Q2) From the derivation of the tensor virial theorem in the notes we have:

$$\frac{d}{dt}\int d^3x\, \rho\, \bar{v}_j\, x_k = 2T_{kj} + \Pi_{kj} + W_{kj}$$

$$= 2T_{kj} + \Pi_{kj} + \left(-\int d^3x \rho\, x_j \partial\phi/\partial x_k\right)$$

Making the substitution $\phi \to \phi + \phi_{ext}$

$$\Rightarrow \frac{d}{dt}\int d^3x\, \rho\, x_k \bar{v}_j = 2T_{kj} + \Pi_{kj} + W_{kj} - \int d^3x \rho x_j \frac{\partial \phi_{ext}}{\partial x_k}$$

Replacing $k \leftrightarrow j$ and adding gives

$$\frac{d}{dt}\int d^3x\, \rho\,(x_k \bar{v}_j + x_j \bar{v}_k) = \Big(2T_{jk} + \Pi_{jk} + W_{jk}$$
$$+ 2T_{kj} + \Pi_{kj} + W_{kj} - \int d^3x \rho\, x_j\, \partial\phi_{ext}/\partial x_k$$
$$- \int d^3x\, \rho\, x_k\, \partial\phi_{ext}/\partial x_k \Big)$$

Since $T_{jk} = T_{kj}$ & $\Pi_{jk} = \Pi_{kj}$ & $W_{jk} = W_{kj}$

$$\Rightarrow \frac{1}{2}\, d/dt \int d^3x\, \rho\,(x_k \bar{v}_j + x_j \bar{v}_k) = 2T_{jk} + \Pi_{jk} + W_{jk}$$
$$- 1/2 \left[\int d^3x\, (x_j\, \partial\phi_{ext}/\partial x_k + x_k\, \partial\phi_{ext}/\partial x_j) \right]$$

Since $\int d^3x\, \rho\,(x_k \bar{v}_j + x_j \bar{v}_k = d/dt\, I_{jk}$

$$\Rightarrow 1/2\, d^2 I_{jk}/dt^2 = 2T_{jk} + \Pi_{jk} + W_{jk} + V_{jk} \quad //$$

(where V_{jk} is as defined in the question)