HW 2 - ASTR404

Daniel George - dgeorge5@illinois.edu

Q1) Sirius A & B

a) Masses

Defining constants

In[29]:=
```
G = Quantity["GravitationalConstant"];
oP = Quantity[49.94, "Years"];
{p, θ} = Quantity[{.37921, 7.61}, "ArcSeconds"];
```

Computing distance to binary

In[32]:=
```
d = UnitConvert[FormulaData["ParallaxDistance", {p → p}][[2]], "m"]
```

Out[32]= $8.1371284512369 \times 10^{16}$ m

Finding semi-major axis of reduced mass

In[33]:=
```
aR = Solve[θ == A / d, A][[1, 1, 2]]
```

Out[33]= $3.0021383017581 \times 10^{12}$ m

Finding distances to center of mass

In[34]:=
```
{aA, aB} = NSolve[{a1 + a2 == aR, a1 / a2 == .466}, {a1, a2}][[1, All, 2]]
```

Out[34]= $\{9.5429498541562 \times 10^{11}$ m , $2.0478433163425 \times 10^{12}$ m $\}$

Finding total mass of system (Kepler's 3rd Law)

In[35]:=
```
M = NSolve[oP^2 == 4 Pi^2 aR^3 / (G m), m][[1, 1, 2]]
```

Out[35]= $6.452828528697 \times 10^{30}$ kg

Finding individual masses

In[36]:=
```
{mA, mB} = NSolve[{m1 aA == m2 aB, m1 + m2 == M}, {m1, m2}][[1, All, 2]]
```

Out[36]= $\{4.4016565680061 \times 10^{30}\ \text{kg},\ 2.0511719606909 \times 10^{30}\ \text{kg}\}$

b) Luminosities

Luminosity-magnitude relation

In[37]:= $L = 10^{0.4\,(4.75-M)}\ 1\,L_\odot$;

Luminosity of Sirius A

In[38]:=
```
LA = L /. M → 1.36
```

Out[38]= $22.698648518838\ L_\odot$

Luminosity of Sirius B

In[39]:=
```
LB = L /. M → 8.79
```

Out[39]= $0.024210290467362\ L_\odot$

c) Radius of Sirius B

Stefan Boltzmann Law

In[40]:= $L == 4\,\mathrm{Pi}\,R\hat{}2\,(1\,\sigma)\,T^4$;

Solving for radius of Sirius B

In[41]:= rB = NSolve[LB == 4 Pi R^2 $(1\,\sigma)\,T^4$ /. T → 24790 K, R][[2, 1, 2]]

Out[41]= $5.867646270211 \times 10^{6}\ \text{m}$

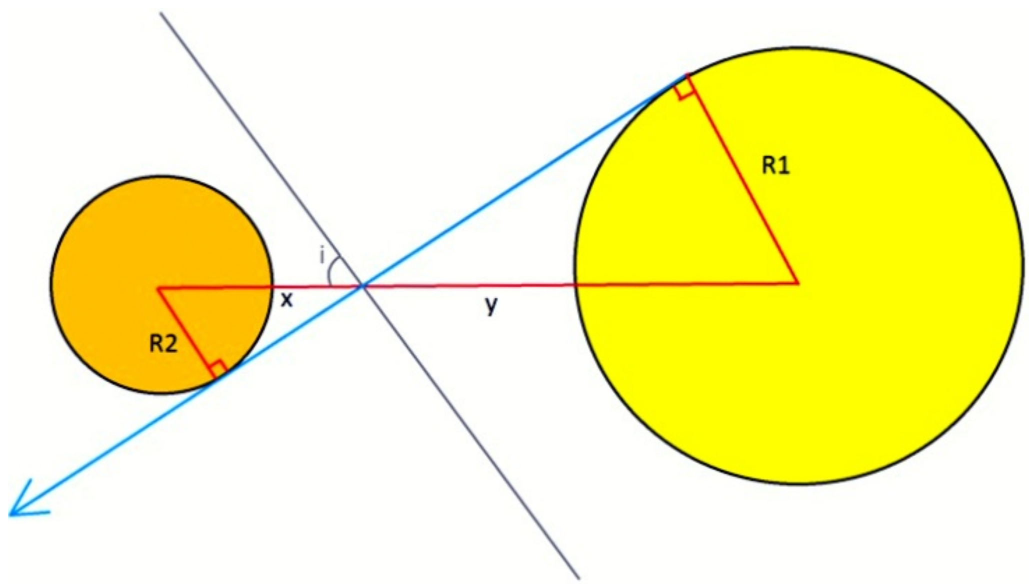
Comparison to Earth and Sun

In[42]:=
```
Transpose@{{"R_Sirius-B", "R_Earth", "R_Sun"}, UnitConvert[
    {rB, Earth (planet) ["Radius"], Sun (star) ["Radius"]}, "km"]} // TableForm
```

