EGGN 517 Lecture 18
March 22

8.0 Modern Control Design
(state Space)

8.1 State F.B

given plant $\dot{x} = Ax + Bu$

$y = Ix$

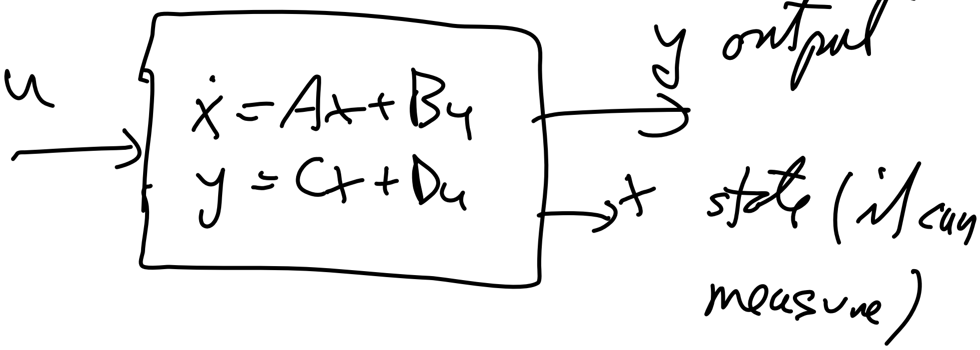
control $u = Kx = Ky$

static (gain)

states

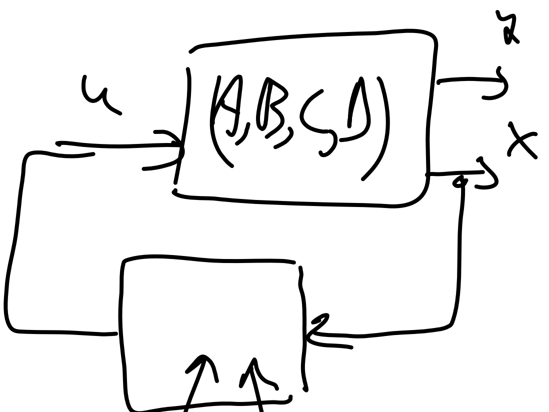
ASIDE

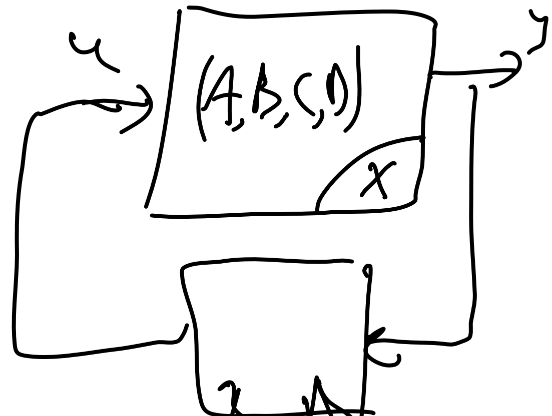
Types of control



state f.b.

output f.b





(1) static

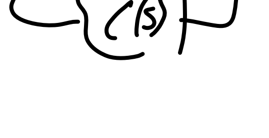
$u = Kx$

(2) dynamic

$\dot{w} = Fw + Hx$

$u = Mw + Nx$

e.g. to



(3) static

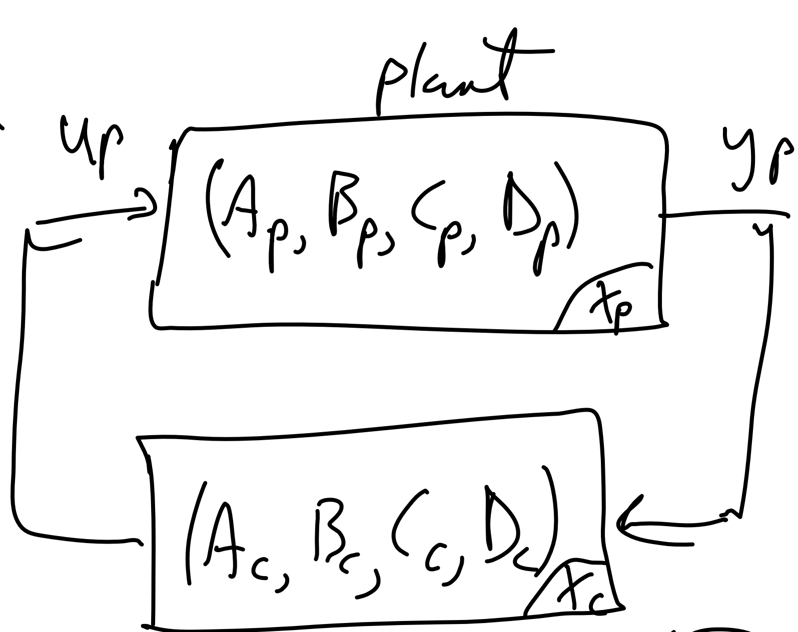
$u = Ky$

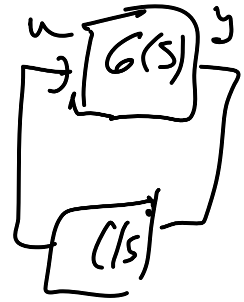
(4) dynamic

$\dot{w} = Fw + Hy$

$u = Mw + Ny$

consider dynamic output f.b





$1+GC=0$ poles

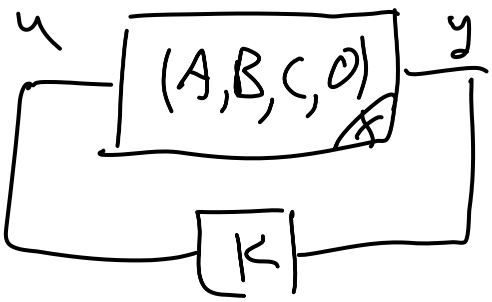
eigs

$$\begin{pmatrix}\dot{x}_p\\ \dot{x}_c\end{pmatrix} = \underbrace{\left[\begin{array}{c|c} A_p + B_p D_c C_p & B_p C_c \\ \hline B_c C_p & A_c \end{array}\right]}_{A_{cl}} \begin{pmatrix} x_p \\ x_c \end{pmatrix}$$

Define $K = \left[\begin{array}{c|c} D_c & C_c \\ \hline B_c & A_c \end{array}\right]$

$$\Longrightarrow A_{cl} = \begin{bmatrix} A_p & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} B_p & 0 \\ 0 & I \end{bmatrix} K \begin{bmatrix} C_p & 0 \\ 0 & I \end{bmatrix}$$

or $\boxed{A_{cl} = \bar{A} + \bar{B} K \bar{C}}$ $\rightarrow$ comes from dynamic output feedback note: equivalent to <u>static output f.b.</u>



$\dot{x} = Ax + Bu$

$y = Cx$

$u = Ky$

$= KCx$

$\dot{x} = (A + BKC)x \Longleftarrow$

$\Longrightarrow$ to find dynamic controller

(A_c, B_c, C_c, D_c)

$\Rightarrow$ form $\bar{A}, \bar{B}, \bar{C}$

solve for K to make A_{cl} stable $\Longrightarrow$ extract A_c, B_c, C_c, D_c from K

$\Longrightarrow$ <u>Fundamental Problem</u>

Given $A_{cl} = A + BKC$ find K to stabilize

C invertible $\Longrightarrow$ eq. to

$A_{cl} = A + B\bar{K}$

static state f.b.

dynamic output f.b. $\xRightarrow{\text{eq. to}}$ static output f.b. $\xRightarrow[\text{eq. to}]{C \text{ invertible}}$ static state f.b. $\xLeftarrow{\text{eq. to}}$ dynamic state f.b.

