ASTR 510 - HW 1

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Q2

Stencil for 4th order centered derivative

```
In[1]:= st4 = {-1, 8, 0, -8, 1} / 12;
```

Expression for 4th order centered derivative

```
In[2]:= d4q = (st4 / dx) . q[n] /@ Range[i + 2, i - 2, -1]
```

Out[2]= $\frac{q[n][-2+i]}{12\,dx} - \frac{2\,q[n][-1+i]}{3\,dx} + \frac{2\,q[n][1+i]}{3\,dx} - \frac{q[n][2+i]}{12\,dx}$

4th order FTCS scheme for the advection equation

```
In[3]:= ftcs4 = (q[n + 1][i] - q[n][i]) / dt + a d4q == 0
```

Out[3]= $a\left(\frac{q[n][-2+i]}{12\,dx} - \frac{2\,q[n][-1+i]}{3\,dx} + \frac{2\,q[n][1+i]}{3\,dx} - \frac{q[n][2+i]}{12\,dx}\right) + \frac{-q[n][i] + q[1+n][i]}{dt} == 0$

Trial solution for von Neumann stability analysis

```
In[4]:= trial = q[n_][j_] -> ξ^n Exp[I k j];
```

Substituting it in the FTCS scheme

```
In[5]:= eq = ftcs4 /. trial
```

Out[5]= $a\left(\frac{e^{i(-2+i)k}\xi^n}{12\,dx} - \frac{2\,e^{i(-1+i)k}\xi^n}{3\,dx} + \frac{2\,e^{i(1+i)k}\xi^n}{3\,dx} - \frac{e^{i(2+i)k}\xi^n}{12\,dx}\right) + \frac{-e^{i\,i\,k}\xi^n + e^{i\,i\,k}\xi^{1+n}}{dt} == 0$

Solving for ξ

```
In[6]:= sol = Solve[eq, ξ][[1]] // FullSimplify
```

Out[6]= $\left\{\xi \to 1 + \frac{i\,a\,dt\,(-4 + Cos[k])\,Sin[k]}{3\,dx}\right\}$

a)

Calculating Dissipation Error = |ξ|

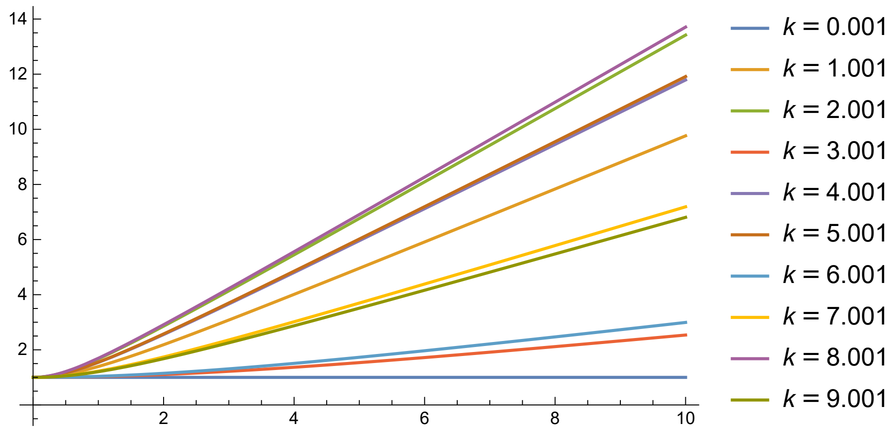
In[7]:=
```
dissE[σ_] = Abs[ξ] /. sol /. {dt → σ dx / a}
```

Out[7]= $\text{Abs}\left[1 + \frac{1}{3}\, i\, \sigma\, (-4 + \text{Cos}[k])\, \text{Sin}[k]\right]$

Plot of dissipation error vs σ

In[8]:=
```
Plot[Evaluate@Table[dissE[σ] /. {k → k1}, {k1, 0.001, 10}],
 {σ, 0, 10}, PlotLegends → Table[k == k1, {k1, 0.001, 10}]]
```

Out[8]=



It can be observed that for all values of k ≠ 0 the dissipation error increases monotonically with σ. It is always greater than 1.

b)

Calculating Dispersion Error = 1 + arg(ξ) / (k σ)

