EGGN 517

Theory and Design of Advanced Control Systems

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Spring 16

http://inside.mines.edu/~kmoore/EEGN517/
(case sensitive)

1.0 Introduction

- Personal Credentials:
  - Kevin L. Moore
  - B.S. from LSU, M.S. from USC
  - 3 years system engineering, control at Hughes Aircraft
  - PhD 1989 in EE - Control Systems at Texas A&M
  - 30 years teaching control and senior design
  - Industrial consulting for ~1/3 of my non-admin career
  - Active researcher, currently doing autonomous mobile robotics, iterative learning control, distributed systems
- Course Organization and Outline of Topics – see syllabus

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1.1 Control Systems

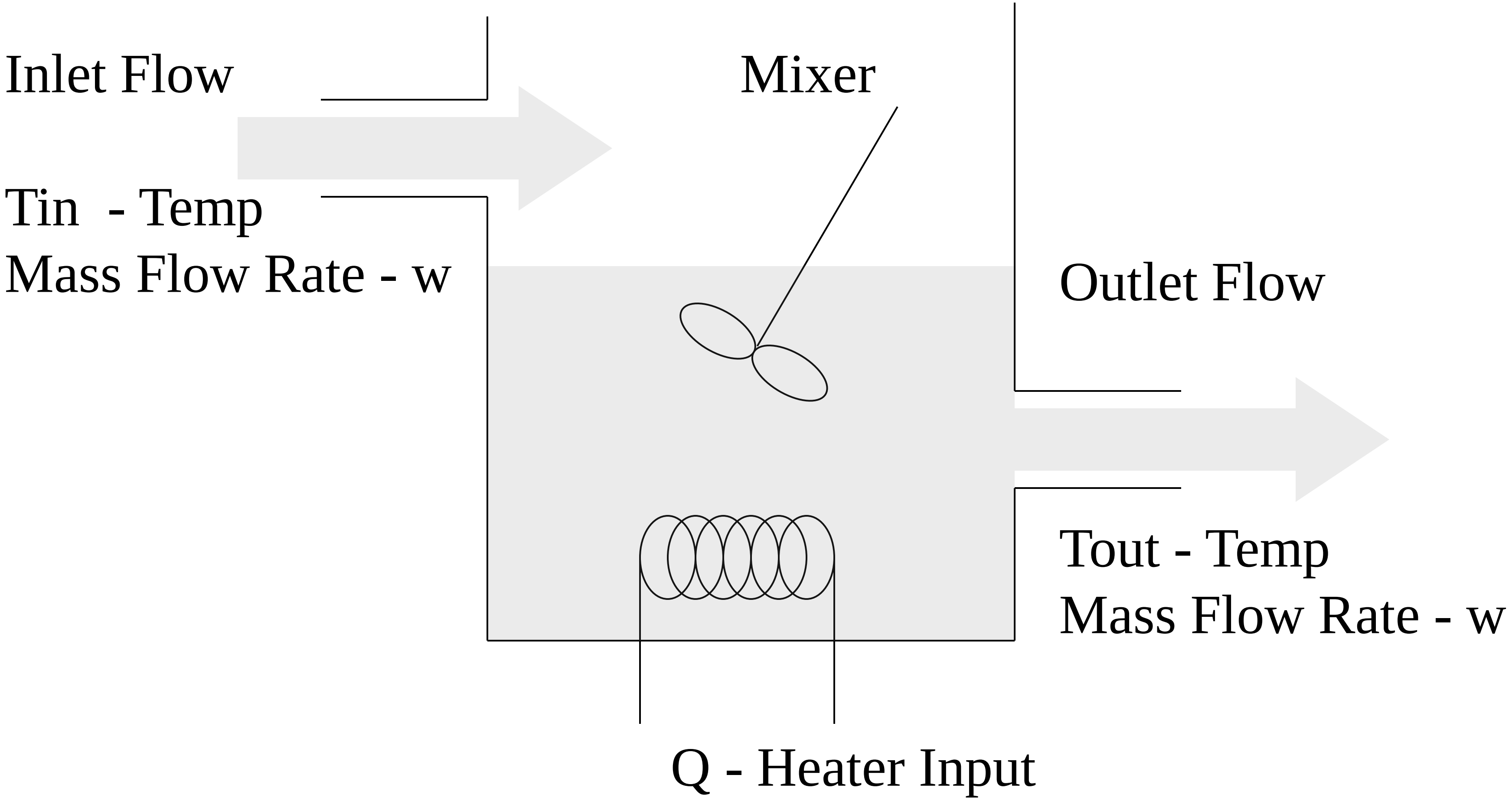
- Purpose: Learn advanced theory for control system design.

But, first, what is a system? Some definitions:

- System – a set of interconnected components working together towards some common objective.

- Systems can be:
  - Physical (robot arm)
  - Biological (cat)
  - Economic (country)
  - Social (church)

Ex. Continuous Stirred-Tank Reactor (CSTR)



For the CSTR, at steady state, we have

$$T_{out} = T_{in} + Q/(wC)$$

where C is the specific heat of the liquid.

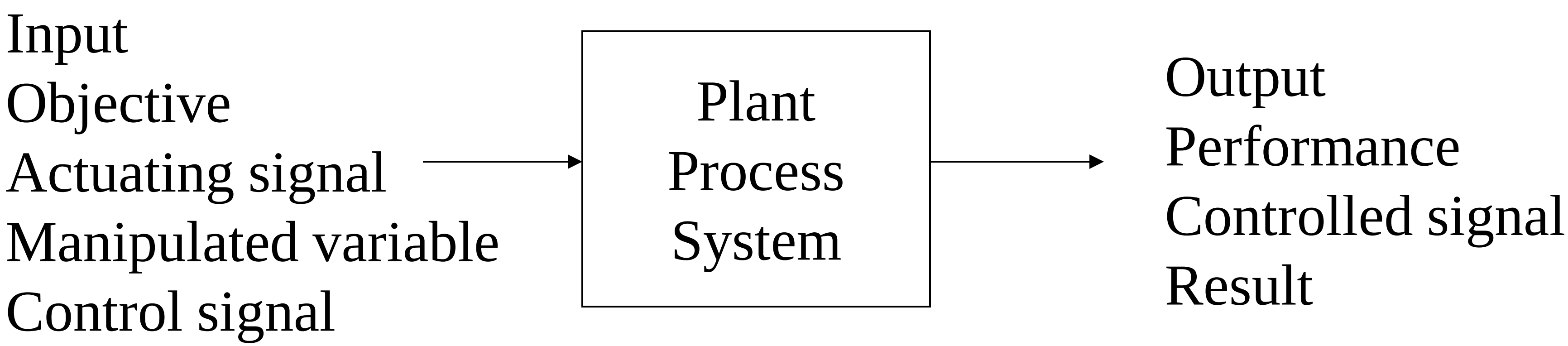
Thus we are also led to the term:

- Process – another name for a system; indicates the idea that the system "processes" or acts on some or all of the system variables to produce or affect other variables in a causal way

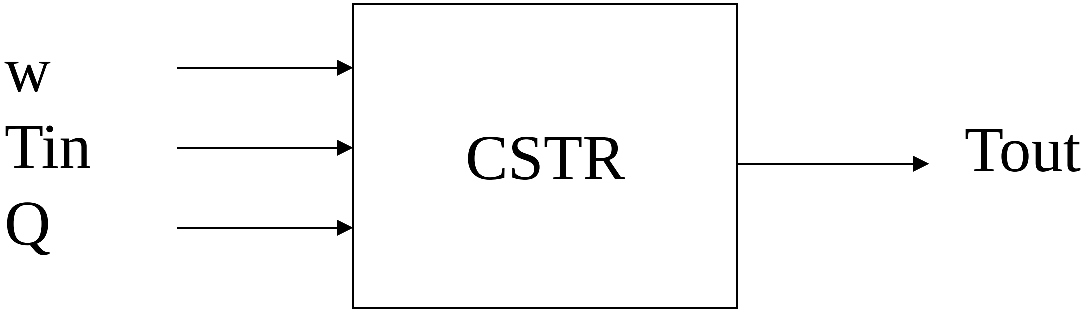
- Plant – another name for a system

- Variables – can be classified as
  - Inputs: variables that **cause** an output
  - Outputs: variables that **result** from/respond to an input
  - Parameters: variables that are fixed and constant due to physical constraints of the system
- Ex. - For the CSTR we can identify
  - Inputs: Tin, w, Q
  - Outputs: Tout
  - Parameters: C

- Note: we often use **block diagrams** to depict systems or processes and their I/O relationships



