All uppercase letter variables are positive integers unless otherwise stated. All fractions containing uppercase letter variables are in lowest terms. NOTA means "None of the Above."

∼∼∼∼∼∼∼∼ *Good luck, and have fun!* ∼∼∼∼∼∼∼∼

1) Evaluate: $\displaystyle\int_2^5 (x-2)(x-5)\,dx$

(A) -10 (B) $-\dfrac{9}{2}$ (C) -7 (D) $-\dfrac{7}{2}$ (E) NOTA

2) If $f(x) = x^3 + 2x^2 + 5x + 6$, then rank the following approximations for the value of $\displaystyle\int_0^1 f(x)\,dx$ in increasing order.

I: Left-hand Riemann sum with 1000 equal subintervals
II: Midpoint sum with 1000 equal subintervals
III: Right-hand Riemann sum with 1000 equal subintervals
IV: Trapezoidal sum with 1000 equal subintervals

(A) I, II, IV, III
(B) I, IV, II, III
(C) I, III, II, IV
(D) II, I, IV, III
(E) NOTA

3) Evaluate: $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\cos 2t}{\cos t - \sin t}\,dt$

(A) 0 (B) 1 (C) $\sqrt{2}$ (D) DNE (E) NOTA

4) Find the maximum value of $\displaystyle\int_a^{a+1} x(1-x)\,dx$.

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$ (E) NOTA

5) Evaluate: $\displaystyle\int_1^{16} \frac{dx}{\sqrt[4]{x}}$

(A) $\frac{28}{3}$ (B) $\frac{32}{3}$ (C) 10 (D) 11 (E) NOTA

6) Evaluate the follwing, where it is defined: $\int \frac{d(\arcsin x)}{\sqrt{1-x^2}}$

(A) $\ln\left(\frac{x}{1-x^2}\right)+C$
(B) $\ln\left(\frac{x+1}{\sqrt{1-x^2}}\right)+C$
(C) $\frac{x}{\sqrt{1-x^2}}+C$
(D) $\arcsin^2 x+C$
(E) NOTA

7) Evaluate: $\int_1^2 \log_{x^2+1}\left(e^{\frac{2x}{x^2+1}}\right)dx$

(A) $\ln\left(\frac{\ln(2)}{\ln(5)}\right)$
(B) $\ln\left(\frac{\ln(5)}{\ln(2)}\right)$
(C) $\frac{\ln(\ln(5))}{\ln(\ln(2))}$
(D) $\frac{\ln(\ln(2))}{\ln(\ln(5))}$
(E) NOTA

8) Evaluate: $\int_1^{2018} \frac{dx}{x^2+8x-20}$

(A) $\frac{1}{12}\ln\left(\frac{2028}{22176}\right)$
(B) $\frac{1}{12}\ln\left(\frac{2016}{22308}\right)$
(C) $\frac{1}{12}\ln\left(\frac{2028}{2016}\right)$
(D) $\frac{1}{12}\ln\left(\frac{2016}{2028}\right)$
(E) NOTA

9) Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\tan^2 x+1}{\tan x+1}\,dx$

(A) $\ln 2$
(B) $\ln\left(1+\sqrt{2}\right)$
(C) 1
(D) $\frac{\pi}{4}+\frac{1}{2}\ln 2$
(E) NOTA

10) Evaluate: $\int_0^{\frac{\pi}{2}} \sin(20x)\sin(17x)\,dx$ *Hint: Product-to-Sum*

(A) $\frac{20}{111}$
(B) $\frac{40}{111}$
(C) $-\frac{20}{111}$
(D) $-\frac{40}{111}$
(E) NOTA

11) The indefinite integral of secant cubed is equal to the average of the derivative and the indefinite integral of which of the following functions?

(A) tangent
(B) cotangent
(C) secant
(D) cosecant
(E) NOTA

12) The force needed to move a certain horizontal non-Hookian spring is $F(x)=kx^3$, where F is in newtons and x is the displacement from rest in meters. If it takes 12 joules of work to displace the spring 2 meters from rest, how much work (in joules) is needed to displace the spring 3 meters from rest?

(A) 18
(B) $\frac{243}{4}$
(C) 81
(D) $\frac{81}{4}$
(E) NOTA

13) A particle has acceleration $a(t) = 2t - 4$. Its velocity at $t = 1$ is 7, and its position at $t = 6$ is 54. What is the position of the particle at the other time its velocity is 7?

(A) -6 (B) 2 (C) 12 (D) 15 (E) NOTA

Sometimes, an unintuitive u-substitution is crucial to solving a hard integral. You can solve Questions 14 and 15 by using the substitution $u = \dfrac{1}{x}$ *at some point.*

14) Evaluate: $\displaystyle\int_0^\infty \frac{x \ln x}{16 + x^4}\, dx$

(A) 0 (B) $\dfrac{\pi}{16}\ln 2$ (C) $\dfrac{\pi}{8}\ln 2$ (D) $\dfrac{\pi}{4}\ln 2$ (E) NOTA

15) Evaluate: $\displaystyle\int_0^\infty \frac{dx}{(1 + x^2)(1 + x^{2018})}$

(A) $\dfrac{\pi}{4036}$ (B) $\dfrac{\pi}{2018}$ (C) $\dfrac{\pi}{4}$ (D) $\dfrac{\pi}{2}$ (E) NOTA

16) Evaluate: $\displaystyle\int_0^1 x^2(x+1)^5\, dx$

(A) $\frac{341}{56}$ (B) $\frac{183}{28}$ (C) $\frac{341}{28}$ (D) $\frac{183}{56}$ (E) NOTA

