Best Practices in Mixture Modeling using Free Open Source Software

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BEST PRACTICES MIXTURE MODELING

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Abstract

Latent class analysis is a popular technique for identifying groups in data based on a

parametric model. Examples of this technique are known as mixture models, latent profile 10

analysis, latent class analysis, growth mixture modeling, and latent class growth analysis. 11

Despite the popularity of this technique, there is limited guidance with respect to best 12

practices in estimating and reporting mixture models. Moreover, although user-friendly 13

interfaces for advanced mixture modeling have long been available in commercial software

packages, open source alternatives have remained somewhat inaccessible. This tutorial 15

describes best practices for the estimation and reporting of latent class analysis, using free

and open source software in R. To this end, this tutorial introduces new functionality for

estimating and reporting mixture mixture models in the tidySEM R-package. These

functions rely on estimation using the OpenMx R-package.

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Best Practices in Mixture Modeling using Free Open Source Software

Latent class analysis has become quite popular across scientific fields, and under a
number of different names. The purpose of this technique is to estimate unobserved group
membership based on a parametric model of one or more observed indicators of group
membership. Despite the popularity of the method, there is a noted absence of guidelines
for estimating and reporting latent class analyses []. This complicates manuscript review
and assessment of the quality of published studies, and introduces a risk of misapplications
of the technique. The present paper seeks to address this gap in the literature by
suggesting updated guidelines for estimation and reporting on latent class analysis, based
on best practice.

Throughout this paper, examples make use of free, open source software for latent class analysis in R.

Defining latent class analysis

Latent class analysis can be understood as a method for estimating unobserved groups
based on a parametric model of observed indicators of group membership. The concept of
latent class analysis can be understood in different ways. Generally speaking, it can be said
that a mixture model assumes that the study population is composed of K subpopulations,
or classes. It further assumes that the observed data are a mixture of data generated by
class-specific models. The simplest univariate "model" is a normal distribution, which can
be described with two parameters: the mean and the variance. Commonly, the same model
is estimated accross all classes, but with different parameters for each class (i.e.,
class-specific means and variances). Mixture modeling then estimates both the parameters
for each class, and the probability for each that an individual belongs to each class.

As an illustrative example, imagine that a detective wants to know if it would be
possible to use mixture modeling to identify the sex of a suspect, based on footprints found

at the crime scene. To test the feasibility of this approach, the detective records the shoe sizes and sex of 100 volunteers. The resulting observed data look like this:

The distribution is evidently bimodal, which bodes well for the intended mixture model. In this case, the number of classes is known a-priori. When estimating a two-class mixture model, the detective observes that the model estimates the mean shoesize of the two groups are equal to 7.25 and 9.22, which is close to the true means of the two groups, namely 9.04 and 6.93. When tabulating estimated group membership against observed (known) group membership, it can be seen that women are classified with a high degree of accuracy, but men are not:

One way to is to conceptualize latent class analysis is by analogy to a measurement
model in structural equation modeling. A mixture model is like confirmatory factor
analysis, except that the continuous latent variable is substituted with a categorical latent
variable. One difference between the two techniques is that factor analysis can be
considered as a way to group observed variables into latent constructs, with factor loadings
indicating which items belong are most indicative of a construct. By contrast, mixture
modeling groups individuals into classes (see
nylund-gibsonTenFrequentlyAsked2018?). In line with this distinction, latent class
analysis is sometimes referred to as a "person-centered" technique, and factor analysis as a
"variable-centered" technique.

When the focus is on the model parameters in each group, then latent class analysis
can be thought of as similar to a multi-group structural equation model. The main
distinction is that group membership is not known a-priori, but is instead estimated - with
measurement error - based on the data. Whereas in a multigroup model, the data are split
by group and treated as independent samples, in a mixture model, all cases contribute to
the estimation of all parameters in all groups. The relative contribution of each case to the
parameters of each group is determined by that case's posterior probability of belonging to

73 that group.

When the focus is on each individual's estimated class membership, latent class
analysis can be thought of as a type of clustering algorithm. In line with this perspective,
mixture modeling is sometimes described as "model-based clustering" (Hennig, Meila,
Murtagh, & Rocci, 2015; Scrucca, Fop, Murphy, & Raftery, 2017). Many clustering
algorithms apply some recursive splitting algorithm to the data. By contrast
"model-based" clustering refers to the fact that latent class analysis estimates cluster
membership based on a parametric model. Specifically, the posterior class probability that
an individual belongs to a latent class can be computed from the likelihood of that
individual's observed data under given the class-specific model.

