1.29

(a) $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$

Let P be the pumping length

We choose string, $S = 0^P 1^P 2^P$ $\quad S \in A_1$ & $|S| \geq P$

String y consists only of 0s, only of 1s, or only of 2s.

String $xyyz$ will not have equal number of 0s, 1s, and 2s

Hence $xyyz$ where $i=2$ is not a member of A_1, which this leads to a contradiction. Therefore, A_1 is not regular.

(b) $A_2 = \{www \mid w \in \{a,b\}^*\}$

let P be the pumping length

We choose string $S = a^P b a^P b a^P b$ $\quad S \in A_2$ $\quad |S| \geq P$

String S can be divided into three pieces $S = xyz$ where $|xy| \leq P$. This means xy contains a's. since $|y| > 0$

let $y = a^K$ $(K>0)$ $(i=2)$ then we have $xy^2z = a^{P+K} b a^P b a^P b$, where $P+K > P$, is not in A_2. Therefore S cannot be pumped.

This is a contradiction, hence A_2 is not regular.

(c) $A_3 = \{a^{2^n} \mid n \geq 0\}$ {Here, a^{2^n} means a string of 2^n a's}

Let p be the pumping length.

Choose string s to be $s = a^{2^p}$ $s \in A_3$ and $|s| \geq p$

So we could divide s into three parts $s = xyz$

Third condition says $|xy| \leq p$, meaning $p < 2^p$, so $|y| < 2^p$

Therefore, $|xyyz| = |xyz| + |y| < 2^p + 2^p = 2^{p+1}$. The second condition requires $|y| > 0$ so $2^p < |xyyz| < 2^{p+1}$.

The length of xyyz cannot be power of 2. Hence, xyz is not member of A_3 ($xyz \notin A_3$), (contradiction) therefore, A is not regular.

1.30

The error is that string $s = 0^p1^p$ can not be pumped.

But, string $s = 0^p1^p$ can actually be pumped.

Let $x, y = 0$ and $z = 0^{p-2}1^p$, then we have

(1) For $\forall i \geq 0$, $xy^iz = 00^i(0^{p-2}1^p) = 0^{p-1+i}1^p \in 0^*1^*$

(2) $|y| = |0| = 1 > 0$

(3) $|xy| = |00| = 2 < p$ therefore S can be pumped and hence S is a regular language.

1.42

Let's define two DFAs $D_A = (Q_A, \Sigma, \delta_A, q_A, F_A)$ and $D_B = (Q_B, \Sigma, \delta_B, q_B, F_B)$ that each recognize A and B respectively. We'll go a head and prove by construction. The diffrence between this question and perfect shuffle is that we may switch from D_A to D_B after each chracter is read. In oreder to have a simplified construction, we design an NFA $N = (Q, \Sigma, \delta, q, F)$ That could recognize the shuffle of A and B. N needs to keep track of current states of D_A and D_B. When the whole string is processed, if both DFAs are in accept states, the input is accepted. otherwise, the input string is rejected. NFA could be defined as the following:

- $Q = (Q_A \times Q_B)$
- $q_0 = (q_A, q_B)$
- $F = F_A \times F_B$
- For $a \in \Sigma$, δ is given as
  - $\delta(q_0, \epsilon) = (q_A, q_B)$, which at start state q, N can make D_A, D_B in q_A and q_B respectively.
  - $(\delta_A(x,a), y) \in \delta((x,y), a)$, which says if current state of D_A is x, the current state of D_B is y, when a is read next, we change the current state of A to $\delta_A(x,a)$, while the state of B is not changed.
  - $(x, \delta_B(y,a)) \in \delta((x,y), a)$

1.46

ⓐ $L = \{0^n 1^m 0^n \mid m,n \geq 0\}$

let p be the pumping length.

we choose a string, $s = 0^p 1 0^p$ $s \in L$ and $|s| \geq p$

xy are composed only of zeros (by $|xy| \leq p$)

let's $y = 0^k$ $(k > 0)$. let $i = 0$, then we have:

$i = 0$ $xy^0z = xz = 0^{p-k} 1 0^p$, this is not in L

therefore a contradiction. Hence L is not a regular language.

ⓑ $\{0^m 1^n \mid m \neq n\}$

Observe that $\overline{B} \cap 0^*1^* = \{0^k 1^k \mid k \geq 0\}$.

If this was the case that language B itself was regulare,

then $\overline{B}$ would be regular, and ultimately $\overline{B} \cap 0^*1^*$ would be

regular as well. But, we already know that $\{0^k 1^k \mid k \geq 0\}$

is not a regular language, so B cannot be regular.

(c) $L = \{w \mid w \in \{0,1\}^*$ is not a plindrome$\}$

Let's prove that the original language is not regular by proving, that its compliment is not regular.

Let p be the pumping length.

choose string $s = 0^p 1 0^p$ $\quad s \in L$ & $|s| \geq p$

xy contains only 0's.

let $y = 0^k$ $(k>0)$

Let's have $i=0$, then we have: $xy^0z = 0^{p-k}10^p$

which is not in a palindrome, therefore it's not in the language L.

Hence, L is not regular.

(d) $\{wtw \mid w,t \in \{0,1\}^+\}$

Let p be the pumping length.

Choose string $s = \underbrace{0^p1}_{w}\,1\,\underbrace{0^p1}_{w}$ $\quad s \in L$ & $|s| \geq p$

By $|xy| \leq p$, xy is composed only of zeros, and $y = 0^k$ $(k>0)$

Let's take $i=2$ then we have $xy^2z = xyyz = 0^{(p+k)}110^p1$

Which it shows that $xy^2z \notin L$. Therefore L is not a regular language.

(1.47) $\Sigma = \{1, \#\}$

$Y = \{w \mid w = x_1 \# x_2 \# \cdots \# x_k$ for $k \geq 0$, each $x_i \in 1^*$, and $x_i \neq x_j$ for $i \neq j\}$

let p be the pumping length.

Choose a string, $S = x_1 \# x_2$ if $k=2$

let's take $x_1 = 1^p 1$

And

$x_2 = 111^p$

so $S = 1^p 1 \# 111^p = xyz$ $S \in Y$ & $|S| \geq p$

If $y = \#$ then xy^2z would have two #'s between

x_1 and x_2, so s won't be in Y.

Let $x = 1^p$

$y = 1$

$z = \#111^p$

Then $xy^2z = 1^p (1)^2 \# 111^p$

$= 1^p 11 \# 111^p$

$= 1^{p+2} \# 1^{p+2}$ which shows x_1 and x_2 are equal

Therefore s is not in the language Y. Hence, Y is not a regular language.

1.55

(e) $(01)^*$

Let s be a string in the language. We could have ϵ but it can not be pumped because its length is 0. We could have xyz such that $x = \epsilon$, $y = 01$, and z is everything that satifies the three condition of pumping lemma.

Hence, the min pumping length is 1.

(f) ϵ

let s be a string in the language. let string $s = \epsilon$. Based on, pumping lemma it can't be pumped. Hence, the min pumping length is 0.

(i) 1011

let string $s = 1011$. let's divide s based on pumping lemma into xy^iz such that $x = 10$, $y = 1$, and z be empty string ϵ.

Hence, the min pumping length is 3.

(9) Σ^*

Let string s be divided to xyz in such a way

$x = \varepsilon$, and $y = 01$ and $z = \varepsilon$; Now ε could not

be pumped. Therefore, ten minimum pumping length is 1.