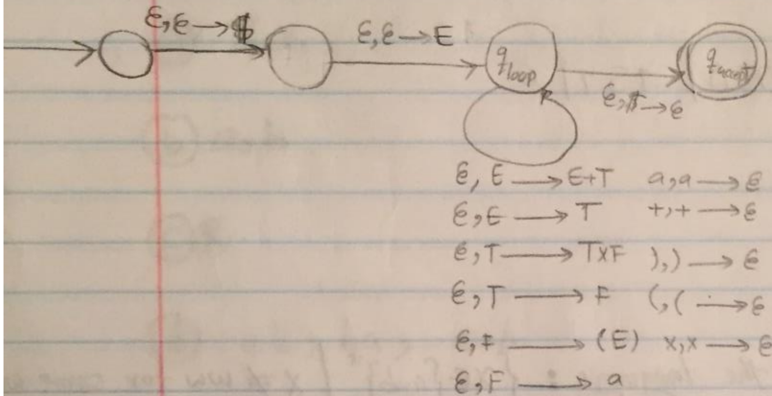
HW #8

2.11 CFG

$E \rightarrow E+T \mid T$

$T \rightarrow T \times F \mid F$

$F \rightarrow (E) \mid a$



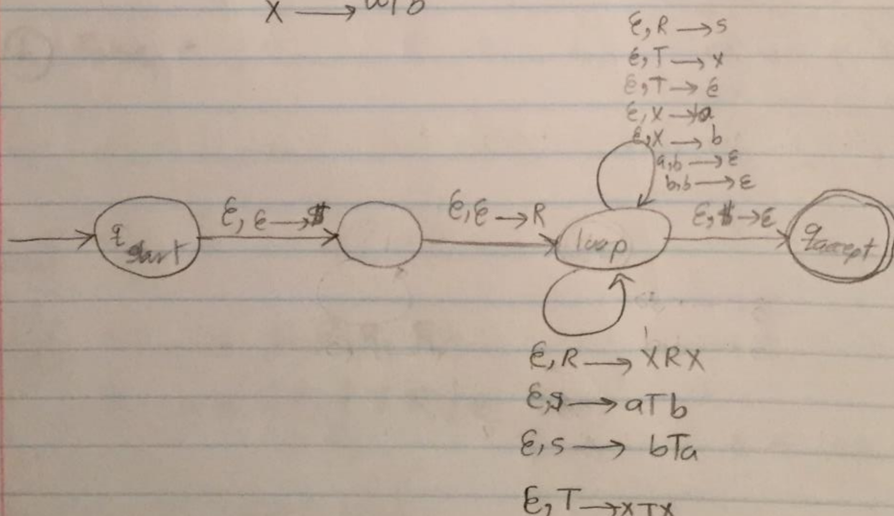
2.12

$R \rightarrow XRX \mid S$

$S \rightarrow aTb \mid bTa$

$T \rightarrow XTX \mid X \mid \varepsilon$

$X \rightarrow a \mid b$



2.14

$A \rightarrow BAB \mid B \mid \epsilon$

$B \rightarrow 00 \mid \epsilon$

- Add start state S

$S \rightarrow A$

$A \rightarrow BAB \mid B \mid \epsilon$

$B \rightarrow 00 \mid \epsilon$

- Remove $B \rightarrow \epsilon$

$S \rightarrow A$

$A \rightarrow BAB \mid A \mid B \mid BA \mid AB \mid \epsilon$

$B \rightarrow 00$

- Remove $A \rightarrow \epsilon$

$S \rightarrow A \mid \epsilon$

$A \rightarrow BAB \mid BA \mid AB \mid BB \mid B \mid A$

$B \rightarrow 00$

- Remove $A \rightarrow A$

$S \rightarrow BAB \mid BA \mid AB \mid BB \mid B \mid \epsilon$

$A \rightarrow BAB \mid BA \mid AB \mid BB \mid B$

$B \rightarrow 0U$

- Remove $A \rightarrow B$

$S \rightarrow BC \mid BA \mid AB \mid UU \mid BB \mid \epsilon$

$A \rightarrow BC \mid BA \mid AB \mid UU \mid BB$

$B \rightarrow UU$

$U \rightarrow 0$

$C \rightarrow AB$

2.30

(a) $\{0^n1^n0^n1^n \mid n \geq 0\}$

Let p be the pumping length.

$s = 0^p1^p0^p1^p$ $\quad s \in L \quad |s| > p$ ✓

$s = uvxyz$, following conditions should hold:

(1) For each $i \geq 0$, uv^ixy^iz is in the language.
(2) $|vy| > 0$
(3) $|vxy| \leq p$

vxy can be the following cases:

(a) 0^p or 1^p

For $i=2$, uv^ixy^iz will lead to more 0s or 1s in first half or the second half of s s.t. Condition (1) of pumping lemma fails.

(b) 0^p1^p in the first half of s

for $i=2$, uv^ixy^iz will lead to more 0s or 1s in the first half.

Condition (1) fails.

(c) 1^p0^p is in the middle of s

for $i=2$, uv^ixy^iz will lead to more 0s or 1s in the middle of s

so condition (1) fails.

② 0^p1^p in the second half of s.

For $i=2$, uv^ixy^iz will lead to more 0s or 1s in the second half, so condition ① fails. ☒

2.30.b

(b) $L = \{0^n \# 0^{2n} \# 0^{3n} \mid n \geq 0\}$

Let p be the pumping length.