all roads lead to here

C not invertible
$\Rightarrow$ do other things

consider $A_{cl} = A + BKC$

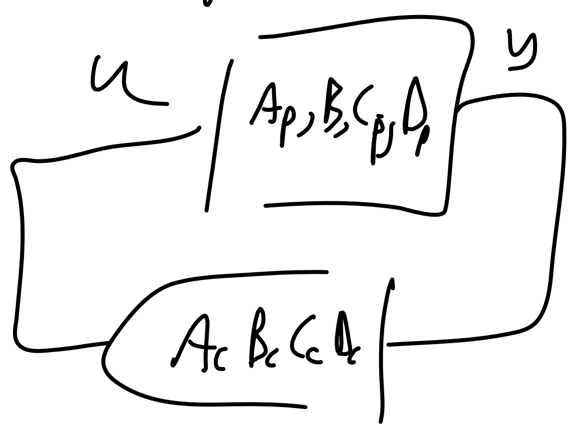
$n \times n$ $n \times r$ $m \times n$

states   # inputs   outputs

Best result to date Kimura 1975

If $n+1 \leq r+m$ then $\exists$ K to arbitrarily assign eig. of A_{cl} if (A,B) cont. and (A,C) obs.

Apply Kimura theorem to dynamic output f.b.



A_p be $n_p \times n_p$

A_c be $n_c \times n_c$

$r = m_p = 1$

$p \Longrightarrow$ SISO

$$\bar{A} = \begin{bmatrix} A_p & 0 \\ 0 & 0 \end{bmatrix} \quad \bar{B} = \begin{bmatrix} B_p & 0 \\ 0 & I \end{bmatrix} \quad \bar{C} = \begin{bmatrix} C_p & 0 \\ 0 & I \end{bmatrix}$$

$$K = \begin{bmatrix} D_c & C_c \\ B_c & A_c \end{bmatrix} \Longrightarrow \bar{A} \in \mathbb{R}^{(n_c+n_p)\times(n_c+n_p)}$$

i.e. $\quad A_{cl} = \bar{A} + \bar{B} K \bar{C}$

$m_p + n_c = m$

$r_p + n_c = r$

$n = n_c + n_p$

$\Rightarrow n+1 \leq r+m$

$$n_c + n_p + 1 \leq \underset{\substack{\uparrow \\ 1}}{r_p} + n_c + \underset{\substack{\uparrow \\ 1}}{m_p} + n_c$$

SISO

$r_p = m_p = 1$

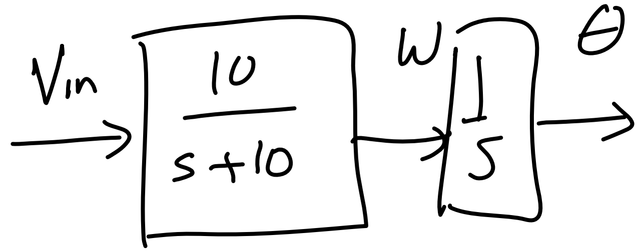
$$\boxed{n_c \geq n_p - 1}$$

state space Kimura result

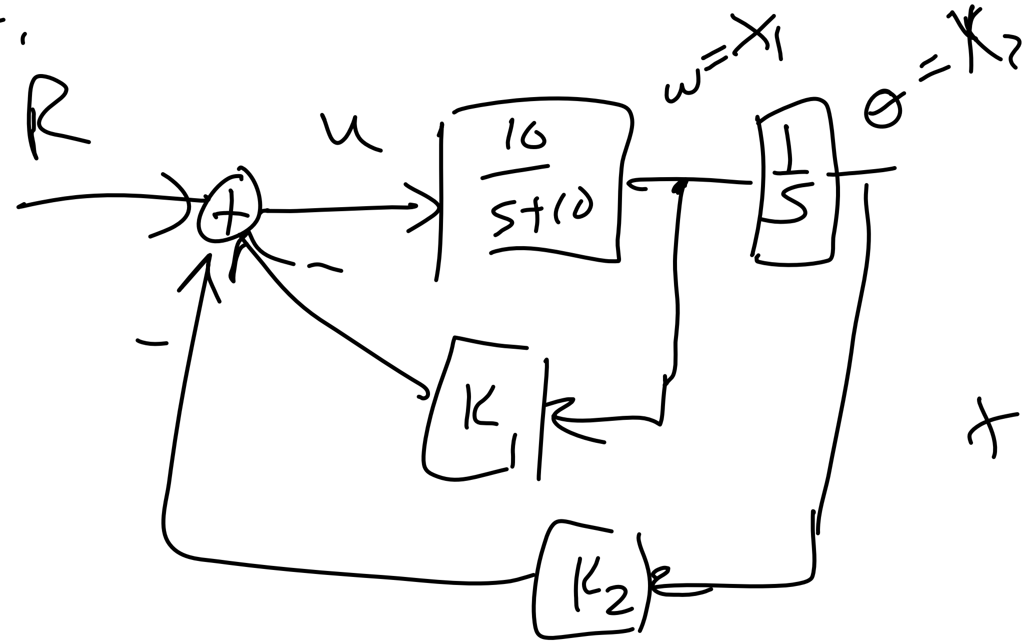
same as we derived using transfer function, except Kimura result covers MIMO

