1	B	11	C	21	D
2	A	12	B	22	A
3	B	13	D	23	B
4	D	14	B	24	A
5	A	15	C	25	D
6	B	16	A	26	A
7	B	17	C	27	C
8	E	18	B	28	B
9	A	19	C	29	A
10	C	20	C	30	B

1)
$$\begin{aligned}\int_2^5 (x-2)(x-5)\ dx &= \int_2^5 (x^2-7x+10)\ dx\\ &= \left(\frac{x^3}{3}-\frac{7x^2}{2}+10x\right)\Bigg]_2^5\\ &= -\frac{9}{2}\\ &\implies \boxed{\text{B}}\end{aligned}$$

2) The function is increasing and concave up on the interval, so I < II < IV < III. $\boxed{\text{A}}$

3)
$$\begin{aligned}\int_0^{\frac{\pi}{4}} \frac{\cos 2t\ dt}{\cos t-\sin t} &= \int_0^{\frac{\pi}{4}} \frac{\cos^2 t-\sin^2 t}{\cos t-\sin t}\ dt\\ &= \int_0^{\frac{\pi}{4}} (\cos t+\sin t)\ dt\\ &= (\sin t-\cos t)]_0^{\frac{\pi}{4}}\\ &= 1\\ &\implies \boxed{\text{B}}\end{aligned}$$

4) The derivative of the integral is $(a+1)(-a)-a(1-a) = -a(a+1+1-a) = -2a = 0 \implies a=0 \implies$ The maximum value is $\int_0^1 (x-x^2)dx = \frac{1}{2}-\frac{1}{3} = \frac{1}{6}$ $\boxed{\text{D}}$

5) $\frac{28}{3}$ $\boxed{\text{A}}$

6) Set $u = \arcsin x$. Then $x = \sin u$ and $du = d(\arcsin x)$.
$$\begin{aligned}\int \frac{d(\arcsin x)}{\sqrt{1-x^2}} &= \int \frac{du}{\sqrt{1-\sin^2 u}}\\ &= \int \sec u\ du\\ &= \ln(\tan u+\sec u)+C\\ &= \ln(\tan(\arcsin x)+\sec(\arcsin x))+C\\ &= \ln\left(\frac{x+1}{\sqrt{1-x^2}}\right)+C\\ &\implies \boxed{\text{B}}\end{aligned}$$

7) $log_{x^2+1}\left(e^{\frac{2x}{x^2+1}}\right) = \dfrac{\frac{2x}{x^2+1}}{\ln(x^2+1)} \implies \int_1^2 \log_{x^2+1}\left(e^{\frac{2x}{x^2+1}}\right)dx = \int_1^2 \dfrac{\frac{2x}{x^2+1}}{\ln(x^2+1)}dx = \int_{\ln(2)}^{\ln(5)} \dfrac{du}{u} = \ln\left(\frac{\ln(5)}{\ln(2)}\right)$. $\boxed{\text{B}}$

8) The integrand has an asymptote at $x=2$ $\boxed{\text{E}}$

9)
$$\begin{aligned}\int_0^{\frac{\pi}{4}} \frac{\tan^2 x + 1}{\tan x + 1}\, dx &= \int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{\tan x + 1}\, dx \\ &= \int_1^2 \frac{du}{u} \\ &= \ln 2 \\ &\implies \boxed{\text{A}}\end{aligned}$$

10) By Product-To-Sum, $\sin(20x)\sin(17x) = \frac{1}{2}\left(\cos(3x) - \cos(37x)\right)$.
$$\begin{aligned}\frac{1}{2}\int_0^{\frac{\pi}{2}} \left(\cos(3x) - \cos(37x)\right)\, dx &= \left(\frac{\sin(3x)}{6} - \frac{\sin(37x)}{74}\right)\Bigg]_0^{\frac{\pi}{2}} \\ &= -\frac{20}{111}\end{aligned}$$

