Best Practices in Mixture Modeling using Free Open Source Software

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BEST PRACTICES MIXTURE MODELING

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Abstract

Latent class analysis is a popular technique for identifying groups in data based on a

parametric model. Examples of this technique are known as mixture models, latent profile 10

analysis, latent class analysis, growth mixture modeling, and latent class growth analysis. 11

Despite the popularity of this technique, there is limited guidance with respect to best 12

practices in estimating and reporting mixture models. Moreover, although user-friendly 13

interfaces for advanced mixture modeling have long been available in commercial software

packages, open source alternatives have remained somewhat inaccessible. This tutorial 15

describes best practices for the estimation and reporting of latent class analysis, using free

and open source software in R. To this end, this tutorial introduces new functionality for

estimating and reporting mixture mixture models in the tidySEM R-package. These

functions rely on estimation using the OpenMx R-package.

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Best Practices in Mixture Modeling using Free Open Source Software

Latent class analysis is an umbrella term that refers to a number of techniques for 23 estimating unobserved group membership based on a parametric model of one or more 24 observed indicators of group membership. This method has become quite popular across 25 scientific fields, and under a number of different names; most notably (finite Gaussian) mixture modeling and latent profile analysis. Despite the popularity of the method, there 27 is a lack of standards for estimating and reporting latent class analyses. This complicates manuscript review and assessment of the quality of published studies, and introduces a risk of misapplications of the technique. The present paper seeks to address this gap in the literature by suggesting updated guidelines for estimation and reporting on latent class analysis, based on current best practice. Importantly, in order to lower the barrier of entry and ensure reproducibility of all examples, this paper exclusively relies on free, open source software for latent class analysis in R. Our goal is to make best-practices latent class analysis widely accessable.

36 Defining latent class analysis

Latent class analysis can be understood as a method for estimating unobserved groups
based on a parametric model of observed indicators of group membership. The concept of
latent class analysis can be understood in different ways. Generally speaking, it can be said
that a mixture model assumes that the study population is composed of K subpopulations,
or classes. It further assumes that the observed data are a mixture of data generated by
class-specific models. The simplest univariate "model" is a normal distribution, which can
be described with two parameters: the mean and the variance. Commonly, the same model
is estimated accross all classes, but with different parameters for each class (i.e.,
class-specific means and variances). Mixture modeling then estimates both the parameters
for each class, and the probability for each that an individual belongs to each class.

As an illustrative example, imagine that a detective wants to know if it would be
possible to use mixture modeling to identify the sex of a suspect, based on footprints found
at the crime scene. To test the feasibility of this approach, the detective records the shoe
sizes and sex of 100 volunteers. The resulting observed data look like this:

The distribution is evidently bimodal, which bodes well for the intended mixture model. In this case, the number of classes is known a-priori. When estimating a two-class mixture model, the detective observes that the model estimates the mean shoesize of the two groups are equal to 7.25 and 9.22, which is close to the true means of the two groups, namely 9.04 and 6.93. When tabulating estimated group membership against observed (known) group membership, it can be seen that women are classified with a high degree of accuracy, but men are not:

One way to is to conceptualize latent class analysis is by analogy to a measurement model in structural equation modeling. A mixture model is like confirmatory factor analysis, except that the continuous latent variable is substituted with a categorical latent variable. One difference between the two techniques is that factor analysis can be considered as a way to group observed *variables* into latent constructs, with factor loadings indicating which items belong are most indicative of a construct. By contrast, mixture modeling groups *individuals* into classes (see Nylund-Gibson & Choi, 2018). In line with this distinction, latent class analysis is sometimes referred to as a "person-centered" technique, and factor analysis as a "variable-centered" technique.

When the focus is on the model parameters in each group, then latent class analysis
can be thought of as similar to a multi-group structural equation model. The main
distinction is that group membership is not known a-priori, but is instead estimated - with
measurement error - based on the data. Whereas in a multigroup model, the data are split
by group and treated as independent samples, in a mixture model, all cases contribute to
the estimation of all parameters in all groups. The relative contribution of each case to the

parameters of each group is determined by that case's posterior probability of belonging to that group.

