PHYS540 HW6
Jiayin Dong

Problem 1

$\mu_{\log P} = 5$

$\sigma_{\log P} = 2.3$

Since $dN/d\log P$ is a normal distribution, $\pm 2\sigma$ is a good cut-off range.

$-0.4 \leq \log P \leq 9.6 \Rightarrow 95\%$ of confidence.

a) Reasonable minimium period $\log P = 10^{0.4} \approx 2.5$ days

b) Max period $\log P = 11.9$ $P = 10^{9.6} = 3.98 \times 10^{9}$ days

c) $w^2 a = \frac{GM}{a^2} \Rightarrow w^2 = \frac{GM}{a^3} = \frac{4\pi^2}{T^2} \Rightarrow T^2 = \frac{4\pi^2}{GM} a^3$

$T = \sqrt{\frac{4\pi^2}{GM} a^3} = 1$ year for $a = 1$ AU
$= 365$ days

For $a < 1$ AU, $T < 1$ yr $= 365$ days

Find those $\log T < \log(365) = 2.562$. The fraction then is 14.46%

$\log P$

d) At $a = 0.4$ AU, $T = \sqrt{0.4^3}\, T_\odot = \sqrt{0.4^3} \times 365 = 92.34$ days $\Rightarrow \log T = 1.965$

At $a = 40$ AU, $T = \sqrt{40^3}\, T_\odot = \sqrt{40^3} \times 365 \approx 92338.5$ days $\Rightarrow \log T = 4.965$

$\log P$

The fraction between 1.965 and 4.965 is 40.04%.

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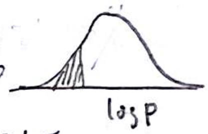

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The fraction between 1.965 and 4.965 is 40.04%.

Problem 2

a. $d = d_9 \text{Gpc}$

FRB energy $\propto$ Jy

Energy $\propto 4\pi d^2$ Jy

$\propto 10 \cdot (\text{Gpc})^2 \cdot$ Jy

$\propto 9.521 \times 10^{25}$ J $= 9.521 \times 10^{32}$ ergs

b. $w_p^2 = 4\pi n_e e^2 / m_e$ $\quad n_e = 0.03$, $e = 4.8 \times 10^{-10}$ esu, $m_e = 9.1 \times 10^{-28}$ g

$$= \frac{4\pi \times 0.03 \times (4.8 \times 10^{-10})^2}{9.1 \times 10^{-28}}$$

$= 9.54 \times 10^7 \ s^{-2}$

$w_p \approx 9769.8 \sim 10^4 s^{-1}$

c. $w^2 = w_p^2 + c^2k^2 \Rightarrow w = (w_p^2 + c^2k^2)^{\frac{1}{2}}$

$V_g = \frac{dw}{dk} = \frac{1}{2}(w_p^2 + c^2k^2)^{-\frac{1}{2}} \cdot 2c^2k$

$$= \frac{c^2k}{(w_p^2 + c^2k^2)^{\frac{1}{2}}} = \frac{c^2k/ck}{(\frac{w_p^2}{c^2k^2} + 1)^{\frac{1}{2}}} = \frac{c}{\sqrt{1 + (\frac{w_p}{ck})^2}} = c\left(1 - \frac{1}{2}\left(\frac{w_p}{ck}\right)^2 + \frac{3}{8}\left(\frac{w_p}{ck}\right)^4 - \cdots\right)$$

$$= c - \frac{c}{2}\frac{w_p^2}{c^2k^2}$$

$$= c - \frac{w_p^2}{2ck^2}$$

positive > 0

$< c$

Without plasma,

$w^2 = c^2k^2 \quad V_g = \frac{dw}{dk} = c$

The presence of plasma lower the group velocity.