Finally, in the context of machine learning, latent class analysis can be considered as
an unsupervised classification problem (figueiredoUnsupervisedLearningFinite2002?).
The term unsupervised refers to the fact that the outcome variable, true class membership,
is not known, and the term classification refers to the fact that the algorithm is predicting
a categorical outcome (class membership).

88 A taxonomy of latent class analyses

In this paper, we use the term latent class analysis to refer to techniques that estimate latent class membership based on a parametric model of observed indicators. From a historical perspective, the term latent class analysis was initially conceived to refer to analyses with categorical (binary) indicators (vermuntj.k.LatentClassAnalysis2004?). Nowadays, there are a number of related techniques, known by distinct names, that serve a similar purpose. The term "latent class analysis" seems most appropriate as an umbrella term for this broader class of models, as it only refers to the purpose of the analysis, and does not imply restrictions to the model used, or the level of measurement of the indicators. Given the abundance of terms in use for closely related classes of models, we

will provide a rudimentary taxonomy of latent class analyses.

One common type of latent class analyses is the finite Gaussian mixture model; a 99 univariate analysis where the observed distribution of a single variable is assumed to result 100 from a mixture of a known number of Gaussian (normal) distributions. The parameters of 101 a finite Gaussian mixture model are the means and variances of these underlying normal 102 distributions. The analysis of shoe sizes presented earlier is a canonical example of this 103 type of analysis. In the multivatiate case, with more than one indicator variable, the 104 parameters of a mixture model are the means, variances, and covariances between the 105 indicators (which can be standardized to obtain correlations). These parameters can be 106 estimated freely, or constrained, across classes. 107

The technique known as *latent profile analysis (LPA)* is a special case of the mixture 108 model, which assumes conditional independence of the indicators. Conditional 109 independence means that, after class membership is accounted for, the 110 covariances/correlations between indicators are assumed to be zero. This can be conceived 111 of as a restricted mixture model with covariances fixed to zero. In some cases, such 112 constraints will be inappropriate; for example, when the cohesion between indicators is 113 expected to differ between classes. As an example, a mixture model analysis of ocean 114 plastic particles found two classes of particles based on length and width: A class of 115 smaller particles with a high correlation between length and width, meaning that these particles were approximately round or square in shape, and a class of larger particles with a low correlation between length and width, meaning that these particles were 118 heterogenous in shape. From a theoretical perspective, this makes sense, because the smaller particles have been ground down to a more uniform shape by the elements.

It is also possible to estimate a mixture model based on latent indicators. This means that, within each class, one or more continuous latent variables are estimated based on the observed indicators. Categorical latent variable membership is then estimated based on

these continuous latent variables. A common application of this approach is in longitudinal research, where the indicators reflect one construct assessed at different time points. 125 Examples of this approach include growth mixture models (GMM) and latent class growth 126 analyses (LCGA). These techniques estimate a latent growth model to describe individual 127 trajectories over time. The growth mixture model is a latent class model where the 128 parameters that indicate class membership are the intercepts and variances (and typically 129 covariances) of the latent growth variables, e.g., a latent intercept and slope. This 130 technique assumes that individuals within a class can have heterogenous trajectories. If the 131 variance of the growth parameters is fixed to zero, it is known as a latent class growth 132 analysis. This latter approach assumes that all individuals within a class share the same 133 identical trajectory, and that any variance in the indicators not explained by the 134 class-specific latent trajectories is due to residual error variance.

When a latent class analysis relies on categorical indicators, typically of a binary or 136 ordinal measurement level, it has a different parameterization than a mixture model. One 137 common assumption is that each categorical variable reflects an underlying standard 138 normal distribution. The parameters in such a model are then "thresholds", corresponding 139 to the quantiles of a standard normal distribution (with $N(\mu = 0, \sigma = 1)$). These 140 thresholds are estimated based on the proportion of individuals in each of the response 141 categories of the indicator variable. For example, a binary indicator has a single threshold that distinguishes the two response categories. If responses are distributed 50/50, then the corresponding threshold would be $t_1 = 0.00$. If the responses are distributed 60/40, then 144 the resulting threshold would be $t_1 = 0.25$. This paper will primarily focus on mixture 145 models and special cases thereof, although most of the suggested guidelines are applicable 146 to latent class analyses.