When the focus is on each individual's estimated class membership, latent class
analysis can be thought of as a type of clustering algorithm. In line with this perspective,
mixture modeling is sometimes described as "model-based clustering" (Hennig, Meila,
Murtagh, & Rocci, 2015; Scrucca, Fop, Murphy, & Raftery, 2017). Many clustering
algorithms apply some recursive splitting algorithm to the data. By contrast
"model-based" clustering refers to the fact that latent class analysis estimates cluster
membership based on a parametric model. Specifically, the posterior class probability that
an individual belongs to a latent class can be computed from the likelihood of that
individual's observed data under given the class-specific model.

Finally, in the context of machine learning, latent class analysis can be considered as an unsupervised classification problem (Figueiredo & Jain, 2002). The term unsupervised refers to the fact that the outcome variable, true class membership, is not known, and the term classification refers to the fact that the algorithm is predicting a categorical outcome (class membership).

89 A taxonomy of latent class analyses

In this paper, we use the term latent class analysis to refer to techniques that
estimate latent class membership based on a parametric model of observed indicators.
From a historical perspective, the term latent class analysis was initially conceived to refer
to analyses with categorical (binary) indicators (Vermunt, J.K. et al., 2004). Nowadays,
there are a number of related techniques, known by distinct names, that serve a similar
purpose. The term "latent class analysis" seems most appropriate as an umbrella term for
this broader class of models, as it only refers to the purpose of the analysis, and does not
imply restrictions to the model used, or the level of measurement of the indicators. Given

the abundance of terms in use for closely related classes of models, we will provide a rudimentary taxonomy of latent class analyses.

One common type of latent class analyses is the finite Gaussian mixture model; a 100 univariate analysis where the observed distribution of a single variable is assumed to result 101 from a mixture of a known number of Gaussian (normal) distributions. The parameters of 102 a finite Gaussian mixture model are the means and variances of these underlying normal 103 distributions. The analysis of shoe sizes presented earlier is a canonical example of this 104 type of analysis. In the multivatiate case, with more than one indicator variable, the 105 parameters of a mixture model are the means, variances, and covariances between the 106 indicators (which can be standardized to obtain correlations). These parameters can be 107 estimated freely, or constrained, across classes. 108

The technique known as *latent profile analysis (LPA)* is a special case of the mixture 109 model, which assumes conditional independence of the indicators. Conditional 110 independence means that, after class membership is accounted for, the 111 covariances/correlations between indicators are assumed to be zero. This can be conceived 112 of as a restricted mixture model with covariances fixed to zero. In some cases, such 113 constraints will be inappropriate; for example, when the cohesion between indicators is expected to differ between classes. As an example, a mixture model analysis of ocean 115 plastic particles found two classes of particles based on length and width: A class of smaller particles with a high correlation between length and width, meaning that these particles were approximately round or square in shape, and a class of larger particles with 118 a low correlation between length and width, meaning that these particles were heterogenous in shape. From a theoretical perspective, this makes sense, because the 120 smaller particles have been ground down to a more uniform shape by the elements. 121

It is also possible to estimate a mixture model based on latent indicators. This means that, within each class, one or more continuous latent variables are estimated based on the

observed indicators. Categorical latent variable membership is then estimated based on 124 these continuous latent variables. A common application of this approach is in longitudinal 125 research, where the indicators reflect one construct assessed at different time points. 126 Examples of this approach include growth mixture models (GMM) and latent class growth 127 analyses (LCGA). These techniques estimate a latent growth model to describe individual 128 trajectories over time. The growth mixture model is a latent class model where the 129 parameters that indicate class membership are the intercepts and variances (and typically 130 covariances) of the latent growth variables, e.g., a latent intercept and slope. This 131 technique assumes that individuals within a class can have heterogenous trajectories. If the 132 variance of the growth parameters is fixed to zero, it is known as a latent class growth 133 analysis. This latter approach assumes that all individuals within a class share the same 134 identical trajectory, and that any variance in the indicators not explained by the 135 class-specific latent trajectories is due to residual error variance.

