# Estimating 7Q10 Confidence Limits from Data: A Bootstrap Approach

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**Abstract:** 7Q10 streamflow estimates used to support modeling and data analysis under the Clean Water Act national pollution discharge elimination system and total maximum daily load programs can have direct environmental and economic impacts. Thus it is important that 7Q10 streamflow always be reported together with confidence limits indicating the reliability of the estimate. In practice this is rarely done. This technical note presents a bootstrap approach for computing 7Q10 confidence limits from data and compares it to an empirical method. A case study using randomly selected subsets of data from five rivers in Idaho is used to evaluate the two methods. While both methods exhibit the expected increase in confidence interval as fewer years of data used, the bootstrap approach generally results in wider confidence intervals than does the empirical method. The opposite appears to be true in cases where fewer than 15 years of data are used or when the data are positively skewed. As most streamflow data are positively skewed short records, the bootstrap approach can generally be thought of as a more conservative means for estimating 7Q10 confidence intervals.

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## Introduction

The 7Q10 streamflow statistic is defined as the "7-day, consecutive low flow with a 10-year return frequency; [or] the lowest streamflow for seven consecutive days that would be expected to occur once in 10 years" (USEPA 1997). 7Q10 is regularly used as a representative low streamflow value for regulatory and modeling purposes, particularly with respect to point-source pollution. Several states and the federal government require use of 7Q10 when determining allowable pollutant levels from point sources under the national pollutant discharge elimination system (NPDES) (for example, see *Federal Register* 1995 and NCDWQ 1999) or from point and nonpoint sources under the total maximum daily load (TMDL) program (for example, *Federal Register* 1998). In these cases, regulated entities are required to reduce pollution discharges such that concentrations in downstream water bodies remain below legally prescribed limits. Given the same pollutant load, lower streamflow results in less dilution and higher pollutant concentration—thus 7Q10 can be used as a benchmark by which the allowable pollutant load is set.

Methods for 7Q10 computation are documented in the report of the ASCE Task Committee on low-flow evaluation, methods, and needs of the Committee on Surface-Water Hydrology of the Hydraulics Division (ASCE 1980). Although this report has been available for several years, less statistically sound solutions continue to be used (e.g., selecting the lowest 7 day average streamflow that occurred in the past 10 years) particularly in cases of limited data (for example DEPLW 2000). Regardless of the method used to estimate the 7Q10, confidence limits can also be derived.

In practice, confidence limits for 7Q10 estimates are rarely provided. For example, of the three approved TMDLs on the EPA.GOV website (USEPA 2005) which use 7Q10 streamflow, none provide an indication of confidence limits. In fact, an informal review of approved TMDL and NPDES reports on state and federal websites indicates that confidence limits are never provided together with 7Q10 estimates. Interestingly, in many cases these reports also do not provide enough information (at least the USGS station number and period used) for a third party to recompute the 7Q10 and derive confidence limits.

This lack of confidence limits is problematic considering that the 7Q10 estimate for any given river can result in economic impacts on regulated entities and environmental impacts downstream from pollution sources. For example, a low 7Q10 estimate may result in industry and agricultural interests being required to alter their operations at some expense (e.g., a wastewater-treatment plant may be required to install new equipment to attain lower effluent pollutant concentration). Similarly, high 7Q10 estimates may imply higher than actual dilution potential and greater pollutant capacity. In the first case, wide confidence limits would weaken the argument for the required operational changes—effectively favoring the polluter. In the second case, wide confidence limits might suggest that the water body has less dilution potential—effectively favoring environmental concerns. The value of formal 7Q10 confidence limits is evident in both cases.

The remainder of this technical note presents a simple bootstrap approach for computing 7Q10 confidence limits. Equations

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J. Water Resour. Plann. Manage., 2006, 132(3): 204-208

and pseudocode are also provided for the approach as are equations for computing confidence intervals using an empirical statistical method. Finally, a case study of five rivers in Idaho is presented to evaluate the results of both methods under different data availability scenarios.

## Methodology

### Log-Pearson Type III Probability Distribution

ASCE (1980) recommends the use of the Log-Pearson Type III probability distribution for low flow estimates. The Pearson distribution was proposed by Karl Pearson as a general equation that fits many distributions, including normal, beta, and gamma, depending on the values selected for its three parameters, $\lambda$, $\beta$, and $\varepsilon$. One form of the Pearson distribution is known as the Pearson Type III (PTIII) distribution. When natural logs of data are fit to the PTIII distribution, it is called the Log-Pearson Type III.

The PTIII probability density function can be represented as

$$p(x) = \frac{\lambda^{\beta}(x-\varepsilon)^{\beta-1}e^{-\lambda(x-\varepsilon)}}{\Gamma(\beta)} \tag{1}$$

where $\lambda$, $\beta$, and $\varepsilon$=parameters of the distribution; and $\Gamma(\beta)$=gamma function evaluated at $\beta$. The three parameters of the PTIII distribution can be estimated from data using the method of moments or the method of maximum likelihood. The U.S. Department of the Interior (1981) recommends using the method of moments for its relative simplicity. Using the method of moments, each of the distribution parameters can be computed from the mean, $\mu$, standard deviation, $\sigma$, and skew, $\gamma$, of the sample data—in this case, the sample data consist of logarithms of the yearly lowest 7 day averages. The following equations, taken from *Guidelines* (U.S. Department of the Interior 1981), show the relationships between the PTIII distribution parameters ( $\lambda$, $\beta$, and $\varepsilon$) and the data statistics ($\mu$, $\sigma$, and $\gamma$)

$$\beta = \left(\frac{2}{\gamma}\right)^2 \tag{2}$$

$$\lambda = \frac{\sqrt{\beta}}{\sigma} \tag{3}$$

$$\varepsilon = \mu - \frac{\beta}{\lambda} \tag{4}$$

For the purpose of estimating the low flow magnitude that has a recurrence probability of one in 10 years, one must evaluate the integral of the PTIII probability density function (the following equation), solving for $x'$ given $P(x \leqslant x')=0.10$

$$P(x \leqslant x_0) = \frac{\lambda^{\beta}}{\Gamma(\beta)} \cdot \int_0^{x'} (x-\varepsilon)^{\beta-1}e^{-\lambda(x-\varepsilon)}dx \tag{5}$$

Kite (1988) shows that $x'$ in Eq. (5) can be approximated by the expression

$$x' \approx \frac{\beta}{\lambda}\left(1 - \frac{1}{9\beta} + t\sqrt{\frac{1}{9\beta}}\right)^3 + \varepsilon \tag{6}$$

where $t$=standard normal deviate corresponding to the value of $P(x \leqslant x')$. Values for $t$ can be read from tables of standard normal deviates found in most statistics texts or by using the Microsoft Excel function NORMINV. For a 10 year return period one would use $P(x \leqslant x')=0.10$ and $t=-1.2816$. Using Eq. (6) and the relationships in Eqs. (2)–(4), one can determine the magnitude of the 7Q10 flow given only the mean, standard deviation, and skew of the logs of the yearly lowest 7 day average flows.

### Empirical Method Confidence Limits

Confidence limits for the 7Q10 statistic can be derived empirically from data by estimating the standard error, $S$, as

$$S \approx \delta\sqrt{\frac{\sigma}{n}} \tag{7}$$

where $\delta$ is dependent on the standard normal deviate, $t$, skew, $\gamma$, slope, $m$, and frequency factor, $K$, computed using the following equations

$$\delta = 1 + \left(1 + \frac{3\gamma^2}{4}\right)\left(\frac{K^2}{2}\right) + K\gamma + 6\left(1 + \frac{\gamma^2}{4}\right)m \times \left[m\left(1 + \frac{5\gamma^2}{4}\right) + \frac{K\gamma}{2}\right] \tag{8}$$

$$m = \left(\frac{t^2-1}{6}\right) + 4\gamma\left(\frac{t^3-6t}{6^3}\right) - 3\gamma^2\left(\frac{t^2-1}{6^3}\right) + 4\gamma^3\left(\frac{t}{6^4}\right) - 5\gamma^4\left(\frac{2}{6^6}\right) \tag{9}$$

$$K = t + \gamma\left(\frac{t^2-1}{6}\right) + 2\gamma^2\left(\frac{t^3-6t}{6^3}\right) - \gamma^3\left(\frac{t^2-1}{6^3}\right) + \gamma^4\left(\frac{t}{6^4}\right) - \gamma^5\left(\frac{2}{6^6}\right) \tag{10}$$

Upper and lower confidence bounds at the 5 and 95% limit can be computed from the standard error, $S$, and the 7Q10 estimate, $\theta$, as

$$C.L. = \exp(\theta \pm S \cdot t_{0.05}) \tag{11}$$

where $t_{0.05}$=standard normal deviate corresponding to the 5th percentile (Kite 1988).

