HW 5 - ASTR540

Created with Wolfram Mathematica 11.0 on October 11, 2016

Daniel George - dgeorge5@illinois.edu

Q1) Distribution of Binary Periods

Probability distribution of periods

Since $\log_{10}$P is a normal distribution, we can find the distribution of P as follows

In[26]:=
```
distP = TransformedDistribution[10^logP , logP ≈ NormalDistribution[5, 23/10]];
```

Period for two solar mass stars separated by d AU

Function to find the period in days, given separation in AU, using Kepler's 3rd law

In[27]:=
```
fP[d_] := Solve[P^2 == (4 π^2)/(G 2 M☉) (d au)^3, P]〚2, 1, 2〛~QuantityMagnitude~"Days"
```

---

a)

Minimum period estimate in days

Finding a P such that probability of period being less than P is 3% or CDF(P) = .03

In[28]:=
```
NSolve[distP~CDF~P == 3/100]〚1, 1〛
```

Out[28]= P → 4.72253

This is larger than the period corresponding to the separation when two solar mass stars touch

In[29]:=
```
fP[2 Sun (star) ["Radius"]~QuantityMagnitude~ "AU"]
```

Out[29]= 0.2317

---

b)

Maximum period estimate in days

Finding a P such that probability of period being greater than P is 3% or CDF(P) = .97

In[30]:=
```
NSolve[distP~CDF~P == 97/100]〚1, 1〛
```

Out[30]= P → 2.117509×10^9

c)

Fraction of stars with smaller separation than 1 AU

Finding the CDF at the period corresponding to this separation

In[31]:=
```
distP~CDF~fP@1
```
Out[31]=
```
0.1303
```

d)

Fraction of stars with separation between 0.4 AU & 40 AU

Subtracting the values of the CDF corresponding to the two periods

In[32]:=
```
Subtract @@ (distP~CDF~fP@# & /@ {40, .4})
```
Out[32]=
```
0.3849052
```

Q2) Fast Radio Bursts

a)

Total energy output

Assuming the width of the spectrum is ~ 1 GHz:

In[33]:=
```
UnitConvert[power * area * time /. {power -> 1 Jy * 1 GHz , area → 4. π Gpc^2 , time -> 1 ms }, "Ergs"]
```
Out[33]= 1.196495×10^{39} ergs

b)

Formula for ω_p

In[34]:=
```
swp := ωp → Sqrt[4 π ne[s] e^2 / me]
```

Finding plasma frequency in CGS

In[35]:=
```
swp /. {e → 4.8 × 10^-10 , me -> 9.109383 × 10^-28, ne[s] → .03}
```
Out[35]=
```
ωp → 9764.777
```

c)

Taylor expansion of group velocity $\left(\frac{\partial\omega}{\partial k}\right)$

In[36]:=
```
sP = cPlasma -> Normal@Series[D[Sqrt[ωp^2 + c^2 k^2], k], {ωp, 0, 2}] // PowerExpand
```

Out[36]= $c_{Plasma} \to c - \frac{\omega p^2}{2\,c\,k^2}$

So we can see that the speed decreases in the presence of plasma.

d)

Difference in pulse arrival times

Let c_{Plasma} be the speed in plasma

$$\Delta t = \int\left(\frac{1}{c_{Plasma}} - \frac{1}{c}\right) ds;$$

First term in Taylor expansion of the integral

In[37]:= $\text{Integrate}\left[\text{Normal@Series}\left[\left(\frac{1}{c_{Plasma}} - \frac{1}{c}\right) \text{/. sP, \{ωp, 0, 2\}}\right] \text{/. swp, s}\right]$

Out[37]= $\frac{2\,e^2\,\pi\int ne[s]\,ds}{c^3\,k^2\,me}$

This is equivalent to the given expression

Substituting the value of ω in the given expression and expanding up to first order

In[38]:= $\text{Series}\left[\frac{2\,e^2\,\pi\int ne[s]\,ds}{c\,\omega^2\,me} \text{/. } \omega \to \text{Sqrt}[\omega p\text{^}2 + c\text{^}2\,k\text{^}2],\ \{\omega p, 0, 1\}\right]$

Out[38]= $\frac{2\,e^2\,\pi\int ne[s]\,ds}{c^3\,k^2\,me} + O[\omega p]^2$

d)

Distance to center of galaxy in parsecs

In[39]:=
```
dCenter = QuantityMagnitude[ Milky Way (galaxy) ["DistanceFromSun"], "Parsecs"]
```

Out[39]= 7611.133

DM on the galactic pole at angle θ

In[40]:= $\text{poleDM}[\theta\_] = \int_0^{\text{dCenter Tan}[\theta]} \frac{ne[z]}{\text{Cos}[\theta]}\,dz \text{ /. } ne[z] \to .03\,\text{Exp}[-z/1000];$

Finding maximum for $0 < \theta < 45\,°$

In[41]:=
```
Maximize[{poleDM[θ], 0 < θ < π / 4}, θ]
```
Out[41]=
```
{42.40541, {θ → 0.7853982}}
```

Therefore the DM is maximum at $\theta \sim 45°$ (0.7853982 radians) and the maximum value is ~ 42.4 pc/cm^3

---

e)

Number density of electrons in ISM in CGS

Source: (Cosmological Effects of Scattering in the Intergalactic Medium - Raymond F Bonn)

In[42]:=
$$ne_{ISM} = 3.2 \times 10\text{^}-8;$$

DM at 1Gpc

In[43]:=
$$ne_{ISM}\ 10\text{^}9\ \text{"pc/cm}^3\text{"}$$
Out[43]=
$$32.\ \text{pc}/\text{cm}^3$$

---

f)

Rate of bursts per steradian

In[44]:=
$$\text{rate} = \text{UnitConvert}\left[\frac{1}{\text{time} * 7 * \Omega}\ //.\ \left\{\Omega \to \frac{\lambda^2}{\text{Area}}\ \text{sr},\ \text{Area} \to \pi \left(\frac{d}{2}\right)^2,\ d \to 305.\ \text{m},\ \text{time} \to 300\ \text{h},\ \lambda \to 20\ \text{cm}\right\},\ \text{per second}\right]$$
Out[44]=
```
0.241606 per second per steradian
```

---

g)

Number density of bursts

In[45]:=
$$\frac{4\,\pi\ \text{sr rate}}{\text{Volume@Ball}\left[\{0, 0, 0\},\ 3\ \text{Gpc}\right]}$$
Out[45]=
$$0.02684511\ \text{per gigaparsec}^3\ \text{per second}$$

---

h)

DM of ionosphere

Source: Tools of Radio Astronomy - Thomas Wilson, Susanne Hüttemeister

In[47]:=
$$ne * \text{width}\ /.\ \left\{ne \to 10\text{^}5 / \text{"cm}^3\text{"},\ \text{width} \to 6.7 \times 10\text{^}-13\ \text{"pc"}\right\}$$
Out[47]=
$$\frac{6.7 \times 10^{-8}\ \text{pc}}{\text{cm}^3}$$