- Steady-State Tracking using state f.b.

Ex: Motor Control (SISO, static state f.b) assume motor



s.f.



$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} w \\ \theta \end{pmatrix}$

$$u = -K_1 w \cancel{+} K_2 \theta = -\begin{pmatrix} K_1 & K_2 \end{pmatrix} \begin{pmatrix} w \\ \theta \end{pmatrix}$$

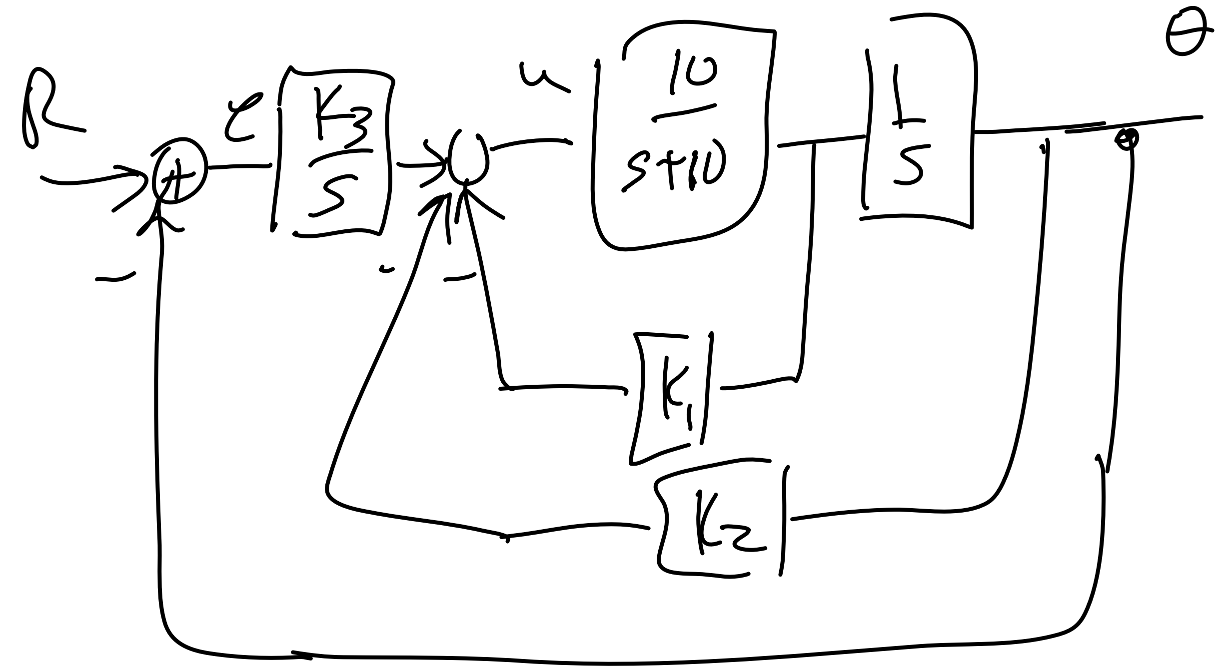
$$u = -Kx$$

$$\frac{\theta}{R}(s) = \frac{10}{s^2 + (10 + 10K_1)s + 10K_2}$$

if $R(s) = \frac{1}{s}$ $\quad \lim_{t \to \infty} \theta(t) = \frac{1}{K_2} \neq 1$

state f.b. screwed up e_{ss}.