### Bootstrap Confidence Limits

Another approach for estimating 7Q10 confidence limits uses the bootstrap concept introduced by Efron (1979) as a general method for assessing the accuracy of a statistic. The bootstrap has been used extensively in hydrology since its inception (e.g., see Rajagopalan and Lall 1999; Chrysikopoulos et al. 2002; and Srinivas and Srinivasan 2005). Applied to 7Q10, the bootstrap approach assumes a vector of yearly low flows, $\mathbf{X_0}=(x_1, x_2, \ldots x_n)$ and a 7Q10 statistic, $\theta_0$, derived from $\mathbf{X_0}$. To compute a confidence interval for $\theta_0$, one generates a large number, $M$, of new "realizations" of the data set, $\mathbf{X_0}$, each populated with data by randomly sampling with replacement from $\mathbf{X_0}$. This results in $M$ data sets $\mathbf{X_1}, \mathbf{X_2}, \ldots \mathbf{X_M}$. For each of these new data sets, $\mathbf{X_j}$, the 7Q10 statistic, $\theta_j$, is then computed. This results in the vector, $\mathbf{\Theta}=(\theta_1, \theta_2, \ldots \theta_m)$ representing the range of possible values of $\theta$ given the realizations of the original data set, $\mathbf{X_0}$. From $\mathbf{\Theta}$, one can estimate confidence limits for $\theta_0$ by sorting and ranking the values in $\mathbf{\Theta}$ and identifying the 5 and 95% confidence limits as the values of $\theta_j$ that lie at 5 and 95% of the ranked data (e.g., $\theta_{50}$ and $\theta_{951}$) thus forming a 90% confidence interval around $\theta_0$.



J. Water Resour. Plann. Manage., 2006, 132(3): 204-208

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MAIN PROGRAM:
   read x(1..n)                       //x is an array of n annual low flows
   for i=1 to n                       //loop through all years
      x(i)=ln(x(i))                   //take the natural log of each flow
   next i
   7Q10=exponent[GET_7Q10(x)]         //compute the initial 7Q10, θ0
   for j=1 to 1000                    //loop through 1000 realizations
      y=GET_RESAMPLE(x)               //create array y by resampling array x
      theta(j)=exponent[GET_7Q10(y)]//compute 7Q10, θj
   next j
   sort(theta)                        //sort θ1..θ1000 in ascending order
   conf_5=theta(50)                   //report θ50 as the 5% confidence limit
   conf_95=theta(951)                 //report θ951 as the 95% confidence limit
END MAIN PROGRAM

FUNCTION GET_7Q10(input array: x(1...n))
   for i=1 to n                       //loop through years
      sum=sum+x(i)
      sum2=sum2+x(i)^2
   next i
   mean=sum/n                         //compute μ
   std_dev=(sum2/n)-(sum/n)^2         //compute σ
   for i=1 to n                       //loop through years
      sum3=sum3+[(x(i)-mean)/std_dev]^3
   next i
   skew=sum3*n/[(n-1)*(n-2)]          //compute γ
   beta=(2/skew)^2                    //compute β
   lamda=SQRT(beta)/std_dev           //compute λ
   epsilon=mean-(beta/lamda)          //compute ε
   //compute the 7Q10 value given these distribution parameters
   7q10=(beta/lamda)*(1-1/(9*beta)-1.2816*SQRT(1/(9*beta)))^3+epsilon
   return 7q10
END FUNCTION GET_7Q10