- Ex: CSTR



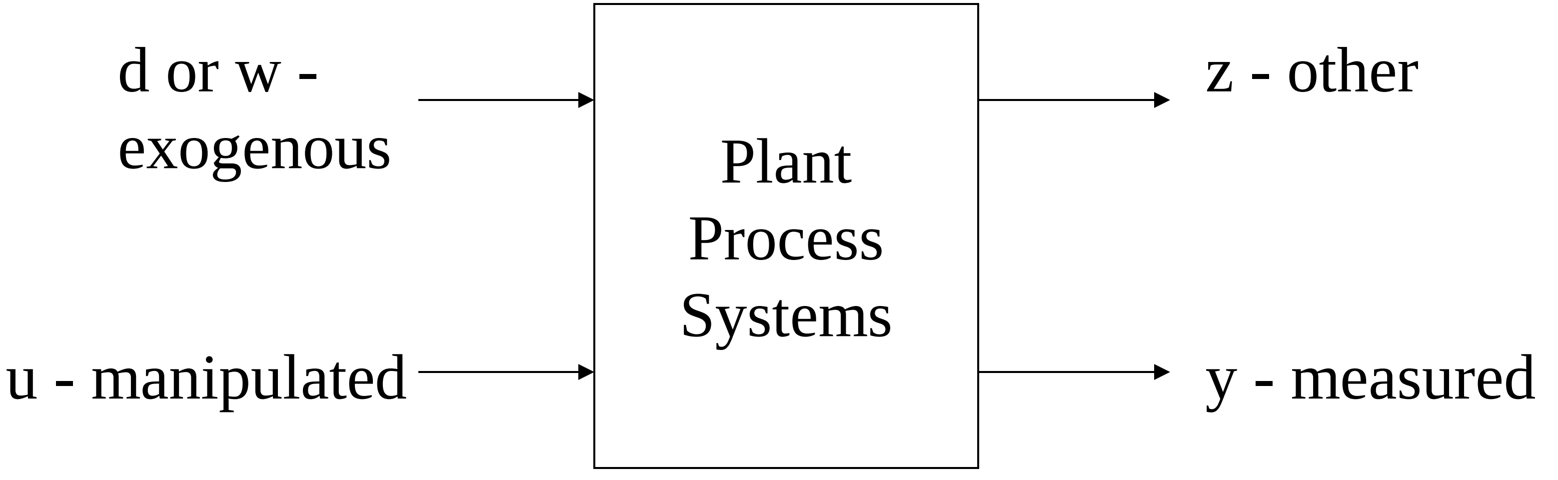
- Note: Nothing is implied in such a diagram about the actual processing that takes place inside the plant.

- Variables – can be also be classified as

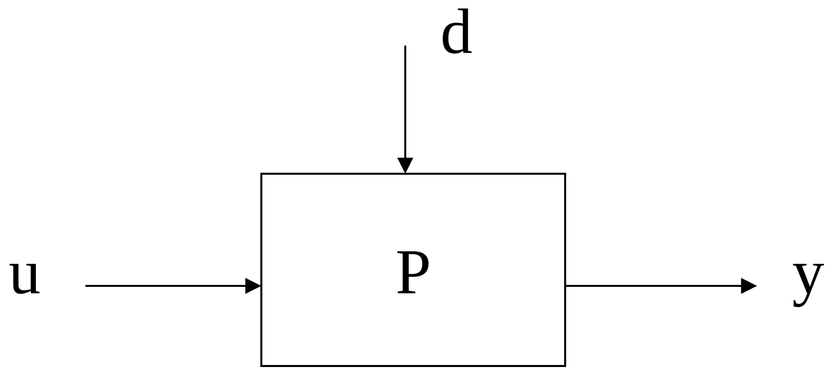
  - Manipulated: inputs that we can adjust
  - Exogenous: inputs that are external to the system and cannot be changed by the controller (disturbances, noise)
  - Measured outputs: outputs that we can sense
  - Other outputs: variables of interest that cannot be measured
- Ex. - For the CSTR we can identify

  - Manipulated variable: Q
  - Exogenous variable: w, Tin
  - Output variable: Tout

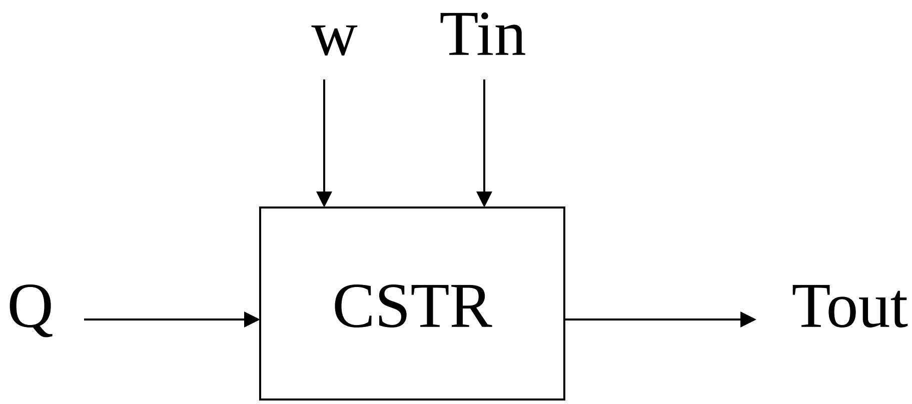
- This classification of variables is depicted in a **block diagram** as



- <u>Alternately</u>, we draw this as **block diagram** as



- <u>Ex</u>: CSTR

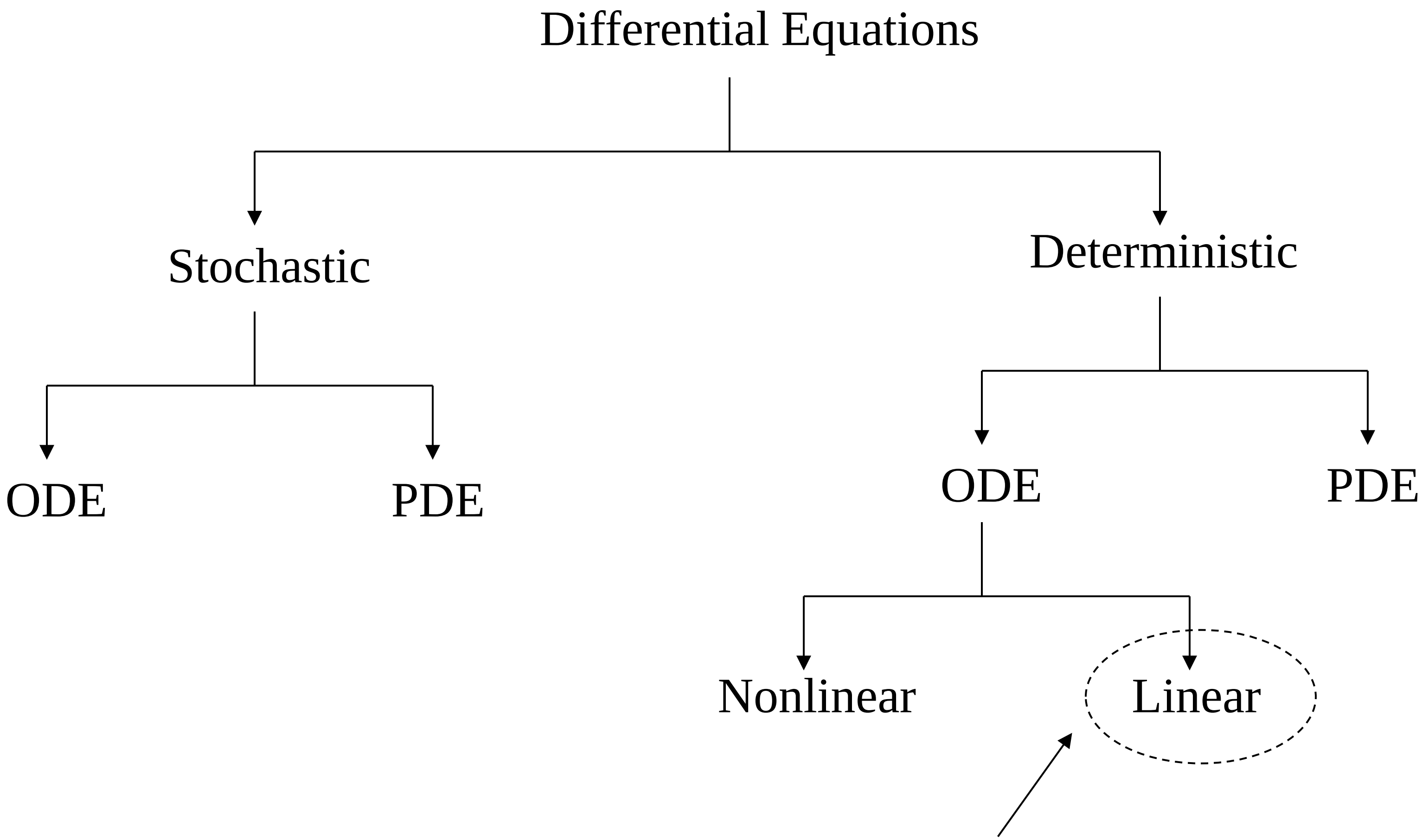


Note: In this course, we are interested not in just any type of "system," but more importantly, "dynamic systems"

- Dynamic – involves differential equations (assumed with respect to time, but could be w.r.t. space),

as opposed to

- Static – involves only algebraic equations



This course will consider primarily linear analysis and design topics.

Now that we have defined a “dynamic system,” what is a “control system?”