The term latent class analysis originally referred to cases where the observed 137 indicators were categorical. Nowadays, it is more commonly used as an umbrella term. To 138 prevent ambiguity, the special case where indicators are of binary or ordinal measurement 139 level might be described as latent class analysis with ordinal indicators. Latent class models 140 with ordinal indicators are parameterized differently from mixture models. One common 141 parameterization assumes that each categorical variable reflects an underlying standard 142 normal distribution. The parameters are "thresholds" that correspond to quantiles of a 143 standard normal distribution (with $N(\mu = 0, \sigma = 1)$). These thresholds are estimated 144 based on the proportion of individuals in each of the response categories of the indicator variable. For example, a binary indicator has a single threshold that distinguishes the two response categories. If responses are distributed 50/50, then the corresponding threshold 147 would be $t_1 = 0.00$. If the responses are distributed 60/40, then the resulting threshold 148 would be $t_1 = 0.25$. This paper will primarily focus on mixture models and special cases 149 thereof, although most of the suggested guidelines are applicable to latent class analyses. 150

Use cases for latent class analysis

There are several use cases for which latent class analyses are suitable. One example 152 is to test a theory that postulates the existence of a categorical latent variable. For 153 example, identity status theory posits that, at any given point in time, adolescents reside in 154 one of four identity statuses. Latent class analysis can be used to identify these four 155 statuses based on observed indicators (e.g., self-reported identity exploration and 156 commitment). If results indicate that the data are better described by a different number 157 of classes, or that the four-class solution does not correspond to the predicted pattern of 158 responses on the indicators, then the theory may be called into question. 159

Another use case is unsupervised learning; when the goal is to restore unobserved class membership based on observed indicators, or to classify individuals. For example, a mixture model can be used as a diagnostic aid when several clinical indicators can be used to distinguish between a fixed number of physical (Baughman, Bisgard, Lynn, & Meade, 2006) or mental (Wu, Woody, Yang, Pan, & Blazer, 2011) health problems. The example of shoe size is a rudimentary illustration of this type of application.

Best practices

In estimation

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The best practices in estimation, as outlined in Table ??, are rooted in existing
recommendations for best practices for estimating specific sub-types of latent class
analyses, including latent class growth analysis

(schootGRoLTSChecklistGuidelinesReporting2017?) and latent class analysis with
ordinal indicators (e.g., Nylund-Gibson & Choi, 2018). These were generalized to be more
relevant to all types of latent class analyses, and updated to current best practices, as
explained below.

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#	Item
1.	Examine observed data
2.	Handling missing data
3.	Alternative model specifications
4.	Starting values
5.	Algorithm

Examine observed data. Examining observed data is essential for any analysis, 176 as it may reveal patterns and violations of assumptions that had not been considered prior 177 to data collection. Pay special attention to level of measurement of the indicators. Mixture 178 modeling (including LPA) is strictly speaking only suitable for continuous variables. 179 Indicators with an ordinal level of measurement are likely to violate the assumption of 180 within-class normal distributions of mixture models (see Vermunt, 2011). Personal 181 experience consulting on latent class analyses and moderating the tidyLPA Google group suggest that the mis-application of mixture models to ordinal (e.g., Likert-type) indicators 183 is the most common source of user error. Whereas it has been argued that some parametric 184 methods are robust when scales with 7+ indicators are treated as continuous (e.g., 185 Norman, 2010), this certainly does not imply that all methods are. It is certainly unlikely 186 that such ordinal variables can be treated as a mixture of multiple normal distributions. 187 The problem becomes egregious when the number of classes estimated equals or exceeds 188 the number of categories; in this case, each class-specific mean could describe a single 189 response category, and a class-specific variance component would be nonsensical. In sum, 190 Likert-type scales are rarely suitable for mixture modeling; latent class analysis with 191 ordinal indicators is more appropriate. 192

Relatedly, a recent publication claimed that an assumption of mixture models is that observed indicators are normally distributed [REF SPURK]. This is incorrect. When the number of classes is greater than one, mixture models assume that the observed indicators

are a mixture of multiple (multivariate) normal distributions.

