HW 1 - ASTR510

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Q1)

a)

Let $\sigma = \sigma_\theta$

In[549]:=
```
eqσ := σ -> 1.22 λ/Diameter
```

b)

In[545]:=
```
Gᵥ = Integrate[Iᵥ θ Exp[-θ^2/(2 σ^2)]/(2 π σ^2), {θ, 0, R/d}, {ϕ, 0, 2 π}] // Expand
```

Out[545]= $I_\nu - e^{-\frac{R^2}{2 d^2 \sigma^2}} I_\nu$

Note that the 2nd term vanishes as $\sigma \to 0$ (small beam), which implies G_ν is independent of d in this limit.

c)

Taylor series about R = 0

In[547]:=
```
Series[Gᵥ, {R, 0, 2}] // Normal
```

Out[547]= $\frac{R^2 I_\nu}{2 d^2 \sigma^2}$

Therefore as R → 0 (corresponding to beam being larger than angular size of source) G_ν is proportional to $\frac{1}{d^2}$

d)

Substituting result from part a in above:

In[552]:=
```
Series[Gν, {R, 0, 2}] /. eqσ // Normal
```

Out[552]=
$$\frac{0.335931\ \text{Diameter}^2\ R^2\ I_\nu}{d^2\ \lambda^2}$$

Therefore G_ν would increase by a factor of 4 if diameter is doubled.

Extra

Expanding up to 4th order in R and solving for d

In[557]:=
```
Solve[Normal[Series[Gν, {R, 0, 4}]] == 0, d][[-1, 1]]
```

Out[557]=
$$d \to \frac{R}{2\,\sigma}$$

This is approximately the distance when the $\frac{1}{d^2}$ contribution is equal to the $\frac{1}{d^4}$ contribution. Farther than this distance, the inverse square law would dominate.

Q2)

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a)

In[561]:=
```
I[ν_] := 2 h ν^3 / c^2 / (Exp[h ν / (k 5780 K)] - 1)
```

In[565]:=
```
I[10^14 Hz] // UnitConvert
```

Out[565]=
$1.13944 \times 10^{-8}\ \text{kg}/\text{s}^2$

$= 1.13944 \times 10^{-5}\ g/s^2$

---

b)

In[571]:=
```
n[ν_] := 1 / (Exp[h ν / (k 5780. K)] - 1)
```

In[573]:=
```
n /@ {10^14 Hz, 10^12 Hz, 10^15 Hz} // UnitConvert
```

Out[573]=
```
{0.772767, 119.936, 0.000247786}
```

---

c)

In[582]:=
```
1 / (4 π) Integrate[θ I[1 × 10^14 Hz], {θ, 0, π / 2}, {ϕ, 0, 2 π}]
```

Out[582]=
$7.0287 \times 10^{-9}\ \text{kg}/\text{s}^2$

d)

Half the energy inside uniform radiation

```
In[598]:= 1/2 × 4/ c σ 5780 K ^4 // UnitConvert

Out[598]= 0.42221 kg/(m s^2)
```

f)

```
In[599]:= 1/(4 π) Integrate[Cos[θ] θ I[ 1 × 10^14 Hz ], {θ, 0, π/2}, {ϕ, 0, 2 π}]

Out[599]= 3.25195 × 10^-9 kg/s^2
```

g)

```
Integrate[Cos[θ] θ I[ν], {θ, 0, π/2}, {ϕ, 0, 2 π}, {ν, 0, ∞}]
```

i)

```
UnitConvert[4/ c σ 2.725 K ^4, "eV/cm^3"]

0.260379 eV/cm^3
```