HW 5 - ASTR501

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Q I

Let Ex and Ey be the x and y component of the electric fields and ExC and EyC be their complex conjugates respectively, then we get

```
In[8]:= sE := {Ex -> ϵ0 (Cos[χ] Cos[β] Exp[-I ω t] + Sin[χ] Sin[β] Exp[-I (ω t + π / 2)]),
   Ey -> ϵ0 (Sin[χ] Cos[β] Exp[-I ω t] - Cos[χ] Sin[β] Exp[-I (ω t + π / 2)]),
   ExC -> ϵ0 (Cos[χ] Cos[β] Exp[I ω t] + Sin[χ] Sin[β] Exp[I (ω t + π / 2)]),
   EyC -> ϵ0 (Sin[χ] Cos[β] Exp[I ω t] - Cos[χ] Sin[β] Exp[I (ω t + π / 2)])}
```

Stokes parameters

```
In[13]:= sI = I → Ex ExC + Ey EyC /. sE // Simplify
Out[13]= I → ϵ0^2

In[12]:= sQ = Q → Ex ExC - Ey EyC /. sE // Simplify
Out[12]= Q → Cos[2 β] Cos[2 χ] ϵ0^2

In[17]:= sU = U → Ex EyC + Ey ExC /. sE // Simplify
Out[17]= U → Cos[2 β] Sin[2 χ] ϵ0^2

In[18]:= sV = V → (Ex EyC - Ey ExC) / I /. sE // Simplify
Out[18]= V → -Sin[2 β] ϵ0^2
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b)

```
In[25]:= {I, Q, U, V} / ϵ0^2 /. {sI, sQ, sU, sV} /. β → π / 4
Out[25]= {1, 0, 0, -1}
```

Therefore it is a circularly polarized wave since Q = U = 0 and V ≠ 0

c)

In[26]:=
```
{I, Q, U, V} / ϵ0^2 /. {sI, sQ, sU, sV} /. {β → 0, χ → π / 4}
```

Out[26]= {1, 0, 1, 0}

$\beta \to 0$, $\chi \to \pi/4$ works.

Q2

Using formula for Rosseland mean opacity and substituting expressions for κ_ν, B_ν, and j_ν

$$1/\kappa_R \text{ -> } h^2\ \pi / (2\ c^2\ k\ T^5\ \sigma)\ \text{Integrate}\left[\frac{1}{\kappa_\nu}\nu^4 \frac{\text{Exp}\left[\frac{h\nu}{kT}\right]}{\left(\text{Exp}\left[\frac{h\nu}{kT}\right]-1\right)^2}\ //.\ \left\{\kappa_\nu \to \frac{j\nu}{\rho B_\nu},\ B_\nu \to \frac{2 h \nu^3/c^2}{\text{Exp}[h\nu/(kT)]-1},\ j\nu \to n^2/\text{Sqrt}[T]\ \text{Exp}[-h\nu/(kT)]\ g_{ff}\right\},\ \{\nu, 0, \infty\},\ \text{Assumptions} \to \{h>0, k>0, T>0, \nu>0, c>0, T>0, \rho>0, g_{ff}>0\}\right]$$

Out[42]= $$\frac{1}{\kappa_R} \to \frac{8 k^7 \pi T^{7/2} \rho \left(\pi^6 + 945\,\text{Zeta}[7]\right)}{3 c^4 h^5 n^2 \sigma g_{ff}}$$

Therefore, since n is proportional to ρ^2, κ_R is proportional to $\rho\, T^{\frac{-7}{2}}$

Q3

Function to find cooling time given temperature

In[67]:=
```
CoolingTime[T_] = 3. × 10^11 s Sqrt[T] / n /. Solve[n T == 3000, n][[1, 1]]
```

Out[67]= $T^{3/2}\left(1. \times 10^8\ \text{s}\right)$

In[68]:=
```
CoolingTime[6000]
```

Out[68]= 4.64758×10^{13} s

In[69]:=
```
CoolingTime[1.1 × 10^4]
```

Out[69]= 1.15369×10^{14} s

In[70]:=
```
CoolingTime[10^6]
```

Out[70]= $1. \times 10^{17}$ s

Q4

For an unpolarized wave moving in the z-axis, after scattering at an angle θ in the y-z plane, the x-component of the electric field (Ex) remains the same since it is perpendicular to the emerging ray.

The y component (Ey) is reduced to the projection on a plane perpendicular to the emerging ray. Therefore, the degree of polarization is

In[36]:=
```
(Ex^2 - Ey^2) / (Ex^2 + Ey^2) /. Ey → Ey Cos[θ]
```

Out[36]=
$$\frac{Ex^2 - Ey^2 \,Cos[\theta]^2}{Ex^2 + Ey^2 \,Cos[\theta]^2}$$

Since Ex = Ey initially we get

$$\frac{1 - Cos[\theta]^2}{1 + Cos[\theta]^2}$$