HW 7 - ASTR404

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Initialization

Importing data for 15M star

In[1]:=
```
data = SemanticImport[NotebookDirectory[] <> "15M_at_ZAMS.dat"];
(*Central density and radius of the star*)
ρc = 10^data[Max, "logRho"]; R = 10^data[Max, "logR"];
```

Solving the Lane-Emden equation

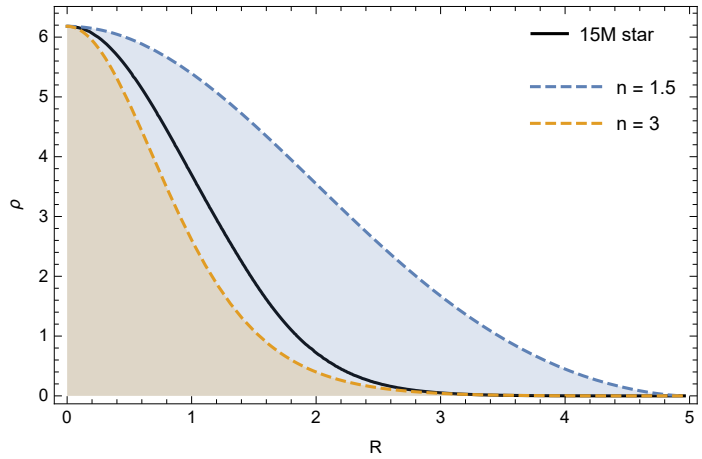
In[3]:=
```
θ@n_ := θ@n = Module[{o = $MachineEpsilon , x, y},
    NDSolveValue[{x^-2 D[x^2 D[y[x], x], x] + y[x]^n == 0, y[o] == 1,
      y'[o] == 0, WhenEvent[Re@y[x] ≤ 0, "StopIntegration"]}, y, {x, o, 10^2}]]
r@n_ := θ[n][[1, 1, 2]] (*Point where θ = 0*)
```

Q1)

Plotting density vs radius

In[5]:=
```
Show[ListLinePlot[data[10^# &, {"logR", "logRho"}], PlotStyle → Black,
  PlotLegends → Placed[{"15M star"}, {Right, Top}], Frame -> True, FrameLabel → {"R", "ρ"}],
 Plot[ρc (θ[#][r@# x / R])^# & /@ {1.5, 3} // Evaluate, {x, 0, R},
  PlotLegends → Placed[{"n = 1.5", "n = 3"}, {Right, Top}], PlotStyle → Dashed, Filling → Bottom]]
```

Out[5]=



a)

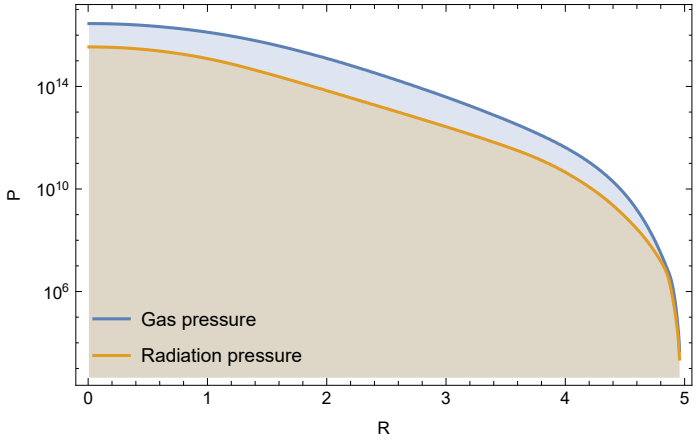
The n = 3 polytrope is a good fit near the radius of the star (r > 3). Neither polytrope fits well in other regions.

Q2)

Gas and radiation pressure vs radius

In[6]:=
```
ListLogPlot[Transpose[{10^Normal@data[ ;; , "logR"], #}] & /@
  {10^Normal@data[ ;; , "logPgas"], 10^Normal@data[ ;; , "logP"] - 10^Normal@data[ ;; , "logPgas"]},
 PlotRange → All, Joined → True, Frame -> True, Filling → Bottom, FrameLabel → {"R", "P"},
 PlotLegends → Placed[{"Gas pressure", "Radiation pressure"}, {Left, Bottom}]]
```

Out[6]=



a)

The radiation pressure never dominates for this star. (The question does not ask why it *doesn't* dominate, however this could be because the radiative temperature gradient assumption).

b)

