

## Assignment 1 – Basic probability

### Stock market

There are  $2^4 = 16$  scenarios for the stock price change in 4 days:

- The price goes up by 25% on all days and will be:

$$256 \cdot \left(\frac{125}{100}\right)^4 = 625\$$$

The probability for this scenario is  $\frac{1}{16}$

- The price goes up by 25% on 3 days and down by 20% on 1 day and will be:

$$256 \cdot \left(\frac{125}{100}\right)^3 \cdot \left(\frac{80}{100}\right) = 400\$$$

The probability for this scenario is  $\frac{4}{16} = \frac{1}{4}$  (we have 4 options to the day of which the price goes down)

- The price goes up by 25% on 2 days and down by 20% on 2 days will be:

$$256 \cdot \left(\frac{125}{100}\right)^2 \cdot \left(\frac{80}{100}\right)^2 = 256\$$$

In this case the option worth is 0.

For the other scenarios – the stock price will be lower than 300\$ which is the option price.

**The option worth:**

$$E(\text{profit}) = \frac{1}{16}(625 - 300) + \frac{1}{4}(400 - 300) = 20.31 + 25 = 45.31\$$$

### Hitchhiker

Probability of no car passing the intersection in 20-minutes window: 0.1

The above actually equals the product:

$$(P \text{ no car passing in } 0 - 5) \cdot (P \text{ no car passing in } 5 - 10) \cdot (P \text{ no car passing in } 10 - 15) \\ \cdot (P \text{ no car passing in } 15 - 20)$$

$$\Rightarrow (P \text{ no car passing in 5 minutes interval})^4 = 0.1$$

$$\Rightarrow P(\text{no car passing in 5 minutes}) = 0.562$$

**Probability of at least one car passing the intersection in a 5-minute window:**

$$1 - 0.562 = 0.438$$

### Credit scoring

We need to calculate  $P(\text{bad} \mid \text{recent application, repaid loan}) =$

$$\frac{P(\text{recent application, repaid loan} \mid \text{bad})P(\text{bad})}{P(\text{recent application, repaid loan} \mid \text{bad})P(\text{bad}) + P(\text{recent application, repaid loan} \mid \text{good})P(\text{good})}$$

where:

$$P(\text{bad}) = 0.1$$

Given that recent application is independent of repaying loan for each type of client:

$$P(\text{recent application, repaid loan} \mid \text{bad})$$

$$= P(\text{recent application} \mid \text{bad}) * P(\text{repaid loan} \mid \text{bad}) = 0.8 \cdot 0.05 = 0.04$$

$$P(\text{good}) = 0.9$$

$$P(\text{recent application, repaid loan} \mid \text{good}) = 0.3 \cdot 0.1 = 0.03$$

**The probability that she is bad:**

$$P = \frac{0.04 \cdot 0.1}{0.04 \cdot 0.1 + 0.03 \cdot 0.9} \approx 0.129$$