Let $s = 0^p \# 0^{2p} \# 0^{3p}$, we want to show that $s = uvxyz$ can't be pumped. We have the following cases: $s \in L$, $|s| > p$

① For each $i \geq 0$, uv^ixy^iz is in the language.

② $|vy| > 0$

③ $|vxy| \leq p$

Case 1 Neither v nor y can contain #, otherwise uv^2xy^2z (when $i=2$) will have more than two #'s.

Therefore, if we divide s into three segments by #'s: 0^p, 0^{2p}, and 0^{3p}, at least one of the segments is not contained within either v or y. Hence uv^2xy^2z is not in B because the 1,2,3 length ratio of the segments is not maintained.

2.30.c

ⓒ $L = \{ w\#t \mid w$ is a substring of t, where $w, t \in \{a,b\}^* \}$

Let p be the pumping length.

$S = a^p b^p \# a^p b^p$, $S \in L$, $|S| \geq p$

Neither v or y can contain #, otherwise uv^0xy^0z does not contain # and therefore is not in L.

If both u and v be on the left side of #, the string uv^2xy^2z (when $i=2$) cannot be in L. (pumped up case)

If both u and v be on the right side of #, the string uv^0xy^0z (when $i=0$) cannot be in L (pumping down case) because it is again longer on the left hand side of #.

If one of v and y is empty (both can't be ε), treat them as if both occurred on the same side of the # as above. ∎

2.30.d

(d) $L = \{t_1\#t_2\# \cdots \#t_k \mid k \geq 2$, each $t_i \in \{a,b\}^*$, and $t_i = t_j$ for some $i \neq j\}$

Let P be the pumping length.

$s = a^P b^P \# a^P b^P$ for $k=2$ in the language. $s \in L$, $|s| > P$

We will have the following cases:

(I) vxy is in either $a^P b^P$ part of s.

In this case, for $i=0$ or $i>1$, the number of as and bs will be unbalanced on both sides of s separated by #, which this violates condition 1 of the pumping lemma.

(II) vxy is in the middle part of s, $b^P \# a^P$

In this case, for $i=0$ or $i>1$, the substring on both sides of separated by # won't be consistent, which this makes condition (1) of the pumping lemma to fail.

Therefore the language is not context-free. ∎

2.31

For Contradiction, let's assume That B is Context-free.

Therefore B has a pumping length P.

$s = 0^P 1^{2P} 0^P$ $s \in B$ & $|s| > P$. Therfore there exists

uvxyz such that:

① For all $i \geq 0$, $uv^ixy^iz \in B$

② $|vy| > 0$

③ $|vxy| \leq P$

we will have the following cases:

Case 1

vxy Consists of 1s only. If $i=2$, we have

$uv^2xy^2z \notin B$, since string s won't have the same number of 0s and 1s.

Case 2

vxy contains at last one 0. If $i=2$ we have $uv^2xy^2z \notin B$ because it is no longer a palindrome. vxy could only contain 0s from the beggining of s or 0s from the end, not both, which this causes s to have different number of 0s before and after 1s.

Since in each case we contradict the first condition of pumping lemma ($\forall i \geq 0 \ uv^i x y^i z \in B$), therefore B is not a context free language. ■

Q.32

$\Sigma = \{1, 2, 3, 4\}$ $\quad C = \{w \in \Sigma^* \mid$ in w, # of 1s = # of 2s, and # of 3s = # of 4s$\}$

diction: Let's assume that C is context free. Let p be the pumping length

$s = 1^p 3^p 2^p 4^p \in C \qquad |s| > p, \ s \in C$

therefore, we have the following cases:

① vxy contains a 1. For $i=2$, we have $uv^2xy^2z \notin C$, because it won't have the same number of 1s and 2s.

② vxy contains a 2. Then $uv^2xy^2z \notin C$ (for $i=2$) because it won't have the same number of 1s and 2s

③ vxy contains a 3. For $i=2$ we have $uv^2xy^2z \notin C$, since s won't have same # of 3s and 4s

④ vxy contains a 4. For $i=2$ we have $uv^2xy^2z \notin C$, since s won't have same # of 3s and 4s. (since vxy can't contain any 3s)

In each case we contradict ①, therefore C is not context free. ■

2.35

Since G is a CFG in chomsky normal form, every derivation can generate at most two non-terminals, so an internal node can have at most two children, which this implies $\Rightarrow$ tree with height h has at most $2^h - 1$ internal nodes.

If G generates some string with derivation having at least 2^b steps, then the parse tree of that string will have at least 2^b internal nodes. Based on what it was mentioned above, height of this parse tree is $b+1$, so there exits a path from root to leaf containing $b+1$ variables. By pigeonhole principle, there is one variable occuring at least twice. So, we could construct infinitely many strings all in L(G) by using pumping Lemma.

- Give an informal description of a PDA that recognizes the language $\{x \in \{a,b\}^* \mid x \neq ww \text{ for some } w \in \{a,b\}^*\}$

- push a \$ symbol to the stack.
- we read the first symbol in the string and push it to the stack.
- While the second symbol read from the string is not the same symbol as the one on the top of the stack, push it to the stack.

By pushing the symbols that are read onto the stack, at each point, nondeterministically guess that the middle of the string has been reached and then change into popping off the stack for each symbol read, checking to see that they are the same. If they were always the same symbol and the stack empties at the same time as the input is finished, accept, otherwise reject.

If $x \in ww^R$ then $x \notin ww$