HW 4 - ASTR404

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Q1)

Equations for pressure and energy

In[1465]:=
```
eqs = {P → n k T + 1/3 a T^4, u → 3/2 n k T + a T^4}; eqs // Column // TraditionalForm
```

Out[1465]//TraditionalForm=

$$P \to \frac{a T^4}{3} + k n T$$
$$u \to a T^4 + \frac{3 k n T}{2}$$

Relation between pressure and energy

In[1466]:=
```
eqγ = u == P/(γ - 1)
```

Out[1466]= $u == \frac{P}{-1 + \gamma}$

Formula for γ

In[1467]:=
```
sγ = Solve[eqγ, γ][[1, 1]] /. eqs // Simplify
```

Out[1467]= $\gamma \to \frac{15 k n + 8 a T^3}{9 k n + 6 a T^3}$

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a) Limit of γ as T→ ∞

In[1468]:=
```
Limit[sγ[[2]], T → ∞]
```

Out[1468]= $\frac{4}{3}$

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b) Limit of γ as T→ 0

In[1469]:=
```
Limit[sγ[[2]], T → 0]
```

Out[1469]= $\frac{5}{3}$

Q2)

Solving for i_{Out} & i_{In}

Equations

In[1470]:=
```
eqs2 = {J == (iOut + iIn) / 2, F == Pi (iOut - iIn) == σ T^4, J == 3 F / (4 Pi) (τν + 2 / 3)} // Echo;
```

» $\left\{J == \frac{1}{2}\left(i_{In} + i_{Out}\right), F == \pi\left(-i_{In} + i_{Out}\right) == T^4 \sigma, J == \frac{3 F\left(\frac{2}{3} + \tau_\nu\right)}{4\pi}\right\}$

Solution

In[1471]:=
```
Quiet@Solve[eqs2, {iIn, iOut, J, F}][[1, ;; 2]]
```

Out[1471]= $\left\{i_{In} \to \frac{3 T^4 \sigma \tau_\nu}{4\pi}, i_{Out} \to \frac{T^4 \sigma\left(4 + 3\tau_\nu\right)}{4\pi}\right\}$

Depth when isotropy is 1%

Equations

In[1472]:=
```
eqs3 = Append[eqs2, 2 (iOut - iIn) / (iOut + iIn) == 1 / 100 // Echo];
```

» $\frac{2\left(-i_{In} + i_{Out}\right)}{i_{In} + i_{Out}} == \frac{1}{100}$

Solution

In[1473]:=
```
NSolve[eqs3, {τν, iIn, iOut, J, F}][[1, 1]]
```

Out[1473]= $\tau_\nu \to 132.66666666667$

Q3)

Solving the radiative transfer equation

In[1474]:=
```
sRad = i[λ] -> iλ0 E^-τλ0 - Integrate[S[λ] E^-τ[λ], {τ[λ], τλ0, 0}]
```

Out[1474]= $i[\lambda] \to e^{-\tau\lambda 0}\, i\lambda 0 - S[\lambda]\left(-1 + \text{Cosh}[\tau\lambda 0] - \text{Sinh}[\tau\lambda 0]\right)$

a) Limit $\tau_{\lambda 0} >> 1$

In[1475]:=
```
Limit[i[λ] /. sRad, τλ0 → Infinity]
```

Out[1475]= $S[\lambda]$

At thermodynamic equilibrium S(λ) = B(λ).

b) Limit $\tau_{\lambda 0} << 1$

Taylor series

In[1476]:=
```
Series[i[λ] /. sRad, {τλ0, 0, 1}]
```

Out[1476]= $i\lambda 0 + (-i\lambda 0 + S[\lambda])\, \tau\lambda 0 + O[\tau\lambda 0]^2$

Thus if S(λ) > $I_{\lambda 0}$, we can see that emission lines will be added to the incident light at specific frequencies and vice versa.

Q4)

Solving the stellar structure equations

Out[1456]=
$$M'(r) = 4\,\pi\,\rho\,r^2$$
$$P'(r) = -\frac{G\,\rho\,M(r)}{r^2}$$

Assuming ρ = constant

In[1477]:=
```
DSolve[{M'[r] == 4 Pi r^2 ρ, D[P[r], r] == -G M[r] ρ/r^2, M[0] == 0, P[R] == 0},
   {M[r], P[r]}, r][[1, 2]] // FullSimplify
```

Out[1477]= $P[r] \to \frac{2}{3} G\,\pi\,(-r^2 + R^2)\,\rho^2$

Q5)

Solving the Lane-Emden equation

$$\frac{\frac{\partial}{\partial\xi}\left(\xi^2 \frac{\partial\theta(\xi)}{\partial\xi}\right)}{\xi^2} + \theta(\xi)^n = 0$$

Assuming n = 0 and solving

In[1478]:=
```
sθ = DSolve[{ξ^-2 D[ξ^2 D[θ[ξ], ξ], ξ] + 1 == 0, θ[0] == 1, θ'[0] == 0}, θ[ξ], ξ][[1, 1]]
```

Out[1478]= $\theta[\xi] \to \frac{1}{6}\,(6 - \xi^2)$

Location of first zero

In[1479]:=
```
Solve[θ[ξ] == 0 && ξ ≥ 0 /. sθ, ξ][[1, 1]]
```

Out[1479]= $\xi \to \sqrt{6}$