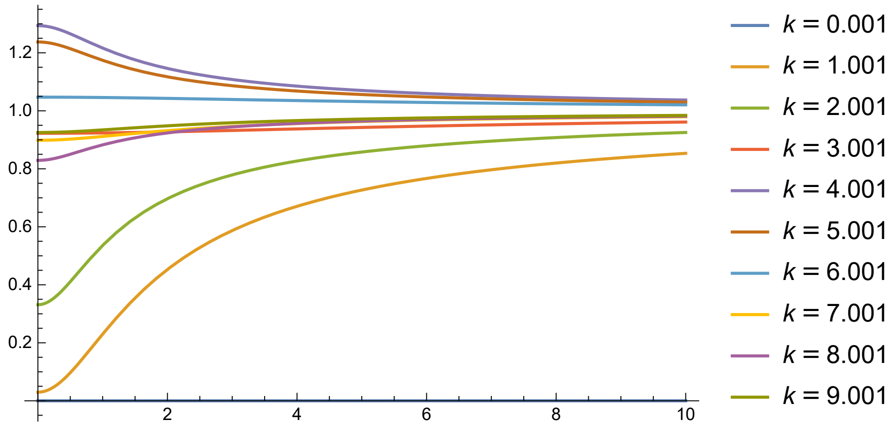
In[9]:=
```
dispE[σ_] = 1 + Arg[ξ] / (k σ) /. sol /. {dt → σ dx / a}
```

Out[9]= $1 + \frac{\text{Arg}\left[1 + \frac{1}{3}\, i\, \sigma\, (-4 + \text{Cos}[k])\, \text{Sin}[k]\right]}{k\, \sigma}$

Plot of dispersion error vs σ

In[10]:=
```
Plot[Evaluate@Table[dispE[σ] /. {k → k1}, {k1, 0.001, 10}],
 {σ, 0, 10}, PlotLegends → Table[k == k1, {k1, 0.001, 10}], PlotRange → All]
```

Out[10]=



It can be observed by varying the values of k that the dispersion error always remains bounded. The dispersion error increases monotonically and asymptotically tends to 1 for all values of k.

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c)

The method is stable if dissipation error is less than 1

In[11]:=
```
Reduce[dissE[σ] < 1 && k ≥ 0 && σ >= 0, σ]
```

Out[11]=
```
False
```

Therefore there is no solution for σ such that the dissipation error is less than 1. Therefore the method is unconditionally unstable.

Q1

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f)

Number of cores used

In[12]:=
```
nP = {1, 2, 4, 8, 12, 24, 36};
```

Grid sizes used

In[13]:=
```
nG = {200, 400, 1600};
```

Time taken for grid size = 200

In[14]:=
```
time[200][1] = 4.5407269001007080;
time[200][2] = 2.2643110752105713;
time[200][4] = 1.0853230953216553;
time[200][8] = 0.67126584053039551;
time[200][12] = 0.65181708335876465;
time[200][24] = 9.5089979171752930;
time[200][36] = 9.6296432018280029;
```

Time taken for grid size = 400

In[21]:=
```
time[400][1] = 69.583978891372681;
time[400][2] = 36.501708030700684;
time[400][4] = 26.051327943801880;
time[400][8] = 15.519846916198730;
time[400][12] = 21.609544992446899;
time[400][24] = 78.850456953048706;
time[400][36] = 79.048186063766479;
```

Time taken for grid size = 1600

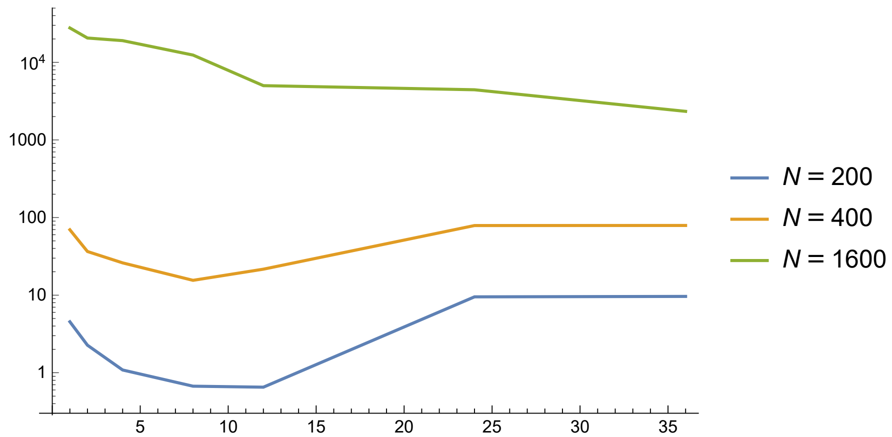
In[28]:=
```
time[1600][1] = 27739.625624895096;
time[1600][2] = 20571.754885911942;
time[1600][4] = 19038.544435977936;
time[1600][8] = 12406.615384101868;
time[1600][12] = 5006.3509409427643;
time[1600][24] = 4432.3666520118713;
time[1600][36] = 2335.3625159263611;
```

g)

Log-linear plots of time vs number of cores for each grid size

In[35]:=
```
ListLogPlot[Function[n, Transpose[{#, time[n] /@ #}] &@nP] /@ nG,
 Joined → True, PlotLegends → Table[N == i, {i, nG}]]
```

