HW 11 - ASTR501

Daniel George - dgeorge5@illinois.edu

Q1)

a)

```
In[1]:= sD = NSolve[MDust / (NDust 4 / 3 Pi a^3) == ρDust, NDust][[1, 1]] /. {MDust -> 0.01 g, ρDust -> 2.2 g/cm³}
```

Out[1]= $\text{NDust} \to \frac{0.00108515\ \text{cm}^3}{a^3}$

b)

```
In[2]:= sα = αν → nDust σDust /. {sD, σDust → Q π a^2, nDust → ρDust / (4 / 3 Pi a^3 ρDust)}
```

Out[2]= $\alpha\nu \to \frac{3\,Q}{4\,a}$

```
In[3]:= sk = kν -> αν / ρDust /. ρDust -> 2.2 g/cm³ /. sα
```

Out[3]= $k\nu \to \frac{Q\left(0.340909\ \text{cm}^3/\text{g}\right)}{a}$

c)

```
In[4]:= sj = jν -> αν Bν /. {sB = Bν → 2 h ν^3 / c² / (Exp[h ν / (k T)] - 1), sα}
```

Out[4]= $j\nu \to \frac{Q\,\nu^3\left(\frac{3}{2}\,h/c^2\right)}{a\left(-1+e^{\frac{\nu\left(1\,h/k\right)}{T}}\right)}$

d)

```
In[5]:= sF = Fν -> Bν τν A / d^2 /. τν → kν ρDisk ds /. A → mDisk / (ρDisk ds)
```

Out[5]= $F\nu \to \frac{B\nu\,k\nu\,\text{mDisk}}{d^2}$

```
In[6]:= sM = Solve[Equal @@ sF, mDisk][[1, 1]]
```

Out[6]= $\text{mDisk} \to \frac{d^2\,F\nu}{B\nu\,k\nu}$

e)

```
In[7]:= UnitConvert[sF[[2]] //. {sk, sB, a -> 10 μm, mDisk -> 0.01 M☉,
           d -> 150 pc, T -> 20 K, Q → 1, ν → peak frequency 20K [[2]]}, "Jy"]
```

Out[7]= 4799.65 Jy

f)

$$\frac{\partial \log(F\nu)}{\partial \log(\nu)}$$

In[8]:=
```
Dt@Log@Fν[ν] / Dt@Log@ν /. Fν'[ν] → D[Fν /. sF /. sB, ν] /. Fν[ν] → Fν //.
  {sB, sF, ν → c / 3 mm , T -> 20 K , sk, a -> 10 μm , mDisk -> 0.01 M☉ , d -> 150 pc , Q → 1}
```

Out[8]= `1.87531`

I have not used the Rayleigh-Jeans approximation, which explains why the value is not exactly 2.

Q2)

a)

In[9]:=
```
sI = Iν → UnitConvert[FormulaData[{"PlanckRadiationLaw", "Frequency"},
     {"T" -> 10^15 K , "ν" -> 22.23508 GHz }][[2]], "g/s^2"]
```

Out[9]= $I\nu \to 0.145999\ \mathrm{g/s^2}$

b)

In[10]:= $\pi\left(\frac{r}{d}\right)^2$

c)

In[10]:=
```
sr = d -> r 1 au / .3 mas rad // UnitConvert
```

Out[10]= $d \to r\left(1.02856\times 10^{20}\ \mathrm{m}\right)$

d)

Brightness of unresolved source

In[11]:=
```
UnitConvert[Pi (r 1 au / d) ^2 Iν /. {sI, sr}, "Jy"]
```

Out[11]= `97027. Jy`

Farthest detectable distance

In[12]:=
```
NSolve[Pi (r 1 au / d) ^2 Iν == 1 mJy /. sI, d] [[2, 1]]
```

Out[12]= $d \to r\left(1.01315\times 10^{24}\ \mathrm{m}\right)$

e)

In[13]:=
```
F -> r^2 UnitConvert[
   FormulaData[{"PlanckRadiationLaw", "Frequency"}, {"T" -> 10^3 K , "ν" -> 22.23508 GHz }][[2]]
    π (1 au / d) ^2 /. d -> 1 kpc , 1 μJy ]
```

Out[13]= $F \to r^2\left(1.12103\ \mu\mathrm{Jy}\right)$