HW 7 - ASTR501

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Initialization

In[26]:=
```
SetOptions[Plot,
  {Frame → True, FrameLabel → {"ν (Hz)", "T_B (K)"}, Filling → Bottom, ImageSize → 400}];
Unprotect@Quantity;
Quantity[0., _] = Quantity[0, _] = 0;
Protect@Quantity;
```

a)

In[28]:=
```
ν0 = 115.272 GHz ;
sB = Solve[h ν0 == k B J (J + 1) / 2 - 0 /. J → 1, B][[1, 1]] // UnitConvert
```

Out[29]= B → 5.53219 K

b)

In[30]:=
```
g[j_] := 2 j + 1;
f[T_][j_] := g[j] Exp[- B / T j (j + 1) / 2] / Sum[g[i] Exp[- B i (i + 1) / 2 / T], {i, 0, 10}] /. sB;
# -> f@ 10 K /@ # &@{0, 1, 2} // Thread
```

Out[31]= {0 → 0.252014, 1 → 0.434797, 2 → 0.239671}

c)

In[32]:=
```
CO[n_] =
  Quiet@Solve[{nCO == .2 nC, nH / nC == solar molar abundance H / solar molar abundance C} /.
      nH → 2 n / cm^3 , {nCO, nC}][[1, 1, 2]];
CO@50
```

Out[32]= 6451.61 per meter^3

d)

In[33]:=
```
sV = Solve[3 × 1/2 mass 1 CO vx^2 == 3/2 k 10 K, vx][[2, 1]]
```
Out[33]= vx → 54.483 m/s

e)

Line profile function

In[34]:=
```
νc = (1 - vz/c) ν0;
```
$$\phi = \frac{\mathrm{Exp}[-(\nu - \nu c)^2/(2\sigma^2)]}{\sqrt{2\pi}\,\sigma};$$

Frequency width

In[36]:=
```
σν[v_] := v/c ν0;
```

Einstein’s coefficients

In[37]:=
```
A21 = 7.166 × 10^-8 Hz;
```
In[38]:=
```
B21 = Solve[A21 == 2 h ν0^3/c^2 b21, b21][[1, 1, 2]]
```
Out[38]= 3.17293×10^{9} s/kg

In[39]:=
```
B12 = Solve[g@0 b12 == g@1 B21, b12][[1, 1, 2]]
```
Out[39]= 9.51879×10^{9} s/kg

Emissivity

In[40]:=
```
n@i_ := f[T][i - 1] CO[nH];
jν[nH_, vz_, T_, σ_] = n@2 A21 h ν / (4 π) ϕ;
```

f) Absorptivity

In[42]:=
```
αν[nH_, vz_, T_, σ_] = h ν / (4 π) (n@1 B12 - n@2 B21) ϕ;
```
Yes.

g) Optical depth

In[43]:=
```
αν[50, 0, 10 K, σν@1 km/s] 1 pc /. ν → ν0
```
Out[43]= 1.27966

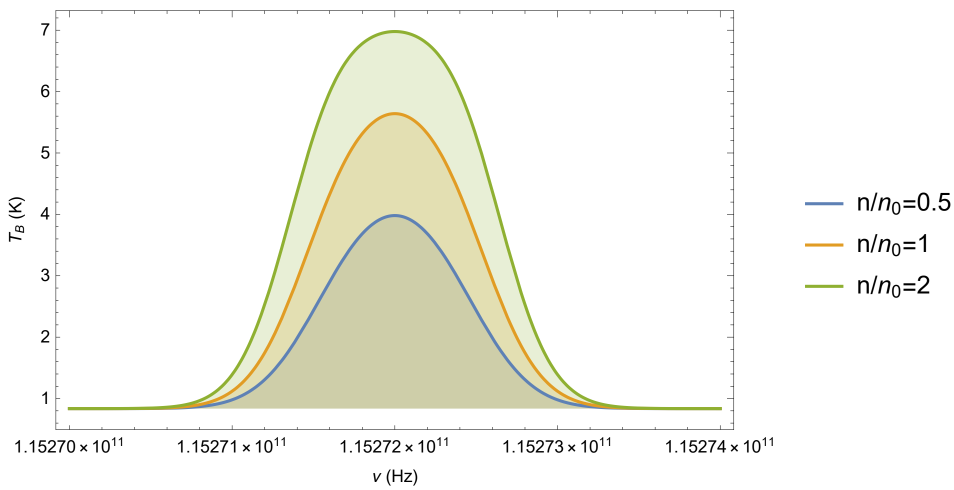
h)

One function to solve them all (including CMB)

In[44]:=
```
BT[nH_: 50, d_: 1, vz_: 0, Δv_: 1 km/s, T_: 10 K, nH2_: 0, vz2_: 0, Δv2_: 1, T2_: 1] :=
 ParametricNDSolveValue[
  UnitConvert@{Iν'@z / 1.0 pc == -(αν[nH, vz, T, σν@Δv] + αν[nH2, vz2, T2, σν@Δv2]) Iν[z] +
       (jν[nH, vz, T, σν@Δv] + jν[nH2, vz2, T2, σν@Δv2]),
     Iν@0 == 2 h ν^3 / c^2 / (Exp[h ν / (k 2.725 K)] - 1)} /. Quantity[x_, _] :→ x,
  QuantityMagnitude@UnitConvert[c^2 / (2 k)] Iν[d] / ν^2, {z, 0, d}, ν]
```

In[45]:=
```
Plot[Evaluate@Table[BT[i 50]@x, {i, {.5, 1, 2}}],
 {x, 115.27 × 10^9, 115.274 × 10^9}, PlotLegends → {"n/n0=0.5", "n/n0=1", "n/n0=2"}]
```