Extensive descriptive statistics (including the number of unique values, variance of 197 categorical variables, and missingness; see next paragraph) can be obtained using the function tidySEM::descriptives(data). Note, however, that sample-level descriptive 199 statistics are of limited value when the goal of a study is to identify sub-samples using 200 latent class analysis. Plots (density plots for continuous variables, and bar charts for 201 categorical ones) may be more diagnostic. Note that density plots can also aid in the 202 choice of the number of classes, as further explained in the section on visualization. 203 Descriptive statistics and plots can be relegated to online supplements, provided that these 204 are readily accessible (consider using a GitHub repository as a comprehensive public 205 research archive, as explained in Van Lissa et al., 2020). 206

Missing data. Previous work has emphasized the importance of examining the 207 pattern of missing data and reporting how missingness was handled 208 (schootGRoLTSChecklistGuidelinesReporting2017?). Three types of missingness 209 have been distinguished in the literature (Rubin, 1976): Missing completely at random 210 (MCAR), which means that missingness is random; missing at random (MAR), which 211 means that missingness is contingent on the *observed* data (and can thus be accounted for); 212 and finally missing not at random (MNAR), which means that missingness is related to 213 unobserved factors. It is possible to conduct a so-called "MCAR" test, for example the 214 non-parametric MCAR test (Jamshidian & Jalal, 2010). But note that the name "MCAR 215 test" is somewhat misleading, as the null-hypothesis of this test is that the data are not 216 MAR, and a significant test statistic indicates that missingness is related to the observed data (MAR). A non-significant test statistic does not distinguish between MCAR or 218 MNAR. As Little's classic MCAR test relies on the comparison of variances across groups with different patterns of missing data, it assumes normality (Little, 1988). This assumption is tenuous in the context of latent class analysis. A non-parametric MCAR 221 test, as provided by Jamshidian and Jalal, may be more suitable (2010). Unfortunately,

this test was removed from the central R-repository CRAN due to lack of maintenance. For this tutorial, I have re-implemented it in the mice package as mice::mcar(), with a fast backend in C++ and new printing and plotting methods.

While we concur that investigating missingness is due dilligence, it is important to 226 emphasize that missingness is adequately handled by default in many software packages for 227 latent class analyses, such as OpenMx (and e.g., Mplus). These packages use Full 228 Information Maximum Likelihood (FIML) estimation, which makes use of all available 220 information without imputing missing values. FIML is a best-practice solution for handling 230 missing data; on par with multiple imputation (Lee & Shi, 2021). FIML estimation 231 assumes that missingness is either MCAR or MAR. Thus, one would typically proceed with 232 FIML regardless of the outcome of an MCAR test. Although FIML does not, by default, 233 handle missingness in exogenous variables - all indicator variables in latent class analysis 234 are endogenous, so this is not a concern. 235

Multiple imputation is less suitable to latent class analyses for two reasons. First, 236 because latent class analyses are often computationally expensive, and conducting them on 237 multiple imputed datasets may be unfeasible. Second, because there is no straightforward way to integrate latent class analysis results across multiple datasets. To conclude; our recommendation is to inspect missingness (e.g., using mice::MCAR()) and report the 240 proportion of missingness per variable (e.g., using tidySEM::descriptives()), before 241 proceeding with FIML. One minor concern is that the K-means algorithm, which tidySEM 242 uses for determining starting values, is not robust to missing values. When it fails, 243 tidySEM automatically switches to hierarchical clustering, unless the user specifies a 244 different clustering algorithm or uses manual starting values. 245

