+\gamma^2}\,\text{Cos}[\theta]}{\gamma}}}{\sqrt{1-\frac{\sqrt{-1+\gamma^2}\,\text{Cos}[\theta]}{\gamma}}}\right\}$$

b)

In[166]:=
```
eqj = jSource1 / jSource2 == (νSource1 / νSource2)^-s /. sν // PowerExpand
```

Out[166]=
$$\frac{\text{jSource1}}{\text{jSource2}} == \left(1-\frac{\sqrt{-1+\gamma^2}\,\text{Cos}[\theta]}{\gamma}\right)^{-s}\left(1+\frac{\sqrt{-1+\gamma^2}\,\text{Cos}[\theta]}{\gamma}\right)^{s}$$

c)

In[167]:=
```
sj =
 Solve[{jObs1 / νObs^2 == jSource1 / νSource1^2, jObs2 / νObs^2 == jSource2 / νSource2^2, eqj},
   {jObs1, jSource1, jSource2}][[1, 1]] /. sν
```

Out[167]= $\text{jObs1} \to \text{jObs2} \left(1 - \frac{\sqrt{-1+\gamma^2}\,\text{Cos}[\theta]}{\gamma}\right)^{-2-s} \left(1 + \frac{\sqrt{-1+\gamma^2}\,\text{Cos}[\theta]}{\gamma}\right)^{2+s}$

d)

Assuming intensity is proportional to emissivity at the point of emission.

In[168]:=
```
sI = Solve[{IObs1 / νObs^3 == ISource1 / νSource1^3, IObs2 / νObs^3 == ISource2 / νSource2^3,
      eqj} /. {ISource1 → k jSource1, ISource2 → k jSource2},
   {IObs1, jSource1, jSource2}][[1, 1]] /. sν
```

Out[168]= $\text{IObs1} \to \text{IObs2} \left(1 - \frac{\sqrt{-1+\gamma^2}\,\text{Cos}[\theta]}{\gamma}\right)^{-3-s} \left(1 + \frac{\sqrt{-1+\gamma^2}\,\text{Cos}[\theta]}{\gamma}\right)^{3+s}$

In[169]:=
```
sI /. {γ → 10., s → .7, θ -> 20°}
```

Out[169]= IObs1 → 283 458. IObs2

Therefore the ratio is 283458.

c)

In[63]:=
```
Solve[(p - 1) / 2 == s, p] /. s → 1
```

Out[63]= {{p → 3}}

d)

Let n_e^{NT} = neNT, integrating to find the constant N

In[132]:=
```
sN = Solve[neNT == Integrate[N γ^-p, {γ, 1, ∞}], N][[1, 1]]
```

Out[132]= N → ConditionalExpression[-neNT + neNT p, Re[p] > 1]

Assuming thermal energy is given by dE = (γ-1) m c^2 dn

In[173]:=
```
Integrate[(γ - 1) m c^2 N γ^-p, {γ, 1, ∞}] /. sN // Simplify
```

Out[173]= $\text{ConditionalExpression}\left[\frac{c^2\,m\,\text{neNT}}{-2+p}, \text{Re}[p] > 2\right]$

Therefore the thermal energy is $\frac{c^2\,m\,\text{neNT}}{-2+p}$.

e)

Finding energy density

In[178]:=
```
su = u -> B^2/(2. μ0) /. B -> 1/10^6 G // UnitConvert
```

Out[178]= $u \to 3.97887 \times 10^{-15}\ \mathrm{kg}/(\mathrm{m\,s^2})$

Substituting in equation in part d) and solving for n_e^{NT} = neNT

In[179]:=
```
Solve[u == (c^2 1 me neNT)/(-2 + p) /. {su, p → 3}, neNT]
```

Out[179]= {{neNT → 0.0485993 per meter³}}

f)

Finding γ using synchrotron frequency equation (in SI units)

In[180]:=
```
sγ = Solve[1 GHz == 3/2 γ^2 1 e B/(1 me) /. B -> 1/10^6 G, γ][[2]]
```

Out[180]= {γ → 6156.639}

Substituting in the cooling time equation

In[160]:=
```
6.5 × 10^7 yr (B/(5/10^6 G))^-2 (γ/10^4)^-1 /. B -> 1/10^6 G /. sγ
```

Out[160]= 2.63943×10^{9} yr

Q2

Equation 8.20

In[231]:=
```
eq1 = dt/dw == 1.1/10^5 s^2 == -4 Pi e^2/(c m w^3) Integrate[n[s], {s, 0, d}]
```

Out[231]= $$\frac{dt}{dw} == 0.000011\, s^2 == -\frac{4\, e^2 \pi \int_0^d n[s]\, ds}{c\, m\, w^3}$$

Derivative of Equation 8.31

In[239]:=
```
eq2 = dθ / dw == 1.9/10^4 s == D[2 Pi e^3 / (m^2 c^2 w^2) Bparallel Integrate[n[s], {s, 0, d}], w]
```

Out[239]= $\frac{d\theta}{dw} == 0.00019\ s == -\frac{4\ \text{Bparallel}\ e^3 \pi \int_0^d n[s]\, ds}{c^2 m^2 w^3}$

Dividing this by the first equation and solving for $B_{||}$

In[240]:=
```
Solve[eq2[[2]] / eq1[[2]] == eq2[[3]] / eq1[[3]] // PowerExpand, Bparallel]
```

Out[240]= $\left\{\left\{\text{Bparallel} \to \frac{c\, m\, (17.2727\ \text{per second})}{e}\right\}\right\}$

Plugging in the values we get, in CGS units, $B_{||} = 10^{-6}$

Q3

a)

Here $\dot{E}$ is proportional to power, which in turn is proportional to γ^2. Therefore a = 2, since E is proportional to γ

b)

Substituting the values of $\dot{E}$ in the given equation and setting the first term to zero.

$$\frac{\partial(-b\, E^2\, N(E))}{\partial E} = q(E)$$

This implies N(E) is proportional to $\frac{\int q(E)}{E^2}$

c)

In[241]:=
```
Integrate[A E1^-α, {E1, E, ∞}] / E^2
```

Out[241]= ConditionalExpression$\left[\frac{A\, E^{-1-\alpha}}{-1+\alpha}, \text{Re}[\alpha] > 1\ \&\&\ ((\text{Im}[E] == 0\ \&\&\ \text{Re}[E] > 0)\ ||\ E \notin \text{Reals})\right]$

Therefore the power index is $\alpha + 1$

d)

The newest electron seems to be from the edge of the jet (hot spots). High energy electrons cool faster initially as the jet moves outwards but are later accelerated by shocks in the hot spots causing the spectrum to flatten.