FUNCTION GET_RESAMPLE(input array: x(1…n))
   declare y(1…n)                     //create an empty array of size n
   for i=1 to n                       //loop through years
      k=random (1, n)                 //generate a random integer from 1 to n
      y(i)=x(k)                       //copy data to the new array
   next i
   return y                           //the function returns the resampled array
END FUNCTION GET_RESAMPLE
```

**Fig. 1.** Pseudocode for implementing bootstrap approach to 7Q10 confidence limits. Syntax presented here is generic and should be easily coded in any programming language that supports function calls and arrays. Comments are indicated by "//."

Fig. 1 shows pseudocode that can be used to implement the bootstrap approach for 7Q10 confidence limits. This code is divided into three blocks with comments marked by "//." The first block contains the main program code. The second block defines a function called "GET_7Q10" used to compute the 7Q10 statistic from an array of yearly low flows. The third block contains a function called "GET_RESAMPLE" that returns an array randomly resampled from an input array. This pseudocode can be encoded in software to assist with estimating 7Q10 confidence limits from data. An implementation of this code using Microsoft Visual Studio NET 2003 is available for download with source code as the "Low Flow Calculator" from ⟨http://www.hydromap.com/⟩.

## Case Study

Five long streamflow records (>80 years) from rivers throughout Idaho (see Fig. 2) were selected for a case study comparing confidence bounds as estimated by both the empirical and bootstrap methods. Table 1 lists each station, years of data, drainage area, and streamflow statistics—including 7Q10 confidence intervals using each method. Interestingly, the bootstrap method results in a wider confidence interval than does the empirical method in four of the five cases. For a decision maker, this can have direct consequences. For example, a wider confidence interval implies less certainty regarding the 7Q10 estimate—a point that can

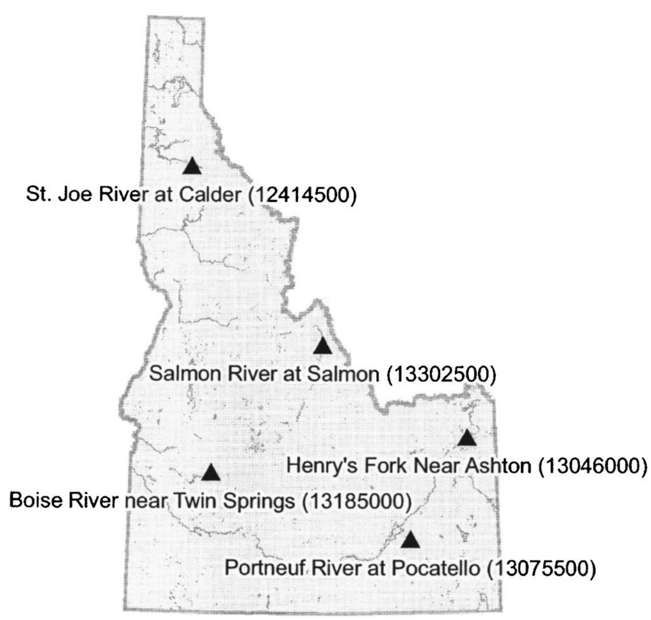


**Fig. 2.** State of Idaho USGS gage station locations used in case study

**Table 1.** USGS gauge stations and statistics for selected rivers in Idaho

| Site | Years | Drainage area ($km^2$) | Mean ($m^3/s$) | Standard deviation ($m^3/s$) | 7Q10 ($m^3/s$) | 90% C.I. empirical ($m^3/s$) | 90% C.I. bootstrap ($m^3/s$) |
|---|---|---|---|---|---|---|---|
| Boise River near Twin Springs (13185000) | 1911–2004 | 2,150 | 33.9 | 41.3 | 6.1 | 5.9–6.4 | 5.8–6.5 |
| Portneuf River at Pocatello (13075500) | 1897–2004 | 3,237 | 7.8 | 6.5 | 0.5 | 0.4–0.5 | 0.3–0.6 |
| Henry's Fork near Ashton (13046000) | 1890–2004 | 2,694 | 43.3 | 20.2 | 16.8 | 15.8–18.0 | 16.0–17.8 |
| Salmon River at Salmon (13302500) | 1912–2004 | 9,738 | 54.6 | 53.1 | 16.1 | 15.4–16.8 | 14.9–17.3 |