solution: add an integrator



$$\frac{\theta}{R} = \frac{10K_3}{s^3 + (10+10K_1)s^2 + 10K_2 s + 10K_3}$$

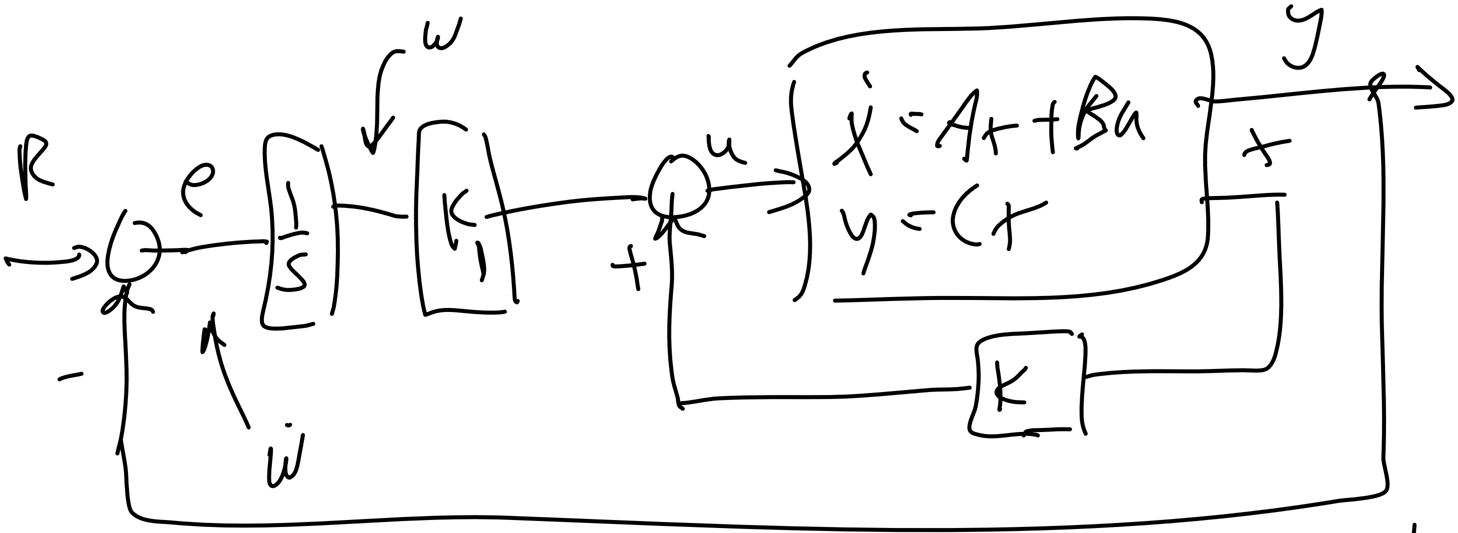
now $\theta(t) \to 1$ if $R(s) = \frac{1}{s}$

This controller looks like

$$u = -K_1 w - K_2 \theta - K_3 e$$

like another state

In general:



mixed with ~~dynamic output fb~~ static stab fb

$$\dot{x} = Ax + Bu$$
$$\dot{w} = -Cx + r$$
$$u = Kx + K_1 w$$

$\dot{w} = e = r - y$

$= r - Cx$

states: w, x

control, plant

put together

$$\begin{pmatrix} \dot{x} \\ \dot{w} \end{pmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{pmatrix} x \\ w \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} \begin{pmatrix} K & K_1 \end{pmatrix} \begin{pmatrix} x \\ w \end{pmatrix} + \begin{pmatrix} 0 \\ B \end{pmatrix} r$$

$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{pmatrix} x \\ w \end{pmatrix}$$

stability depends on eig A_{cl}

$$A_{cl} = \begin{pmatrix} A & 0 \\ -C & 0 \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} \begin{pmatrix} K & K_i \end{pmatrix}$$

or $$A_{cl} = \underbrace{\bar{A} + \bar{B}\bar{K}}_{\text{solve static state f-b problem}}$$

The augmented system $(\bar{A}, \bar{B})$ embeds model of integrator

Acker