Alternative model specifications. The different types of latent class models
have different parameters. For example, mixture models and latent profile analyses
typically have class-specific means, variances, and covariances. Latent growth analyses
have the same parameters, but with respect to the latent growth variables. Latent class

analyses with ordinal indicators have thresholds. All of these parameters can be freely 250 estimated across classes, or constrained, or fixed (e.g., to zero). The total number of 251 parameters thus scales with the number of estimated classes. Consequently, latent class 252 analyses have a potentially very high number of parameters. As any of these parameters 253 could be mis-specified, it is important to consider alternative model specifications. 254 However, alternative model specifications may be approached differently depending on 255 whether an analysis is data driven (exploratory), or theoretically driven (confirmatory). 256 This distinction has remained underemphasized in prior writing. 257

Prior literature on latent class analysis has emphasized exploratory applications of 258 the method (see Nylund, Asparouhov, & Muthén, 2007). In exploratory analyses, a large 259 number of models are typically estimated in batch, with varying numbers of classes and 260 model specifications. The "correct" model specification is then determined based on a 261 combination of fit indices, significance tests, and interpretability. For latent profile analysis, 262 the function tidySEM::mx profiles(classes, variances, covariances) largely 263 automates this process. The argument classes indicates which class solutions should be 264 estimated (e.g., 1 through 6). The argument variances specifies whether variances should 265 be "equal" or "varying" across classes. The argument covariances specifies whether 266 covariances should be constrained to "zero", "equal" or "varying" across classes. The 267 means are free to vary across classes by default, although the more general function 268 tidySEM::mx mixture() could be used to circumvent this. After all models have been 260 estimated, the function tidySEM::table fit() can be used to obtain a model fit table 270 suitable for determining the optimal model according to best practices. Note however that this table does not include the bootstrapped likelihood ratio test (BLRT) by default, 272 because this test is very computationally expensive. It is recommended to use the function tidySEM::BLRT() to compare a shortlist of likely candidate models based on other fit indices. Fit indices typically used for determining the optimal number of classes include 275 the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). Both

information criteria are based on the -2 log likelihood (which is lower for better fitting models), and add a penalty for the number of parameters (thus incentivizing simpler models). This helps balance fit and model complexity. The BIC usually applies a stronger penalty for complexity that scales logarithmically with the sample size.

Fit indices may occasionally contradict each other, so it is important to identify a suitable strategy to reconcile them. One option is to select a specific fit index before analyzing the data. Another option is to always prefer the most parsimonious model that has best fit according to any of the available fit indices. Yet another option is to incorporate information from multiple fit indices using the analytic hierarchy process (Akogul & Erisoglu, 2016).

Confirmatory analyses typically require less comprehensive alternative model specifications. For example, in the context of preregistered analyses, the main models of interest may have been specified a priori.

Van de Schoot and colleagues (2017) argue that there are many choices to be made in
the specification of latent trajectory models, and that alternative

* Consider alternative parameterizations; e.g., free variances/covariances

* See Nylund

294 Software

Many software packages are available for the estimation of latent class analyses.

Some of these packages have limited functionality, or implement specific innovations. Other packages implement latent class analyses in the context of a more flexible structural equation modeling framework. The most notable examples of the latter are the commercial programs Mplus and Latent GOLD, and the free open source R-package OpenMx. The commercial packages stand out because they offer relatively user-friendly interfaces and implement sensible defaults for complex analyses, including latent class analysis. This

lowers the threshold for applied researchers to adopt such methods. Commercial software
also has several downsides, however. One such downside is that use of the software is
restricted to those individuals and institutions who can afford a license. A second downside
is that the source code, being proprietary, cannot be audited, debugged, or enhanced by
third parties. This incurs the risk that mistakes in the source code may go unnoticed, and
curbs progress as software developers cannot add new functionality.

Conversely, the free open source program OpenMx is very flexible, but not very user-friendly.

New functionality in the R-package tidySEM seeks to lower the threshold for latent class analysis using OpenMx. It adheres to best practices in estimation and reporting, as described in this paper. The user interface is simple, making use of the model syntax of the widely used lavaan R-package. This syntax offers a human-readable way to specify latent variable models. Minor enhancements are made to simplify the specification of latent class analysis.

