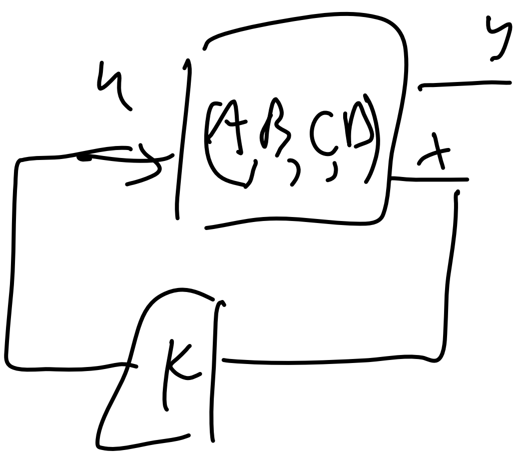
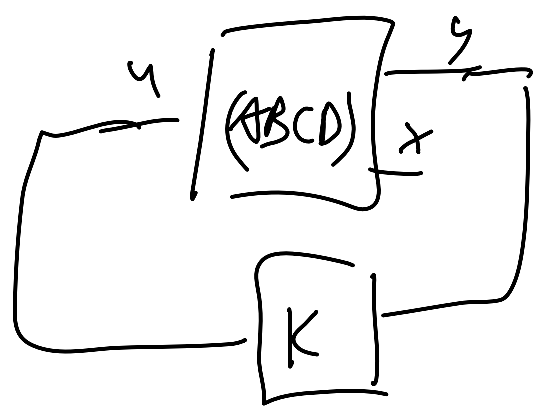
EGGN 517
Lecture 20
3-29-12

8.3 Observers (or State Estimation)

recall: 2 times ago:



state f.b.



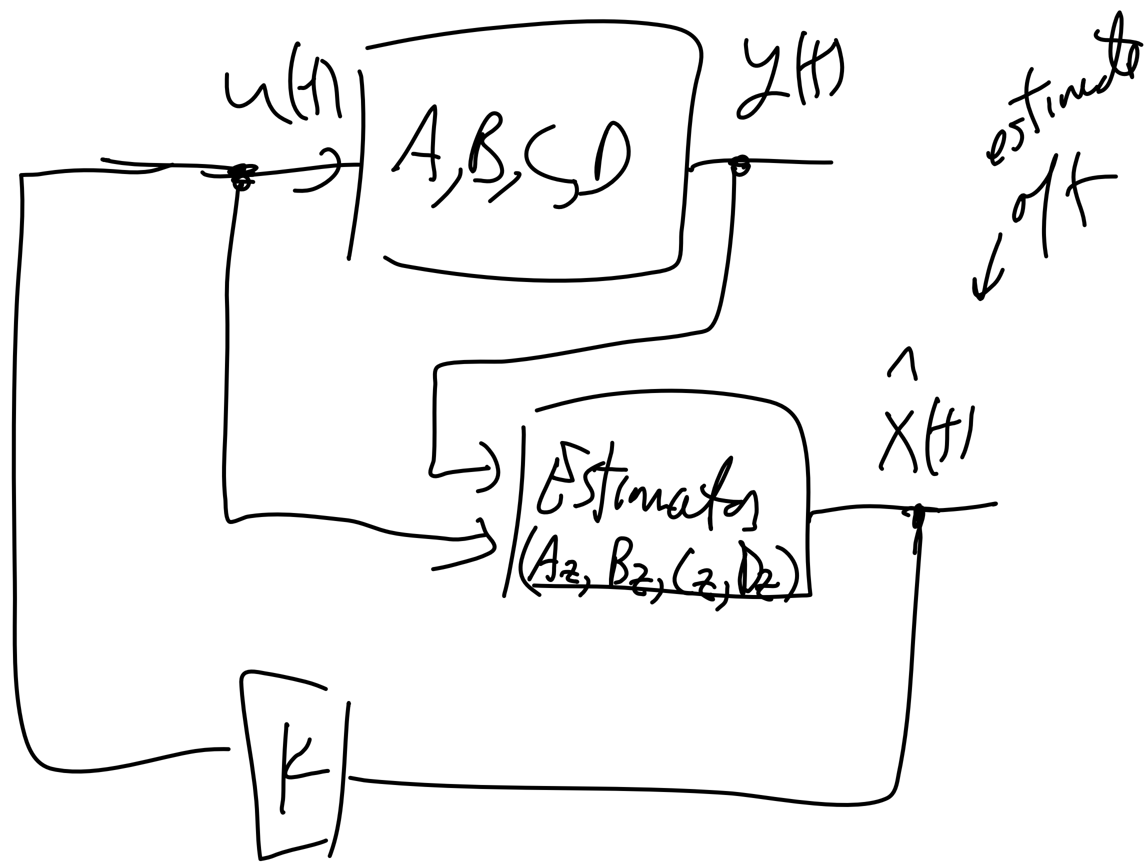
output f.b.

