

HW 10 - ASTR501

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a)

```
In[1]:= sθ = θ -> ArcTan[H / r] /. H -> .04 r (*Angle with respect to mid plane in radians*)
```

```
Out[1]= θ -> 0.0399787
```

```
In[2]:= ArcTan[D[n r^(9/7), r]] - ArcTan[H / r] /. {H -> .04 r, r -> 1 au, n -> .04 r / r^(9/7)}
(*Angle with surface of disk in radians*)
```

```
Out[2]= 0.0114046
```

b)

```
In[3]:= peak wavelength 5000K [[2]] < 2 π 0.1 microns
```

```
Out[3]= True
```

```
In[4]:= mdust = (4/3 Pi a^3 3 g/cm^3);
```

```
sΣ = Solve[{1 == Pi a^2 Q Ndust, Ndust == .01 Σgas / mdust}, {Σgas, Ndust}] [[1, 1]] /.
{a -> 0.1 μm, Q -> If[λ < 2 Pi a, 1, 2 Pi a / λ],
```

```
λ -> peak wavelength 5000K [[2]] // Echo}
```

```
» λ -> 5.7955458000000000 × 10-7 m
```

```
Out[4]= Σgas -> 0.04 kg/m2
```

I don't like to be really precise, since this is Astronomy where everything is order of magnitude.

c)

```
In[5]:= Solve[1000 g/cm^2 .01 / mdust / H π a^2 z 1 au / Sin@θ == 1, z] [[1, 1, 2]] /.
{H -> .04 r, r -> 1 au, sθ, a -> 0.1 μm }
```

```
Out[5]= 6.39489 × 10-9
```

Ratio with respect to column in b)

```
In[6]:= Σgas / 1000 g/cm^2 /. sΣ
```

```
Out[6]= 4. × 10-6
```

d)

$$\text{In[7]:= eqL} = L / (4 \pi r^2) \pi a^2;$$

e)

$$\text{In[8]:= eqR} = 4 \pi a^2 \sigma T_{gr}^4 Q;$$

f)

$$\text{In[9]:= Echo@Solve[eqL == eqR, Tgr][[-1, 1]] /. \{r \rightarrow 1 \text{ au}, a \rightarrow 0.1 \mu\text{m},$$

$$L \rightarrow 2 * \text{Sun}(\text{star})[\text{ luminosity}], Q \rightarrow 1, \sigma \rightarrow \sigma\} // \text{UnitConvert}$$

$$\gg T_{gr} \rightarrow \frac{L^{1/4}}{2 \pi^{1/4} Q^{1/4} \sqrt{r} \sigma^{1/4}}$$

$$\text{Out[9]:= Tgr} \rightarrow 331.42 \text{ K}$$

g)

$$\text{In[10]:= } 1000 \text{ g/cm}^2 \cdot 0.1 / \text{mdust} Q \pi a^2 /. \{a \rightarrow 0.1 \mu\text{m}, Q \rightarrow 2 \pi a / \lambda, \lambda \rightarrow 2.2 \mu\text{m}\}$$

$$\text{Out[10]:= } 71399.8$$

h)

$$\text{In[11]:= } 1000 \text{ g/cm}^2 \cdot 0.1 / \text{mdust} Q \pi a^2 /. \{a \rightarrow 0.1 \mu\text{m}, Q \rightarrow 2 \pi a / \lambda, \lambda \rightarrow 40 \mu\text{m}\}$$

$$\text{Out[11]:= } 3926.99$$