# HW 1 - ASTR404

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# Q3) Binary star data

## Importing data in solar units

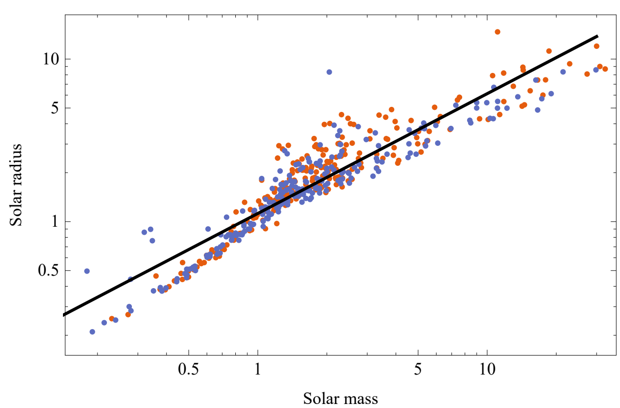
In[1]:=
```
tSun = QuantityMagnitude@ Sun (star) ["EffectiveTemperature"];
SetDirectory["C:\\Users\\dan7g\\Google Drive\\Acads\\ASTR404\\"];
dataS =
  SemanticImport["eker_2014_simple.dat"][All, {"T1" → (# / tSun &), "T2" → (# / tSun &)}];
```

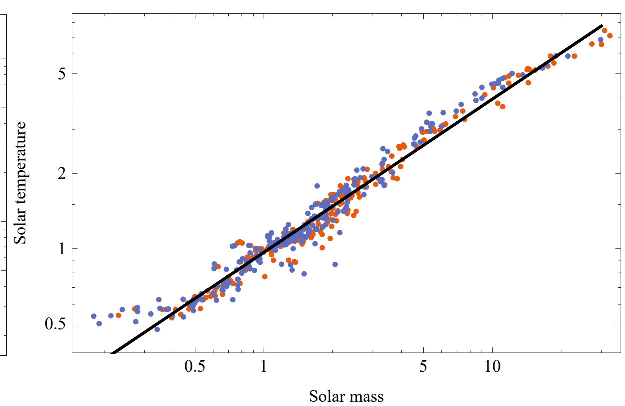
## a) & b) Radius and temperature vs mass

### Scatter plots and best fits

In[3]:=
```
SetOptions[ListLogLogPlot, PlotStyle → PointSize[.01],
  PlotTheme → "Scientific", Frame → True, PlotRange → All];
fit[s_] := LinearModelFit[Sort[Join @@ (Log10@Values@Normal@dataS[All, #] & /@
       {{"M1", s <> "1"}, {"M2", s <> "2"}})], {1, x}, x];
Show[ListLogLogPlot[Values@Normal@dataS[All, #] & /@ {{"M1", #1 <> "1"}, {"M2", #1 <> "2"}},
     FrameLabel → {"Solar mass", #2}, ImageSize → 320],
    LogLogPlot[10^fit[#1][Log10[M]], {M, 0, 30}, PlotStyle → Black]] & @@@
  {{"R", "Solar radius"}, {"T", "Solar temperature"}} // Row
```

Out[5]=




**We can see that both the radius and the temperature increase monotonically with increase in mass of the stars. Moreover, they seem to fit a power law relationship since a linear fit to the log-log plot matches closely.**

**There is a larger spread in the radius when mass increases whereas temperature appears to converge to the fit with increasing mass.**

### Fitting functions in solar units

In[6]:=
```
# == 10^fit[#][Log10@M] & /@ {"R", "T"} // Column
```

Out[6]=
$R == 1.1222945049359\, M^{0.73689311038015}$
$T == 0.96705300226301\, M^{0.6120798178381}$

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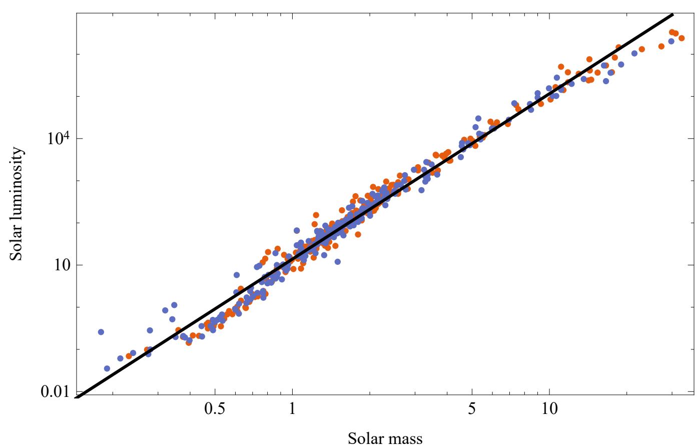
## c) Luminosity vs mass

### Scatter plot along with best fit

In[7]:=
```
dataL = Transpose[{Normal@dataS[All, "M" <> #],
      Normal@dataS[#^4 &, "T" <> #] * Normal@dataS[4 Pi #^2 &, "R" <> #]}] & /@ {"1", "2"};
fitL = LinearModelFit[Join @@ Log10@dataL, {1, x}, x];
Show[ListLogLogPlot[dataL, FrameLabel → {"Solar mass", "Solar luminosity"}],
 LogLogPlot[10^fitL[Log10@M], {M, 0, 30}, PlotStyle → Black]]
```

