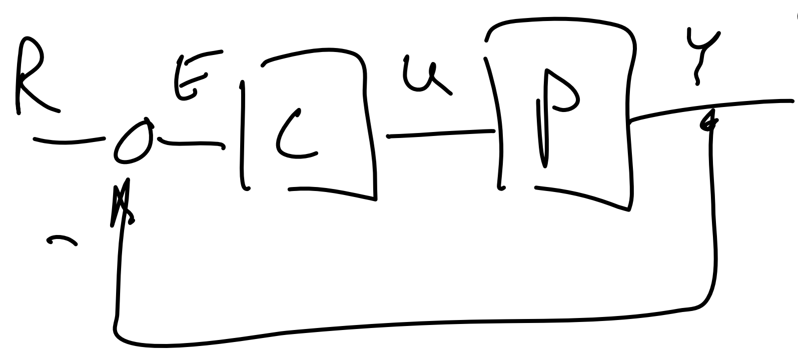
EGGN 517 Lecture 15

Mar 1, 2012

7.0 Classical SISO Design

7.1 Intro

LODE
STATE SPACE
Transfer Function
Matrix



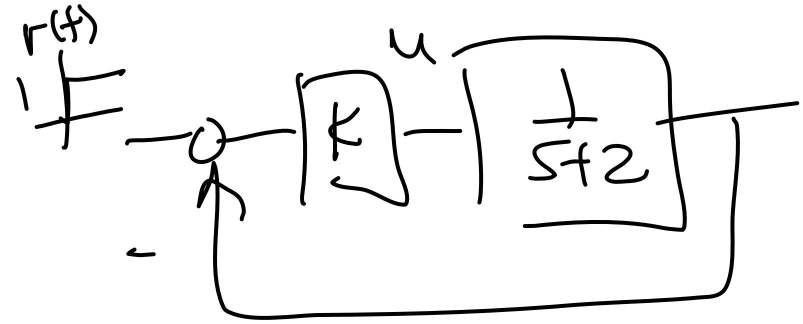
for SISO $\frac{Y}{R} = \frac{CP}{1+CP}$ $\frac{E}{R} = \frac{1}{1+CP}$

$\frac{U}{R} = \frac{C}{1+CP}$

Given P, Find C, so these have desired properties

* Desired properties

- transient



$$\frac{Y}{R} = \frac{K}{s+(2+K)} \Rightarrow$$ "set" settling time by picking K

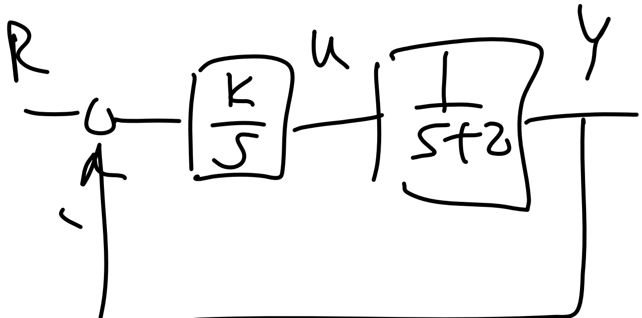
$$y(t) = \frac{K}{2+K}\left(1-e^{-(2+K)t}\right)$$ when $r(t)=1$

- steady-state

$$y_{ss} = \frac{K}{2+K} \neq 1$$

$\Rightarrow$ DC error $1 - \frac{K}{2+K}$

solution: Different structure:



$$\frac{Y}{R} = \frac{K}{s^2+2s+K}$$

poles $-1 \pm \frac{1}{2}\sqrt{4-4K}$

$e^{-t}\cos(\omega t+\phi)$

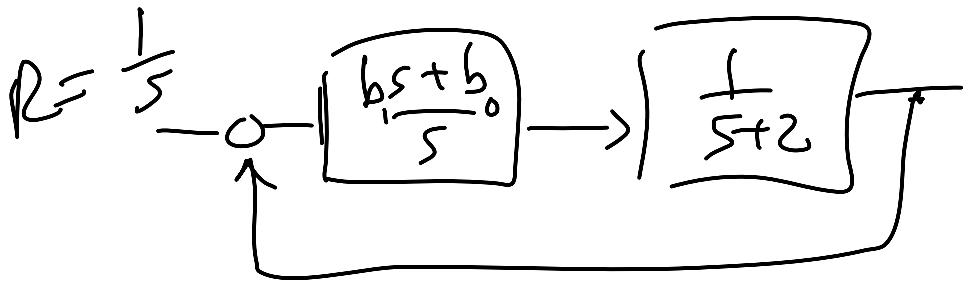
if $R(s)=\frac{1}{s}$ $\quad$ $y(t)\to 1$ $\quad$ slower than 5