Because of the limitations in ex tools, we set out to develop a tool that a) provided
sensible defaults and were easy to use, but provided the option to access and modify all of
the inputs to the model (i.e., low barrier, high ceiling), b) interfaced to existing tools, and
are able to translate between what existing tools are capable of and what researchers and
analysts carrying-out person-oriented analyses would like to specify,

fully-reproducible analyses and

322 Best practices in estimation

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Algorithm. Mixture model parameters and model fit statistics can be estimated in a variety of ways. The choice of the estimator depends on the presence of missing values, sample size, number of indicators, and available computational resources (Weller, Bowen & Faubert, 2020). A commonly used technique is maximum likelihood (ML) estimation with

the expectation-maximization (EM) algorithm as a local optimizer. Imagine we are 327 estimating two parameters, e.g. the class-specific means μ_c on a continuous indicator 328 (ignoring the variance for now). The EM algorithm will attempt to find a combination of 329 values for these two parameters that maximizes the likelihood (LL) of all observed data. In 330 practice, instead of maximizing LL, often -2*LL is minimized, as this offers 331 computational advantages. We can think of this optimization problem as a 332 three-dimensional landscape: The X and Y dimensions are determined by the class-specific 333 means, so $X = \mu_1$ and $Y = \mu_2$ - and the Z-dimension is determined by Z = -2 * LL. The 334 optimizer must find the deepest "valley" in this landscape, which reflects the combination 335 of μ_1 and μ_2 that maximizes the likelihood of the data. The EM optimizer behaves 336 somewhat like a marble, dropped in this landscape. It is dropped at some random point in 337 space, and will roll into the nearest valley. The problem is that, once EM rolls into a valley, 338 it will settle on the bottom of that valley (this is known as "convergence"). It cannot climb out again. Thus, if their are multiple valleys, the risk is that the optimizer gets stuck in a 340 shallower valley (a "local optimum"), and never discovers the deepest valley (the "global 341 optimum", or best solution). One solution to this problem is to drop many marbles at 342 random places, compare their final -2*LL values, choose the solution with the lowest 343 -2*LL, and make sure that several marbles replicated this solution. This is the "random 344 starts" approach. 345

One problem with the random starts approach is that it is computationally expensive to run this many replications. Moreover, because the algorithm begins with random starting values, many of the marbles are likely to be very far away from a "good enough" solution. Two innovations may improve the estimation procedure. The first is that, instead of picking random starting values, a "reasonable solution" may be used for the starting values. For example, if we assume that the different classes are likely to have different mean values on the indicators, then the K-means clustering algorithm can be used to determine these cluster centroids. We can compute the expected values of all model

parameters by treating the K-means solution as a known class solution, and use these as starting values for a mixture model. One remaining concern is that this approach may result in starting values close to a local optimum, and that the EM algorithm will thus never find the global optimum. A second innovation addresses this concern.

Instead of using EM, it is possible to use an optimizer that can climb out of a valley.

Simulated annealing iteratively considers some "destination" in the landscape, and

compares its likelihood to the current one. If the destination likelihood is higher, the

estimator moves there. If the destination likelihood is *lower*, the estimator still moves there

occasionally, based on probability. This latter property allows it to escape local optima,

and find the global optimum.

By default, tidySEM employs this solution of deriving starting values using K-means clustering, and identifying the global optimum solution using simulated annealing. Once a solution has been found, simulated annealing is followed up with a short run of the EM algorithm, as EM inherently produces an asymptotic covariance matrix for the parameters that can be used to compute standard errors. Note that these defaults can be manually overridden.

One recent paper suggested maximum likelihood with robust standard errors should
be used when the observed indicators are not normally distributed (Spurk et al., 2020).
This statement is incorrect, and may lead readers to believe that they must use commercial
software, as robust maximum likelihood is currently only implemented in Mplus and
latentGOLD. As explained before, mixture modeling assumes that observed data are a
mixture of (multivariate) normal distributions; thus, the observed indicators will likely not
be normally distributed.

