HW 10 - ASTR540

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Q1

Distance for obtaining flux f0

In[36]:=
```
sd = Last@Solve[f0 == L / (4 π d^2), d]
```

Out[36]= $\left\{d \to \frac{\sqrt{L}}{2 \sqrt{f0} \sqrt{\pi}}\right\}$

Volume of sphere with radius d:

In[38]:=
```
Volume@Ball[{0, 0, 0}, d] /. sd
```

Out[38]= $\frac{L^{3/2}}{6\, f0^{3/2} \sqrt{\pi}}$

Given that N(f>f0) is proportional to the above.

Q3

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a)

Solving for M(r) in terms of velocity dispersion:

In[220]:=
```
sM = Solve[vc^2 == G M[r] / r, M[r]][[1, 1]]
```

Out[220]= $M[r] \to \frac{r\, vc^2}{G}$

Equating kinetic energy to potential energy to find escape velocity:

In[233]:=
```
eqv = 1 / 2 ve^2 == Integrate[G M[r] / r^2 /. sM, {r, r0, R}, Assumptions → {G > 0, R > r0 > 0}] +
   Integrate[(G M[r] /. sM /. r → R) / r^2, {r, R, ∞}, Assumptions → {G > 0, R > r0 > 0}]
```

Out[233]= $\frac{ve^2}{2} == vc^2 + vc^2 \, \mathrm{Log}\left[\frac{R}{r0}\right]$

This is the required relation.

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b)

Lower limit of R:

In[196]:=
```
sR = R -> UnitConvert[NSolve[Vesc^2 == 2 Vc^2 (1 + Log[R / r]) /.
    {Vc -> 240 km/s , r -> 8 kpc , Vesc -> 440 km/s }, R][[1, 1, 2]], "Kiloparsecs"]
```

Out[196]= $R \to 15.7998\ \text{kpc}$

Lower limit of galaxy mass:

In[204]:=
```
UnitConvert[Solve[Vesc^2 == 2 G M / R /. sR /. Vesc -> 440 km/s , M][[1, 1, 2]], "SolarMass"]
```

Out[204]= $3.55611 \times 10^{11}\ M_{\odot}$

Q4

In[247]:=
```
sn = n -> M / (m π r^2 2 h)
```

Out[247]= $n \to \frac{M}{2\, h\, m\, \pi\, r^2}$

In[248]:=
```
sσ = σ -> π (θ d)^2 /. θ^2 → 4 G m / c^2 d / (l d)
```

Out[248]= $\sigma \to \frac{4\, d^2\, G\, m\, \pi}{c^2\, l}$

Since l = 2 d = r

In[251]:=
```
Num = n σ l /. sn /. sσ /. l → 2 d /. d → r / 2
```

Out[251]= $\frac{G\, M}{2\, c^2\, h}$

Q5

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a)

In[186]:=
```
sθ = θ → Sqrt[4 G M / c^2 d / (d 2 d)]
```

Out[186]= $\theta \to \sqrt{2}\ \sqrt{\frac{G\, M}{c^2\, d}}$

In[188]:=
```
sd = Last@Solve[vc^2 == G M / (d θ) /. sθ, d]
```

Out[188]= $\left\{d \to \frac{c^2\, G\, M}{2\, vc^4}\right\}$

In[205]:=
```
sθ2 = sθ /. sd // PowerExpand
```

Out[205]= $\theta \to \frac{2\, vc^2}{c^2}$

Numerical value of θ

In[212]:=
```
sθ3 = sθ2 /. {vc -> 300 km/s , c -> c} // N
```

Out[212]= $\theta \to 2.00277 \times 10^{-6}$

Value of R

In[211]:=
```
R = d θ /. sθ3 /. d -> 0.5 Gpc
```

Out[211]= 1.00139×10^{-6} Gpc

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b)

In[215]:=
```
n σ l /. {l → 1 Gpc , σ → π R^2, R -> 1 kpc , n -> 0.01/Mpc^3 }
```

Out[215]= 0.000031503