$(r(t)=1)$

settling time

error $\to 0$

try $C(s)=\frac{b_1 s+b_0}{s}=b_1+\frac{b_0}{s}$ $\boxed{PI}$



$$\frac{Y}{R}=\frac{b_1 s+b_0}{s^2+(2+b_1)s+b_0}$$

closed-loop DC gain $=\frac{b_0}{b_0}=1\Rightarrow e_{ss}=0$

$e(t)\to 0$

settling time $\frac{5}{(2+b_1)/2}$

poles $-\frac{2+b_1}{2}\pm\sqrt{(2+b_1)^2-4b_0}$

consider $\frac{E}{R} = \frac{1}{1+CP}$

case 1

$C = K$

$E = \frac{1}{1+K\left(\frac{1}{s+2}\right)} \cdot \frac{1}{s}$

$E = \frac{s+2}{s+2+K} \cdot \frac{1}{s}$

case 2

$E = \frac{1}{1+CP} \cdot R$ $\quad C = \frac{b_1 s + b_0}{s}$

$E = \frac{1}{1+\left(\frac{b_1 s+b_0}{s}\right)\left(\frac{1}{s+2}\right)} \cdot \frac{1}{s}$

$E = \frac{s(s+2)}{s^2+(2+b_1)s+b_0} \cdot \frac{1}{s}$

apply F.V.T

$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$

if limit exists

$e_{ss} = \lim_{s\to 0} \cancel{s} \cdot \frac{s+2}{s+2+K} \cdot \frac{1}{\cancel{s}}$

$= \frac{2}{2+K}$

$e_{ss} = \lim_{s\to 0} s \cdot \frac{\cancel{s}(s+2)}{s^2+(2+b_1)s+b_0} \cdot \frac{1}{\cancel{s}}$

$= 0$

"s" from F.V.T. cancelled "s" from R

"s" from F.V.T make $e_{ss}=0$

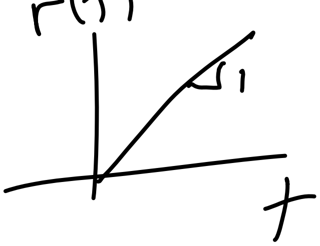
"s" from controller cancelled "s" from R

$\Longrightarrow$ leads to <u>internal model principal</u>

controller should have in its denominator all the unstable poles of the reference

Ex.

$$R(s) = \frac{1}{s^2}$$

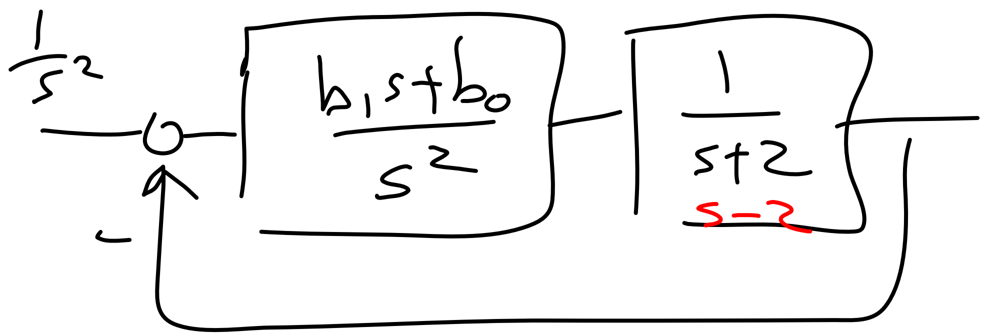


$$E = \frac{s(s+2)}{s^2+(2+b_1)s+b_0} \cdot \frac{1}{s^2}$$

$$e_{ss} = \lim_{s\to 0} s \, \frac{s(s+2)}{s^2+(2+b_1)s+b_0} \cdot \frac{1}{s^2}$$

$$= \frac{2}{b_0} \neq 0$$

to fix it?



