HW 4 - ASTR501

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Q1

The distance traveled through the atmosphere of height h at latitude θ is given by

In[166]:=
```
d == h / Sin[θ]
```

Therefore the visual extinction (proportional to distance) is

In[167]:=
```
h / Sin[θ] * constant
```

Q2

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a)

In[168]:=
```
sλ = λ -> FormulaData[{"PhotonWavelength", "Energy"}, {"E" -> 10.2 eV}][[2]]
```

Out[168]= $\lambda \to 1.21553 \times 10^{-7}$ m

---

b)

In[169]:=
```
sB = NSolve[{B12 / B21 == g2 / g1, A / B21 == 2 h ν^3 / c^2} /. {g2 → 6, g1 → 2,
    A → 6.2 × 10^8 per second} /. Solve[h ν == 10.2 eV, ν], {B12, B21}][[1]]
```

Out[169]= $\{B21 \to 2.80274 \times 10^{12}$ s/kg, $B12 \to 8.40823 \times 10^{12}$ s/kg$\}$

c)

In[170]:=
```
Solve[{x == N2 / (N1 + N2), N2 / N1 == g2 / g1 Exp[-E12 / (k T)]} /.
    {g2 → 6, g1 → 2, T -> 5000 K, E12 -> 10.2 eV}, {x, N1}][[1, 1, 2]]
```
Out[170]= 1.57026×10^{-10}

d)

Cross-section from textbook:

In[196]:=
```
sσ = σ -> B12 h ν0 / (4 π) ϕ[ν] //. {ν0 -> c / λ, sλ} /. sB // UnitConvert
```
Out[196]= $\sigma \to \left(1.09347 \times 10^{-6}\ \mathrm{m}^2/\mathrm{s}\right) \phi[\nu]$

e)

Substituting value of B12 in cross-section above:

In[172]:=
```
B12 h ν0 / (4 π) ϕ[ν] /.
  Solve[{B12 / B21 == g2 / g1, A / B21 == 2 h ν0^3 / c^2}, {B12, B21}][[1]] /. ν0 → c / λ
```
Out[172]= $\dfrac{A\, g2\, \lambda^2 \left(\frac{1}{8\pi} h\right) \phi[\nu]}{g1\, h}$

f)

Function

In[173]:=
```
f[ν_] = Exp[-(ν - ν0)^2 / (2 σ^2)] /. σ → ν0 5 km/s / c /. ν0 → c / λ /. sλ // UnitConvert
```
Out[173]= $e^{\left(\nu + -2.46635\times10^{15}\ \text{per second}\right)^2 \left(-2.95503\times10^{-22}\ \mathrm{s}^2\right)}$

Normalization

In[174]:=
```
Integrate[Exp[-a x^2], {x, -∞, ∞}, Assumptions → a > 0]
```
Out[174]= $\dfrac{\sqrt{\pi}}{\sqrt{a}}$

Therefore normalization constant = $1 \Big/ \sqrt{\pi/a}$, where -a is the coefficient of ν^2

In[175]:=
```
const = 1/Sqrt[π/(2.955*10^-22)]
```
Out[175]= 9.69848×10^{-12}

Value at line center

In[181]:=
```
ϕ0 = const f[c/λ /. sλ] /Hz
```
Out[181]= 9.69848×10^{-12} /Hz

---

g)

Collision strength (f)

In[177]:=
```
sf12 = Solve[A == 8 π^2 e^2 ν^2/(m_e c^3) g1/g2 f12 /.
    {g2 → 6, g1 → 2, A → 6.2×10^8 per second} /. ν → c/λ /. sλ, f12][[1, 1]]
```
Out[177]= f12 → 3.70292×10^{9} m/F

Finding density of atoms (N1) given $\tau = 1$

In[195]:=
```
Solve[1 == π e^2/(m_e c^2) λ^2 N1 ϕ0 f12 /. sf12 /. sλ, N1][[1, 1]] // UnitConvert
```
Out[195]= N1 → 1.91328×10^{39} per meter^3 per second

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h)

In[200]:=
```
1/(n σ) /. {n -> 1/cm^3, sσ} /. ϕ[ν] → ϕ0 // UnitConvert
```
Out[200]= 9.42955×10^{10} m