Use cases for latent class analysis

There are several use cases for which latent class analyses are suitable. One example 149 is to test a theory that postulates the existence of a categorical latent variable. For 150 example, identity status theory posits that, at any given point in time, adolescents reside in 151 one of four identity statuses. Mixture modeling can be used to identify these four statuses 152 based on observed indicators (e.g., self-reported identity exploration and commitment). If 153 results indicate that the data are better described by a different number of classes, or that 154 the four-class solution does not correspond to the predicted pattern of responses on the 155 indicators, then the theory may be called into question. 156

Another use case is unsupervised learning; when the goal is to restore unobserved class membership based on observed indicators, or to classify individuals. For example, a mixture model can be used as a diagnostic aid when several clinical indicators can be used to distinguish between a fixed number of physical

(baughmanMixtureModelAnalysis2006?) or mental

(wuAbuseDependencePrescription2011?) health problems. The example of shoe size is a rudimentary illustration of this type of application.

Best practices

65 In estimation

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The best practices in estimation, as outlined in Table ??, are inspired by prior recommendations for best practices for estimating specific sub-types of latent class analyses, including latent class growth analysis (schootGRoLTSChecklistGuidelinesReporting2017?) and latent class analysis with ordinal indicators (e.g., nylund-gibsonTenFrequentlyAsked2018?). These were generalized to be more relevant to all types of latent class analyses, and updated to current best practices, as explained below.

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#	Item
1.	Examine observed data
2.	Handling missing data
3.	Alternative model specifications
4.	Starting values
5.	Algorithm

Examine observed data. Examining observed data is essential for any analysis, 174 as it may reveal patterns and violations of assumptions that had not been considered prior 175 to data collection. Pay special attention to level of measurement of the indicators. 176 Indicators with an ordinal level of measurement are likely to violate the assumption of 177 within-class normal distributions (see vermuntKmeansMayPerform2011?). Personal 178 experience consulting on latent class analyses and moderating the tidyLPA Google group 179 suggest that the mis-application of mixture models to ordinal (e.g., Likert-type) indicators is perhaps the most common source of user error. Whereas it has been argued that some 181 parametric methods are robust when scales with 7+ indicators are treated as continuous 182 (e.g., normanLikertScalesLevels2010?), this certainly does not imply that all methods 183 are. It is certainly unlikely that such ordinal variables can be treated as a mixture of 184 multiple normal distributions. The problem becomes eggregious when the number of 185 classes estimated equals or exceeds the number of categories; in this case, each 186 class-specific mean could describe a single response category, and a class-specific variance 187 component would be nonsensical. In sum, Likert-type scales are rarely suitable for mixture 188 modeling; a latent class analysis with ordinal indicators may be more appropriate. 189

Extensive descriptive statistics (including the number of unique values, variance of categorical variables, and missingness; see next paragraph) can be obtained using the function tidySEM::descriptives(data). Note, however, that sample-level descriptive statistics are of limited value when the goal of a study is to identify sub-samples using

latent class analysis. Plots (density plots for continuous variables, and bar charts for
categorical ones) may be more diagnostic. Note that density plots can also aid in the
choice of the number of classes, as further explained in the section on visualization.

Descriptive statistics and plots can be relegated to online supplements, provided that these
are readily accessible (consider using a GitHub repository as a comprehensive public
research archive, as explained in vanlissaWORCSWorkflowOpen2020?).

Missing data. Previous work has emphasized the importance of examining the 200 pattern of missing data and reporting how missingness was handled 201 (schootGRoLTSChecklistGuidelinesReporting2017?). Three types of missingness 202 have been distinguished in the literature (rubinInferenceMissingData1976?): Missing 203 completely at random (MCAR), which means that missingness is random; missing at 204 random (MAR), which means that missingness is contingent on the observed data (and can 205 thus be accounted for); and finally missing not at random (MNAR), which means that 206 missingness is related to unobserved factors. It is possible to conduct a so-called "MCAR" 207 test, for example the non-parametric MCAR test 208 (jamshidianTestsHomoscedasticityNormality2010?). But note that the name 200 "MCAR test" is somewhat misleading, as the null-hypothesis of this test is that the data 210 are not MAR, and a significant test statistic indicates that missingness is related to the 211 observed data (MAR). A non-significant test statistic does not distinguish between MCAR 212 or MNAR. As Little's classic MCAR test relies on the comparison of variances across 213 groups with different patterns of missing data, it assumes normality 214 (littleTestMissingCompletely1988?). This assumption is tenuous in the context of 215 latent class analysis. A non-parametric MCAR test, as provided by Jamshidian and Jalal, may be more suitable (jamshidianTestsHomoscedasticityNormality2010?). Unfortunately, this test was removed from the central R-repository CRAN due to lack of 218 maintenance. For this tutorial, I have re-implemented it in the mice package as 219 mice::MCAR(), with a fast backend in C++ and new printing and plotting methods.