$\boxed{\text{C}}$

11)
$$\begin{aligned}\int \sec^3 x\, dx &= \int \sec^2 x \cdot \sec x\, dx \\ &= \sec x \tan x - \int \sec x \tan^2 x\, dx \\ &= \sec x \tan x - \int \sec x(\sec^2 x - 1)\, dx \\ &= \sec x \tan x - \int (\sec^3 x - \sec x)\, dx \\ &= \sec x \tan x + \ln(\sec x + \tan x) - \int \sec^3 x\, dx\end{aligned}$$

Adding the $\int \sec^3 x\, dx$ to each side and dividing by 2, $\int \sec^3 x\, dx$ is equal to the average of $\sec x \tan x$ and $\ln(\sec x + \tan x) + C$, which are respectively the derivative and integral of $\sec x$. $\boxed{\text{C}}$

12) $\int_0^2 kx^3 dx = 12 \implies \frac{k}{4}(16) = 4k = 12 \implies k = 3$ Therefore $\int_0^3 3x^3 dx = \frac{3}{4}(3^4) = \frac{243}{4}$ $\boxed{\text{B}}$

13) $v(t) = t^2 - 4t + 10$, which is equal to 7 at $t = 1$ and $t = 3$. $x(t) = \frac{t^3}{3} - 2t^2 + 10t - 6$. $x(3) = 15$. $\boxed{\text{D}}$

14)
$$\begin{aligned}
\int_0^\infty \frac{x\ln x}{16+x^4}\ dx &= \frac{1}{2}\int_0^\infty \frac{x\ln(x^2)}{16+x^4}\ dx \\
&= \frac{1}{4}\int_0^\infty \frac{\ln u}{16+u^2}\ du \\
&= \frac{1}{16}\int_0^\infty \frac{\ln(4w)}{1+w^2}\ dw \\
&= \frac{1}{16}\int_0^\infty \frac{\ln 4+\ln w}{1+w^2}\ dw \\
&= \frac{\pi\ln 2}{16}+\frac{1}{16}\int_0^\infty \frac{\ln w}{1+w^2}\ dw \\
&= \frac{\pi\ln 2}{16}+\frac{1}{16}\int_0^{\frac{\pi}{2}} \ln(\tan\theta)\ d\theta \\
&= \frac{\pi\ln 2}{16}+\frac{1}{16}\int_0^{\frac{\pi}{2}} (\ln(\sin\theta)-\ln(\cos\theta))\ d\theta \\
&= \frac{\pi\ln 2}{16} \\
&\implies \boxed{\text{B}}
\end{aligned}$$

15)
$$\begin{aligned}
\int_0^\infty \frac{dx}{(1+x^2)(1+x^{2018})} &= \int_0^\infty \frac{u^{2018}}{(u^2+1)(u^{2018}+1)}\ du \\
&= \frac{1}{2}\int_0^\infty \left(\frac{1}{(u^2+1)(u^{2018}+1)}+\frac{u^{2018}}{(u^2+1)(u^{2018}+1)}\right)\ du \\
&= \frac{1}{2}\int_0^\infty \frac{du}{1+u^2} \\
&= \frac{\pi}{4} \\
&\implies \boxed{\text{C}}
\end{aligned}$$

16)
$$\begin{aligned}
\int_0^1 x^2(x+1)^5\ dx &= \int_1^2 (u-1)^2u^5\ du \\
&= \int_1^2 (u^7-2u^6+u^5)\ du \\
&= \left(\frac{u^8}{8}-\frac{2u^7}{7}+\frac{u^6}{6}\right)\Bigg]_1^2 \\
&= \frac{341}{56}
\end{aligned}$$

$\boxed{\text{A}}$

17) The limit at ∞ makes this the average value of $\arctan^2 x$ over $\mathbb{R}^+$. Because $\arctan^2 x$ asymptotically approaches $\frac{\pi^2}{4}$, the average value of it over $\mathbb{R}^+$ is $\frac{\pi^2}{4}$. $\boxed{\text{C}}$