$$e_{ss} = \lim_{s\to 0} s \, \frac{\cancel{s}^2(s+2)\,(s-2)}{s^3+2s^2+b_1 s+b_0} \cdot \frac{1}{\cancel{s}^2}$$

-2

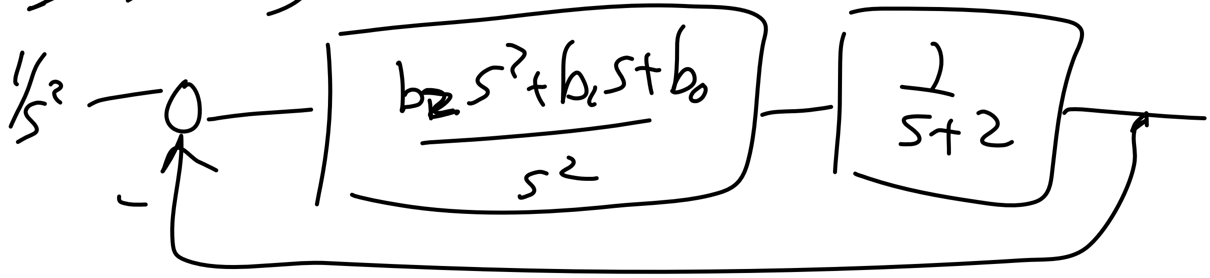
$\stackrel{?}{=} 0$ if limit exists

controller "s^2" cancels $r(t)$ "s^2". F.V.T. makes $e_{ss} \to 0$

if $\frac{1}{s+2}$ was $\frac{1}{s-2}$ $\Longrightarrow$ system unstable

$\Longrightarrow$ need another parameter

Try ~~PID~~ "D"



$$e_{ss} = \lim_{s \to 0} s \, \frac{\cancel{s^2}(s+2)}{s^3 + (2+b_2)s^2 + b_1 s + b_0} \cdot \frac{1}{\cancel{s^2}}$$

can independently set all three coefficients of $\delta(s)$

$\Longrightarrow$ can always make it stable

i.e. can place poles arbitrarily

words

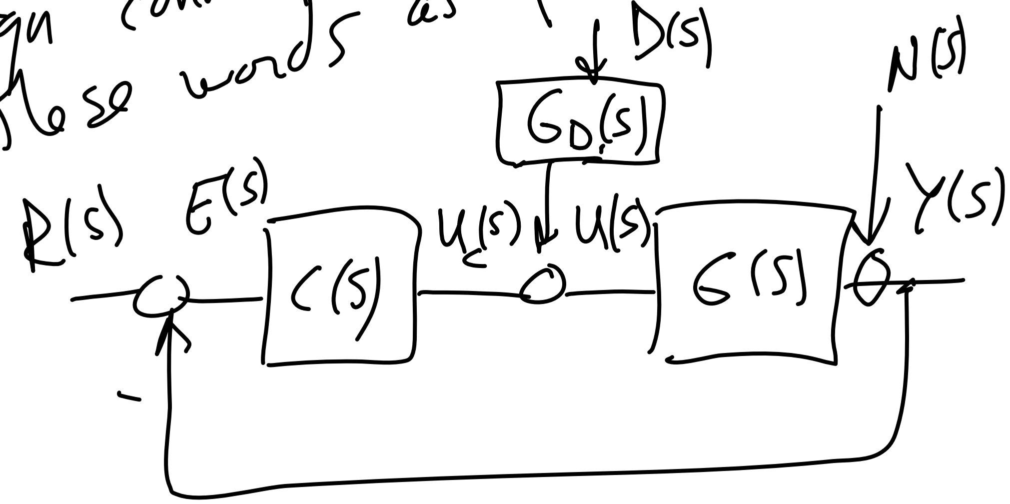
PID
Routh Hurwitz
stable
Root locus

Transient
irreducible
minimal etc

settling time
error steady-state
poles

others:
Bode
Freq

context of design connects all these words as follows:



$$G(s) = \frac{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

$$C(s) = \frac{\beta_p s^p + \beta_{p-1} s^{p-1} + \cdots + \beta_1 s + \beta_0}{s^p + \alpha_{p-1} s^{p-1} + \cdots + \alpha_1 s + \alpha_0}$$

common C(s):

