HW 10 - ASTR404

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Daniel George - dgeorge5@illinois.edu

Q1)

Binding energy of core

Assuming the core is a sphere of constant density.

```
In[39]:= coreBE = -3/5 G 1 M☉ ^2/ 10000 km // UnitSimplify
```

```
Out[39]= -1.583 × 10^43 J
```

Total radius of the star

Assuming density is constant and solving for total radius.

```
In[40]:= totalR = NSolve[1 M☉ / 10000 km ^3 == 10 M☉ /R^3, R][[1, 1, 2]]
```

```
Out[40]= 2.15443 × 10^7 m
```

Initial binding energy of the shell

Subtracting the core binding energy from the total binding energy.

```
In[41]:= shellBE = -3/5 G 10 M☉^2/totalR - coreBE // UnitSimplify
```

```
Out[41]= -7.191 × 10^44 J
```

This must be equal to the energy absorbed from the neutrinos, which is 1% of total energy released.

Computing final radius

Equating final and initial total energies.

```
In[106]:= NSolve[-3/5 G 1 M☉ ^2/Rf - 100 shellBE == coreBE + shellBE, Rf][[1, 1]]
```

```
Out[106]= Rf → 2180. m
```

Comparing to Schwarzschild radius

schwarschild radius solar mass

Out[96]= r == 2953.190051923170 m

Therefore, the core is probably a black-hole.

Q2)

Number of nuclei at time t

```
In[122]:= nN[t_] := 2^(-t/τ) n0
```

Relation between flux, number of nuclei, and magnitude

$F2 / F1 == N2 / N1 == 10^{0.4\,\Delta m}$

Solving for change in magnitude after a day

```
In[128]:= Quiet@Solve[nN[t + 1] / nN[t] == 10^(.4 Δm) /. τ → 77.7 // Simplify, Δm][[1, 1]]
Out[128]= Δm → -0.00968565
```

This is rate of decay in units of magnitudes per day.

Q3)

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a) Linear dimensions

Assuming x and y are the breadth and length respectively.

```
In[59]:= sd = First@NSolve[{x / 2 kpc == 2', y / 2 kpc == 4'}, {x, y}]
```

Out[59]= $\{x \to 3.590349 \times 10^{16}\ \mathrm{m},\ y \to 7.180698 \times 10^{16}\ \mathrm{m}\}$

---

b) Age

Solving for time given mean radius and assuming constant velocity.

```
In[114]:= NSolve[1500 km/s == Mean[{x, y} / 2] / t /. sd, t][[1, 1]]
```

Out[114]= $t \to 1.795174 \times 10^{10}\ \mathrm{s}$

Q4)

Neutron degeneracy pressure at center

Assuming a degenerate non-relativistic neutron gas.

In[75]:= h ^2/ (20 1 m_n ^8/3) (3/π)^2/3 ρc^5/3 /. ρc -> 1.5 × 10^18 kg/m^3 // UnitSimplify

Out[75]= 1.057529×10^{34} Pa

Central pressure of Sirius B

Assuming an n = 1.5 polytrope.

In[76]:= .206 (4 π)^1/3 G M^2/3 ρc^4/3 /. {M -> .98 $M_\odot$, ρc → 5.991 × 3 0.98 $M_\odot$ / (4 π 0.0086 $\mathcal{R}_\odot^N$ ^3)} // UnitSimplify

Out[76]= 1.526403×10^{23} Pa

This is much smaller than that of the neutron star.

Q5)

Finding change in radius

The moment of inertia is proportional to the radius and angular velocity is inversely proportional to time period. Then conservation of angular momentum implies:

In[111]:= NSolve[{R1^2 / P1 == (R1 - ΔR)^2 / P2, (P1 - P2) == 10^-8 P1} /. R1 -> 10 km , {ΔR, P2}][[1, 1]]

Out[111]= ΔR → 0.00005 m