HW 5 - ASTR404

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Q1) Solving Lane-Emden Equation

a) Euler's method

Function to update one step

In[1]:=
```
next[Δξ_, n_][{ξ_, θ1_, θ2_}] := {ξ + Δξ, θ1 + θ2 Δξ, (ξ / (ξ + Δξ))^2 (θ2 - θ1^n Δξ)}
```

Initial conditions at $\xi = \Delta\xi$

In[2]:=
```
θ11@Δξ_ := 1 - 1/6 Δξ^2; θ21@Δξ_ := -1/3 Δξ
```

Function to iterate with n = 1.5

In[3]:=
```
steps@Δξ_ := {{"ξ", "θ", "dθ/dξ"}, {0, 1, 0}}~Join~
  NestWhileList[next[Δξ, 1.5], {Δξ, θ11@Δξ, θ21@Δξ}, #[[2]] > 0 &]
```

b) $\Delta\xi = \pi / 4$

Table of steps

In[4]:=
```
Grid[steps[π / 4.], Frame → All]
```

Out[4]=

ξ	θ	dθ/dξ
0	1	0
0.785398	0.897192	-0.261799
1.5708	0.691575	-0.232312
2.35619	0.509117	-0.304005
3.14159	0.270353	-0.331489
3.92699	0.0100016	-0.282812
4.71239	-0.212118	-0.196943

Finding first zero

In[5]:=
```
sξ11 = Quiet@Solve[Interpolation[steps[π / 4][[-2 ;;, ;; 2]]]@ξ11 == 0, ξ11][[1, 1]]
```

Out[5]= ξ11 → 3.96236

c) $\Delta\xi = \pi / 8$

Table of steps

In[6]:=
```
Grid[steps[π / 8.], Frame → All]
```

Out[6]=

ξ	θ	dθ/dξ
0	1	0
0.392699	0.974298	-0.1309
0.785398	0.922894	-0.127139
1.1781	0.872966	-0.211247
1.5708	0.79001	-0.298995
1.9635	0.672595	-0.367834
2.35619	0.528147	-0.405868
2.74889	0.368763	-0.408927
3.14159	0.208178	-0.380413
3.53429	0.0587897	-0.330045
3.92699	-0.0708186	-0.27187

Finding first zero

In[7]:=
```
sξ12 = Quiet@Solve[Interpolation[steps[π / 8][[-2 ;;, ;; 2]]]@ξ12 == 0, ξ12][[1, 1]]
```

Out[7]= ξ12 → 3.71242

d) Richardson extrapolation

First order expression for ξ_1

In[8]:=
```
sξ1 = Simplify@Solve[{ξ1 == ξ11 + α Δξ1, ξ1 == ξ12 + α Δξ2}, {ξ1, α}][[1, 1]]
```

Out[8]= $\xi 1 \to \frac{-\Delta\xi 2\ \xi 11 + \Delta\xi 1\ \xi 12}{\Delta\xi 1 - \Delta\xi 2}$

This is equivalent to the given expression. Therefore this gives an estimate of ξ_1 which is valid up to first order.

Substituting $\xi^{(1)}{}_1$ and $\xi^{(2)}{}_1$

In[9]:=
```
sξ1 /. {sξ12, sξ11, Δξ1 → π / 4, Δξ2 → π / 8}
```

Out[9]= ξ1 → 3.46248

This is an estimate of the true zero with error of order $(\Delta\xi)^2$.

Q2) Convective Stability

Solution for n = 1 (given)

In[10]:=
```
sθ := θ → Sin@ξ / ξ
```

Equations for pressure & density

In[11]:=
```
sPρ := {P → K ρc^2 θ^2, ρ → ρc θ}
```

Finding temperature of ideal gas

In[12]:=
```
sT = T → P μ mH / (ρ kB) /. sPρ /. sθ // Simplify
```

Out[12]= $T \to \frac{\mathrm{K}\,\mu\,\rho_c\,m_H \sin(\xi)}{\xi\,k_B}$

Finding $\partial T/\partial$r (LHS)

In[13]:=
```
LHS = D[T /. sT, ξ] / α // Simplify
```

Out[13]= $\frac{\mathrm{K}\,\mu\,\rho_c\,m_H\,(\xi\cos(\xi)-\sin(\xi))}{\alpha\,\xi^2\,k_B}$

Finding adiabatic $\partial T/\partial$r (RHS)

In[14]:=
```
RHS = -(1 - 1/γ) μ mH G ρc α^3 / kB Integrate[4 π ξ Sin@ξ, {ξ, 0, ξ}] / (α^2 ξ^2) /. γ → 5/3
```

Out[14]= $-\frac{8\,\pi\,\alpha\,G\,\mu\,\rho_c\,m_H\,(\sin(\xi)-\xi\cos(\xi))}{5\,\xi^2\,k_B}$

Showing |LHS| > |RHS|

Subtracting RHS from LHS:

In[15]:=
```
Abs@LHS - Abs@RHS /. α -> Sqrt[2 K / (4 π G)] // Simplify
```

Out[15]= $\frac{1}{5}\sqrt{2\,\pi}\left|\frac{G\sqrt{\frac{\mathrm{K}}{G}}\,\mu\,(\xi\cos(\xi)-\sin(\xi))\,m_H\,\rho_c}{\xi^2\,k_B}\right|$

This is always positive, therefore |$\partial T/\partial$r| > |$\partial T/\partial$r| $_{\text{adiabatic}}$. Hence the solution is convectively unstable.

Q3) Energy Released in Reactions

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a) Exothermic

In[16]:=
```
-UnitConvert[
  (magnesium-24 (isotope) [ atomic mass ] - 2 carbon-12 (isotope) [ atomic mass ]) c ^2, "MeV"]
```

Out[16]= 13.9336 MeV

b) Endothermic

In[17]:=
```
-UnitConvert[(oxygen-16 (isotope) [atomic mass] +
    2 helium-4 (isotope) [atomic mass] - 2 carbon-12 (isotope) [atomic mass]) c^2, "MeV"]
```

Out[17]= -0.11283 MeV

c) Exothermic

In[18]:=
```
-UnitConvert[(oxygen-16 (isotope) [atomic mass] + helium-4 (isotope) [atomic mass] -
    hydrogen (isotope) [atomic mass] - fluorine-19 (isotope) [atomic mass]) c^2, "MeV"]
```

Out[18]= 8.113670 MeV

Q4) Range of Stellar Lifespans

a) Lowest mass star

In[19]:=
```
UnitConvert[.007 * .072 Sun (star) ["Mass"] c^2 / (10^-4.3 Sun (star) ["Luminosity"]), "Years"]
```

Out[19]= 1.48095×10^{14} yr

b) Highest mass star

In[20]:=
```
UnitConvert[.007 * .1 * 85 Sun (star) ["Mass"] c^2 / (10^6.006 Sun (star) ["Luminosity"]), "Years"]
```

Out[20]= 864 228. yr

Q5) Lifespan of Sun

Energy output divided by luminosity

In[21]:=
```
UnitConvert[Sun (star) ["Mass"] / hydrogen (element) [atomic mass]
   10 eV / Sun (star) ["Luminosity"], "Years"]
```

Out[21]= 1.569×10^{5} yr

Therefore most of the Sun's energy cannot be from chemical reactions since we know that the lifespan of the sun is many orders of magnitudes greater than the above result.