HW 2 - ASTR540

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Q1)

a)

Let R be the radius of the Sun and d be the distance from the earth to the Sun, then:

In[95]:=
```
eqΩ = Ω == Pi R^2/d^2
```

Out[95]= $\Omega == \frac{\pi R^2}{d^2}$

Let FR be the flux at the surface of the sun, then flux at distance r from the Sun is given by:

In[96]:=
```
F[r_] = FR R^2/r^2
```

Out[96]= $\frac{FR\, R^2}{r^2}$

Flux at earth (Fd) is:

In[97]:=
```
eqFd = Fd == F[d]
```

Out[97]= $Fd == \frac{FR\, R^2}{d^2}$

Solving for FR and substituting the value of R in terms of Ω:

In[98]:=
```
sFRd = Solve[eqFd, FR][[1, 1]] /. Solve[eqΩ, R][[1]]
```

Out[98]= $FR \to \frac{Fd\, \pi}{\Omega}$

b)

Given angular diameter to be .57°:

In[99]:=
```
eqA = 2 R/d == .57 °
```

Out[99]= $\frac{2 R}{d} == 0.0099483767363677$

Solid angle subtended by sun:

In[100]:=
```
sΩ = Ω → Last[eqΩ /. Solve[eqA, R][[1]]]
```
Out[100]= $\Omega \to 0.000077731013066585$

c)

Substituting the value of flux at Earth:

In[101]:=
```
sFR = sFRd /. Fd -> 1.4 kW/m^2 /. sΩ
```
Out[101]= $FR \to 56\,582.688704419\ \mathrm{kW/m^2}$

Using Stefan-Boltzmann law and solving for T:

In[102]:=
```
Solve[T > 0 && FR == (1 σ) T^4 /. sFR, T]
```
Out[102]= $\{\{T \to 5620.4113614522\ \mathrm{K}\}\}$

d)

We have the following formula:

In[103]:=
```
Equal @@ sFRd
```
Out[103]= $FR == \frac{Fd\,\pi}{\Omega}$

Flux at the surface of Sun is given by:

In[104]:=
```
sF = FR -> Pi FormulaData[{"PlanckRadiationLaw", "Wavelength"}][[2]]
```
Out[104]= $FR \to \frac{2\,\pi\,h\,c^2}{\left(-1 + e^{\frac{1\,h\,c/k}{T\,\lambda}}\right)\lambda^5}$

Substituting above:

In[105]:=
```
eqT = Equal @@ sFRd /. sF
```
Out[105]= $\frac{2\,\pi\,h\,c^2}{\left(-1 + e^{\frac{1\,h\,c/k}{T\,\lambda}}\right)\lambda^5} == \frac{Fd\,\pi}{\Omega}$

Therefore we can solve the above equation for temperature.

Q3)

Defining number density (nd):

In[106]:=
```
nd = 0.1/pc^3 ;
```

Finding maximum distance (d):

In[107]:=
```
sd = NSolve[0.01" == 1 au / d, d][[1, 1]]
```

Out[107]= $d \to 2.062648062471 \times 10^7$ au

Number of stars in sphere of radius d:

In[108]:=
```
nd 4 / 3 Pi d^3 /. sd
```

Out[108]= 418 879.02048848

Q4)

e)

Defining function to find ratio of specific intensities:

In[109]:=
```
iRatio[τ_, b_] := (1 - Exp[-τ Sqrt[1 - b^2]]) / (1 - Exp[-τ])
```

Plotting ratio vs b for different τ:

Out[110]=