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Class enumeration. In LCA, a sequence of models is fitted to the data with each additional model estimating one more class than the previous model. The final model

called the final class solution is chosen based on both theoretical and statistical criteria.

Theory should drive the selection of indicator variables, inform the expectations and reflect on the findings. In addition to this, there are several statistical criteria to consider in model selection. These include but are not limited to likelihood ratio tests, information criteria, and the Bayes factor (Weller, Bowen & Faubert, 2020).

Relative model fit can be examined using the likelihood ratio test. This is only appropriate when the two models we wish to compare are nested. The likelihood ratio test statistic is computed as the difference in maximum log likelihoods of the two models, with the new degrees of freedom being the difference in their degrees of freedom. This statistic also follows the χ^2 distribution. Similar to the LR χ^2 goodness-of-fit test, we want the test statistic to be non-significant in order to give support to the simpler model. The likelihood ratio test can only compare two models at a time (Lanza et al., 2003).

Model fit indices. If we wish to simultaneously compare multiple models based on 392 their relative fit, this can be done through a comparison of multiple information criteria. 393 Examples include the Akaike information criterion (AIC), the Consistent Akaike 394 Information Criterion (CAIC), the Bayesian information criterion (BIC), and the 395 Sample-size Adjusted Bayesian Information Criterion (SABIC). Information criteria are a 396 sum of a measure of fit (usually a form of the converged maximum log likelihood value) 397 and a penalty for model complexity (combination of sample size and number of modeled 398 parameters). As a general rule, the lower the value of an information criterion, the better 399 the model fits the data. When examining several competing models, each with differing 400 number of classes, we expect the information criteria values to drop with each successive model until the final class solution is reached. Further models with additional classes should show worse fit as information criteria reach an optimum before their values rise again with an increasing number of classes. Multiple information criteria should be compared when choosing the final class solution. This can be done by means of an elbow 405 plot (for an example see Nylund-Gibson and Choi, 2018). Advantages of using information

criteria are that we can compare multiple models at a time, and these models need not be nested (Masyn, 2013).

Classification Diagnostics. Best models will divide the sample into subgroups 409 which are internally homogeneous and externally distinct. Classification diagnostics give us 410 a way to assess the degree to which this is the case. They are separate from the absolute 411 and relative goodness-of-fit as a model can fit the data well but show poor latent class 412 separation (Masyn, 2013). A fundamental concept when examining classification precision 413 and accuracy are the posterior class probabilities. A probability of belonging to each latent 414 class is computed for each individual. The highest posterior class probability is then 415 determined and the individual is assigned to the corresponding class. We want each 416 individual's posterior class probabilities to be high for one and low for the remaining latent classes. This is considered a high classification accuracy and means that the classes are distinct. Three important classification diagnostic measures are entropy, the average posterior probability and the modal class assignment proportion. 420

Entropy is a summary measure of posterior class probabilities across classes and individuals. It ranges from 0 (model classification no better than random chance) to 1 (perfect classification). As a rule of thumb, values above .80 are deemed acceptable and those approaching 1 are considered ideal. Entropy should not be used to select a particular class solution. An appropriate use of entropy is that it can disqualify certain solutions if class separability is too low or if one of the latent classes is too small to be meaningful or to calculate descriptive statistics.

The average posterior probability is a measure of classification uncertainty for each latent class.

The modal class assignment proportion

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Interpreting the results. Class names should be chosen to accurately reflect group membership. Overly simplified and generalized class names may prove misleading to

both audiences and researches alike leading to what is known as a naming fallacy (Weller,
Bowen & Faubert, 2020).

Label switching. The final class solution will usually discover and enumerate
several classes. The class ordering however is completely arbitrary. The class labeled as
Class 1 in one solution may become Class 2 or Class 3 in another model, even when the
only difference between the models is in their starting values. Label switching is something
to be mindful of when comparing different LCA models (Masyn, 2013).