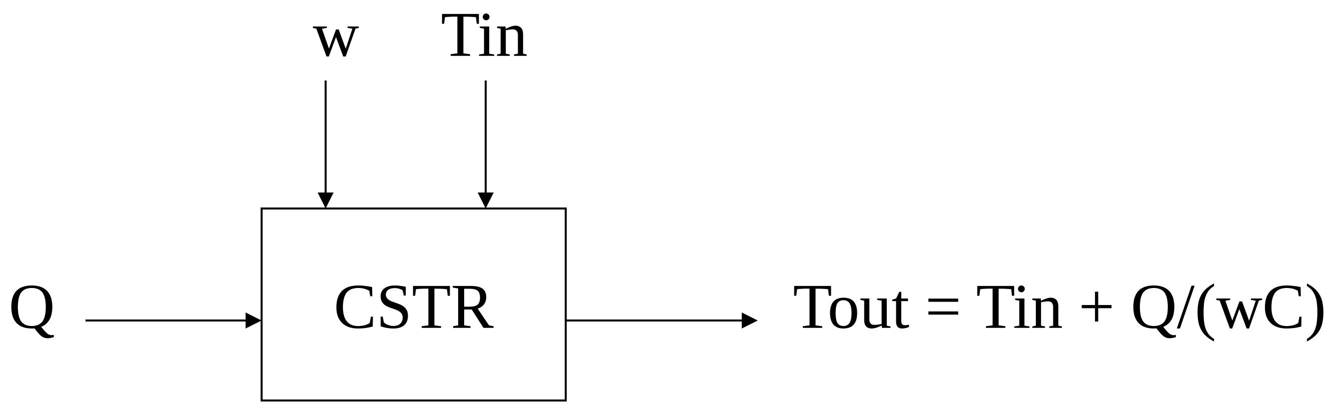
- Control – to maintain desired (output) conditions in a (physical) system by adjusting selected (input) variables
- Control system – a system whose components have been configured to provide a desired system behavior
- Control system engineering – developing appropriate configurations of system components to meet performance objectives
- Objective of control system engineering: To change or modify (with the use of feedback) a given system so the new system has desired properties.

Feedback Systems

- Key word in previous definition was “feedback”
- Feedback – use of an output of the system to influence an input of the same system

Feedback is the essential feature of an effective control system

- Ex: CSTR



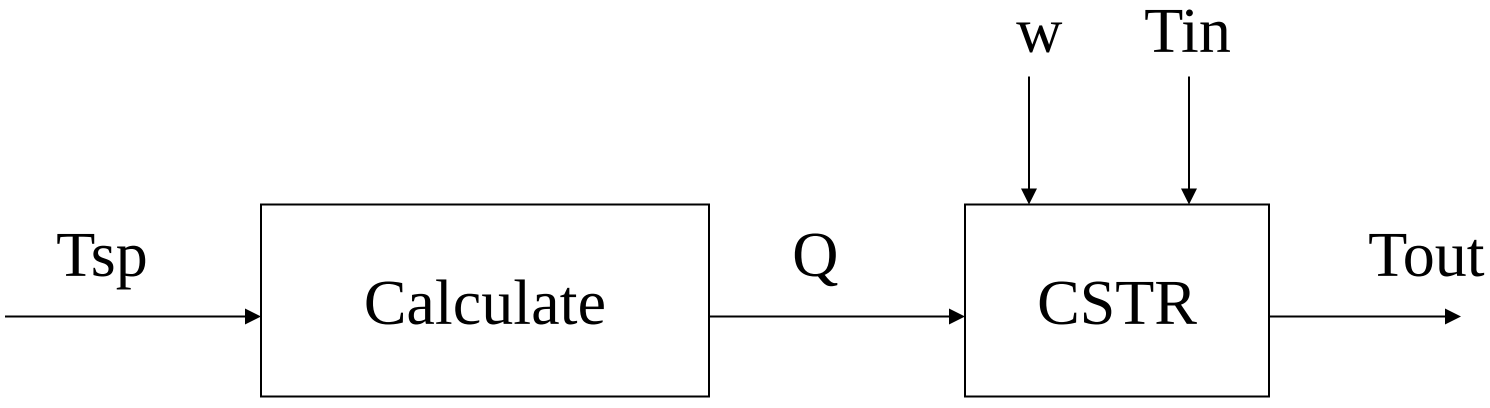
- Suppose our goal is to have

$$T_{out}=T_{sp}$$

where subscript “sp” means “setpoint”

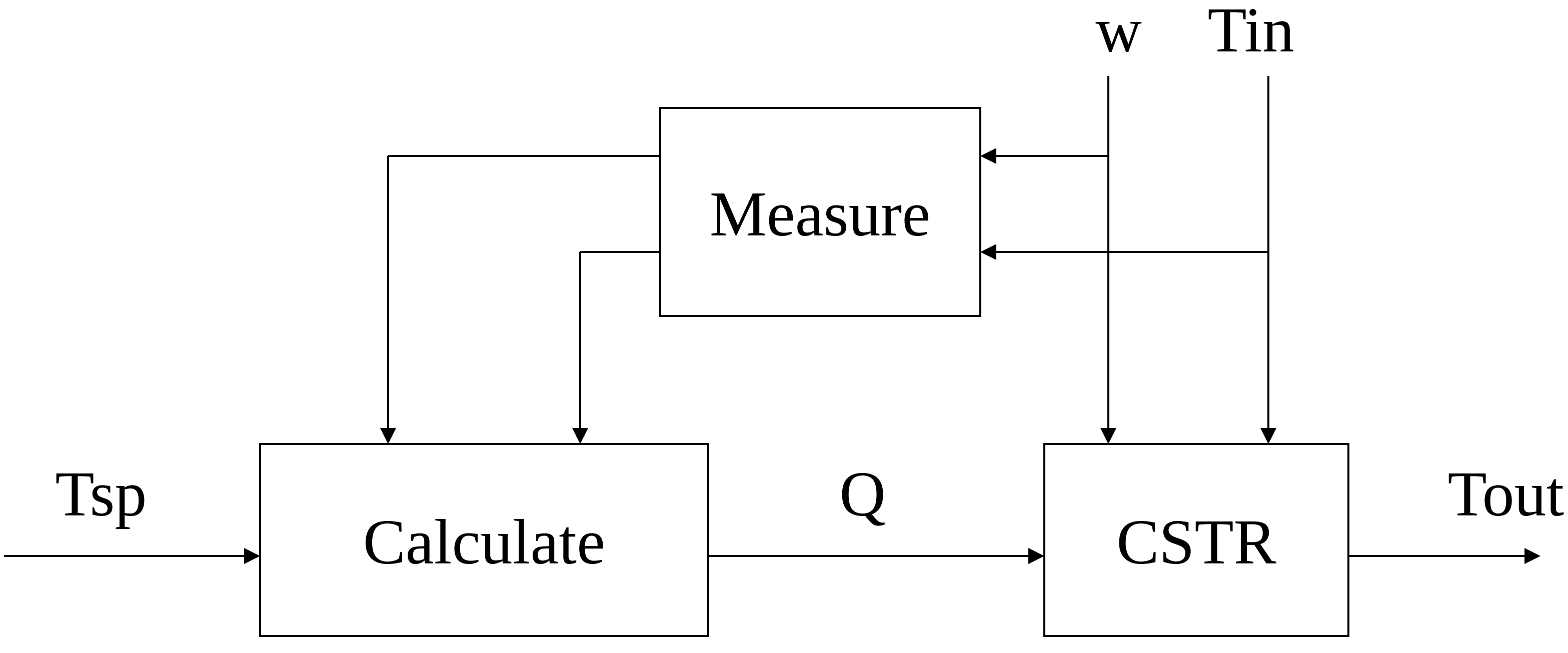
- Suggest three solutions

- <u>Solution 1</u>: Given Tin and w, let Q=wC(Tsp-Tin)



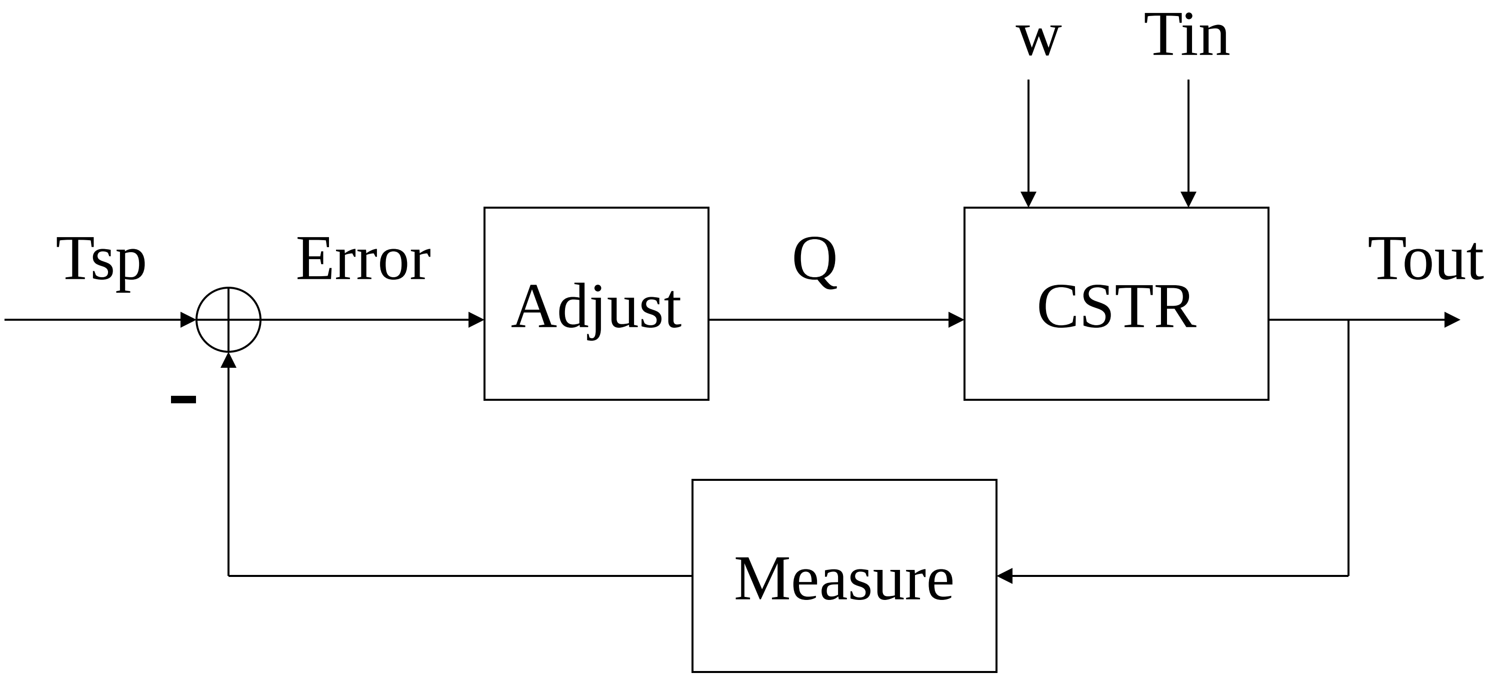
- This “open loop” approach works until Tin and/or w change(s)
- This suggests the second solution