17) Evaluate: $\displaystyle\lim_{x\to\infty} \frac{\int_0^x \arctan^2 t\, dt}{x}$

(A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{2}$ (C) $\dfrac{\pi^2}{4}$ (D) $\dfrac{\pi^2}{2}$ (E) NOTA

18) Evaluate: $\displaystyle\int_0^{25} \sqrt{4 + \sqrt{x}}\, dx$

(A) $\frac{1011}{15}$ (B) $\frac{1012}{15}$ (C) $\frac{1013}{15}$ (D) $\frac{1014}{15}$ (E) NOTA

19) Evaluate: $\displaystyle\int_1^e \left[\left(\frac{e}{x}\right)^x + \left(\frac{x}{e}\right)^x\right] \ln x\, dx$

(A) $\dfrac{1}{e}$ (B) e (C) $e - \dfrac{1}{e}$ (D) $e + \dfrac{1}{e}$ (E) NOTA

20) Find the volume of the solid formed when the region bounded by $y = e^{-x^2}$, the coordinate axes, and the line $x = 1$ is rotated about the y-axis.

(A) $\dfrac{e}{\sqrt{\pi}}$ (B) $\dfrac{\pi}{e-1}$ (C) $\dfrac{(e-1)\pi}{e}$ (D) $e\sqrt{\pi}$ (E) NOTA

21) Evaluate: $\displaystyle\int_0^\infty \ln\left(1 - e^{-x}\right)\, dx$

(A) $-\dfrac{\pi^2}{2}$ (B) $-\dfrac{\pi^2}{3}$ (C) $-\dfrac{\pi^2}{4}$ (D) $-\dfrac{\pi^2}{6}$ (E) NOTA

22) A particle is traveling along the path defined by $(x, y) = (3t^2 - 3, t^3 - 3t)$ for $t \geq 0$. How far along the path does the particle travel between $t = 0$ and $t = 3$?

(A) 36 (B) $\frac{100}{3}$ (C) $9\sqrt{13}$ (D) $\frac{144}{5}$ (E) NOTA

23) If $\int_{-1}^{1} \frac{x^4 + 1}{e^x + 1}\, dx = \frac{U}{V}$, find $U + V$ if U and V are coprime positive integers.

Hint: $\int_{-k}^{k} f(x)\, dx = \int_{0}^{k} (f(x) + f(-x))\, dx$

(A) 9 (B) 11 (C) 14 (D) 17 (E) NOTA

24) Daniel and Vlad are investigating the function $f(x) = x^2$ over the range $[4, 16]$. Daniel finds the value of c that satisfies the Mean Value Theorem for Derivatives over the range and Vlad finds the value of c that satisfies the Mean Value Theorem for Integrals over the range. Whose value is smaller?

(A) Daniel

(B) Vlad

(C) The two values are equal

(D) One of the two values does not exist

(E) NOTA

25) Find the value of $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^2}{k^3 + n^3}$

(A) $\frac{1}{2}\ln(3)$ (B) $\frac{1}{3}\ln(3)$ (C) $\frac{1}{2}\ln(2)$ (D) $\frac{1}{3}\ln(2)$ (E) NOTA

26) If the area bounded by the coordinate axes and the curve $x^4 + y^3 = x^2 y$ in the first quadrant is equal to $\frac{M}{N}$, find $M + N$ if M and N are coprime positive integers.

(A) 109 (B) 111 (C) 113 (D) 115 (E) NOTA

27) A continuous *probability density function* is a non-negative function bounded by one for which the area between it and the x-axis is one over its domain. If $P(x) = e^{-k|x|}$ is a probability denisity function over the real numbers, find k.

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 4 (E) NOTA

28) A random real number is chosen using on the probability density function described in question 27. It is twice as likely that this number will be within the interval $[-1, 1]$ than it is it will be within the interval $[A, \infty)$. Find A.

(A) $\frac{1}{2}\ln(1-e^{-2})$ (B) $-\frac{1}{2}\ln(1-e^{-2})$ (C) $\ln(1-e^{-2})$
(D) $-\ln(1-e^{-2})$ (E) NOTA

The following two integrals use the Weierstrass substitution, which can be used to simplify an integral by applying the substitution $t = \tan\left(\frac{x}{2}\right)$.

29) Using the Weierstrass substitution, which of the following integrals is equivalent to $\int \csc\theta \, d\theta$, given that $t = \tan\left(\frac{\theta}{2}\right)$?

(A) $\displaystyle\int \frac{dt}{t}$ (B) $\displaystyle\int \frac{4t}{(1+t^2)^2}\, dt$ (C) $\displaystyle\int \frac{1+t^2}{2t}\, dt$
(D) $\displaystyle\int (1-t^2)\, dt$ (E) NOTA

30) Richard Feynman once said about a method that could be used to solve the following integral, "When guys at MIT or Princeton had trouble doing a certain integral, it was because they couldn't do it with the standard methods they had learned in school... I got a great reputation for doing integrals, only because my box of tools was different from everybody else's, and they had tried all their tools on it before giving the problem to me." (Quote from *Surely You're Joking, Mr. Feynman!*)

If $\displaystyle\int_0^\infty \frac{e^{2/(1+x^2)}\cos\left(\frac{2x}{1+x^2}\right)}{1+x^2}\, dx = \frac{\pi^A e^B}{C}$, with A, B, and C positive integers, find $A+B+C$.

(A) 3 (B) 4 (C) 6 (D) 8 (E) NOTA