While we concur that investigating missingness is due dilligence, it is important to 221 emphasize that missingness is adequately handled by default in many software packages for 222 latent class analyses, such as OpenMx (and e.g., Mplus). These packages use Full 223 Information Maximum Likelihood (FIML) estimation, which makes use of all available 224 information without imputing missing values. FIML is a best-practice solution for handling 225 missing data; on par with multiple imputation (leeComparisonFullInformation2021?). 226 FIML estimation assumes that missingness is either MCAR or MAR. Thus, one would 227 typically proceed with FIML regardless of the outcome of an MCAR test. Although FIML 228 does not, by default, handle missingness in exogenous variables - all indicator variables in 220 latent class analysis are endogenous, so this is not a concern. 230

Multiple imputation is less suitable to latent class analyses for two reasons. First, 231 because latent class analyses are often computationally expensive, and conducting them on 232 multiple imputed datasets may be unfeasible. Second, because there is no straightforward 233 way to integrate latent class analysis results across multiple datasets. To conclude; our 234 recommendation is to inspect missingness (e.g., using mice::MCAR()) and report the 235 number of missings per variable (e.g., using tidySEM::descriptives()), before 236 proceeding with FIML. One minor concern is that the K-means algorithm, which tidySEM 237 uses for determining starting values, is not robust to missing values. When it fails, 238 tidySEM automatically switches to hierarchical clustering, unless the user specifies a different clustering algorithm or uses manual starting values.

Alternative model specifications. The different types of latent class models
have different parameters. For example, mixture models and latent profile analyses
typically have class-specific means, variances, and covariances. Latent growth analyses
have the same parameters, but with respect to the latent growth variables. Latent class
analyses with ordinal indicators have thresholds. All of these parameters can be freely
estimated across classes, or constrained, or fixed (e.g., to zero). The total number of
parameters thus scales with the number of estimated classes. Consequently, latent class

analyses have a potentially very high number of parameters. As any of these parameters
could be mis-specified, it is important to consider alternative model specifications.

However, alternative model specifications may be approached differently depending on
whether an analysis is data driven (exploratory), or theoretically driven (confirmatory).

This distinction has remained underemphasized in prior writing.

Prior literature on latent class analysis has emphasized exploratory applications of 253 the method (see nylundDecidingNumberClasses2007?). In exploratory analyses, a large number of models are typically estimated in batch, with varying numbers of classes 255 and model specifications. The "correct" model specification is then determined based on a 256 combination of fit indices, significance tests, and interpretability. For latent profile analysis, 257 the function tidySEM::mx profiles(classes, variances, covariances) largely 258 automates this process. The argument classes indicates which class solutions should be 259 estimated (e.g., 1 through 6). The argument variances specifies whether variances should 260 be "equal" or "varying" across classes. The argument covariances specifies whether 261 covariances should be constrained to "zero", "equal" or "varying" across classes. The 262 means are free to vary across classes by default, although the more general function 263 tidySEM::mx mixture() could be used to circumvent this. After all models have been 264 estimated, the function tidySEM::table_fit() can be used to obtain a model fit table 265 suitable for determining the optimal model according to best practices. Note however that 266 this table does not include the bootstrapped likelihood ratio test (BLRT) by default, 267 because this test is very computationally expensive. It is recommended to use the function 268 tidySEM::BLRT() to compare a shortlist of likely candidate models based on other fit indices. Fit indices typically used for determining the optimal number of classes include 270 the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). Both information criteria are based on the -2 log likelihood (which is lower for better fitting models), and add a penalty for the number of parameters (thus incentivizing simpler 273 models). This helps balance fit and model complexity. The BIC usually applies a stronger

275 penalty for complexity that scales logarithmically with the sample size.

Fit indices may occasionally contradict each other, so it is important to identify a suitable strategy to reconcile them. One option is to select a specific fit index before analyzing the data. Another option is to always prefer the most parsimoneous model that has best fit according to any of the available fit indices. Yet another option is to incorporate information from multiple fit indices using the analytic hierarchy process (akogulComparisonInformationCriteria2016?).

Confirmatory analyses typically require less comprehensive alternative model specifications. For example, in the context of preregistered analyses, the main models of interest may have been specified a priori.