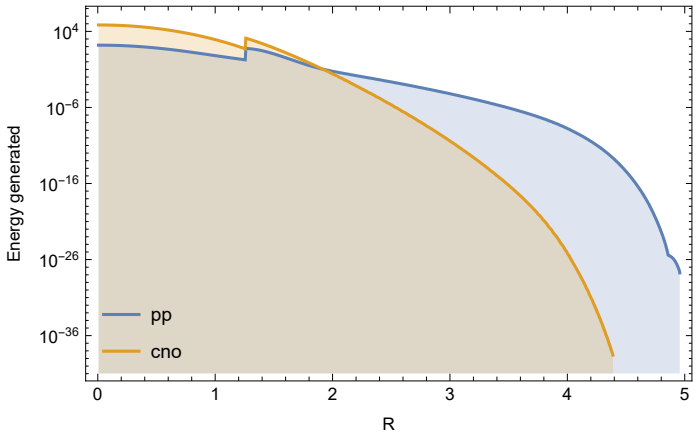
n ~ 1.5 corresponds to an ideal gas with low radiation whereas n ~ 3 corresponds to high radiation. However the star is closer to an n = 3 polytrope, despite the radiation pressure not dominating, because the Eddington model assumes a constant radiative temperature gradient.

Q3)

Energy generated in PP and CNO cycles

In[7]:=
```
ListLogPlot[Transpose[{10^Normal@data[All, "logR"], Normal@data[All, #]}] & /@ #,
  PlotRange → All, Filling → Bottom, Frame -> True, FrameLabel → {"R", "Energy generated"},
  PlotLegends → Placed[#, {Left, Bottom}], Joined → True] &@{"pp", "cno"}
```

Out[7]=



a)

Point where total energy generation reduces by 50%

In[8]:=
```
energy = Interpolation[
   Transpose@{10^Normal[data[All, "logR"]], Normal[Total /@ data[All, {"cno", "pp"}]]}];
FindRoot[energy@r == .5 energy@energy[[1, 1, 1]], {r, Sequence @@ energy[[1, 1]]}]
```

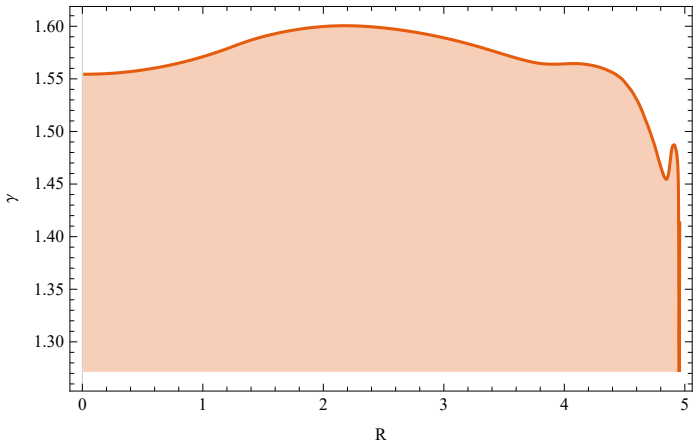
Out[9]=
```
{r → 0.4084886}
```

The energy generation is primarily due to the CNO cycle in this region as seen in the figure above.

Q4)

In[10]:=
```
ListLinePlot[Transpose[{10^Normal@data[All, "logR"], Normal@data[All, #]}],
  PlotRange → All, PlotTheme → "Scientific", Filling → Bottom, Frame -> True,
  PlotTheme → "Scientific", FrameLabel → {"R", "γ"}] &@"gamma1"
```

Out[10]=



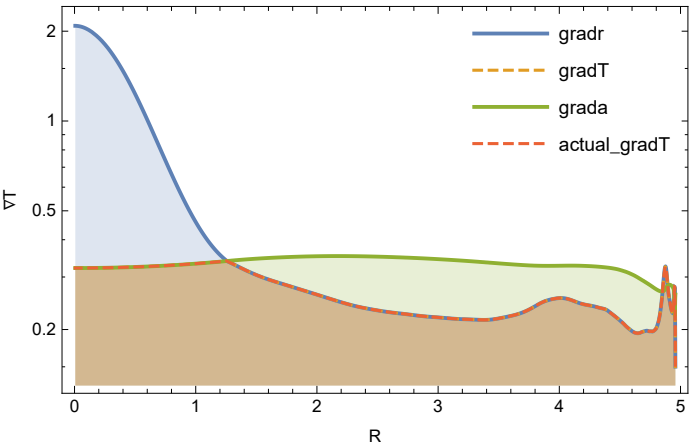
a)

The adiabatic index is always less than 5/3 for this star.
This is means that the equation of state is relativistic (more radiation).

Q5)

In[11]:=
```
ListLogPlot[Transpose[{10^Normal@data[All, "logR"], Normal@data[All, #]}] & /@ #,
  PlotRange → All, Filling → Bottom, PlotStyle → {Thick, Dashed}, Frame -> True,
  FrameLabel → {"R", "∇T"}, PlotLegends → Placed[#, {Right, Top}], Joined → True] &@
 {"gradr", "gradT", "grada", "actual_gradT"}
```

Out[11]=



a)

Yes, for r less than about 1.1 the star is convective as can be seen from the plot above (grada ~ gradT).
This is because the CNO cycle is dominant for this region (see Q3).

Where are all these physicists??

In[35]:=