For all questions, E "NOTA" means none of the above answers is correct.

1. The highway department of Hawaii plans to construct a new road between towns Alpha and Beta. Town Alpha lies on a long abandoned road running east west. Town Beta lies 3 miles north and 5 miles east of Alpha. Instead of building a road directly between Alpha and Beta, the department proposes renovating part of the abandoned road (from Alpha to some point P) and then building a new road from P to Beta. If the cost of restoring each mile of old road is $200,000 and the cost per mile of a new road is $400,000, how much of the old road (in miles) should be restored in order to minimize costs?

a) $5\pm\sqrt{3}$ b) $5+\sqrt{3}$ c) $5-\sqrt{3}$ d) $4+\sqrt{3}$ e) NOTA

2. A mouse is sitting in a toy car hooked to a spring launching device on a negligibly small turntable. The car has no way to turn, but the mouse can control when the car is launched and when the car stops (the car has brakes). When the mouse chooses to launch, the car will immediately leave the turntable on a straight trajectory at 1 m/s. Suddenly someone turns on the turntable; it spins at 30 rpm. Consider the set of points the mouse can reach in his car within 1 second after the turntable is set in motion. What is the area of this set?

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{5\pi}{6}$ d) $\frac{2\pi}{3}$ e) NOTA

3. Find the volume of an hourglass constructed by revolving the graph of $y=\sin^2 x+\frac{1}{10}$ from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$ about the x-axis.

a) $\frac{\pi^2}{100}$ b) $\frac{\pi^2}{200}$ c) $\frac{97\pi^2}{100}$ d) $\frac{97\pi^2}{200}$ e) NOTA

4. The base of a solid is the region between the parabolas $x=y^2$ and $2y^2=3-x$. Find the volume of the solid if the cross-sections perpendicular to the x-axis are equilateral triangles.

a) $\frac{2\sqrt{3}}{3}$ b) $\frac{3\sqrt{3}}{2}$ c) $\frac{3\sqrt{2}}{2}$ d) $\frac{3\sqrt{3}}{4}$ e) NOTA

5. Sam wants to watch a movie of her trip to nationals last year at Wash U on her movie screen. The lower edge of the screen, which is 30 feet high, is 6 feet above eye level. How far from the screen should the observer sit to obtain the most favorable view, i.e. to maximize the visual angle (the observed angle between the top and bottom of the screen)?

a) $2\sqrt{6}$ b) $6\sqrt{6}$ c) $4\sqrt{3}$ d) $6\sqrt{3}$ e) NOTA

6. A boat springs a leak at time t = 0, with water coming in at constant rate. At a time, t = r > 0 hours, someone notices that there is a leak and starts to record the distance the boat travels. The boat's speed is inversely related to the amount of water in the boat. If the boat travels twice as far in the first hour as in the second hour, what is r?

a) $\dfrac{\sqrt{5}+1}{2}$ b) $\dfrac{\sqrt{5}-1}{2}$ c) $\dfrac{1-\sqrt{5}}{2}$ d) $\dfrac{\sqrt{5}\pm 1}{2}$ e) NOTA

7. Find the area enclosed by the graph given by the parametric equations $x=\sin t$ and $y=\sin(2t)$.

a) $\dfrac{2}{3}$ b) $\dfrac{4}{3}$ c) $\dfrac{5}{3}$ d) $\dfrac{8}{3}$ e) NOTA

8. Tangent lines are drawn at the points of inflection for the function f (x) = cos x on [0, 2π]. The lines intersect with the x-axis so as to form a triangle. What is the area of this triangle?

a) $\dfrac{\pi}{4}$ b) $\dfrac{\pi^2}{4}$ c) $\dfrac{\pi}{2}$ d) $\dfrac{\pi^2}{2}$ e) NOTA

9. Find the maximum value of $\int_{\frac{-\pi}{2}}^{\frac{3\pi}{2}} \sin(x) f(x)dx$ subject to the constraint $|f(x)| \le 5$.