Van de Schoot and colleagues

(vandeschootGRoLTSChecklistGuidelinesReporting2017?) argue that there are
many choices to be made in the specification of latent trajectory models, and that
alternative

* Consider alternative parameterizations; e.g., free variances/covariances

290 * See Nylund

$_{^{291}}$ Software

Many software packages are available for the estimation of latent class analyses.

Some of these packages have limited functionality, or implement particular innovations.

Other packages implement latent class analyses in the context of a more flexible structural equation modeling framework. The most notable examples of the latter are the commercial programs Mplus and Latent GOLD, and the free open source R-package OpenMx. The commercial packages stand out because they offer relatively user-friendly interfaces and implement sensible defaults for complex analyses, including latent class analysis. This somewhat lowers the threshold for applied researchers to adopt such methods. Commercial

software also has several downsides, however. One such downside is that use of the software is restricted to those individuals and institutions who can afford a license. A second downside is that the source code, being proprietary, cannot be audited, debugged, or enhanced by third parties. This incurs the risk that mistakes in the source code may go unnoticed, and curbs progress as software developers cannot add new functionality.

Conversely, the free open source program OpenMx is very flexible, but not very user-friendly.

New functionality in the R-package tidySEM seeks to lower the threshold for latent class analysis using OpenMx. It adheres to best practices in estimation and reporting, as described in this paper. The user interface is simple, making use of the model syntax of the widely used lavaan R-package. This syntax offers a human-readable way to specify latent variable models. Minor enhancements are made to simplify the specification of latent class analysis.

Because of the limitations in ex tools, we set out to develop a tool that a) provided
sensible defaults and were easy to use, but provided the option to access and modify all of
the inputs to the model (i.e., low barrier, high ceiling), b) interfaced to existing tools, and
are able to translate between what existing tools are capable of and what researchers and
analysts carrying-out person-oriented analyses would like to specify,

fully-reproducible analyses and

Best practices in estimation

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Algorithm. Mixture model parameters and model fit statistics can be estimated in a variety of ways. The choice of the estimator depends on the presence of missing values, sample size, number of indicators, and available computational resources (Weller, Bowen & Faubert, 2020). A commonly used technique is maximum likelihood (ML) estimation with the expectation-maximization (EM) algorithm as a local optimizer. Imagine we are

estimating two parameters, e.g. the class-specific means μ_c on a continuous indicator 325 (ignoring the variance for now). The EM algorithm will attempt to find a combination of 326 values for these two parameters that maximizes the likelihood (LL) of all observed data. In 327 practice, instead of maximizing LL, often -2*LL is minimized, as this offers 328 computational advantages. We can think of this optimization problem as a 329 three-dimensional landscape: The X and Y dimensions are determined by the class-specific 330 means, so $X = \mu_1$ and $Y = \mu_2$ - and the Z-dimension is determined by Z = -2 * LL. The 331 optimizer must find the deepest "valley" in this landscape, which reflects the combination 332 of μ_1 and μ_2 that maximizes the likelihood of the data. The EM optimizer behaves 333 somewhat like a marble, dropped in this landscape. It is dropped at some random point in 334 space, and will roll into the nearest valley. The problem is that, once EM rolls into a valley, 335 it will settle on the bottom of that valley (this is known as "convergence"). It cannot climb out again. Thus, if their are multiple valleys, the risk is that the optimizer gets stuck in a shallower valley (a "local optimum"), and never discovers the deepest valley (the "global 338 optimum", or best solution). One solution to this problem is to drop many marbles at 339 random places, compare their final -2*LL values, choose the solution with the lowest 340 -2*LL, and make sure that several marbles replicated this solution. This is the "random 341 starts" approach. 342

One problem with the random starts approach is that it is computationally expensive 343 to run this many replications. Moreover, because the algorithm begins with random 344 starting values, many of the marbles are likely to be very far away from a "good enough" 345 solution. Two innovations may improve the estimation procedure. The first is that, instead of picking random starting values, a "reasonable solution" may be used for the starting values. For example, if we assume that the different classes are likely to have different 348 mean values on the indicators, then the K-means clustering algorithm can be used to 349 determine these cluster centroids. We can compute the expected values of all model 350 parameters by treating the K-means solution as a known class solution, and use these as 351

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starting values for a mixture model. One remaining concern is that this approach may result in starting values close to a local optimum, and that the EM algorithm will thus never find the global optimum. A second innovation addresses this concern.

Instead of using EM, it is possible to use an optimizer that can climb out of a valley.

Simulated annealing iteratively considers some "destination" in the landscape, and

compares its likelihood to the current one. If the destination likelihood is higher, the

estimator moves there. If the destination likelihood is *lower*, the estimator still moves there

occasionally, based on probability. This latter property allows it to escape local optima.

By default, tidySEM employs this solution of deriving starting values using K-means clustering, and identifying the global optimum solution using simulated annealing. Once a solution has been found, simulated annealing is followed up with a short run of the EM algorithm, as EM inherently produces an asymptotic covariance matrix for the parameters that can be used to compute standard errors. Note that these defaults can be manually overridden.

In the case of Latent Profile Analysis, it has been suggested that when tests of 366 multivariate normality indicate a violation, maximum likelihood with robust standard 367 errors should be used (Spurk et al., 2020). This statement is problematic for two reasons. Firstly, multivariate normality will always be violated in mixture models by definition. We 360 assume that the population consists of several groups each characterized by its own normal 370 distribution. When two or more normal distributions with different means and standard 371 deviations comprise a mixed population distribution, this combined distribution is no longer normal. For example, you can imagine a bimodal distribution when dealing with 373 two latent classes. If this were not the case and our mixed population distribution were normal, there would be no latent classes to extract as the population would consists of a single class only. For this reason, multivariate normality will always be violated when the 376 population is comprised of several latent groups. Secondly, the claim that maximum

likelihood with robust standard errors should be preferred implicitly implies that only software packages with this estimation technique can be used.

The EM algorithm needs starting values for each parameter it Starting values. 380 wants to estimate. If it were provided a single starting value for each parameter, the EM 381 optimizer would be at risk of finding a local optimum instead of the global optimum. In 382 other words, it would converge on a suboptimal solution resulting in biased parameter and 383 model fit estimates. To prevent this, we should give the algorithm a sufficiently large set of 384 random starting values for each parameter in order to maximize our chances of finding the 385 deepest valley that is the global optimum (McLachlan & Peel, 2004). Hipp and Bauer 386 (2006) recommend using minimally 50-100 sets of random starting values. Also, we should 387 ensure the number of initial stage iterations is large for optimization to be successful 388 (Geiser, 2012). 389

Furthermore, the choice of the starting values can impact the speed of the algorithm's convergence or in case of having poor initial values the algorithm might diverge altogether (McLachlan & Peel, 2004). The key is to use a large number of starting value sets. Geiser (2013) recommends at least 500 sets of initial values in the first optimization step for simple models, and a greater number of sets for more complex models. A large number of starting values ensures that we find the true maximum and estimate the model parameters and fit accurately.

[(nylund-gibsonTenFrequentlyAsked2018?);]

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 $_{
m 398}$ In many circumstances and especially in a cluster analysis setting, the mixture component

Assessing Estimation Results. In LCA, a sequence of models is fitted to the
data with each additional model estimating one more class than the previous model. The
final model called the final class solution is chosen based on both theoretical and statistical
criteria. Theory should drive the selection of indicator variables, inform the expectations

and reflect on the findings. In addition to this, there are several statistical criteria to consider in model selection. These include but are not limited to likelihood ratio tests, information criteria, and the Bayes factor (Weller, Bowen & Faubert, 2020).

Absolute model fit can be assessed using the likelihood ratio (LR) χ^2 goodness-of-fit 406 test. It can compare two nested models when they have the same number of classes (Lanza 407 et al., 2003). Typically, we compare our final class solution (a constrained model) to an 408 unconstrained one (a model characterized by the perfect fit). The former model is a 409 special, less complex version of the latter. In this case, we compare our model-based 410 response pattern frequencies to those in the dataset. Therefore, we make a statement 411 about how well our model fits the data. The perfect fit would be indicated by the $\chi^2 = 0$ 412 and p = 1. Ideally, the LR χ^2 test will be non-significant as this implies that while our 413 constrained model is more parsimonious, it fits the data just as well as the unconstrained 414 model. The underlying assumption of this procedure is that the test statistic is 415 asymptotically distributed as a χ^2 meaning that it can be approximated using this 416 distribution. This is the case when the sample size is adequate (Lanza et al., 2003). The 417 dependence of the LR χ^2 test statistic on sample size has its downsides (Masyn, 2013). 418 Very large samples will often effortlessly reject the null even when the final class solution is a good fit, and small samples will often be underpowered resulting in non-significant χ^2 statistic. In fact, small samples might not be well approximated using the χ^2 distribution, 421 which is a problem shared with large yet sparse datasets. In such cases, special caution is 422 advised when interpreting the resulting p-values (Masyn, 2013). 423

Relative model fit can be examined using the likelihood ratio test. This is only appropriate when the two models we wish to compare are nested. The likelihood ratio test statistic is computed as the difference in maximum log likelihoods of the two models, with the new degrees of freedom being the difference in their degrees of freedom. This statistic also follows the χ^2 distribution. Similar to the LR χ^2 goodness-of-fit test, we want the test statistic to be non-significant in order to give support to the simpler model. The likelihood

ratio test can only compare two models at a time (Lanza et al., 2003).