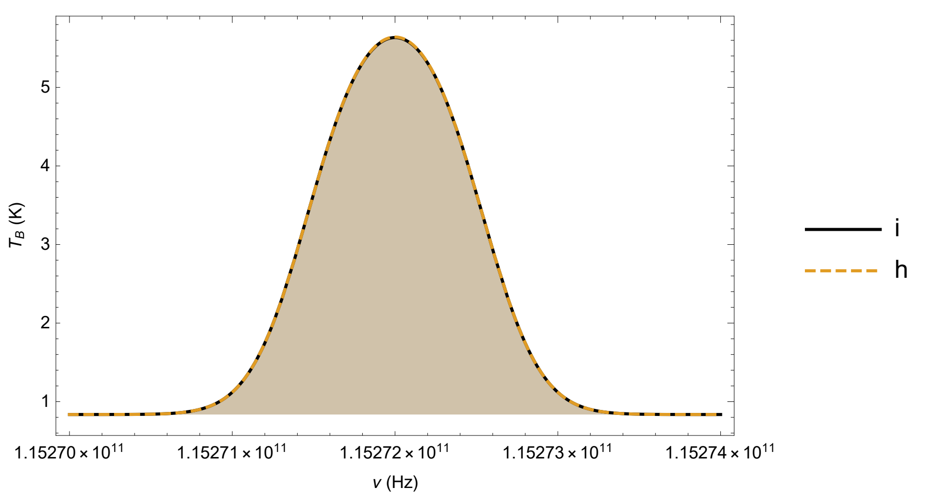
Out[45]=



i)

In[46]:=
```
Plot[Evaluate@{BT[50 / (2 √(2 π)) (Exp[-(z - 5)^2 / 2] + Exp[-(z - 8)^2 / 2]), 16]@x, BT[]@x},
 {x, 115.27 × 10^9, 115.274 × 10^9}, PlotStyle → {Black, Dashed}, PlotLegends → {"i", "h"}]
```

Out[46]=

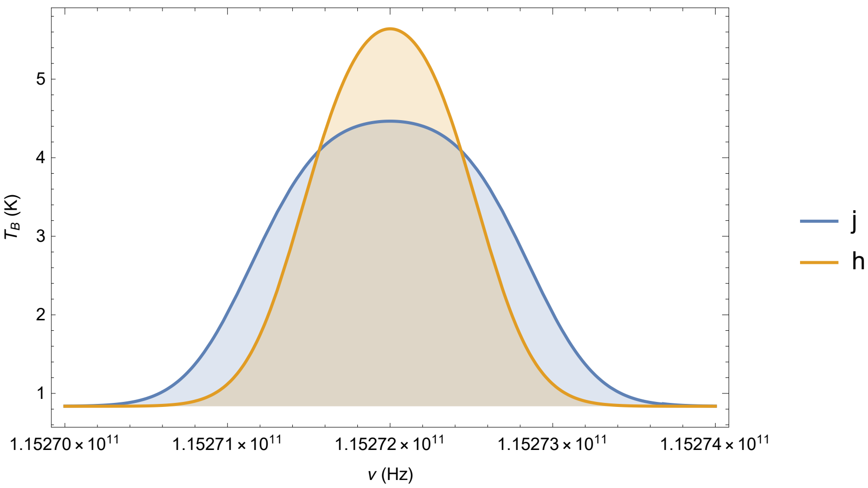


No. They look identical.

j)

In[47]:=
```
Plot[Evaluate@{BT[50, 1, 1.5 Sin[2 π z] 1 km/s ]@x, BT[]@x},
 {x, 115.27 × 10^9, 115.274 × 10^9}, PlotLegends → {"j", "h"}]
```

Out[47]=

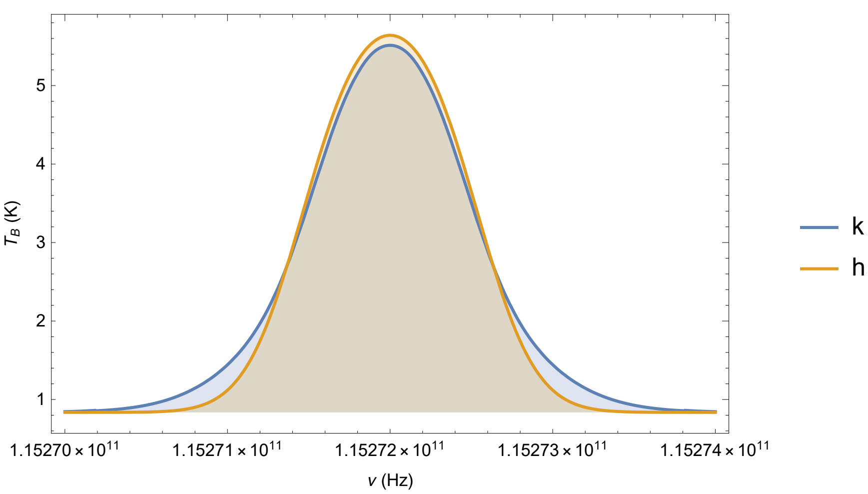


Yes. They can be distinguished.

k)

In[48]:=
```
Plot[Evaluate@{BT[50/(2 √(2 π)) Exp[-(z - 8)^2/2], 16, 0, 1.5 km/s ,
    12 K , 50/(2 √(2 π)) Exp[-(z - 5)^2/2], 0, .8 km/s , 8 K ]@x, BT[]@x},
 {x, 115.27 × 10^9, 115.274 × 10^9}, PlotLegends → {"k", "h"}]
```

Out[48]=



Extra

Yes, I did. I should since the CMB brightness temperature would be comparable to the brightness temperatures we got. It shifts everything up by about 1 K. Furthermore, it changes the relative difference between the two curves in part k)