Out[9]=



**We can see that the luminosity also increases monotonically with increase in mass of the stars. Moreover, the luminosity appears to fit a power law relationship with mass since the linear fit to the log-log plot matches closely. The spread of the points appear to be higher for low and high masses.**

### Fitting function in solar units

In[10]:=
```
"L" == 10^fitL[Log10[M]]
```

Out[10]= $L == 13.842846029206\, M^{3.9221050549238}$

# Q2) Properties of Deneb

### Defining parameters of Deneb

In[11]:=
```
Rs = Quantity[7.72 × 10^10, "Meter"];
Ts = Quantity[8525., "Kelvin"];
ds = Quantity[440., "Parsecs"];
```

## a) Luminosity

$L = 4\pi R^2 \sigma T^4$

### SI units

In[14]:=
```
Ls = UnitConvert[4 Pi Rs^2 σ Ts^4, "SI"]
```

Out[14]= $2.2430201949436 \times 10^{31}$ W

### Solar units

In[15]:=
```
UnitConvert[Ls, "SolarLuminosity"]
```

Out[15]= 58 610.873655353 $L_{\odot}$

## b) Parallax and distance modulus

### Parallax

Parallax = Diameter (in AU) / Distance (in Parsecs)

In[16]:=
```
Quantity[QuantityMagnitude[2 Rs, "AU"] / QuantityMagnitude[ds, "Parsecs"], "ArcSeconds"]
```

Out[16]= 0.0023456823901778"

### Distance modulus

$\mu = 5 \log_{10}(d \text{ in parsecs}) - 5$

In[17]:=
```
5 Log10[QuantityMagnitude[ds, "Parsecs"]] - 5
```

Out[17]= 8.2172633824309

## c) Radiant flux at star’s surface

**Flux is luminosity divided by surface area of star:**

In[18]:=
```
UnitConvert[Ls / (4 Pi Rs^2), "SI"]
```

Out[18]= $2.9949440880982 \times 10^{8}\ \mathrm{N/(m\,s)}$

## d) Radiant flux at earth’s surface

**Luminosity divided by area of sphere with radius equal to distance to earth:**

In[19]:=
```
UnitConvert[Ls / (4 Pi ds^2), "SI"]
```

Out[19]= $9.6831548346327 \times 10^{-9}\ \mathrm{N/(m\,s)}$

## e) Peak wavelength

### Looking up Wien’s displacement law

In[20]:=
```
eqW = FormulaData[{"WienDisplacementLaw", "Wavelength"}]
```

Out[20]= $$\lambda_{\mathrm{max}} == \frac{1\,b}{T}$$

### Substituting value of temperature

In[21]:=
```
eqW[[1]] == UnitConvert[eqW[[2]] /. T → Ts, "meters"]
```

Out[21]= $\lambda_{\mathrm{max}} == 3.3991470967742 \times 10^{-7}\ \mathrm{m}$

# Q1) Planck’s law limits

## a) Rayleigh-Jeans law

### Defining Planck function

In[22]:=
```
Bv = 2 h c^2 / (Exp[h c / k / T / λ] - 1) / λ^5
```

Out[22]= $$\frac{2\,c^2\,h}{\left(-1 + e^{\frac{c\,h}{k\,T\,\lambda}}\right)\lambda^5}$$

### Taylor expansion for $\lambda^{-1}$

In[23]:=
```
TE1 = Normal@Series[Bv /. λ → 1 / x, {x, 0, 10}] /. x → 1 / λ
```

Out[23]= $$\frac{c^7\,h^6}{15120\,k^5\,T^5\,\lambda^{10}} - \frac{c^5\,h^4}{360\,k^3\,T^3\,\lambda^8} + \frac{c^3\,h^2}{6\,k\,T\,\lambda^6} - \frac{c^2\,h}{\lambda^5} + \frac{2\,c\,k\,T}{\lambda^4}$$

## Substituting $\lambda$ = n*h*c/(k*T)

In[24]:= `TE1 /. λ → n * h * c / (k * T)`

Out[24]= $$\frac{k^5 T^5}{15120\, c^3 h^4 n^{10}} - \frac{k^5 T^5}{360\, c^3 h^4 n^8} + \frac{k^5 T^5}{6\, c^3 h^4 n^6} - \frac{k^5 T^5}{c^3 h^4 n^5} + \frac{2 k^5 T^5}{c^3 h^4 n^4}$$

**Therefore for n >> 1, we can approximate the equation using just the last term which corresponds to the following term in the Taylor series:**

$$\frac{2 c k T}{\lambda^4}$$

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# b) Wien's function

## Substituting $\lambda$ = n*h*c/(k*T)

In[25]:= `Bv /. λ → n * h * c / (k * T)`

Out[25]= $$\frac{2 k^5 T^5}{c^3 \left(-1 + e^{\frac{1}{n}}\right) h^4 n^5}$$

## Taking limit

**In the limit n << 1, $e^{1/n}$ - 1 ~ $e^{1/n}$. Therefore we can ignore the 1 in the denominator. This gives:**

$$\frac{2 c^2 e^{-\frac{c h}{k T \lambda}} h}{\lambda^5}$$

**Therefore the constants are:**

$$a = 2 c^2 h$$

$$b = \frac{c h}{k}$$