DETERMINATION OF INITIAL STATES

(I.E., FINDING $\underline{x}(0)$)

Consider
$$y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 \dot{y} + a_0 y = b_n u^{(n)} + \cdots + b_1 \dot{u} + b_0 u$$

with initial conditions:

$$y(0), \dot{y}(0), \text{ ---- }, y^{(n-1)}(0)$$
$$u(0), \dot{u}(0), \text{ --- }, u^{(n-1)}(0)$$

<u>Problem</u> Given an equivalent state space description
$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Find $\underline{x}(0)$, using the initial conditions above

<u>Solution</u> Can show (you do this!)

$$\begin{bmatrix} y(0) \\ \dot{y}(0) \\ \vdots \\ y^{(n-1)}(0) \end{bmatrix} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \underline{x}_0 + \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & & \vdots \\ CAB & CB & \ddots & \\ \vdots & \vdots & D & 0 \\ CA^{n-2}B & CA^{n-3}B & \cdots\, CB & D \end{bmatrix} \begin{bmatrix} u(0) \\ \dot{u}(0) \\ \vdots \\ u^{(n-1)}(0) \end{bmatrix}$$

<u>Example</u>
$$\left.\begin{aligned} &\ddot{y} + 3\dot{y} + 2y = 2\ddot{u} + 7\dot{u} + 7u \\ &y(0) = \dot{y}(0) = u(0) = \dot{u}(0) = 1 \end{aligned}\right\} \quad A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = [3 \;\; 1] \quad D = [2]$$

from above
$$\begin{pmatrix} y(0) \\ \dot{y}(0) \end{pmatrix} = \begin{bmatrix} C \\ CA \end{bmatrix} \underline{x}(0) + \begin{bmatrix} D & 0 \\ CB & D \end{bmatrix} \begin{pmatrix} u(0) \\ \dot{u}(0) \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix} \underline{x}(0) + \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \underline{x}(0) = \begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}^{-1} \cdot \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$$

$$\boxed{\underline{x}(0) = \begin{pmatrix} 1 \\ -4 \end{pmatrix}}$$