K or C(s)

↑ like this one: nice numerical tools

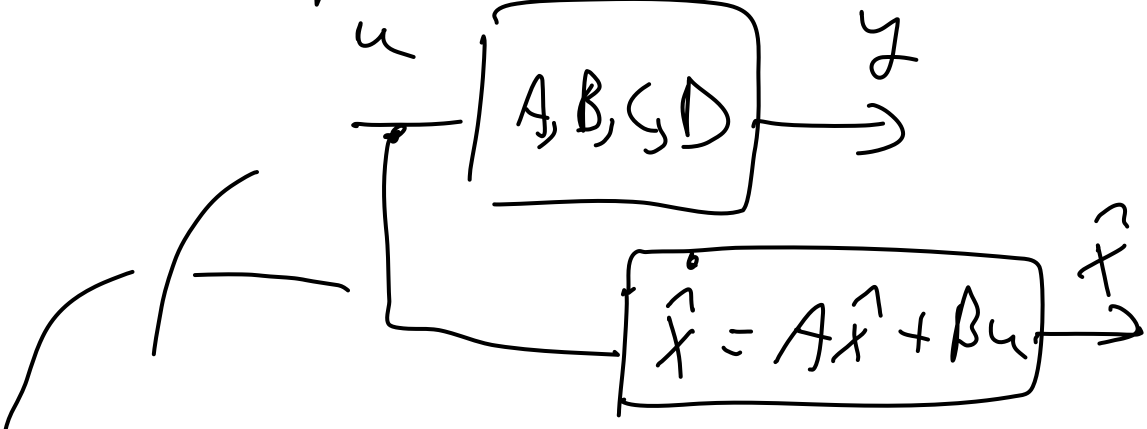
But: suppose can't measure x

$\Longrightarrow$ estimate it



- **Estimator**

  - open-loop estimator



$D=0$

plant $x(t)=e^{At}x(0)+\int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$

