MODELING ANATYSIS AND CONTROL OF THE CONTINUOUS FLOW BOILING SYSTEM

SECTION 1

1. INTRODUCTION

The aim of this project is to study the modeling analysis and control of a continuous flow boiling system (SISO system and MIMO system approach).

The project is organized into five sections, a reference and an appendix parts. The outline of the project is as follows:

Section 2, describes the procedure for developing the non-linear mathematical model for a continuous flow boiling system employing the mass and heat balance equations and the linearization of the identified model around the operating point using a Taylor approximation. The results are presented by simulation of the linear model using MATLAB SIMULINK.

Section 3 discusses the analysis of continuous flow boiling system. The stability (internal, and BIBO stability), canonical forms, and frequency response are introduced.

Section 4 discusses the control of continuous flow boiling system in single input single output (SISO system) approach. In this section the design of the controller using error feedback, state feedback, output feedback, Sylvester matrix technique, and a parametization of all stabilizing controllers are made.

In section 5 control of continuous flow boiling system in multi input multi output (MIMO system) approach is presented. This chapter describes the procedure of controller design using a state feedback controller with integral error feedback, a state feedback controller with integral error feedback with state feedback implemented via observer system, and decentralized controller, and finally the optimal gain method.

Section 6 concludes the results of my work.

SECTION 2

MODELLING OF THE CONTINUOUS FLOW BOILING SYSTEM

This section describes the procedure for developing the non-linear mathematical model for a continuous flow boiling system employing the mass and heat balance equations and the linearization of the non-linear model around the operating point using a Taylor approximation and Simulink method.

2.1 MATHEMATICAL MODEL

2.1.1 MODEL DESCRIPTION [1]

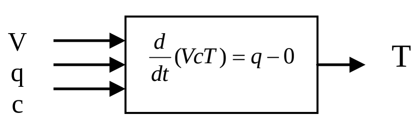
Suppose that a container of fluid is heated at a rate q (PCU/time). Then, the heat balance equation would state:

Rate of change of heat content = heat in - heat out

$$\frac{d}{dt}(VcT) = q - 0 \text{ (no heat loss)} \quad , \qquad (2\text{-}1)$$

where: V represents the volume, and c the specific heat.

Since V, q, and c are known, this equation can be used to establish the temperature (Fig.2-1) by supplying V, q, and c.



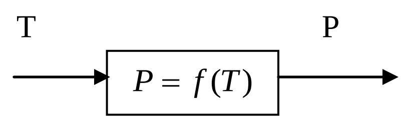


Fig.2-1. Heat Balance Model. **Fig.2-2.** