HW 2 - ASTR501

Created with Wolfram Mathematica 11.0 on February 1, 2016

Daniel George - dgeorge5@illinois.edu

Q1

Approximately flux is given by

In[78]:= $F \to I_\nu \operatorname{Cos}[\theta]\, \Delta\Omega$ /. $\Delta\Omega \to \frac{dA \operatorname{Cos}[\theta]}{r^2}$ /. $dA \to \frac{d^2 \pi}{4}$ /. $r \to L / \operatorname{Cos}[\theta]$ /. $L \to f / d$

Out[78]= $F \to \frac{d^4 \pi \operatorname{Cos}[\theta]^4 I_\nu}{4 f^2}$

Q2

---

a)

Luminosity

$4 \pi R_* \sigma T_*^4$

---

b)

Brightness

sB := Brightness -> $\frac{\sigma T_*^4}{\pi}$

Flux

Using spherical coordinates at a point on the disk with the axis passing through the center of the star.

In[82]:=
```
sF = Flux -> Integrate[Brightness Sin[θ] Cos[ϕ] Sin[θ], {θ, 0, ArcSin[R* / r]}, {ϕ, -π/2, π/2}]
```

Out[82]= $\text{Flux} \to \text{Brightness} \left(\text{ArcSin}\left[\frac{R_*}{r}\right] - \frac{R_* \sqrt{1 - \frac{R_*^2}{r^2}}}{r} \right)$

Solving for T_{disk}

In[84]:=
```
sT = Solve[sF /. sB /. Flux → σ T_disk^4 /. Rule → Equal, T_disk][[-1, 1]]
```

Out[84]= $T_{disk} \to \left(\frac{\text{ArcSin}\left[\frac{R_*}{r}\right] T_*^4}{\pi} - \frac{R_* \sqrt{\frac{r^2 - R_*^2}{r^2}}\, T_*^4}{\pi\, r} \right)^{1/4}$

Series expansion about R_*

In[28]:=
```
Series[T_disk /. sT, {R*, 0, 1}] // Simplify
```

Out[28]= $\left(\frac{2}{3\pi}\right)^{1/4} \left(\frac{T_*^4}{r^3}\right)^{1/4} R_*^{3/4} + O[R_*]^{7/4}$

---

c)

Equating energy absorbed to emitted

In[90]:=
```
Solve[(4 π σ R*^2 T*^4) / (4 π r^2) π R^2 == (4 π R^2) (σ T_eff^4), T_eff][[-1, 1]]
```

Out[90]= $T_{eff} \to \frac{\sqrt{R_*}\, T_*}{\sqrt{2}\, \sqrt{r}}$

The scaling law is different because the planet has more effective surface area.

---

d)

Twice the energy emitted by each side of the disk divided by luminosity of star.

In[95]:=
```
2 Integrate[2 π r T_disk^4 σ /. sT, {r, R*, ∞}, Assumptions → {R* > 0}] / ((4 π R*^2) σ T*^4)
```

Out[95]= $\frac{1}{4}$

---

e)

Larger. Bigger disk implies more absorption.

Q3

a)

In[97]:=
```
s0 = Series[Assuming[α > 0 && ν0 > 0 && ν_c > Δν / 2 && Δν > 0,
    Integrate[I_0 (ν/ν0)^α, {ν, ν_c - Δν / 2, ν_c + Δν / 2}]], {Δν, 0, 1}]
```

Out[97]= $\nu 0^{-\alpha}\, I_0\, \nu_c^{\alpha}\, \Delta\nu + O[\Delta\nu]^2$

b)

Observed Intensity

In[101]:=
```
sI = Solve[I_source / ν_source^3 == I_obs / ν^3 /. I_source -> I_0 (ν_source/ν0)^α /. ν_source → ν (1 + z), I_obs][[1, 1]] //
  PowerExpand
```

Out[101]= $I_{obs} \to (1+z)^{-3+\alpha}\, \nu^{\alpha}\, \nu 0^{-\alpha}\, I_0$

Surface brightness

Series expansion about $\Delta\nu$

In[104]:=
```
sZ = Series[Assuming[α > 0 && ν0 > 0 && ν_c > Δν / 2 && Δν > 0,
    Integrate[I_obs /. sI, {ν, ν_c - Δν/2, ν_c + Δν/2}]], {Δν, 0, 1}]
```

Out[104]= $(1+z)^{-3+\alpha}\, \nu 0^{-\alpha}\, I_0\, \nu_c^{\alpha}\, \Delta\nu + O[\Delta\nu]^2$

c)

In[61]:=
```
-2.5 Log10[s0 / sZ /. {z → 7, α → -1}]
```

Out[61]= $-9.0309 + O[\Delta\nu]^1$

Q4

First integrand

Substituting expression for specific intensity

In[106]:= $d\Omega\, d\nu\, I_\nu$ /. $I_\nu \to \frac{dE}{c\, dA\, dt\, d\nu\, d\Omega}$

Out[106]= $\frac{dE}{c\, dA\, dt}$

Second integrand

Substituting expressions for distribution function, energy, and volume

In[109]:= $d3p\, f\, \mathbb{E}$ /. $f \to \frac{dN}{d3p\, d3x}$ /. $dN \to \frac{dE}{h\, \nu}$ /. $\mathbb{E} \to h\, \nu$ /. $d3x \to c\, dA\, dt$

Out[109]= $\frac{dE}{c\, dA\, dt}$

Thus they are both identical.