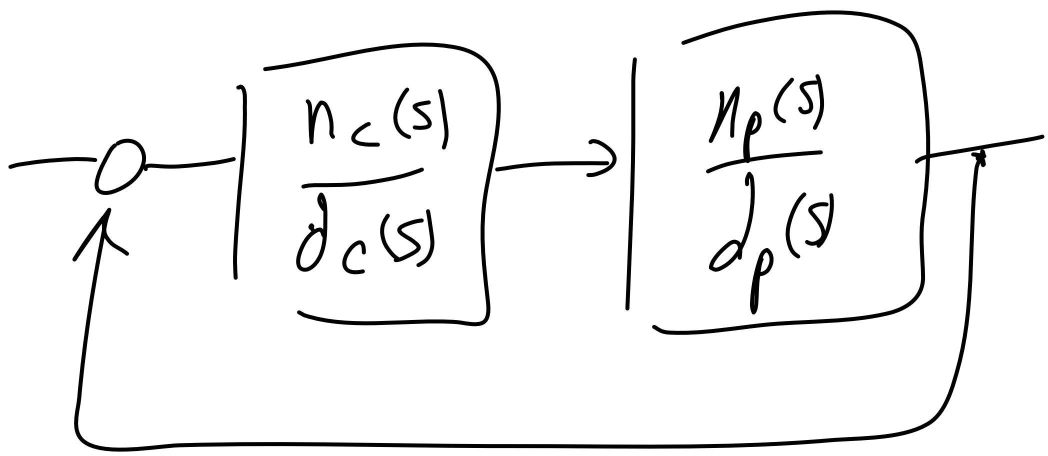
PID: $C(s) = K_p + \frac{K_I}{s} + K_D s$

$= \frac{K_D s^2 + K_p s + K_I}{s}$ ✓ violates a rule

Constant $C(s) = K_p$

Lead/lag $\quad C(s) = K \dfrac{s+b}{s+a}$

7.2 Arbitrary Pole Placement



$$n_c = \beta_p s^p + \cdots \beta_1 s + \beta_0$$

$$d_c = s^p + \cdots + \alpha_1 s + \alpha_0$$

$$n_p = b_n s^n + \cdots + b_1 s + b_0$$

$$d_p : s^n + \cdots + a_1 s + a_0$$

char. polynomial

let

$$\delta(s) = n_c(s) \cdot n_p(s) + d_c(s) d_p(s) \qquad q = n+p$$

$$= s^q + \delta_{q-1} s^{q-1} + \cdots + \delta_1 s + \delta_0$$

Then

$$\begin{pmatrix} 1 \\ \delta_{q-1} \\ \delta_{q-2} \\ \vdots \\ \vdots \\ \delta_1 \\ \delta_0 \end{pmatrix} = \underbrace{\left[\begin{array}{cccc|cccc} 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ a_{n-1} & 1 & \ddots & 0 & b_{n-1} & 1 & \ddots & 1 \\ a_{n-2} & a_{n-1} & \ddots & 1 & b_{n-2} & b_{n-1} & \ddots & 0 \\ \vdots & a_{n-2} & \ddots & a_{n-1} & \vdots & b_{n-2} & \ddots & 1 \\ a_1 & \vdots & \ddots & a_{n-2} & \vdots & \vdots & \ddots & b_{n-1} \\ a_0 & a_1 & & \vdots & b_1 & \vdots & & b_{n-2} \\ 0 & a_0 & \ddots & \vdots & b_0 & b_1 & \ddots & \vdots \\ \vdots & 0 & \ddots & \vdots & 0 & b_0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & a_1 & \vdots & 0 & \ddots & b_1 \\ 0 & 0 & \cdots 0 & a_0 & 0 & 0 & \cdots 0 & b_0 \end{array}\right]}_{} \begin{pmatrix} 1 \\ \alpha_{p-1} \\ \vdots \\ \alpha_1 \\ \alpha_0 \\ 1 \\ \beta_{p-1} \\ \vdots \\ \beta_1 \\ \beta_0 \end{pmatrix}$$

$$\delta = A x$$

$\Longrightarrow$ Given δ (desired)

A (plant)

$\Longrightarrow$ $x = A^{-1}\delta$ (control)

A^{-1} exists if $G = \dfrac{n_p}{d_p}$

is irreducible (no cancellations)

A is called <u>Sylvester matrix</u>