a) 5 b) 10 c) 25 d) 50 e) NOTA

10. There is a unique positive real number a such that the tangent line to $y = x^2 + 1$ at $x = a$ goes through the origin. Compute a.

a) 1 b) 2 c) 3 d) 4 e) NOTA

11. The efficiency of an automobile engine is given by the continuous function $r(c)$ where r is measured in liters/kilometer and c is measured in kilometers. What are the units of $\int_0^5 r(c)dc$?

a) liters b) kilometers c) liter-kilometers d) liters/kilometers e) NOTA

12. A particle is in the xy-plane so that at any time t its coordinates are $x = \alpha \cos \beta t$ and $y = \alpha \sin \beta t$, where α and β are constants. The $y-$component of the acceleration of the particle at any time t is

a) $-\beta^2 y$ b) $-\beta^2 x$ c) $-\alpha\beta \sin \beta t$ d) $-\alpha\beta \cos \beta t$ e) NOTA

13. A certain type of fish grows from x million to $x(15-x)$ million each year. In order to sustain a steady catch each year, a limit of $x(15-x)-x$ million are to be caught, leaving x million fish to reproduce each year. What is the number of fish (in millions) which should be left to reproduce each year so that the maximum catch may be sustained from year to year?

a) 5 b) 7 c) 7.5 d) 10 e) NOTA

14. A ninja warrior is attached to the end of a 30-foot hanging rope that weighs 0.2 lb/ft. The ninja climbs the rope to the top. How much work has the ninja done (in ft-lbs)?

a) 90 b) 150 c) 200 d) 240 e) NOTA

15. Sand is being poured onto a conical pile at a constant rate of 50 cubic ft. per minute. The height of the pile is always equal to the radius of its base. How fast is the height of the pile increasing when the sand is 5 ft. deep (in ft. per minute)?

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{3}{\pi}$ d) $\frac{2}{\pi}$ e) NOTA

16. Ethel is attempting to climb a rope that is not securely fastened. If she pulls herself up x feet at once, then the rope slips x^3 feet down. How many feet at a time must she pull herself up to climb as efficiently as possible?

a) $\frac{\sqrt{2}}{2}$ b) 2 c) $\frac{\sqrt{3}}{3}$ d) 3 e) NOTA

17. Lucy is lost in the densely-forested Cartesian plane. Starting from the origin she walks a sinusoidal path in search of home; that is, after t minutes she is at position (t, sin t). Five minutes after she sets out, Desi enters the forest at the origin and sets out in search of Lucy. He walks in such a way that after he has been in the forest for m minutes, his position is (m, cos t). What is the greatest distance between Lucy and Desi while they are walking along these paths?

a) $\sqrt{29}$ b) $\sqrt{30}$ c) 6 d) $\sqrt{42}$ e) NOTA

18. Paul escapes from prison (where he was unjustly sent) to get back to his home in Alabama. He stopped to rest at a place 1,875 feet from the prison and was spotted by a guard with a crossbow. The guard fired an arrow with an initial velocity of 100 ft/s. At the same time, Paul started running away with an acceleration of 1 ft/s^2. Assuming that air resistance causes the arrow to decelerate at 1 ft/s^2 and that it does hit Paul, how fast was the arrow moving at the moment of impact (in ft /s)?

a) 25 b) 50 c) 100 d) 150 e) NOTA

19. A cube with sides 1 m in length is filled with water, and has a tiny hole through which the water drains into a cylinder of radius 1 m. If the water level in the cube is falling at a rate of 1 cm/s, at what rate is the water level in the cylinder rising (in cm/s)?

a) $\frac{1}{\pi}$ b) $\frac{1}{2\pi}$ c) $\frac{2}{\pi}$ d) $\frac{2}{3\pi}$ e) NOTA

20. A rectangle has sides of length sin x and cos x for some x. What is the largest possible area of such a rectangle?

a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) 1 e) NOTA

21. A rectangle of length $\frac{\pi}{4}$ and height 4 is bisected by the x-axis and is in the 1st and 4th quadrants. The graph of y = sin(x) + C divides the area of the rectangle in half. What is C?