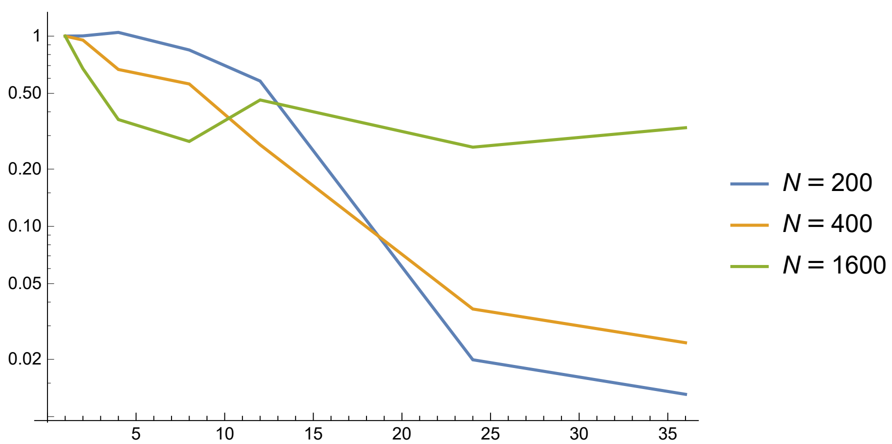
Out[35]=



Log-linear plots of parallel efficiency vs number of cores

In[36]:=
```
ListLogPlot[Function[n, Transpose[{#, time[n][1] / (# * (time[n] /@ #))}] &@nP] /@ nG,
 Joined → True, PlotLegends → Table[N == i, {i, nG}]]
```

Out[36]=



It can be observed that there are deviations from ideal scaling i.e. parallel efficiency decreases from 1 as number of cores increase, especially when number of grid points are small. The main reason for the decrease in parallel efficiency seems to be load imbalance since the parallel efficiency gets better for larger number of grid points. Other factors such as communication delays, data/control dependencies, etc. could also further decrease the parallel efficiency.

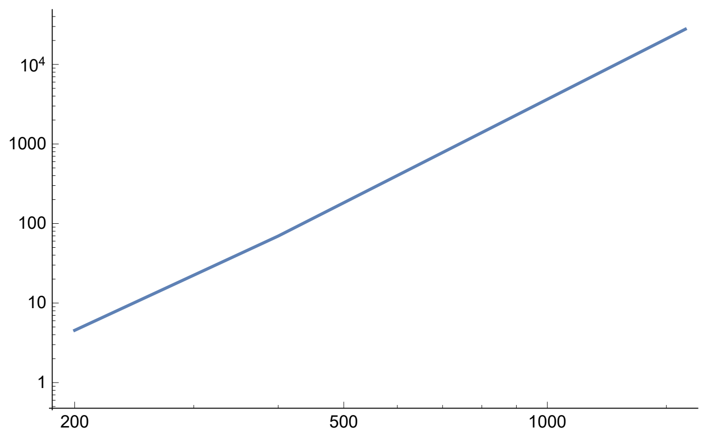
h)

Time taken for a single processor vs grid points

In[37]:=
```
coretimes = Transpose[{nG, time[#][1] & /@ nG}]
```
Out[37]=
```
{{200, 4.5407269001}, {400, 69.5839788914}, {1600, 27739.6256249}}
```

Log-log plot of time vs grid points

In[38]:=
```
ListLogLogPlot[coretimes, Joined → True]
```
Out[38]=



Thus it can be seen that the log-log plot of the data is almost a straight line. Therefore it can be extrapolated to estimate the core hours required for a 3200x3200 grid.

Finding a formula by extrapolating the log of the data

In[39]:=
```
ft[x_] = Fit[Log@coretimes, {1, x}, x]
```
Out[39]=
```
-20.8706704726 + 4.21042336683 x
```

Estimate for time required for 3200x3200 grid

In[40]:=
```
UnitConvert[Exp@ft[Log[3200]] s , "Hours"]
```
Out[40]=
```
137.35543616 h
```

Thus based on the data it would take about **137** core-hours for the calculation on a 3200x3200 grid assuming maximum parallel efficiency.

With up to 32 processors we can see that the total time taken decreases with increase in number of processors. Therefore we expect for a 3200x3200 we should be able to use as many processors as are available in the cluster to run a feasible calculation. (Although with too many processors the time may increase but this cannot be inferred from the data available).

i)

Importing data from mheat.out file

In[41]:=
```
data = Partition[Flatten@Import[
    "C:\\Users\\dan7g\\Google Drive\\Acads\\ASTR510\\mheat.txt", "Table"], 400];
```

Creating a contour plot of the temperature field

In[42]:=
```
ListContourPlot[data, Contours → 10,
 ColorFunction → "Rainbow", PlotLegends → Automatic]
```

Out[42]=