1.51

For every string z, $xz \in L$ iff $yz \in L$.

(1) Reflexivity : xRx

$\forall$ strings z, $xz \in L$ iff $xz \in L$ $\Rightarrow$ therefore $x I s$ is true.

(2) Symmetry : $xRy \Rightarrow yRx$

If xly is true then ; $\forall z$, $xz \in L$ iff $yz \in L$, which this is equivalent to $\forall z$, $yz \in L$ iff $xz \in L$. Therefore yRx is true.

(3) Transitivity: If aRb and bRc then aRC.

we have : $\forall z$, $az \in L$ iff $bz \in L$, and $bz \in L$ iff $cz \in L$

therefore for $\forall z$, $az \in L$ iff $cz \in L$. Therefore aRC is true.

• Use of Myhill-Nerode Theorem.

1.29

ⓐ $A_1 = \{0^n 1^n 2^n \mid n \geq 0$

Assume $0^n 1^n 2^n$ is regular

$\Rightarrow$ There are $\leq n$ equivalency classes

$S = \{\epsilon, 0, 00, 000, \ldots 0^{n-1}, 0^n\}$ $\quad |S| = n+1$ $\quad \epsilon$

Due to the pigeonhole principle two strings in S must be equivalent.

Let $0^{|x|}$ and $0^{|y|}$ be strings where $0^{|x|} \neq 0^{|y|}$ $\quad |x| \neq |y|$

$z = 1^x 2^x$ then $xz = 0^x 1^x 2^x \in L$, but $yz = 0^{|y|} 1^{|x|} c^{|x|} \notin L$

one is in the language, but one is not in the language.

(b) $A_2 = \{www \mid w \in \{a,b\}^*\}$

Let $S = \{0^i 1 \mid i \geq 1\}$

Assume A is a regular language

Therefore it has $< n$ equivalence classes

Find S such that $S = \{0^i 1 \mid i \geq 0\} \subseteq \Sigma^*$

has infinite cardinality

Let $x = 0^d 1$

where $x \neq y \Rightarrow d \neq c$

let $y = 0^c 1$

Let $z = 0^d 1 0^d 1$

we have:

$xz = 0^d 1 \underbrace{0^d 1 0^d 1}_{z} \in L$ ✓

$yz = 0^c 1\, 0^d 1 0^d 1 \notin L$ x

$\forall x,y \in \Sigma^* \; x R_L y$ iff $\forall z \in \Sigma^* \; (xz \in L \Leftrightarrow yz \in L)$

which we have shown that $xz \in L$ but $yz \notin L$, therefore

A_2 is not a regular language.

ⓒ $A_3 = \{a^{2^n} \mid n \geq 0\}$

Assume A_3 is regular.

let $S = \{a^{2^i} \mid i \geq 0\}$ → has infinite cardinality

$\Rightarrow$ there are ∞ equivalence classes.

Let $x = a^{2^c}$

$x \neq y \Rightarrow c \neq d$

Let $y = a^{2^d}$

$z = a^{2^d}$

Then, we have: $xz = a^{2^c} a^{2^d} \notin A_3$

$yz = a^{2^d} a^{2^d} = a^{2^{d+1}} \in A_3$

⊛ $c<d$

$2^c < 2^d < 2^d + 2^c < 2^d + 2^d = 2^{d+1}$

These are two consecutive power of 2, therefore, there is no way that we could have another power of 2 in between them therefore $2^d + 2^c \notin A_3$

If $c \neq d$, then we have:

- $c<d$ ✓ has shown
- $c>d$

- $c>d$

$2^c > 2^d > 2^d + 2^c > 2^d + 2^d = 2^{d+1}$

again with the same reasoning, shown for $(c<d)$ $2^d + 2^c \notin A_3$

$\forall x,y \in \Sigma^*$ $x R_L y$ $\forall z \in \Sigma^*$ $(xz \in L \Leftrightarrow yz \in L)$

which we have shown that $xz \notin$, but $yz \in L$,

$\therefore$ A_3 is not regular language.