- Solution 2: Measure Tin and w, then calculate Q=wC(Tsp-Tin)



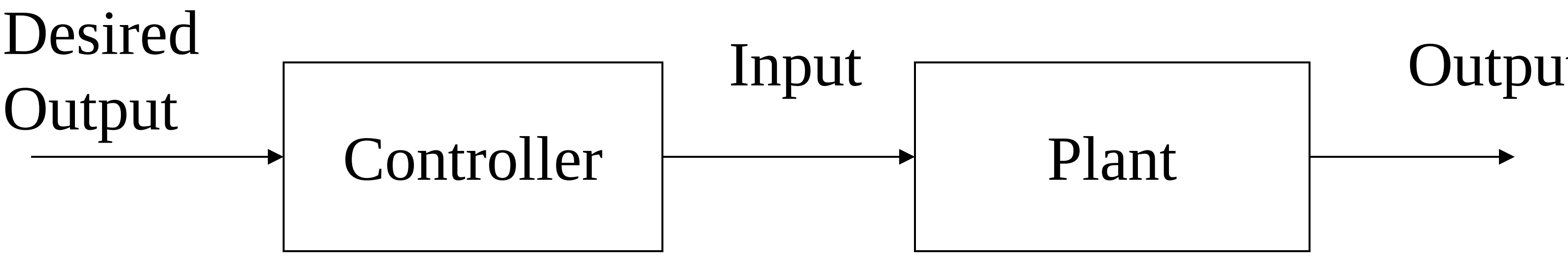
- This "feedforward" configuration is better, but now we ask: what if C changes?

- <u>Solution 3</u>: Measure Tout and adjust Q until Tout=Tsp



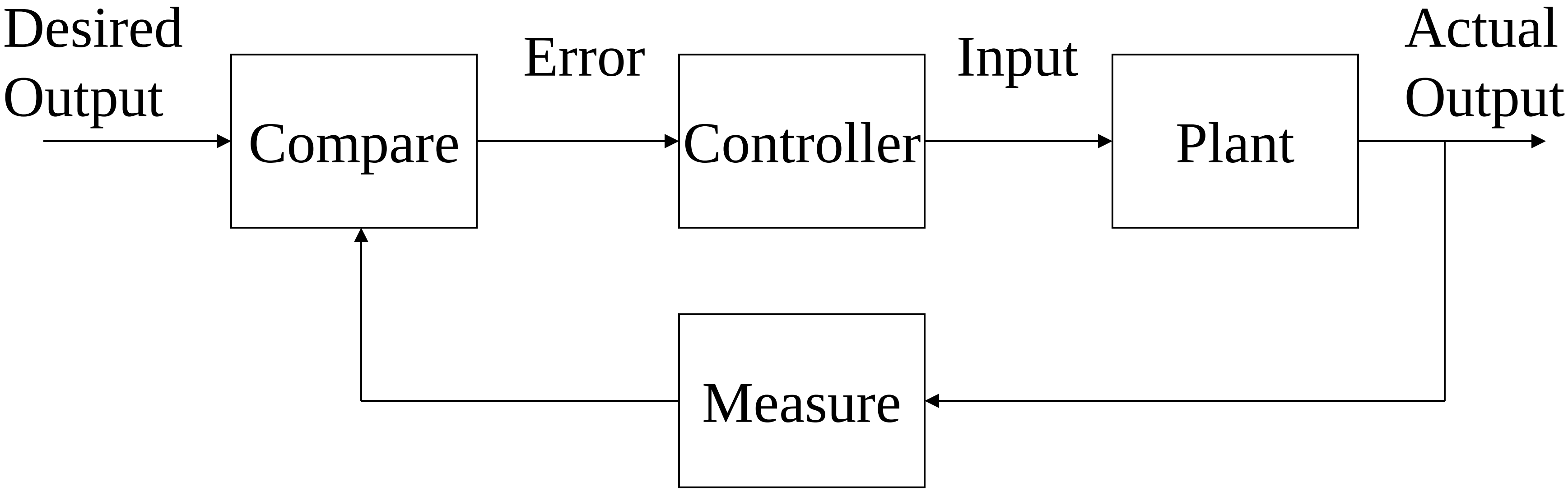
- Error = Tsp-Tout
- This is called “feedback”

- Solutions 1 and 2 are Open-Loop Control approaches



In open-loop control the output has no effect on the input

- Solution 3 is called a Closed-Loop Control approach



In closed-loop control the output is used to derive the input to the plant

- Question: Should we use open-loop or closed-loop control?

- Answer: Depends on the knowledge you have of the system you want to control

- Ex: Suppose we are given a plant defined by

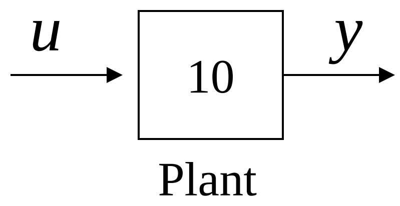
$$y = 10u$$

where y is the output
u is the input

Further suppose we want $y_{sp} = 1$

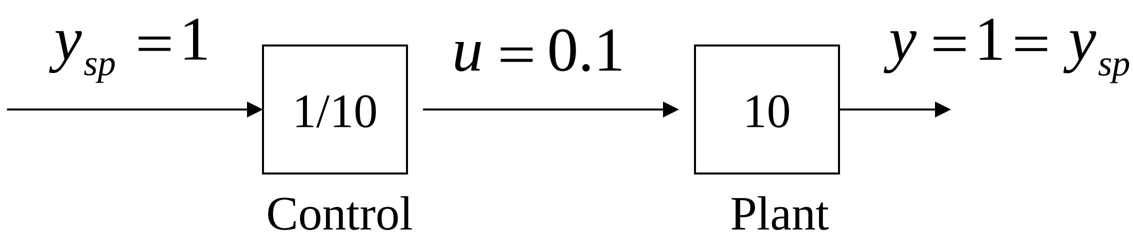
Example

Given:



$y_{sp} = 1$

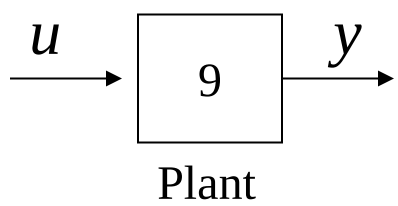
Open-loop Solution:



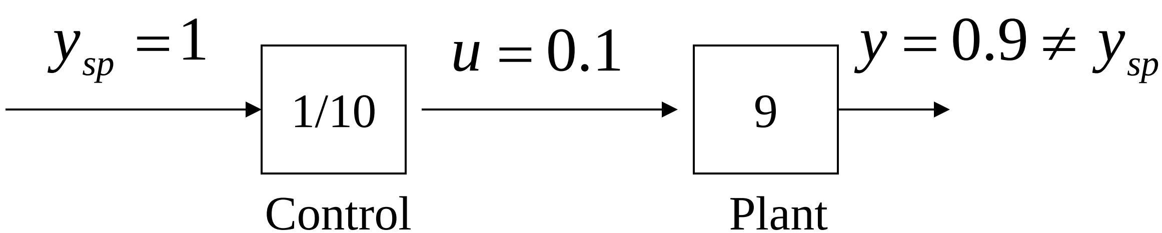
Seems fine, but …

Example

<u>Suppose</u>: Over time the plant changes as shown,
but we did not know about the change



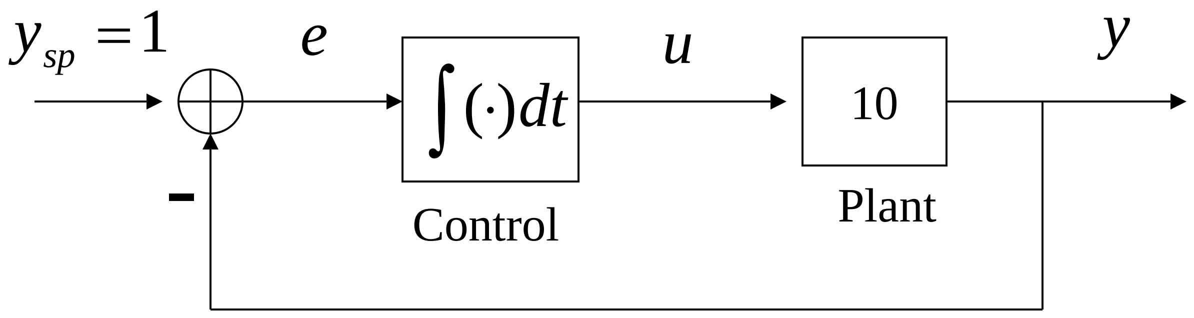
<u>Now our open-loop solution gives</u>:



So, the open-loop approach has some limitations

Example

<u>Consider a Closed-loop Solution</u>:



Now, for this setup we have

$$y = 10u = 10\int e(t)dt$$

$$e = y_{sp} - y = y_{sp} - 10\int e(t)dt$$

Example

So, for $y_{sp} = 1$ we can write

$$\frac{d}{dt}e(t) = \frac{d}{dt}\left\{y_{sp} - 10\int e(t)dt\right\}$$

or

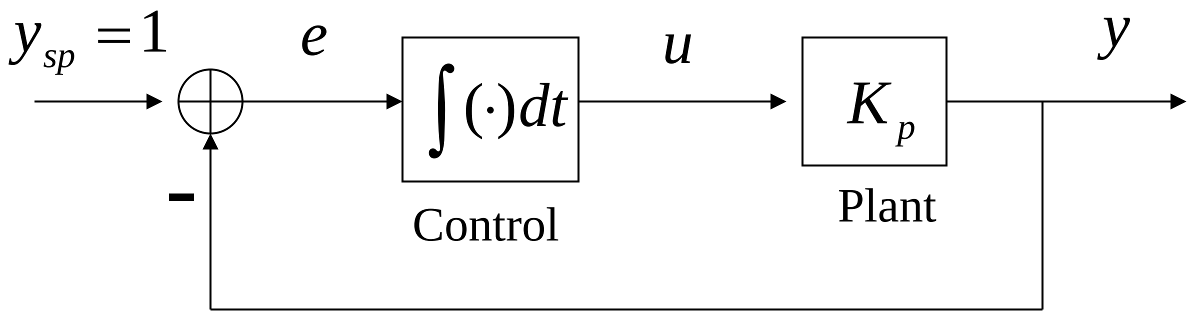
$$\frac{d}{dt}e(t) + 10e(t) = 0$$

Thus

$$e(t) \rightarrow 0; y \rightarrow y_{sp}$$

Example

- Note: The same analysis hold for $y = K_p u$



i.e., $y \to y_{sp}$,no matter what the value of K_p is

- Thus we can say the closed-loop system is robust with respect to
  - Changes in the plant
  - Uncertainty in our knowledge about the plant

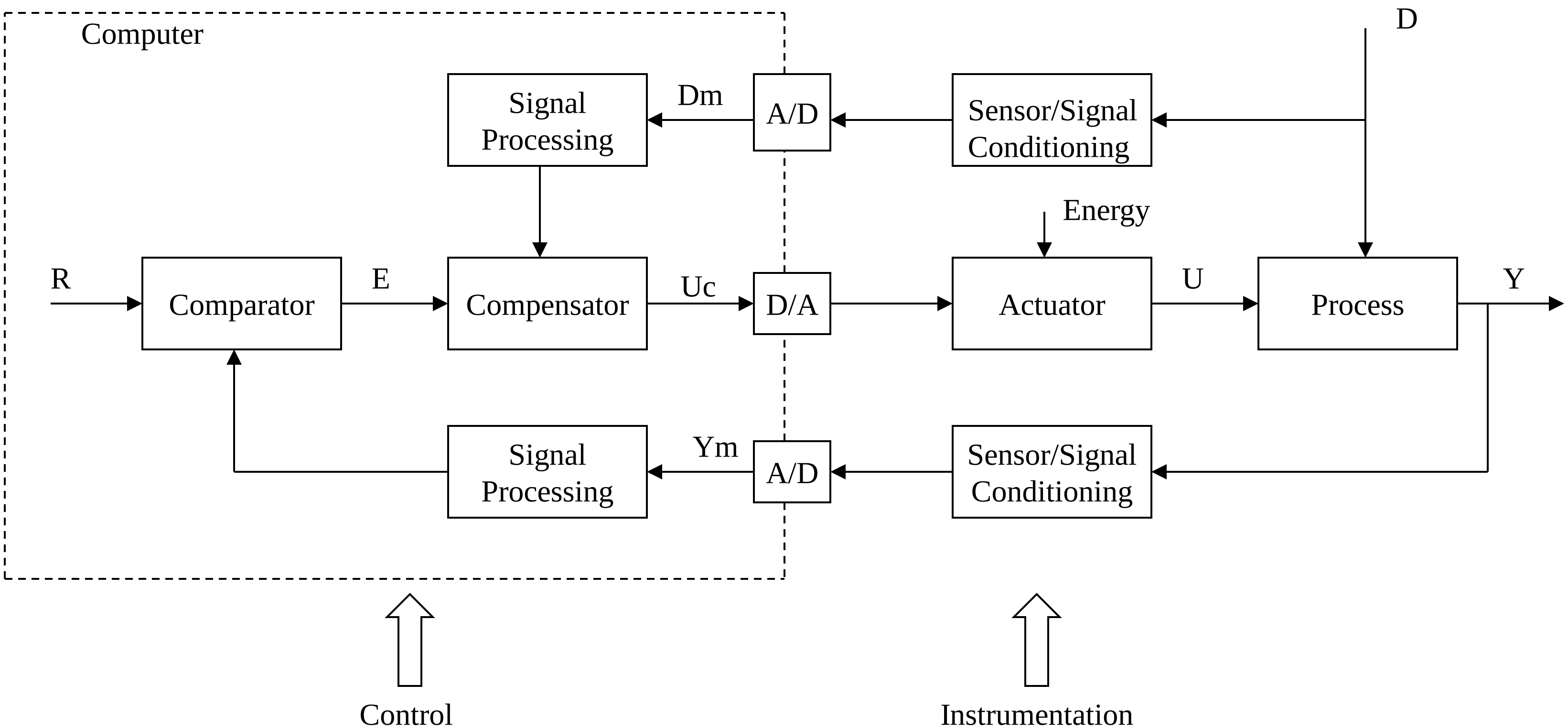
- **To Repeat:**

  - <u>Feedback</u> is the essential feature of an effective control systems (it helps us deal with uncertainty in our knowledge of the plant and changes in the plant behavior)

  - <u>Main Objective</u> of control systems engineering is to develop a feedback controller for a given plant so the closed-loop system satisfies some given set of desired performance requirements

1.2 Control System Engineering and Design

- Figure on the next page shows a generic control system
- Note:
  - All signals can be vectors
  - Signals and elements have several names
  - Not all components are always present
  - Often lump the actuator with the plant
  - Often lump the sensor with the plant
  - Often view DSP as part of the controller
  - Usually the comparator is a simple subtraction
  - There are a variety of other configurations



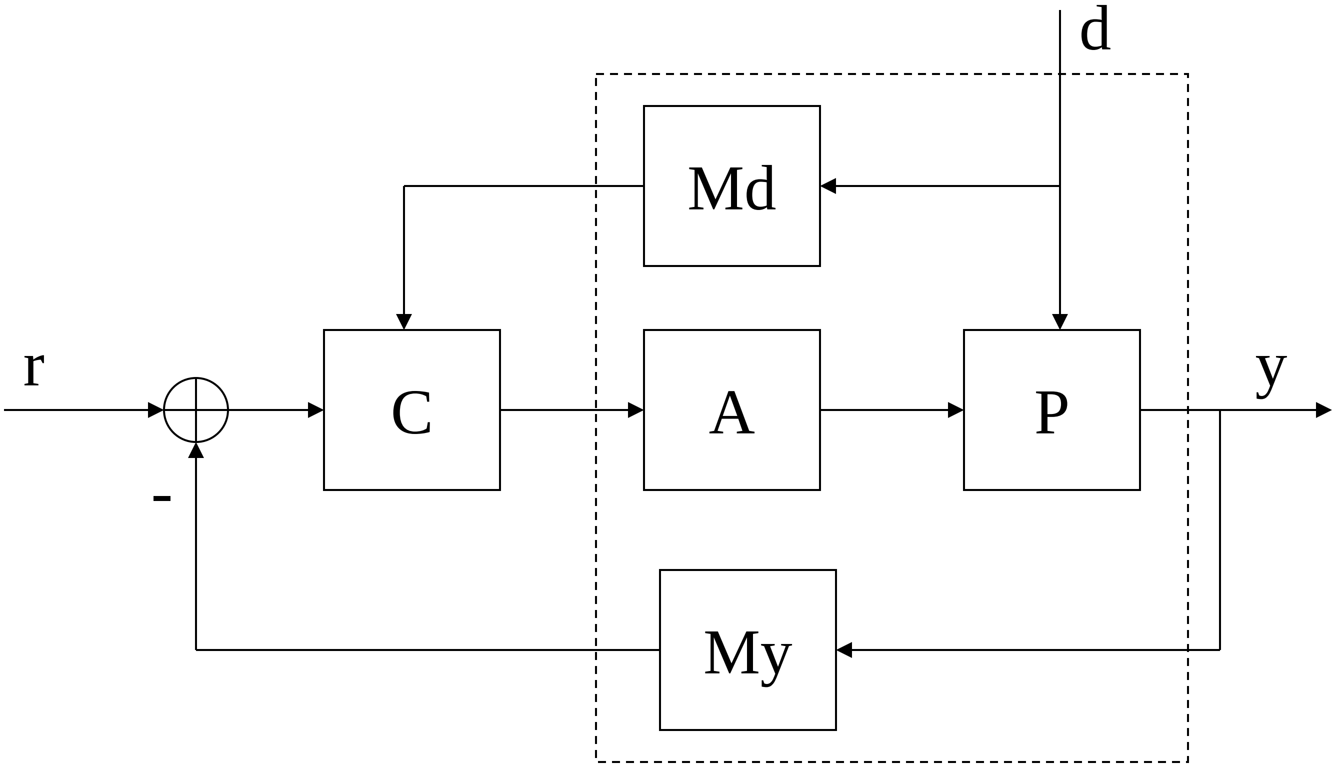
Elements

Comparater
Compensator
Software Signal Processing
A/D and D/A Signal Conversions
Hardware Signal Conditioning
Sensor/Transducer
Actuator (Final Control Element)
Process, Plant, or System