| St. Joe River at Calder (12414500) | 1911–2004 | 2,668 | 66.0 | 86.1 | 6.6 | 6.3–7.0 | 6.2–7.1 |



be argued in favor of any entity being negatively affected by resulting regulation (or lack thereof).

The annual low 7 day average streamflows for each data set are shown in Fig. 3. Note that the Portneuf, Boise, and St. Joe rivers exhibit relative stationarity over the 100 year period while minimum flows in the Salmon and Henry's Fork have experienced strong upward and downward trends over various decades. The impact of this nonstationarity on 7Q10 estimates in the Salmon and Henry's Fork rivers is not accounted for in the current study.

To compare the bootstrap and empirical methods for computing confidence limits, annual low flows from each river were randomly decimated to generate 3,500 simulated sparse data sets per river. These simulated data sets were created by randomly removing 1 year from the full record, then randomly removing 2 years from the full record, and so forth until 70 years were removed from the record. This procedure was repeated for each river 50 times. Next, the 7Q10 and the 5 and 95% confidence limits were computed for each of the 3,500 simulated sparse data sets for each river using both the bootstrap and empirical methods.

The results of this analysis indicate that both methods produce similar confidence limits for the same data set; and as expected, when more data are used the confidence bounds are generally tighter. This is illustrated in Figs. 4(a and b). Here, 7Q10 confidence intervals for the St. Joe River are plotted against the number of years of data used. Fig. 4(a) shows that the bootstrap method results in wider confidence bounds in almost all cases except for when fewer than 15 years of data are used. Note that the relationship between percentage increase in confidence interval and number of years of data used is strongly linear with an $R^2$ value of 0.80 and 0.85 for the empirical and bootstrap methods, respectively. Similar results were observed for each of the other rivers in the case study.

Another outcome of the analysis is the observation that the bootstrap method will generally result in wider confidence intervals for negatively skewed data. While each of the rivers in the analysis exhibited this outcome, it is demonstrated by the St. Joe River as shown in Fig. 5. Here, the difference between the confidence intervals as computed using both methods

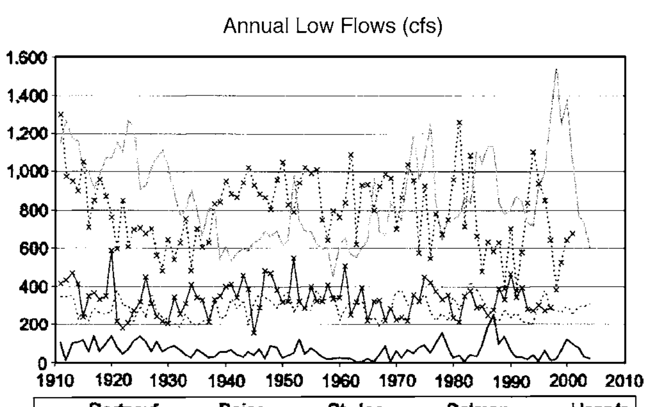


**Fig. 3.** Annual low flows for five streams in Idaho

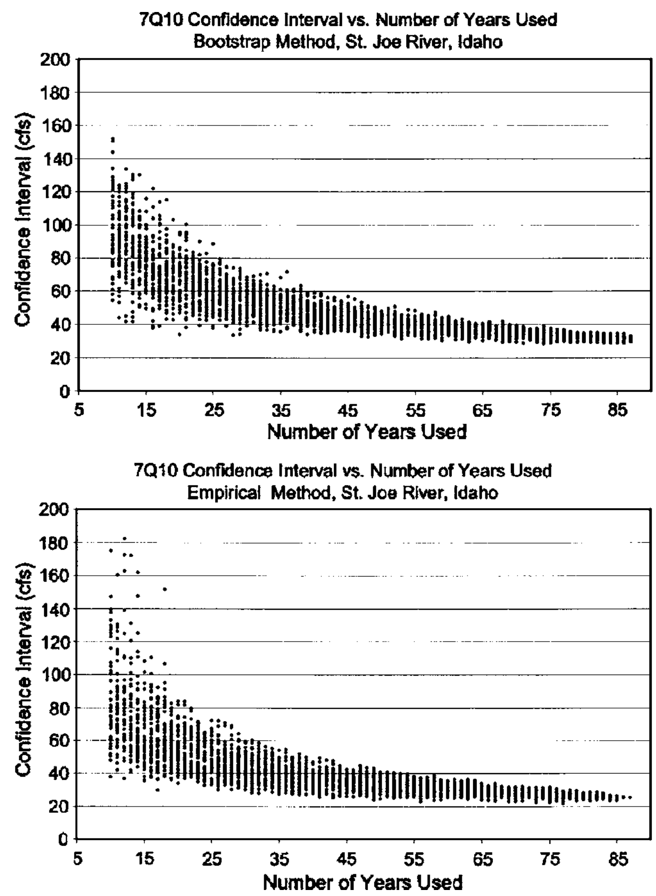


**Fig. 4.** Confidence interval versus number of years used in 7Q10 computation. Note that bootstrap tends to create wider confidence intervals except for when fewer than 15 years of data are used.



J. Water Resour. Plann. Manage., 2006, 132(3): 204-208

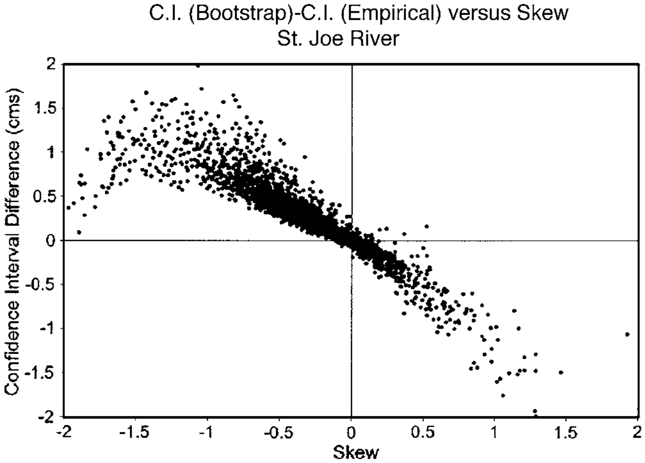


