HW 11 - ASTR404

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I)

Velocity of Alice and distance to τ Ceti

In[1]:=
```
v = 0.95 c ;
d = 11.7 ly ;
```

Lorentz factor of Alice

In[3]:=
```
γ = FormulaData["LorentzFactor", {"v" -> 0.95 c }] // Last
```
Out[3]= 3.202563

Function to convert time to years

In[4]:=
```
yr = UnitConvert[#, "Years"] &;
```

---

a)

In[5]:=
```
d / v // yr
```
Out[5]= 12.32422 yr

---

b)

In[6]:=
```
d / γ / v // yr
```
Out[6]= 3.848238 yr

c)

In[7]:=
```
2 d / γ / v // yr
```
Out[7]=
```
7.696476 yr
```

d)

In[8]:=
```
2 d / v - 2 d / γ / v // yr
```
Out[8]=
```
16.95197 yr
```

2)

a)

Radius of curvature of photon

In[9]:=
```
sRc = Rc -> c ^2 / g //. {g -> G M / R^2, M -> 1.4 M☉ , R → 14 km } // UnitConvert
```
Out[9]=
```
Rc → 94812.73 m
```

Ratio with radius of neutron star

In[10]:=
```
Rc / 14 km /. sRc
```
Out[10]=
```
6.772338
```

The radius of curvature is within an order of magnitude of the radius of the star. So gravitational effects could be important.

b)

In[11]:=
```
Solve[Δt == Δt∞ Sqrt[1 - 2 G M / (R c ^2)] /. {Δt -> 1 h , M -> 1.4 M☉ , R → 14 km }, Δt∞][[1, 1]]
```
Out[11]=
```
Δt∞ → 1.191252 h
```

3)

Equations for a binary system

oP is the orbital period, mS and mC are the masses of the primary and secondary, dS and dC are the distances to the center of mass, vS and vC are the actual velocities, whereas vSr and vCr are the observed radial velocities.

In[12]:=
```
eqs = {2 π dC / oP == vC (*Time period of secondary*),
    2 π dS / oP == vS (*Time period of primary*),
    vS mS == vC mC (*Conservation of momentum*),
    mS * vS^2 / dS == G mC mS / (dS + dC) ^2 (*Centripetal force equals gravity*)
   } /. {vS -> vSr / Sin@i, vC -> vCr / Sin@i};
```

Solving the equations

In[13]:=
```
sol = Quiet@Solve[eqs /. {oP → .3226 days , vSr → 457 km/s , vCr → 43 km/s }, {mS, mC, dS, dC}][[2]]
```

Out[13]= $\{mS \to Csc[i]^3\ (7.145215 \times 10^{29}\ kg),\ mC \to Csc[i]^3\ (7.593868 \times 10^{30}\ kg),$
$dS \to Csc[i]\ (2.027283 \times 10^{6}\ km),\ dC \to Csc[i]\ (190750.9\ km)\}$

---

a)

In[14]:=
```
mC^3 Sin[i]^3 / (mS + mC) ^2 /. sol // Simplify
```

Out[14]= 6.343887×10^{30} kg

This means that the mass of the compact object must be greater than the above result.

---

b)

In[15]:=
```
mC /. sol /. i → 90 °
```

Out[15]= 7.593868×10^{30} kg

---

c)

In[16]:=
```
mC /. sol /. i → 45 °
```

Out[16]= 2.14787×10^{31} kg

4)

a)

Relation between time intervals

In[17]:=
```
st := Δt -> Δt∞ Sqrt[1 - 2 G M / (R c^2)]
```

Relation between frequencies

In[18]:=
```
sν := ν -> ν∞ / Sqrt[1 - 2 G M / (R c^2)]
```

Ratio of luminosities

Since the number of photons emitted is the same:

In[19]:=
```
sL = Solve[L / L∞ == (n h ν / Δt) / (n h ν∞ / Δt∞) /. st /. sν, L∞][[1, 1]] // Simplify
```

Out[19]= $L\infty \to L - \frac{2\,G\,L\,M}{c^2\,R}$

b)

Since T is inversely proportional to λ, which in turn is inversely proportional to ν:

In[20]:=
```
sT = Solve[T / T∞ == ν / ν∞ /. sν, T∞][[1, 1]]
```

Out[20]= $T\infty \to \sqrt{1 - \frac{2\,G\,M}{c^2\,R}}\;T$

c)

Taking ratio of luminosities using Stefan-Boltzmann law and solving for R∞:

In[21]:=
```
Solve[L / L∞ == 4 π R^2 σ T^4 / (4 π R∞^2 σ T∞^4) /. sT /. sL, R∞][[2, 1]]
```

Out[21]= $R\infty \to \frac{c\,R^{3/2}}{\sqrt{-2\,G\,M + c^2\,R}}$

This is equivalent to the given expression.