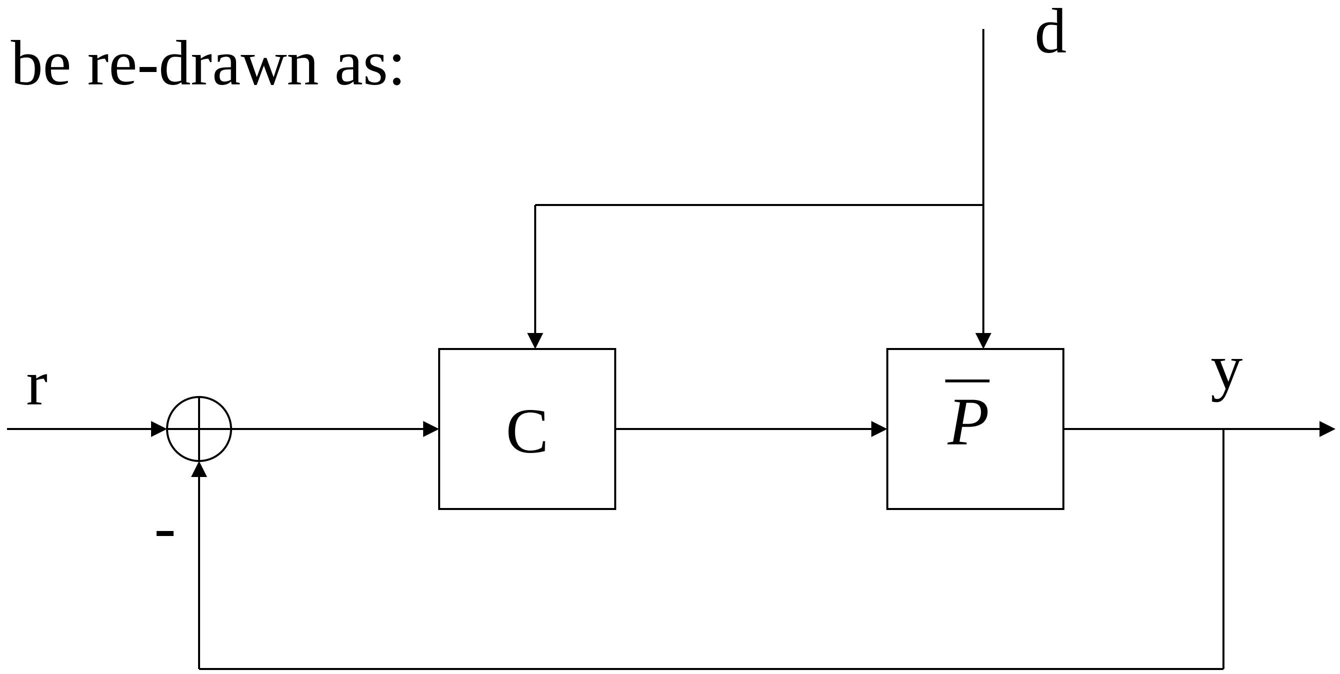
Signals

R: reference, set point,
E: error; often E=R-Ym
Uc: commanded actuator signal
U: plant input
D: disturbance
Y: plant output
Ym: measured output
Dm: measured disturbance

- Consider that this picture:

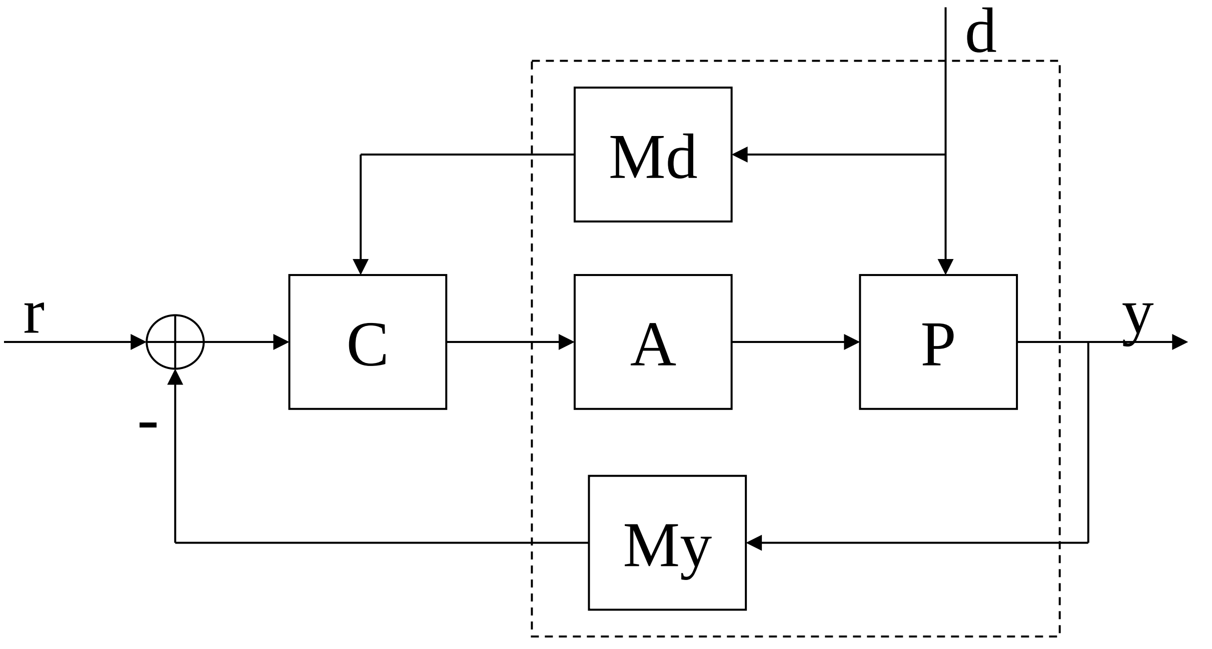


can be re-drawn as:



where $\bar{P}$ represents the combination of A, P, Md, and My in the previous picture

- **Problem #1 - Control system engineering problem**:

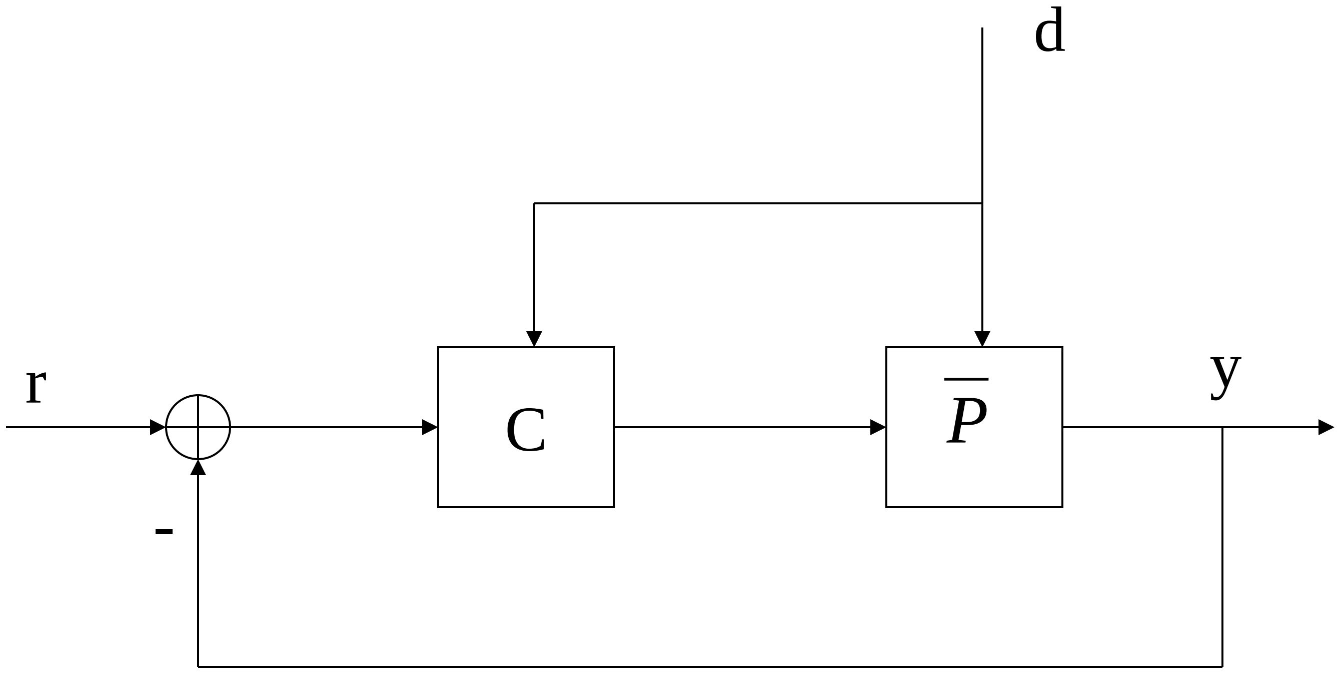


Given: P

Determine: A, Md, My, and C

So that: y has desired properties

- **Problem #2 - Control system design problem**:



