HW 9 - ASTR404

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Q1)

a) Equation for $\dot{M}$

In[1]:= `eqR = D[M[t], t] == -4. × 10^-13 η (L / LSun) ((M[t] / R^2) / (MSun / RSun^2))^-1 (R / RSun)^-1 MSun`

Out[1]= $M'[t] == -\frac{4. \times 10^{-13}\, L\, MSun^2\, R\, \eta}{LSun\, RSun\, M[t]}$

b) Solving for M(t)

In[2]:= `solM = DSolve[{eqR, M[0] == M0}, M[t], t][[2, 1]]`

Out[2]= $M[t] \to \left(8.944272 \times 10^{-7} \sqrt{\left(1.25 \times 10^{12}\, LSun\, M0^2\, RSun - 1.\, L\, MSun^2\, R\, t\, \eta\right)}\right) / \left(\sqrt{LSun}\, \sqrt{RSun}\right)$

c) Finding t such that M(t) = .6 $M_\odot$

In[3]:= `t → Solve[ .6 M⊙ == solM[[2]] /. {LSun -> 1 L⊙N, RSun -> 1 R⊙N, MSun -> 1 M⊙, L → 7000 L⊙N, R -> 310 R⊙N, M0 -> 1 M⊙, η → 1}, t][[1, 1, 2]] yr`

Out[3]= $t \to 368663.6\ yr$

Q2)

a) Finding central density and temperature

In[4]:= `s2 = {ρc -> UnitConvert[5.991 × 3 .98 M⊙ / (4 π .0086 R⊙N ^3)], Tc -> 4 × 10^7 K ((.056 L⊙N / 1 L⊙N) / (.98 M⊙ / 1 M⊙))^(2/7)}`

Out[4]= $\{\rho_c \to 1.301328 \times 10^{10}\ kg/m^3,\ T_c \to 1.765657 \times 10^7\ K\}$

b)

Luminosity as a function of X

In[5]:= $sL = L == 10^{-36} \left(.98\, M_\odot \middle/ \left(4/3\, \pi\, .0086\, \mathcal{R}_\odot^{N}\,{}^\wedge 3\right)\right) X^\wedge 2\, T_c{}^\wedge 4\; .98\, M_\odot\; W \middle/ \left(kg^2/m^3\; K^4\right)$ /. s2

Out[5]= $L == X^2 \left(4.113865 \times 10^{32}\, W\right)$

Upper bound of X

In[6]:= Solve[sL /. L -> $.056\, \mathcal{L}_N^\odot$, X][[2, 1, 2]]

Out[6]= 0.0002288688

Q3)

a) Velocity at which $\gamma = 1.1$

In[7]:= find velocity if lorentz factor is 1.1

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FormulaData["LorentzFactor", {"γ" -> 1.1}]
```

Out[7]= $v == -1.248929 \times 10^8\, m/s \;||\; v == 1.248929 \times 10^8\, m/s$

(Soon I'll be able to solve every question like this ☺)

b) Density of CO white dwarf

Solving for ρ such that momentum is Fermi momentum

In[8]:= $s\rho = \text{Solve}\left[\gamma\, m_e\, v == \left(\frac{3\, n\, h^3}{8\, \pi}\right)^{1/3} /.\; n \to \frac{\rho}{\mu\, m_p} /.\; \{\gamma \to 1.1,\; v \to 1.248929 * 10^\wedge 8\, m/s,\; \mu \to 2\},\; \rho\right][[1, 1]]$

Out[8]= $\rho \to 1.88815 \times 10^8\, kg/m^3$

c) Mass of the white dwarf

Solving the mass-volume relation for M

In[9]:=
```
sM = Solve[(M/ 1 M☉) M/ (4/3 π ρ)/(1 R☉^N)^3 == 1.98 × 10^-6 /. sρ, M][[2, 1]]
```

Out[9]= $M \to 1.023962 \times 10^{30}$ kg

Q4)

Computing Eddington Luminosity

Assuming opacity ~ .02 and using previous mass:

In[10]:=
```
LEdd = UnitConvert[4 π G M c / 0.02 m^2/kg /. sM, "Watts"]
```

Out[10]= 1.287288×10^{31} W

Time span

Dividing energy produced by luminosity:

In[11]:=
```
UnitConvert[10^-4 * .007 1 M☉ c^2/LEdd, "Years"]
```

Out[11]= 308.1544 yr

This is much greater than a couple of years. Therefore not all the Hydrogen gets fused.