If we wish to simultaneously compare multiple models based on their relative fit, this 431 can be done through a comparison of multiple information criteria. Examples include the 432 Akaike information criterion (AIC), the Consistent Akaike Information Criterion (CAIC), 433 the Bayesian information criterion (BIC), and the Sample-size Adjusted Bayesian 434 Information Criterion (SABIC). Information criteria are a sum of a measure of fit (usually 435 a form of the converged maximum log likelihood value) and a penalty for model complexity 436 (combination of sample size and number of modeled parameters). As a general rule, the 437 lower the value of an information criterion, the better the model fits the data. When 438 examining several competing models, each with differing number of classes, we expect the 439 information criteria values to drop with each successive model until the final class solution 440 is reached. Further models with additional classes should show worse fit as information 441 criteria reach an optimum before their values rise again with an increasing number of classes. Multiple information criteria should be compared when choosing the final class 443 solution. This can be done by means of an elbow plot (for an example see Nylund-Gibson 444 and Choi, 2018). Advantages of using information criteria are that we can compare 445 multiple models at a time, and these models need not be nested (Masyn, 2013).

When estimators which require sets of starting values are used, each fitted model will 447 have a corresponding log likelihood value. Ideally, the largest log likelihood value of the 448 final class solution will be replicated a large number of times with each log likelihood value 440 generated by a different starting value set. Successful replications of the largest log 450 likelihood value across different starting values instill confidence that the estimation was 451 successful and we indeed found the global maximum (Masyn, 2013). The model with the highest replicated likelihood will be chosen as our best-fit. If the largest log likelihood value is not replicated many times, the model may be unidentified (Geiser, 2012). Such a model does not have a unique solution. For a discussion on model identification for LCA, 455 see Masyn (2013). Nylund-Gibson and Choi (2018) recommend that the highest log

likelihood should be replicated in at least 3-10% of the models initialized with different starting values in the first step of the optimization. This consideration does not apply in case of models which use simulated annealing since these do not have starting values.

Classification Diagnostics. Best models will divide the sample into subgroups 460 which are internally homogeneous and externally distinct. Classification diagnostics give us 461 a way to assess the degree to which this is the case. They are separate from the absolute 462 and relative goodness-of-fit as a model can fit the data well but show poor latent class 463 separation (Masyn, 2013). A fundamental concept when examining classification precision and accuracy are the posterior class probabilities. A probability of belonging to each latent class is computed for each individual. The highest posterior class probability is then determined and the individual is assigned to the corresponding class. We want each individual's posterior class probabilities to be high for one and low for the remaining latent classes. This is considered a high classification accuracy and means that the classes are 469 distinct. Three important classification diagnostic measures are entropy, the average posterior probability and the modal class assignment proportion. 471

Entropy is a summary measure of posterior class probabilities across classes and individuals. It ranges from 0 (model classification no better than random chance) to 1 (perfect classification). As a rule of thumb, values above .80 are deemed acceptable and those approaching 1 are considered ideal. Entropy should not be used to select a particular class solution. An appropriate use of entropy is that it can disqualify certain solutions if class separability is too low or if one of the latent classes is too small to be meaningful or to calculate descriptive statistics.

The average posterior probability is a measure of classification uncertainty for each latent class.

The modal class assignment proportion

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Conditional item probabilities indicate the probability of a positive score on an item

given that the person belongs to a particular latent class. However, if many conditional item probabilities in estimation are found to be extreme (as in exactly 0 or 1), we are observing boundary parameter estimates. This can be a sign of an invalid solution, warn us that too many classes were extracted, or indicate a local optimum. Boundary parameter estimates are more likely to happen when working with sparse data and should be interpreted with caution (Geiser, 2012).

An important outcome of LCA are unconditional class probabilities. These
communicate the probability that an individual belongs to each class. A limitation is that
our final class solution need not reflect true class membership. Once we do accept a
particular class solution, special care also needs to be taken when naming the classes. Class
names should be chosen to accurately reflect group membership. Overly simplified and
generalized class names may prove misleading to both audiences and researches alike
leading to what is known as a naming fallacy (Weller, Bowen & Faubert, 2020).