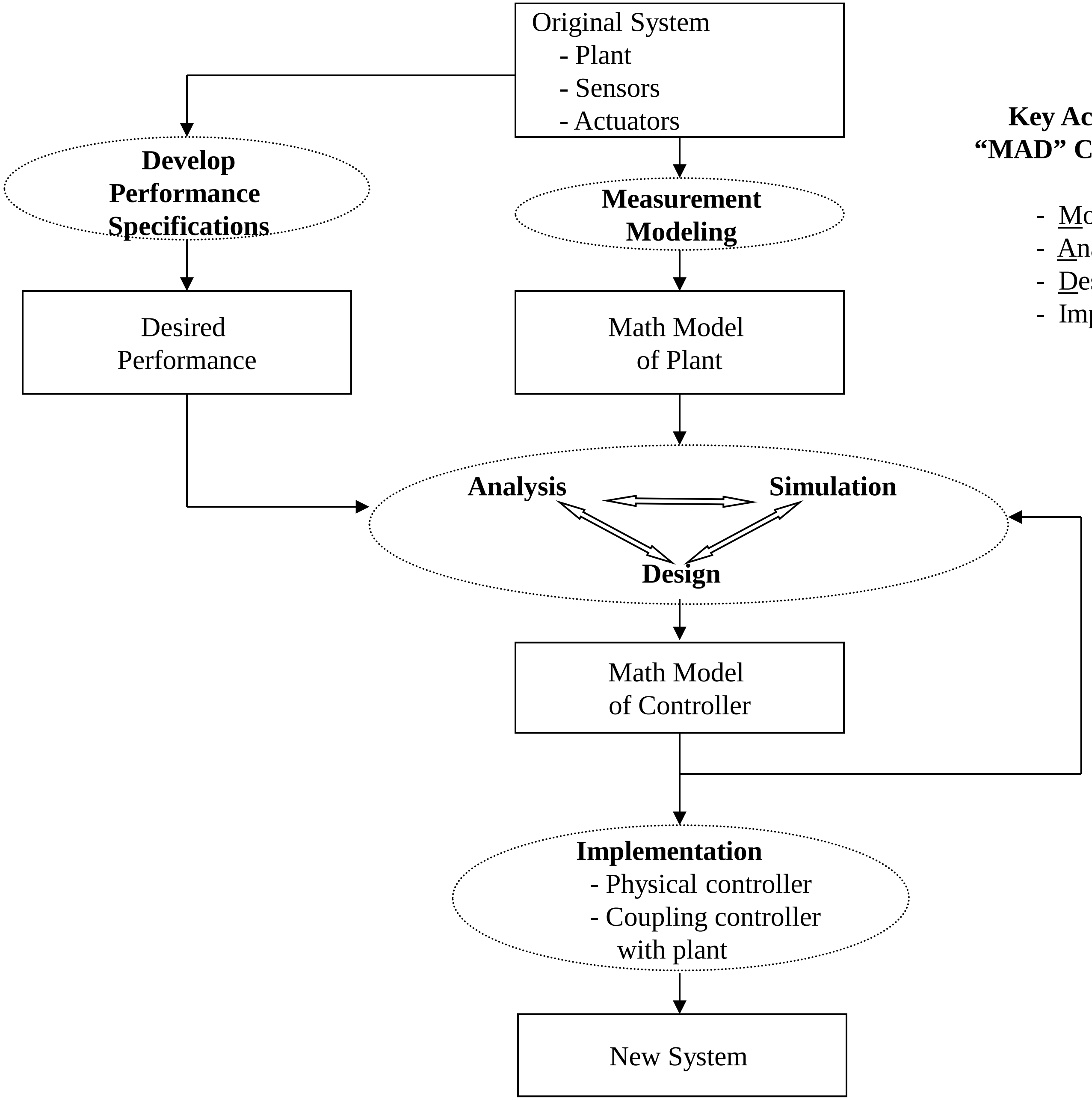
Given: $\overline{P}, d$

Determine: C

So that: y has desired properties

Control System Design Process

- Diagram on the next page gives a flowchart of the control system design process



Key Activities of the "MAD" Control Engineer:

- Modeling
- Analysis
- Design
- Implementation

Control System Design Process

- Hidden in this chart are three important elements :

  1. *Modeling* the system (using mathematics)
  2. *Analysis* techniques for describing and understanding the system's behavior
  3. *Design* techniques for developing control algorithms to modify the system's behavior

- Modeling, analysis, and design = the MAD control theorist
- A fourth key element is *Implementation*

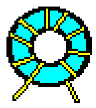
Modeling is the key!

- The single most important element in a control system design and development process is the formulation of a model of the system.
- A framework for describing a system in a precise way makes it possible to develop rigorous techniques for analyzing the system and designing controllers for the system

Note a subtle distinction:

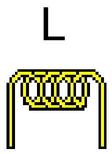
- One can make a subtle distinction between model vs. mathematical description:

  Consider a piece of wire with some loops in it (an inductor):

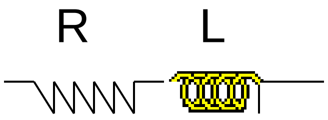


- Three different models:

  1. Simple Inductance



  2. Simple Inductance with resistive dissipation



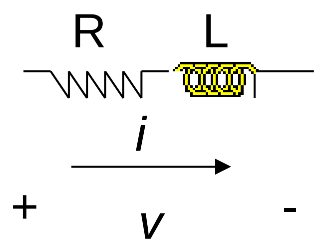
  3. Maxwell’s Equations

Models vs. Mathematical Descriptions:

- So, we can have different models for a given physical reality

- But, a single model can also have different mathematical descriptions

- Consider two different mathematical descriptions of the “simple inductor plus resistive dissipation” model of a coil of wire:



LODE description: $$v = L\frac{di}{dt} + Ri$$

Transfer function description: $$V(s) = (Ls + R)I(s)$$

- In this course we won’t make a distinction between models and mathematical descriptions, but you should be aware of the difference

Factors that Influence Controller Design -1

- Performance measures (what do you want it to do):
  - Do you simply want stability or also optimization
  - Stability requirements (will all signals be bounded?)
  - Do you want to track time-varying or static inputs or do you want to reject disturbances
  - Can you formulate a cost or performance index
  - Can you express the objective in terms of signal norms
  - Does the cost index lead to a solvable problem
  - Steady-state error requirements to a specific input or set of inputs
  - Transient response characteristics requirements (such as rise time, settling time, and overshoot)
  - Robust stability or performance to uncertainty in our models

Factors that Influence Controller Design -2

- Input-output issues (context):
  - Is this a local problem or a global problem?
  - Does the system operate around a set point?
  - Is the system subject to load disturbances?
  - Can you measure the aspects of the process you want to control?
  - Is there noise in the measurements? What kind?

Factors that Influence Controller Design -3

- Modeling issues:

  - Is the model dynamic or static?
  - Is the model linear or nonlinear?
  - Is the model finite dimensional?
  - Does the model change or degrade with time?
  - What kind of uncertainty do you have in the model: structured or unstructured?
  - Is the model stochastic?
  - Is the model qualitative or linguistic?
  - Do human operators perform well controlling the system manually?

Control Methodologies -1

A variety of methods have been developed to deal with the factors that influence controller design. Each is distinguished by the assumptions made about the performance requirements, the system to be controlled (model) , and available measurements:

- Classical Control
  - Laplace transforms: frequency-domain methods ({\it a la} Bode), PID
  - State space: pole placement, controllability, observability
- Nonlinear Control Theory
  - Lyapunov techniques
  - Sliding-mode methods
  - Differential geometry, feedback linearization

Control Methodologies -2

- Optimal Control
  - Linear-quadratic regulator (LQR, H_2)
  - Stochastic Estimation and Control
  - Linear-quadratic Gaussian (LQG)
  - Kalman filtering
  - Risk sensitive
  - Nonlinear
- Robust Control
  - QFT and loop-shaping
  - H_∞ and l_1
  - Kharitonov-based methods

Control Methodologies -3

- Adaptive Control
  - Learning control
  - Model-reference adaptive control
  - Direct and indirect adaptive control
  - Self-tuning regulator
- Process Control
  - Assume first-order with a time delay, stability is primary concern
  - Feedforward control, ratio control
  - Generalized model predictive control (Smith predictor)
  - Auto-tuning PID's
- Intelligent Control
  - Learning-based systems
  - Expert systems
  - Fuzzy logic
  - Artificial neural networks

1.3 Scope of the course -1

- The concepts we have described so far have been very generic and are applicable to a wide variety of systems.