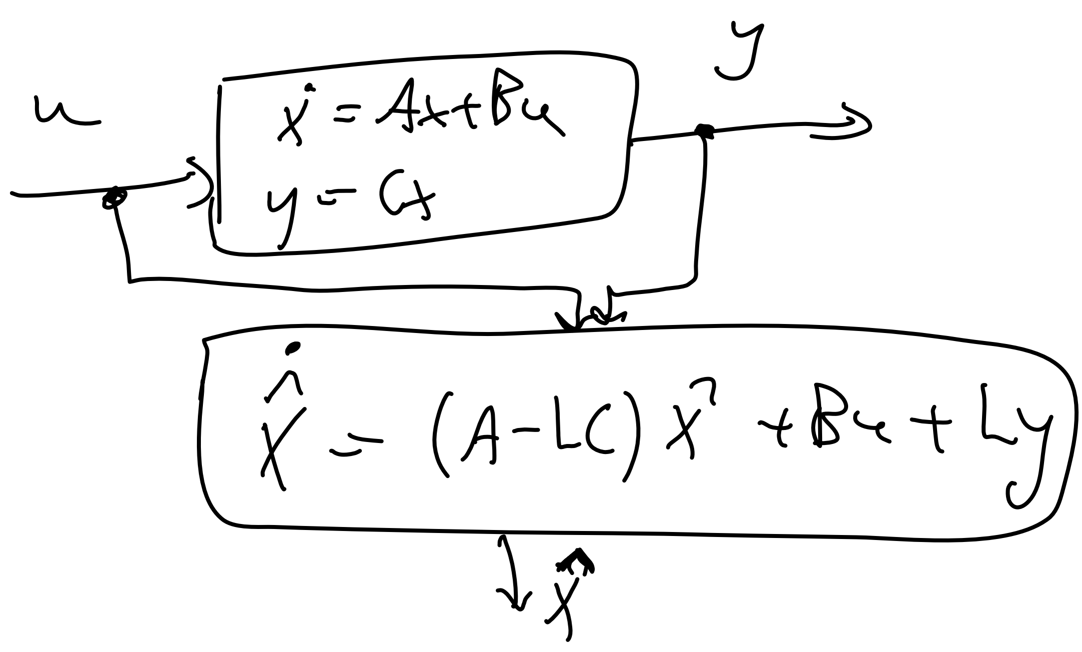
estimate $\hat{x}=e^{At}\hat{x}(0)+\int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$

observer error $\tilde{x}=x(t)-\hat{x}(t)$

$$\tilde{x}(t)=e^{At}\tilde{x}(0)$$

if system unstable observer error can blow up

- <u>full-order observer</u>



$$A_z = A - LC$$
$$B_z = \begin{bmatrix} B & L \end{bmatrix}$$

observer input: $\begin{pmatrix} u \\ y \end{pmatrix}$

consider error dynamics

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}}$$

$$= (Ax + Bu) - \left[(A - LC)\hat{x} + Bu + Ly \right]$$

$$= Ax + Bu - A\hat{x} + LC\hat{x} - Bu - LCx$$

$$\dot{\tilde{x}} = (A - LC)\,\tilde{x}$$

use L to make $(A-LC)$ stable

then $\tilde{x} \to 0 \Rightarrow \hat{x} \to x$

Notes (1) need to solve $(A-LC)$ for L to make $(A-\underset{\uparrow \text{inside}}{L}C)$ stable

compare to state fb $-$ $A + B\underset{\uparrow \text{outside}}{K}$

② to do this, work with $A^T - C^T L^T$

$\Rightarrow$ stab f-b. problem (unknown: L^T)

③ to solve $A^T - C^T L^T$

need (A^T, C^T) controllable

④ or (A, C) observable

minimal order observer

- full-order obs has order n if A is $n \times n$
- full-order obs gives us n measurement/estimates as $\hat{x} \in \mathbb{R}^n$
- But, we already have some

actual measurements from y

- suppose $y = Cx \in \mathbb{R}^q$
  i.e. q outputs
- can build obs. of order $n-q$

- **Ex:**
  - plant $\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} x + \begin{pmatrix} 3 \\ 2 \end{pmatrix} u$