a) $\frac{\sqrt{2}-4}{\pi}$ b) $\frac{2\sqrt{2}-4}{\pi}$ c) $\frac{\sqrt{2}-2}{\pi}$ d) $\frac{2\sqrt{2}-2}{\pi}$ e) NOTA

22. A soccer ball is flawed and hence inflates in the shape of a perfect cube. It is being inflated at a rate such that at time t fortnights, it has surface area $6t$ square furlongs. How many cubic furlongs per fortnight is the air being pumped in when the surface area is 144 square furlongs?

a) $\sqrt{3}$ b) $6\sqrt{3}$ c) $\sqrt{6}$ d) $3\sqrt{6}$ e) NOTA

23. Fred and Wilma are standing on a Cartesian plane. Fred starts at (−1, 1), and Wilma starts at (1, 1). They both walk to the right along the parabola $y = x^2$ such that their midpoint moves along the line $y = 1$ with constant speed 1. When Fred first hits the line $y = \frac{1}{2}$, what is his speed?

a) $2\sqrt{3}+3$ b) $3\sqrt{3}+3$ c) $2\sqrt{3}-3$ d) $3\sqrt{3}-3$ e) NOTA

24. The equation $\frac{d}{dx}\left[F(x)\right] = f(x)$ means

a) F is an antiderivative of f. b) f is an antiderivative of F.

c) F is the derivative of f. d) $f'(x) = F(x)$ e) NOTA

25. If $f(x)$ is a continuous function, then the Second Fundamental Theorem of Calculus says that

a) The function $F(a) = \int_a^x f(t)dt$ is an antiderivative of $f(x)$.

b) The function $F(x) = \int_x^a f(t)dt$ is an antiderivative of $f(x)$.

c) The function $F(x) = \int_a^x f(x)dx$ is an antiderivative of $f(x)$.

d) The function $F(x) = \int_a^x f(t)dt$ is an antiderivative of $f(x)$.

e) NOTA

26. The function $v(t)$ is a velocity function on the time interval $[a,b]$. Which selection below is the best interpretation of the integral $\int_a^b v(t)dt$?

a) The total distance traveled from $t = a$ to $t = b$.
b) The change in position between $t = a$ and $t = b$.
c) The fastest speed attained on $[a,b]$.
d) The area of the velocity function on $[a,b]$.
e) NOTA

27. Assume that $s(t)$ is a position function with velocity $v(t) = s'(t)$ on the interval $[a,b]$. Which of the following statements is true?

a) $\int_a^b v(t)dt = s(b) - s(a)$ b) $\int_a^b v(t)dt = s(a) - s(b)$ c) $\int_a^b |v(t)|\,dt = s(b) - s(a)$

d) $\int_a^b |v(t)|\,dt = s(a) - s(b)$ e) NOTA

28. Kanye is trying to paint the interval [0, 5] using red and green paints. Painting at the point x using red paint costs $2x$ dollars per unit length and using green paint costs x^2 dollars per unit length. What is the minimum amount of money Kanye needs to spend to paint the entire interval if he's allowed to change colors as he paints?

a) $\frac{28}{3}$ b) $\frac{54}{3}$ c) $\frac{71}{3}$ d) $\frac{121}{3}$ e) NOTA

29. At the math convention in Utah two years ago there was an elf named Dobby who had an infinite amount of time. That year, he walked continuously at a speed of $\frac{1}{1+t^2}$, starting at time t = 0. If he continued to walk for an infinite amount of time, how far would he have walked then?

a) $\frac{\pi}{4}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{2}$ d) $\frac{3\pi}{2}$ e) NOTA

30. A 13-foot-tall alien is standing on a very small spherical planet with radius 156 feet. It sees a bug crawling along the horizon. If the bug circles the alien exactly once, always staying on the horizon, how far will it travel (in feet)?

a) 12π b) 60π c) 120π d) 180π e) NOTA