18) Let $u = \sqrt{4+\sqrt{x}}$. Then $x = u^4 - 8u^2 + 16$ and $dx = (4u^3 - 16u)\ du$. Then the integral becomes $\displaystyle\int_2^3 (4u^4 - 16u^2)\ du = \frac{1012}{15}$. $\boxed{\text{B}}$

19) Note that $\left(\frac{e}{x}\right)^x = e^{x - x\ln x}$ and $\left(\frac{x}{e}\right)^x = e^{x\ln x - x}$.

$$\begin{aligned}
\int_1^e \left[\left(\frac{e}{x}\right)^x + \left(\frac{x}{e}\right)^x\right] \ln x\ dx &= \int_1^e (e^{x - x\ln x} + e^{x\ln x - x}) \ln x\ dx \\
&= \int_{-1}^0 (e^{-u} + e^u)\ du \\
&= e - \frac{1}{e} \\
&\implies \boxed{\text{C}}
\end{aligned}$$

20)

$$\begin{aligned}
2\pi \int_0^1 xe^{-x^2}\ dx &= \pi \int_0^1 e^{-u}\ du \\
&= -\pi e^{-u}\Big]_0^1 \\
&= \frac{(e-1)\pi}{e} \\
&\implies \boxed{\text{C}}
\end{aligned}$$

21)

$$\begin{aligned}
\int_0^\infty \ln(1 - e^{-x})\ dx &= -\int_0^\infty \sum_{n=1}^\infty \frac{e^{-nx}}{n}\ dx \\
&= -\sum_{n=1}^\infty \int_0^\infty \frac{e^{-nx}}{n}\ dx \\
&= -\sum_{n=1}^\infty \frac{1}{n^2} \\
&= -\frac{\pi^2}{6} \\
&\implies \boxed{\text{D}}
\end{aligned}$$

22) $\dfrac{dx}{dt} = 6t$ and $\dfrac{dy}{dt} = 3t^2 - 3$.

$$\begin{aligned}
\int_0^3 \sqrt{(6t)^2 + (3t^2 - 3)^2}\ dt &= \int_0^3 \sqrt{9t^4 + 18t^2 + 9}\ dt \\
&= \int_0^3 (3t^2 + 3)\ dt \\
&= 36 \\
&\implies \boxed{\text{A}}
\end{aligned}$$

23) Set $f(x) = \dfrac{x^4+1}{e^x+1}$. Using the hint, the integral is equal to $\displaystyle\int_0^1 (f(x)+f(-x))\ dx$. $f(x)+f(-x) = \dfrac{x^4+1}{e^x+1} + \dfrac{(-x)^4+1}{e^{-x}+1} = \dfrac{x^4+1}{e^x+1} + \dfrac{e^x(x^4+1)}{1+e^x} = x^4+1$. Thus, the integral transforms to $\displaystyle\int_0^1 (x^4+1)\ dx = \frac{6}{5}$. $U=6$ and $V=5$, so $U+V=11$. $\boxed{\text{B}}$

24) $\dfrac{f(16)-f(4)}{16-4} = \dfrac{240}{12} = 20$. The value of c that satisfies the Mean Value Theorem for Derivatives is the value such that $2c = 20 \Rightarrow c = 10$.