**Fig. 5.** Difference between confidence intervals computed by bootstrap and empirical methods versus skew for St. Joe River

($C.I._{bootstrap} - C.I._{empirical}$) is plotted against the skew of the data, showing that the bootstrap method almost always (98% of the time) produces wider confidence intervals for negatively skewed data. In light of these observations, one could conclude that for short positively skewed data sets, the bootstrap approach is a more conservative means for estimating confidence intervals as it tends to be tighter than the empirical method in these cases. This has implications for permitting, modeling, and management under the NPDES and TMDL programs since tighter confidence intervals suggest a greater reliability of the 7Q10 estimate. Such reliability can lend credibility to regulatory actions.

## Conclusions

Some may argue that, for regulation of pollution sources, the 7Q10 low streamflow is an arbitrary statistic. Indeed, some states have selected other low flow indicators such as the 4Q3 or 7Q2 (see for example TNRC 1998) to use for water quality management. However, regardless of the low flow statistic used, the argument remains that a reliable estimate of its confidence limits should be provided to help ensure that unintended economic and environmental impacts are minimized. Such confidence limits on low flow estimates are rarely provided, and only in limited cases do published TMDL and NPDES reports present enough information for a third party to estimate confidence limits.

This technical note presented a bootstrap method for computing 7Q10 confidence limits from data and compared it to an empirical statistical method using a case study of five rivers in Idaho. Results of the case study showed that in most cases the bootstrap approach generated wider confidence intervals than did the empirical method. Exceptions to this occurred when the data were positively skewed or were short records of less than 15 years of data. Because many streamflow data sets are in fact short and positively skewed, the bootstrap approach can be considered a generally conservative method for computing 7Q10 confidence intervals.

Implications for permitting, modeling, and management are not particularly explicit, though it should be clear that a tighter confidence interval can have the effect of reinforcing the validity of the 7Q10 estimate, subsequent water quality concentration calculations, and potential regulatory actions. When considering environmental and ecological concerns, a wide confidence interval could be used to argue that a river's dilution and assimilation capacity is potentially less than expected. Likewise, regulated individuals might use the confidence bounds to argue against mandated regulatory actions. In either case, the information provided by an estimate of the 7Q10 confidence interval can be valuable if used in the decision making process.

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J. Water Resour. Plann. Manage., 2006, 132(3): 204-208