1.46 a $B = \{0^n 1^m 0^n \mid m, n \geq 0\}$

Assume B is regular, so B has $\leq n$ equivalence classes.

Let's define set s such that $S = \{0^k \mid k \geq 0\}$

↳ it has infinite cardinality

Let $x = 0^c$

where $x \neq y \Rightarrow c \neq d$

Let $y = 0^d$

Let's define z as $z = 1^p 0^c$

so, we have:

$$xz = 0^c 1^p 0^c \in L \checkmark$$

$$yz = 0^d 1^p 0^c \notin L$$

$\forall x, y \in \Sigma^* \; x R_L y$ iff $\forall z \in \Sigma^* (xz \in L \Longleftrightarrow yz \in L)$

which we have shown that $xz \in L$ but $yz \notin L$, therefore

language B is not regular.

(b) $B = \{0^m 1^n \mid m \neq n\}$

Assume $0^m 1^n$ is regular

$\Rightarrow$ There are n equivalency classes

Let's define set s such that $S = \{0^p \mid p \geq 0\}$ $|s|$ is infinite

Let's consider the following:

$x = 0^c$

where $x \neq y \Rightarrow c \neq d$

$y = 0^d$

Let z be $z_1 = 1^d$

we have:

$xz = 0^c 1^d \longrightarrow c \neq d \Rightarrow xz \in L$

$yz = 0^d 1^d \longrightarrow d = d \Rightarrow yz \notin L$

$\forall x, y \in \Sigma^* \quad xR_i y$ iff $\forall z \in \Sigma^* \quad xz \in L \Longleftrightarrow yz \in L$

we have shown that $xz \in L$, but $yz \notin L$, therefore B is not a regular language.

(c) $B = \{ w \mid w \in \{0,1\}^* \text{ is not a palindrome} \}$

Assume B is a regular language.

$\Rightarrow$ There are n equivalence classes.

Let's define set S such that $S = \{ 0^k 1 \mid k \geq 1 \}$ $|S|$ is infinite

let's consider the following:

$x = 0^i 1$

where $x \neq y \Rightarrow i \neq j$

$y = 0^j 1$

Let z be $z = 0^j$

We have:

$xz = 0^i 1 0^j$ $xz \in L$

$yz = 0^j 1 0^j$ $yz \notin L$

$\forall x,y \in \Sigma^*$ $x R_L y$ iff $\forall z \in \Sigma^*$ $xz \in L \Leftrightarrow yz \in L$

We have shown that $xz \in L$, but $yz \notin L$, therefore B is not a regular language.

(d) $B = \{wtw \mid w, t \in \{0,1\}^+\}$

Assume B is a regular language

$\Rightarrow$ There are n equivalence classes

Let's define set s, such that $S = \{0^k 1 \mid k \geq 1\}$ $|S|$ is infinite

Let's consider the following:

$x = 0^d$

where $x \neq y \Longrightarrow c \neq d$

$y = 0^c$

Let z be defined as $z = 10^d$

so, we have:

$xz = 0^d \underbrace{10^d}_{z} \Rightarrow xz \in L$

$yz = 0^c 1 0^d \Rightarrow yz \notin L$

$\forall x,y \in \Sigma^* \quad x \equiv_L y \quad$ iff $\quad \forall z \in \Sigma^* \quad xz \in L \Longleftrightarrow yz \in L$

we have shown that $xz \in L$, but $yz \notin L$.

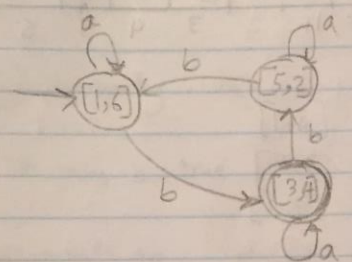
$\therefore$ B is not a regular language.