    $y = (0 \ 1) x$

  - $\dot{\hat{x}} = (A - LC)\hat{x} + Bu + Ly$

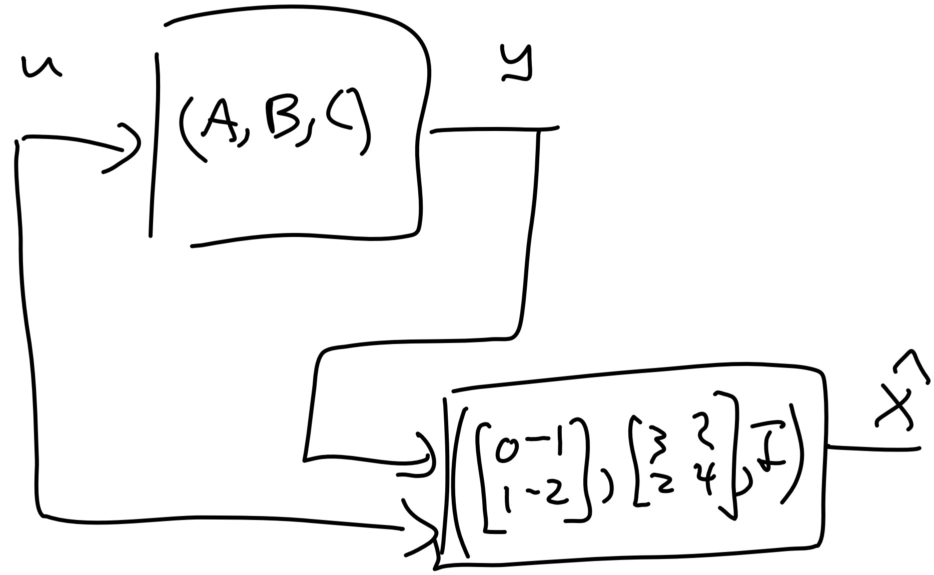
    set poles of $A-LC$ at $s=-1$

    $\Rightarrow L = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

    $(A - LC) = \begin{pmatrix} 0 & -1 \\ 1 & -2 \end{pmatrix}$

- observer is:

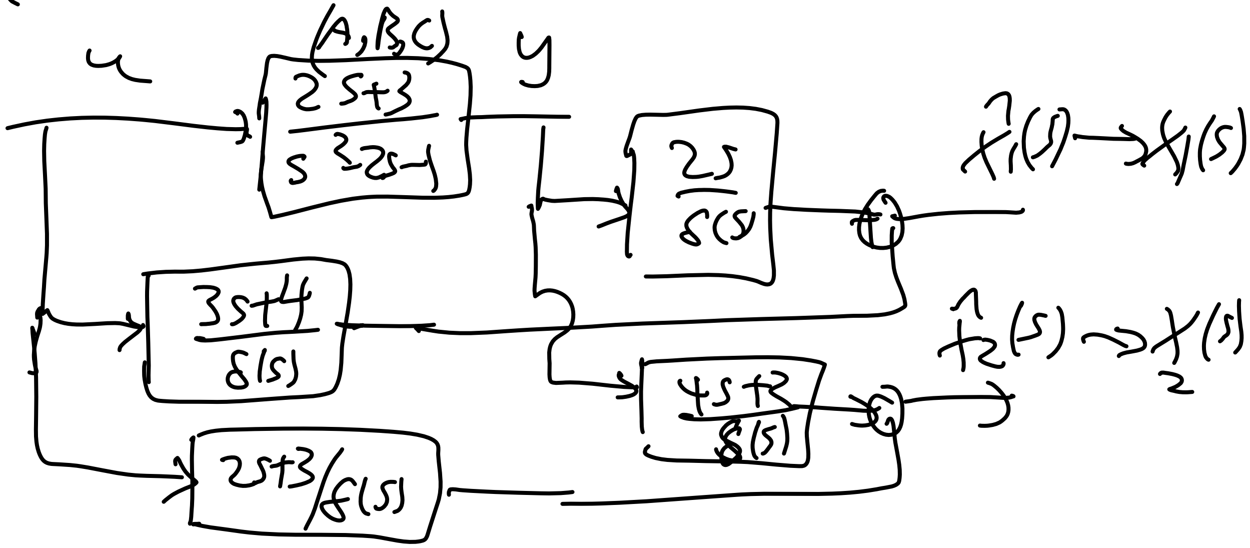
$$\dot{\hat{x}} = \begin{bmatrix}0 & -1\\ 1 & -2\end{bmatrix}\hat{x} + \begin{array}{cc} B & L \\ \downarrow & \downarrow \end{array}\!\!\!\!\!\!\!\!\begin{bmatrix}3 & 2\\ 2 & 4\end{bmatrix}\begin{pmatrix}u\\ y\end{pmatrix}$$