Out[42]//TableForm=

$R_{Sirius-B}$	5867.646270211 km
R_{Earth}	6367.4447 km
R_{Sun}	6.955×10^{5} km

Q2) Eclipsing binaries



a) Analytic solution

Solving inequalities

In[43]:=
```
eqi = Reduce[
    (*Equations*)
    {Abs@Cos[i] < R1/(y) == R2/(x),
     (x + y) == a,
     (*Constraints*)
     0 ≤ i ≤ Pi, x > R2 > 0 m, y > R1 > 0 m}
    , {i, x, y}, Reals] // Simplify
```

Out[43]= $x == \frac{a\,R2}{R1+R2}$ && $y == \frac{a\,R1}{R1+R2}$ && $\left(\frac{\pi}{2} < i < \text{ArcCos}\left[-\frac{R1+R2}{a}\right] \;||\; \left(\text{ArcCos}\left[\frac{R1+R2}{a}\right] < i \;\&\&\; 2\,i \le \pi\right)\right)$ && $0\,m < R1 < a$ && $0\,m < R2$ && $R1 + R2 < a$ && $a > 0\,m$

Thus we can see from the above expression that the angle i must satisfy the following constraint:

$$\cos^{-1}\left(\frac{R1+R2}{a}\right) < i < \cos^{-1}\left(-\frac{R1+R2}{a}\right)$$

b) Numerical solution

Substituting given values

In[44]:=
```
eqi /. {a -> 3 au, R1 → 2 Sun (star) ["Radius"], R2 -> Sun (star) ["Radius"]} //
  Simplify // Last
```

Out[44]= 1.566147 < i < 1.575445

Q3) Boltzmann equation

Energy of nth state of hydrogen

In[45]:=
```
e[n_] := -13.6/n^2 eV
```

Number of distinct states in nth shell

In[46]:=
```
g[n_] := 2 n^2
```

Ratio of electrons in different shells

In[47]:=
```
eqB[i_, j_] := n[i] / n[j] == g[i] / g[j] Exp[(e[j] - e[i]) / (k T)]
```

T at which n_2 / n_1 = 1%

In[48]:=
```
Solve[1 / 100 == eqB[2, 1][[2]], T]
```

Out[48]= {{T → 19 755.79160745 K}}

T at which n_2 / n_1 = 10%

In[49]:=
```
Solve[10 / 100 == eqB[2, 1][[2]], T]
```

Out[49]= {{T → 32 087.284631675 K}}

Q4) Ideal gas properties

a) Most probable velocity

Maxwell-Boltzmann distribution

This is proportional to the probability density function for velocities:

In[50]:=
```
nv[v_] = 4 Pi v^2 n (m / (2 Pi k T)) ^ (3 / 2) Exp[-m v^2 / (2 k T)]
```

Out[50]= $e^{-\frac{m v^2}{2 k T}} n \sqrt{\frac{2}{\pi}} \left(\frac{m}{k T}\right)^{3/2} v^2$

Finding extrema

We equate the derivative of the distribution function to zero and solve for the velocity.

In[51]:=
```
vMP = Assuming[k > 0 && T > 0 && m > 0 && n > 0,
  Solve[D[nv[v], v] == 0 && v > 0, v, Reals] // Simplify]
```

Out[51]= $\left\{\left\{v \to \sqrt{2} \sqrt{\frac{k T}{m}}\right\}\right\}$

This value of velocity is either a maxima or minima of the distribution function since the derivative is zero at this point.

Checking whether this is a maxima

In[55]:=
```
D[n_v[v], {v, 2}] /. vMP[[1]]
```

Out[55]= $$-\frac{4\,n\,\sqrt{\frac{2}{\pi}}\,\left(\frac{m}{k\,T}\right)^{3/2}}{e}$$

The second derivative is negative at this value. Therefore this must be a maxima. Hence this is the most probable velocity.

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b) RMS velocity

Definition of RMS velocity

$$v_{RMS}{}^{2} = \frac{\int_0^\infty v^2\,n_v(v)\,dv}{\int_0^\infty n_v(v)\,dv}$$

Computing value

In[53]:=
```
vRMS = v -> Assuming[k > 0 && T > 0 && m > 0 && n > 0,
   Sqrt[Integrate[v^2 n_v[v], {v, 0, Infinity}] / Integrate[n_v[v], {v, 0, Infinity}]]]
```

Out[53]= $v \to \sqrt{3}\,\sqrt{\frac{k\,T}{m}}$

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c) Pressure

Formula for pressure

$$P = \frac{1}{3}\int_0^\infty m\,v^2\,n_v(v)\,dv$$

Computing value

In[54]:=
```
P -> 1/3
  Integrate[m n_v[v] v^2, {v, 0, Infinity}, Assumptions → k > 0 && T > 0 && m > 0 && n > 0]
```

Out[54]= $P \to k\,n\,T$