EENG 517

In Class Problem 1

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Derive a state-space controller form for

$$\ddot{y} + a_1\dot{y} + a_o y = b_2\ddot{u} + b_1\dot{u} + b_o u \tag{1.1}$$

From examples and literature found, the first step is to take the Laplace transform of the above equation.

$$s^2Y(s) + a_1 sY(s) + a_o Y(s) = b_2 s^2 U(s) + b_1 sU(s) + b_o U(s) \tag{1.2}$$

To calculate the transfer function for the above system, we solve for the ratio $\frac{Y(s)}{U(s)}$

$$\frac{Y(s)}{U(s)} = \frac{b_2 s^2 + b_1 s + b_o}{s^2 + a_1 s + a_o} \tag{1.3}$$

We now multiply equation 0.3 by $\frac{Z(s)}{Z(s)}$, then write expressions for Y(s) and U(s)

$$\begin{aligned} \frac{Y(s)}{U(s)} &= \frac{b_2 s^2 + b_1 s + b_o}{s^2 + a_1 s + a_o}\left(\frac{Z(s)}{Z(s)}\right) \\ Y(s) &= b_2 s^2 Z(s) + b_1 sZ(s) + b_o Z(s) \\ U(s) &= s^2 Z(s) + a_1 sZ(s) + a_o Z(s) \end{aligned} \tag{1.4}$$

Now take the inverse Laplace Transform and we find:

$$\begin{aligned} y &= b_2\ddot{z} + b_1\dot{z} + b_o z \\ u &= \ddot{z} + a_1\dot{z} + a_o \end{aligned} \tag{1.5}$$

Now we can define the state variables

$$\begin{aligned} x_1 &= z & \dot{x}_1 &= \dot{z} = x_2 \\ x_2 &= \dot{z} & \dot{x}_2 &= \ddot{z} = u - a_1\dot{z} - a_o z \\ x_3 &= \ddot{z} \end{aligned}$$

We can now write the output as

$$
\begin{aligned}
y = b_2 x_3 + b_1 x_2 + b_o x_1 \Longrightarrow & b_2 \ddot{z} + b_1 \dot{z} + b_o z \\
= & b_2 (u - a_1 \dot{z} - a_o z) + b_1 \dot{z} + b_o z \\
= & \dot{z}(b_1 - a_1) + z(b_o - a_o) + b_2 u
\end{aligned}
$$

The state space-equations are:

$$
\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_o & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{1.6}
$$

$$
\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}u \Rightarrow \begin{bmatrix} (b_o - a_o) & (b_1 - a_1) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_2 \end{bmatrix} u \tag{1.7}
$$