- Distinction in this course will be in how we model the systems of interest

- Models will be
  - Linear
  - Deterministic vs. stochastic
  - Lumped (ODE models) vs. distributed (PDE models)
  - Continuous vs. discrete
  - Time-invariant vs. time-varying (mostly)

Scope of the course -2

Relative to modeling:

- Assume knowledge of state-space/transfer function descriptions of linear systems
  - Aside: Brief review

Two approaches to LTI systems, or LODEs -1

- Approach 1: Classical
  - Input-output or external approach
  - Based on Laplace transforms
  - Dates from 1800's

- Approach 2: Modern
  - State-variable or internal approach
  - Based on vector-matrix math
  - Dates from 1960's

- Can also consider classical differential equations theory as a third approach

Linear Ordinary Differential Equations (LODEs):

$$\ddot{y} + 3\dot{y} + 2y = 2\dot{u} + 3u$$

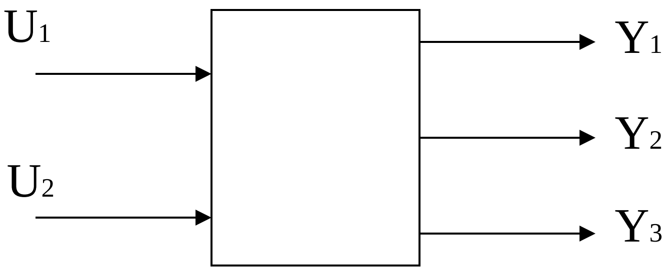
SISO, Linear, Time-Invariant (LTI)
Constant coefficient, ordinary diff. eq.
(LODE)

$$\dot{y} + (3t)y = u$$

SISO, Linear, Time-varying (TV)
ordinary diff. eq.

$$\ddot{y}_1 + 2\dot{y}_1 + 3y_2 = u_1 + u_2$$
$$y_2 + \dot{y}_3 = u_1$$
$$\dot{y}_3 + 2y_1 + 3y_3 = \dot{u}_1 + 3u_2$$

MIMO, LTI LODE



Three Basic Math Descriptions of Linear Systems:

1. LODE $\ddot{y} + 3\dot{y} + 2y = 2\dot{u} + 3u$

2. Laplace Transform $\dfrac{Y(s)}{U(s)} = \dfrac{2s+3}{s^2+3s+2} = \dfrac{n(s)}{d(s)} = d^{-1}(s)n(s)$

3. State Space

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 3 & 2 \end{bmatrix} x$$

Note: Can also talk about "transfer functions" for MIMO Systems

For example, this:

$$\ddot{y}_1 + 2\dot{y}_1 + 3y_2 = u_1 + u_2$$
$$y_2 + \dot{y}_3 = u_1$$
$$\dot{y}_3 + 2y_1 + 3y_3 = \dot{u}_1 + 3u_2$$

can be written as:

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \\ Y_3(s) \end{bmatrix} = \begin{bmatrix} s(s+2) & 3 & 0 \\ 0 & 1 & s \\ 2 & 0 & 3+s \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ s & 3 \end{bmatrix} \cdot \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

$$\vec{Y}(s) = D^{-1}(s) N(s) \vec{U}(s)$$

So, we have:

LODE

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

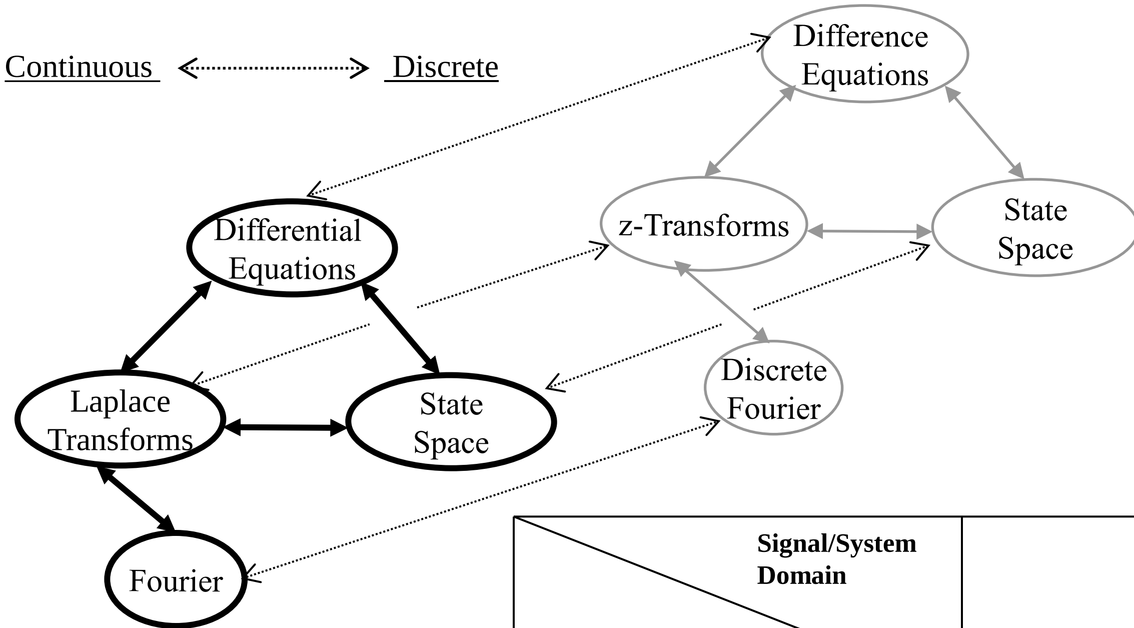
State Space Approach

$$\vec{Y}(s) = D^{-1}(s)N(s)\vec{U}(s)$$

Transfer Matrix Approach

- For each approach there are separate modeling, analysis and design techniques
- Each approach is useful in its own right
- Each approach can be shown to be equivalent to the other

Taxonomy of Essential Signals and Systems Topics (Deterministic) for Undergraduate EE/Control Systems



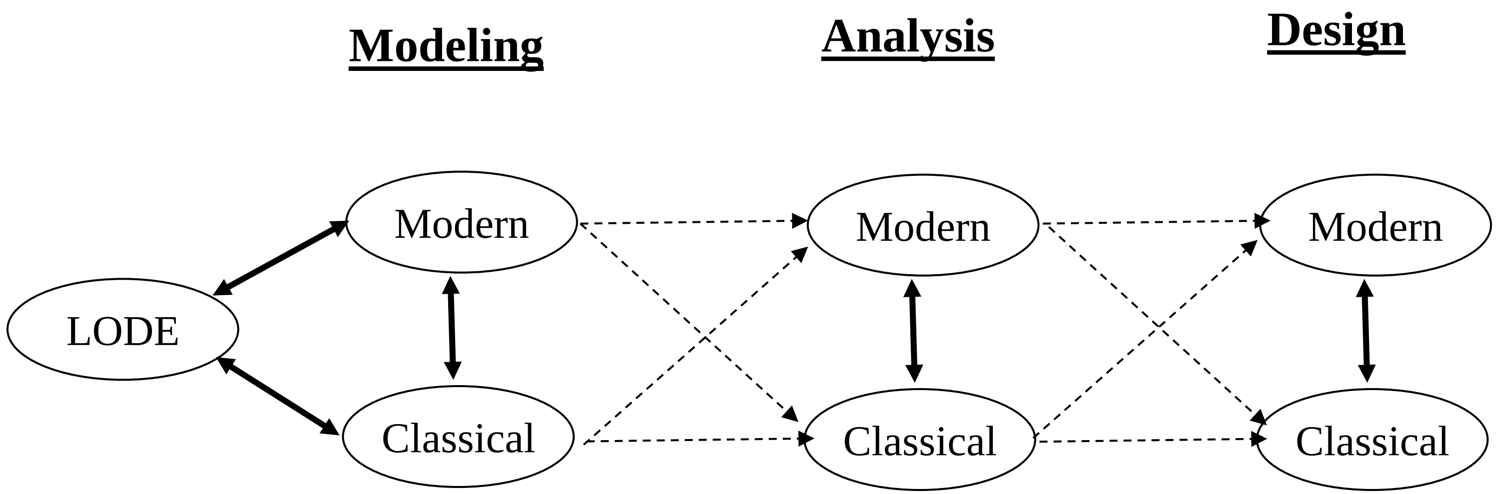
Can do this for SISO or MIMO versions of these topics

Representational Techniques \ **Signal/System Domain**	Continuous	Discrete
Scalar, Classical Differential/Difference Equation (Time-Domain)	**Differential**	**Difference**
Operator Methods	**Laplace, Fourier**	**z-Transform DFT, FFT**
Vector-Matrix Time-Domain	**Continuous-Time State-Space**	**Discrete-Time State-Space**

Variations:

1. Sampled-data
2. Multi-rate
3. Stochastic signals and systems

- Control problems are solved using a combination of modern and classical approaches



Scope of the course -3

Relative to analysis:

- Classical:
  - MIMO transfer functions, poles, zeros
  - MIMO frequency response
- Modern:
  - Controllability/Observability
  - ~~Lyapunov analysis (nonlinear systems)~~

Scope of the course -4

Relative to design:

- Classical:
  - Parameterization of all stablilizing controllers
  - PID/Lead-lag
  - Multi-loop control
  - H-infinity
- Modern:
  - State feedback, observer-based designs
  - Linear quadratic regulator
  - ~~Lyapunov-based design (nonlinear systems)~~
  - ~~Exact feedback linearization (nonlinear systems)~~

Scope of the course -5

Relative to applications:

- Adaptive control
- Aerospace
- Robotics
- Visual servoing
- Automotive
- Neural networks
- Power systems
- … depends on you …!