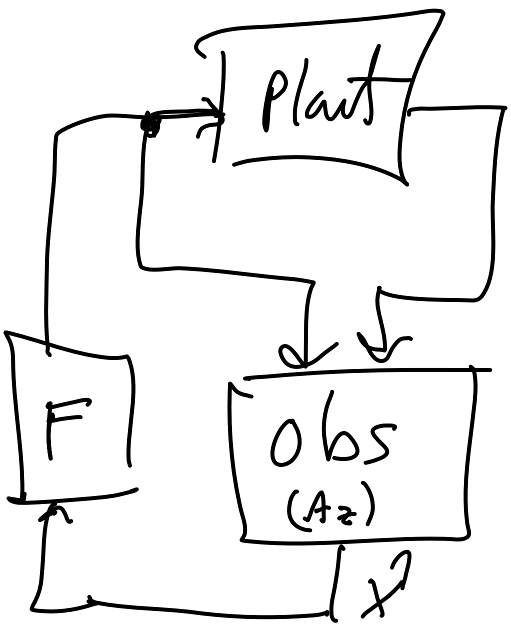
$$\hat{x}(s) = I\left[sI - (A - LC)\right]^{-1}\begin{bmatrix}B & L\end{bmatrix}\cdot\begin{pmatrix}U(s)\\ Y(s)\end{pmatrix}$$

$$\begin{pmatrix}\hat{x}_1\\ \hat{x}_2\end{pmatrix} = \frac{1}{s^2+2s+\cancel{4}1}\begin{bmatrix}3s+4 & 2s\\ 2s+3 & 4s+2\end{bmatrix}\begin{pmatrix}U(s)\\ Y(s)\end{pmatrix}$$

$\delta = s^2+2s+1$



8.4 Observer with State F.B.



plant $\dot{x} = Ax + Bu$

$y = Cx$

obs $\dot{z} = A_z z + B_u u + B_y y$

$\hat{x} = C_z z + 0\, u + D_y y$

control $u = F\hat{x}$

for full-order obs $C_z = I$

$D_y = 0$

closed-loop system:

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \underbrace{\left[\begin{array}{c|c} A + BFD_yC & BFC_z \\ \hline B_yC + B_yFD_yC & A_z + B_uFC_z \end{array}\right]}_{A_{cl}} \begin{pmatrix} x \\ z \end{pmatrix}$$

Theorem: Separation Principle: The

eig. of A_{cl} =

union $\{ \text{eig}(A_z), \text{eig}(A+BF) \}$

observer poles   eig of s.f.b design

will post example of full-order observer with state-f.b.

8.5 Optimal Control: Where to put the poles

Linear Quadratic Regulator (LQR)

find K for $\dot{x} = Ax + Bu$ $e = r - y$ $= r - Cx$

$y = Cx$

so as to $u = Kx$ (state f.b.)

minimize cost function

$$J(u) = \int_0^\infty (y^T Q y + u^T R u)\,dt$$

Ex, $u, y \in \mathbb{R}^2$
~~suppose~~ pick $Q = \begin{pmatrix} 10^5 & 0 \\ 0 & \cancel{10^5}\,1 \end{pmatrix}$ $R = \begin{pmatrix} .01 & 0 \\ 0 & .01 \end{pmatrix}$

$$\text{so } J(u) = \int_0^\infty (\underbrace{10^5 \|y\|_2^2}_{10^5 y_1^2 + y_2^2} + .01 \|u\|_2^2)\,dt$$

so if $J(u) \ll 1 \Rightarrow \|y\|_2 \ll \|u\|_2$

read help in Matlab on lqr