HW 8 - ASTR501

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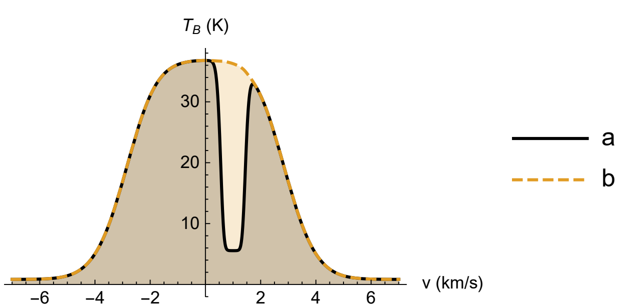
In[12]:=
```
Unprotect@Quantity; Quantity[0. | 0, _] = 0; Protect@Quantity;
SetOptions[Plot, {Filling → Bottom, ImageSize → 250, AxesLabel → {"v (km/s)", "T_B (K)"}}];

CO[nH_] := 2 nH 6.6 × 10^-5 /cm^3 ; ν0 = 115.272 GHz ;

B0 = Solve[h ν0 == k B J (J + 1) / 2 - 0 /. J → 1, B][[1, 1, 2]];
g[j_] := 2 j + 1;
f[T_, j_] := g[j] Exp[- B0 / T j (j + 1) / 2] / Sum[g[i] Exp[- B0 i (i + 1) / 2 / T], {i, 0, 8}];
ϕ = Exp[- (ν - (1 + vz / c) ν0) ^2 / (2 σ^2)] / (Sqrt[2 π] σ);

n@i_ := f[T, i - 1] CO[nH];   σν[v_] := v / c ν0;   A21 = 7.166 × 10^-8 Hz ;

{B21, B12} =
  NSolve[{A21 == 2 h ν0^3 / c ^2 b21, g@0 / g@1 == b21 / b12}, {b12, b21}][[1, ;; , 2]];
jν[nH_, vz_, T_, σ_] = n@2 A21 h ν / (4 π) ϕ;
αν[nH_, vz_, T_, σ_] = h ν / (4 π) (n@1 B12 - n@2 B21) ϕ;
TB[d_, nH_, vz_, Δv_, T_, nH2_: 0, vz2_: 1, Δv2_: 1, T2_: 1] := ParametricNDSolveValue[
    UnitConvert@{Iν'@z / 1.0 pc == - (αν[nH, vz, T, σν@Δv] + αν[nH2, vz2, T2, σν@Δv2]) Iν@z +
          (jν[nH, vz, T, σν@Δv] + jν[nH2, vz2, T2, σν@Δv2]),
        Iν[-d] == 2 h ν^3 / c ^2 / (Exp[h ν / (k 2.725 K)] - 1)} /. Quantity[x_, _] :→ x,
    QuantityMagnitude@UnitConvert[c ^2 / (2 k) / ν^2] Iν@0, {z, -d, 0 }, ν,
    MaxStepFraction → 0.0001] @QuantityMagnitude[(1. + # 1. km/s / c) ν0, 1 Hz ] &
```

a) and b)

In[23]:=
```
Plot[Evaluate@
  {TB[8, 500 UnitBox[(z + 5) / 6], 0, 1.5 km/s , 40 K , 50 UnitBox[z + 1], 1 km/s , .2 km/s , 8 K ]@x,
   TB[8, 500 UnitBox[(z + 3) / 6], 0, 1.5 km/s , 40 K , 50 UnitBox[z + 7], 1 km/s , .2 km/s , 8 K ]@x},
 {x, -7, 7}, PlotStyle → {Black, Dashed}, PlotLegends → {"a", "b"}]
```

Out[23]=



Absorption occurs when the colder cloud is between the observer and the hot cloud.

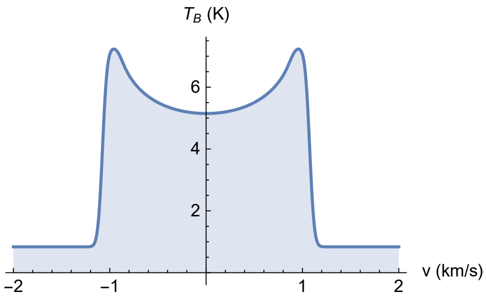
c) and d)

In[24]:=
```
σv = Solve[3 × 1 / 2 mass 1 CO vx^2 == 3 / 2 k 10 K , vx][[2, 1, 2]]
```

Out[24]= 54.483 m/s

In[25]:=
```
Plot[Evaluate@TB[1, 50, Sin[2 π z] 1. km/s , σv, 10 K ]@x, {x, -2, 2}]
```

Out[25]=



In[26]:=

The peaks correspond to the extrema in the sine curve, where most of the velocities are concentrated.

e) No. There can also be dips due to absorption of cool gas or peculiar motion of low density gas.

f)

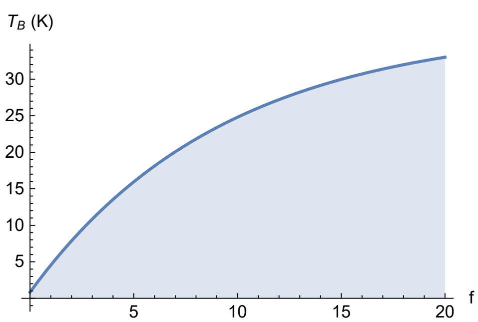
In[27]:=
```
c^2/(2 k)/ν^2 2 h ν^3/c^2/(Exp[h ν/(k 2.725 K)] - 1) /. ν -> 115 GHz // UnitConvert
```

Out[27]=
```
0.838913 K
```

g)

In[28]:=
```
Plot[TB[1, x 50, 0, 1 km/s , 40 K ]@0, {x, 0, 20}, AxesLabel → {"f", "T_B (K)"}]
```

Out[28]=

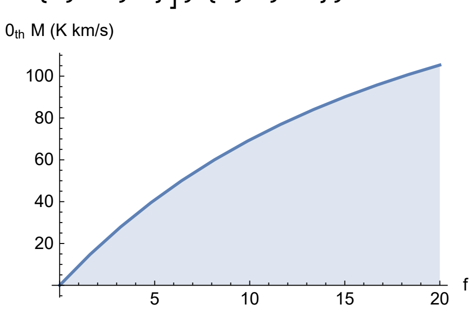


Initially the profile is linear but then saturates below the thermal temperature due to high optical depth.

h)

In[29]:=
```
Plot[NIntegrate[Evaluate@(TB[1, x 50, 0, 1 km/s , 40 K ]@v - TB[1, 50, 0, 1 km/s , 40 K ]@5),
   {v, -5, 5}], {x, 0, 20}, AxesLabel → {"f", "0th M (K km/s)"}, PlotPoints → 4, MaxRecursion → 2]
```

Out[29]=



The curve of growth is initially linearly proportional to the column density of gas but then saturates due to high optical depth.