$\displaystyle\frac{1}{16-4}\int_4^{16} x^2\ dx = 112$. The value of c that satisfies the Mean Value Theorem for Integrals is the value such that $c^2 = 112 \Rightarrow c = 4\sqrt{7}$. Daniel's value is smaller. $\boxed{\text{A}}$

25) $\dfrac{k^2}{k^3+n^3} = \dfrac{1}{n}\dfrac{\frac{k^2}{n^2}}{\frac{k^3}{n^3}+1}$ So the limit of the sum is the same as $\int_0^1 \dfrac{x^2}{x^3+1}dx = \frac{1}{3}\ln(2)$ $\boxed{\text{D}}$

26) Convert to polar. Set $x = r\cos\theta$ and $y = r\sin\theta$. Then $r^4\cos^4\theta + r^3\sin^3\theta = r^3\cos^2\theta\sin\theta$, so $r = \tan\theta\sec\theta(1-\tan^2\theta)$. This is equal to 0 when $\theta = 0$ and $\theta = \dfrac{\pi}{4}$.

$$\begin{aligned}\frac{1}{2}\int_0^{\frac{\pi}{4}} \tan^2\theta\sec^2\theta(1-\tan^2\theta)^2\ d\theta &= \frac{1}{2}\int_0^1 u^2(1-u^2)^2\ du \\ &= \frac{1}{2}\int_0^1 (u^6 - 2u^4 + u^2)\ du \\ &= \frac{4}{105}\end{aligned}$$

$M = 4$ and $N = 105$, so $M+N = 109$. $\boxed{\text{A}}$

27) $\int_{-\infty}^{\infty} e^{-k|x|}dx = 1 \implies \int_0^{\infty} e^{-kx}dx = \frac{1}{2} \implies \frac{1}{2} = \frac{1}{k} \implies k = 2$ $\boxed{\text{C}}$

28) $\int_{-1}^{1} e^{-2|x|}dx = 2\int_0^1 e^{-2x}dx = 2(\frac{1}{2} - \frac{1}{2}e^{-2}) = 1 - e^{-2}$. So we need $\frac{1}{2} - \frac{1}{2}e^{-2} = \int_A^{\infty} e^{-2x}dx = \frac{1}{2}e^{-2A} \implies 1 - e^{-2} = e^{-2A} \implies A = -\frac{1}{2}\ln(1-e^{-2})$ $\boxed{\text{B}}$

29) If $t = \tan\left(\dfrac{\theta}{2}\right)$, then $\cos\theta = \dfrac{1-t^2}{1+t^2}$, $\sin\theta = \dfrac{2t}{1+t^2}$, and $d\theta = \dfrac{2\ dt}{1+t^2}$.

$$\begin{aligned}\int \csc\theta\ d\theta &= \int \frac{d\theta}{\sin\theta} \\ &= \int \frac{1+t^2}{2t} \times \frac{2}{1+t^2}\ dt \\ &= \int \frac{dt}{t} \\ &\implies \boxed{\text{A}}\end{aligned}$$

30) $$\int_0^\infty \frac{e^{2/(1+x^2)}\cos\left(\frac{2x}{1+x^2}\right)}{1+x^2}\,dx = \frac{e}{2}\int_0^\infty e^{(1-x^2)/(1+x^2)}\cos\left(\frac{2x}{1+x^2}\right)\times\frac{2}{1+x^2}\,dx.$$

Applying the Weierstrass substitution transforms the integral to $\frac{e}{2}\int_0^\pi e^{\cos t}\cos(\sin t)\,dt$.

Richard Feynman was famous for popularizing the trick of differentiating under the integral sign, which can be done here. Consider the function $I(a) = \int_0^\pi e^{a\cos t}\cos(a\sin t)\,dt$. $I'(a) = \int_0^\pi e^{a\cos t}(\cos(t)\cos(a\sin t) - \sin(t)\sin(a\sin t))\,dt$. Integrating, this is equal to $I'(a) = \frac{1}{a}e^{a\cos t}\sin(a\sin t)\Big|_0^\pi = 0$. Thus, $I(a)$ is a constant function. By examination, $I(0) = \pi$, so $I(a) = \pi$. This means $I(1) = \pi$. Multiplying by the scalar, the integral is $\frac{\pi e}{2}$. $A = 1$, $B = 1$, and $C = 2$, so $A + B + C = 4$. $\boxed{\text{B}}$