440 Best practices in reporting

Among studies using LCA, reporting practices vary significantly (Weller, Bowen & 441 Faubert, 2020). Various authors have tried to better and standardize ways of reporting LCA (e.g. Masyn, 2013; Weller, Bowen & Faubert, 2020). Building on their work, we provide guidelines with a strong emphasis on scientific reproducibility and transparency better known as the open science framework (OSF). Open science framework promotes 445 scientific openness, reproducibility and integrity through an establishment of guidelines for 446 the scientific process (Foster & Deardorff, 2017) To this end, van Lissa and colleagues 447 (2020) developed WORCS, a workflow for open reproducible code in science. WORCS 448 consists of step-by-step guidelines for research projects based on the TOP-guidelines 449 developed by Nosek and colleagues (2015). It can be easily implemented in R in form of an 450 R package which facilitates preregistration, article drafting, version control, citation and 451 formatting, among others (Van Lissa et al., 2020)

- \star Use comprehensive citation; this includes referencing the software used.
- * Share code and (if possible) data. Van Lissa and colleagues suggest sharing synthetic
 - 55 * Ideally, make the entire research project reproducible so that others may download it

As the open science movement is gaining momentum, researchers are becoming increasingly aware how important it is that analyses can be reproduced and audited. In

line with open science principles, one of the suggested reporting standards relates to reproducible code. In this context, it is important to note that user-friendly methods for 459 estimating latent class analyses have predominantly been available in commercial software 460 packages (e.g., Mplus and Latent GOLD). A potential downside of commercial software is 461 that it restricts the ability to reproduce analyses to license holders, and prevents auditing 462 research because the underlying source code is proprietary. To overcome these limitations, 463 the present paper introduces new user-friendly functions in the tidySEM R-package that 464 can be used to estimate a wide range of latent class analysis models using the free, 465 open-source R-package OpenMx. The reporting guidelines described in this paper are 466 adopted in tidySEM by default. The tidySEM R-package thus makes advanced mixture 467 modeling based on best practices widely accessible, and facilitates the adoption of the 468 estimation and reporting guidelines described in this paper.

470 Best practices in visualization

471 Tutorial

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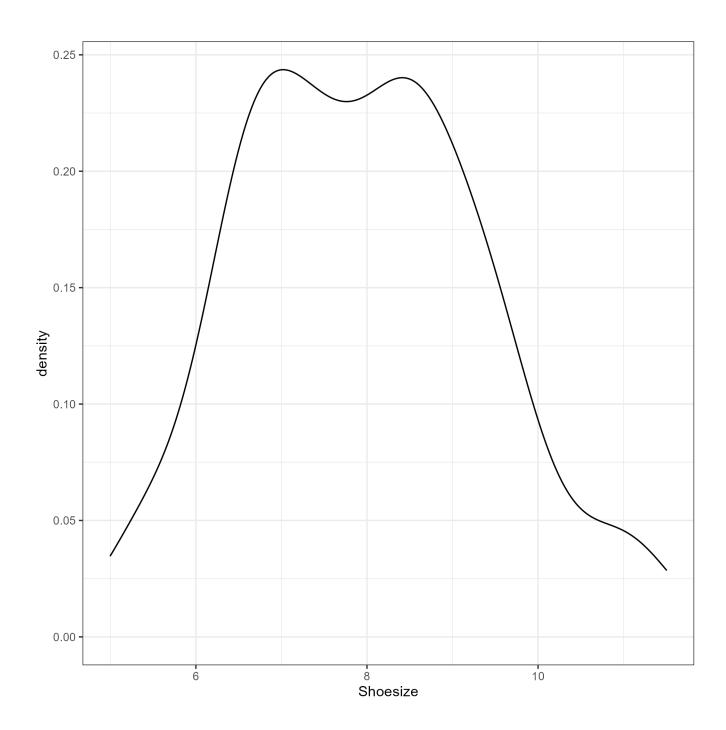
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Table 1

Observed group membership by estimated class membership.

Observed	Class 1	Class 2
Man	21	28
Woman	51	0



 $Figure\ 1.$ Kernel density plot of shoe sizes.