A.

	a	b
1	6	3
2	5	6
3F	4	5
4F	3	2
5	2	1
6	1	4

1					
X	2				
X	X	3			
X	X	_	4		
X	_	X	X	5	
_	X	X	X	X	6

[1,6]
[5,2]
[3,4]

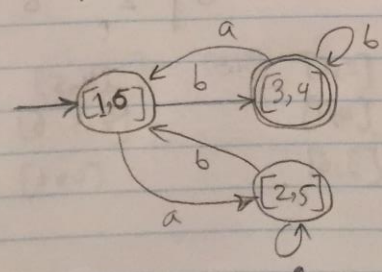


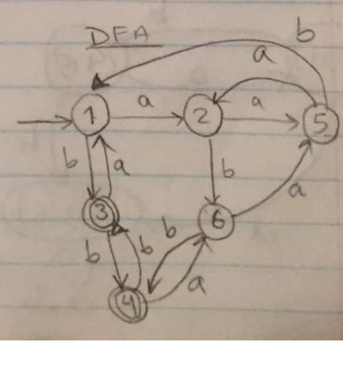
B.

	a	b
1	2	3
2	5	6
3F	1	4
4F	6	3
5	2	1
6	5	4

1	O					
2	2	O				
3	1	1	O			
4	1	1	O	O		
5	2	O	1	1	O	
6	O	2	1	1	2	O
	1	2	3	4	5	6

[1,6]
[3,4]
[2,5]

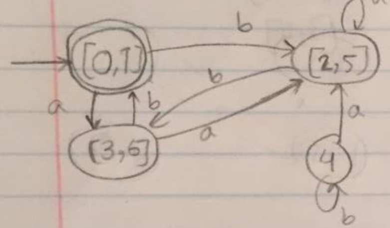




(c)

	a	b
0F	3	2
1F	3	5
2	2	6
3	2	1
4	5	4
5	5	3
6	5	0

	0F	1F	2	3	4	5	6
0F	0						
1F	0	0					
2	1	1	0				
3	1	1	2	0			
4	1	1	3	1	0		
5	1	1	0	2	2	0	
6	1	1	2	0	2	2	0



[0,1]
[2,5]
[3,6]
4

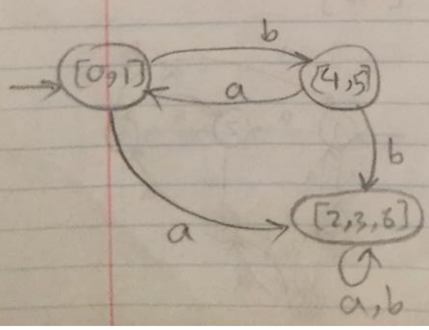
(d)

	0	1	2	3	4F	5F	6
0	0						
1	0	0					
2	2	2	0				
3	2	2	0	0			
4F	1	1	1	1	0		
5F	1	1	1	1	0	0	
6	2	2	0	0	1	1	0

	a	b
0	3	5
1	2	4
2	6	3
3	6	6
4F	0	2
5F	1	6
6	2	6

[2,6] [4,5]
≠ [0,1]
[3,6]

[4,5]
[2,3,6]
[0,1]



• Prove any language of your choice is regular with the Myhill-Nerode Theorem.

$L = \{0^n \mid n \text{ is even}\}$ is a regular language.

proof:

Consider the following finite list of subsets of Σ^*: $A_1 = L$, $A_2 = \{0^n \mid n \text{ is odd}\}$. Claim the members of A_1 are all equivalent to each other, because for any two members, say, $0^i, 0^j$ where $i \neq j$ and both are even it is the case that $0^i R_L 0^j$, since, for any $z \in \Sigma^*$, say, 0^m, both $0^i 0^m$ and $0^j 0^m$ are in L if m is even and both are not in L if M is odd. The members of A_2 are all equivalent by a similar argument with the roles of even and odd switched. Now clearly every string in Σ^* is either even or odd in length, so $A_1 \cup A_2 = \Sigma^*$. QED

* Pumping lemma is to show that a language not regular.