d. $\Delta t = \int \frac{ds}{V_g'} - \frac{ds}{V_g}$ $\quad$ (V_g' in vacuum, V_g in plasma)

$$= \int ds \left(\frac{1}{c} - \frac{1}{c - \frac{w_p^2}{2ck}}\right)$$

Substitude $a = w_p/ck$

$$\Delta t = \int ds \left(\frac{1}{c} - \frac{1}{c - \frac{c}{2}a^2}\right)$$

$$= \int ds \left(\frac{V_g' - V_g}{V_g V_g'}\right)$$

$$= \int ds \cdot \frac{\frac{c}{2}a^2}{(c - \frac{c}{2}a^2) \cdot c}$$

$$= \int ds \cdot \frac{a^2}{2c(1 - \frac{1}{2}a^2)}$$

$$= \int ds \cdot \frac{1}{c(\frac{2}{a^2} - 1)}$$

$$= \int ds \cdot \frac{a^2}{2c}$$

$$= \int ds \cdot \left(\frac{w_p}{ck}\right)^2 \cdot \frac{1}{2c}$$

$$= \int ds \ \frac{w_p^2}{w^2} \cdot \frac{1}{2c}$$

$$= (2\pi e^2 / m_e w^2 c) \int ds \cdot n_e(s)$$

Problem 2 ③

2d.

$DM = \int ds\, n_e \qquad n_e = 0.03 e^{-z/1000pc}$

$= \int \frac{dz}{\cos\theta} \cdot 0.03 e^{-z/1000} \qquad z = \cos\theta\, s$

$ds = \frac{dz}{\cos\theta}$

$= \frac{-30}{\cos\theta} \cdot e^{-z/1000} \Big|_0^{\infty}$

$= \frac{30}{\cos\theta}$. I use $z=0$ to $z=\infty$ for the integral. This is not reasonable for our galaxy.

To maximize DM, let $\cos\theta$ be small as possible $(0 \leq \theta \leq 45°)$. $\Rightarrow \cos\theta = \cos 45° = \frac{\sqrt{2}}{2}$.

$DM = \frac{30}{\sqrt{2}} = 21.21$.

e. From the book, Cosmological Effects of Scattering in the Intergalactic Medium Page 501, $\langle \rho_e \rangle = 3.2 \times 10^{-8}\, cm^{-3}$. Assume $\langle \rho_e \rangle$ is a constant in the intergalatic medium,

$DM = \int ds\, n_e = \underbrace{\int_{nearby}^{far\ away} ds\, n_e}_{IGM} + \underbrace{\int_0^{nearby} ds\, n_e}_{our\ galaxy} + \underbrace{\int_{far\ away}^{far\ away} ds\, n_e}_{host\ galaxy}$

$\sim \underbrace{3.2 \times 10^{-8}}_{\rho_e} \times \underbrace{3.086 \times 10^{27}}_{1\, Gpc} \sim 32\ pc/cm^3$

f. $300\ hours = 1.08 \times 10^6\, sec$

$\Omega = \frac{\lambda^2}{A} = \frac{4\lambda^2}{\pi D^2} = \frac{4 \times 0.12^2}{\pi \times 305^2} = 5.47 \times 10^{-7}$ steradian

$Rate = \frac{1}{time \cdot 7 \cdot \Omega} = 0.242$ burst/sec/ster

g. $V = \frac{4}{3}\pi (3Gpc)^3 = 36\pi\, Gpc^3$

Rate Density $4\pi \cdot Rate/V = \frac{4\pi \times 0.242}{36\pi} = 0.027$ burst/sec/Gpc³

h. In the ionosphere (50 ~ 1000 km), $n_e \sim 10^5/cm^3$.

$D_m = D_m' + D_{m\text{-}ionosphere}$

$= 32 pc/cm^3 + 10^6 m \times 10^5/cm^3$

$= 32\ pc/cm^3 + 10^{11} m/cm^3$

Problem 2

2d. $DM = \int ds\, n_e \qquad n_e = 0.03 e^{-z/1000pc}$

$$= \int \frac{dz}{\cos\theta} \cdot 0.03 e^{-z/1000} \qquad z = \cos\theta\, s \qquad ds = \frac{dz}{\cos\theta}$$

$$= \frac{-30}{\cos\theta} \cdot e^{-z/1000} \Big|_0^{\infty}$$

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$$= 32 pc/cm^3 + 10^6 m \times 10^5/cm^3$$

$$= 32\ pc/cm^3 + 10^{11} m/cm^3$$