Label switching. The final class solution will usually discover and enumerate
several classes. The class ordering however is completely arbitrary. The class labeled as
Class 1 in one solution may become Class 2 or Class 3 in another model, even when the
only difference between the models is in their starting values. Label switching is something
to be mindful of when comparing different LCA models (Masyn, 2013).

Best practices in reporting

Among studies using LCA, reporting practices vary significantly (Weller, Bowen & Faubert, 2020). Various authors have tried to better and standardize ways of reporting LCA (e.g. Masyn, 2013; Weller, Bowen & Faubert, 2020). Building on their work, we provide guidelines with a strong emphasis on scientific reproducibility and transparency better known as the open science framework (OSF). Open science framework promotes scientific openness, reproducibility and integrity through an establishment of guidelines for the scientific process (Foster & Deardorff, 2017) To this end, van Lissa and colleagues

(2020) developed WORCS, a workflow for open reproducible code in science. WORCS
consists of step-by-step guidelines for research projects based on the TOP-guidelines
developed by Nosek and colleagues (2015). It can be easily implemented in R in form of an
R package which facilitates preregistration, article drafting, version control, citation and
formatting, among others (Van Lissa et al., 2020)

* Use comprehensive citation; this includes referencing the software used.

* Share code and (if possible) data. Van Lissa and colleagues suggest sharing synthetic

* Ideally, make the entire research project reproducible so that others may download it

As the open science movement is gaining momentum, researchers are becoming 517 increasingly aware how important it is that analyses can be reproduced and audited. In 518 line with open science principles, one of the suggested reporting standards relates to 519 reproducible code. In this context, it is important to note that user-friendly methods for 520 estimating latent class analyses have predominantly been available in commercial software 521 packages (e.g., Mplus and Latent GOLD). A potential downside of commercial software is that it restricts the ability to reproduce analyses to license holders, and prevents auditing 523 research because the underlying source code is proprietary. To overcome these limitations, the present paper introduces new user-friendly functions in the tidySEM R-package that can be used to estimate a wide range of latent class analysis models using the free, 526 open-source R-package OpenMx. The reporting guidelines described in this paper are 527 adopted in tidySEM by default. The tidySEM R-package thus makes advanced mixture 528 modeling based on best practices widely accessible, and facilitates the adoption of the 529 estimation and reporting guidelines described in this paper. 530

Best practices in visualization

Tutorial

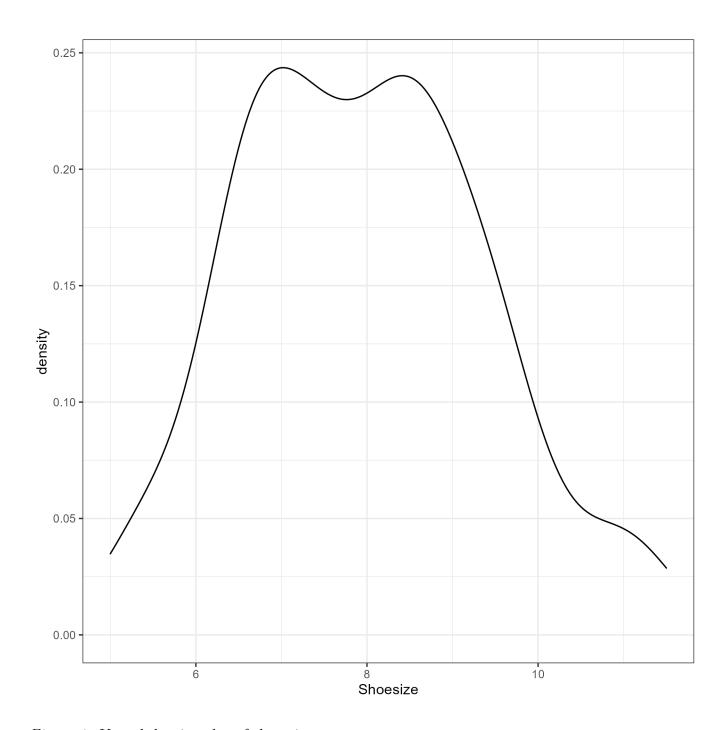
$$->->->$$

545 Starting values.

Table 1

Observed group membership by estimated class membership.

Observed	Class 1	Class 2
Man	21	28
Woman	51	0



 $Figure\ 1.$ Kernel density plot of shoe sizes.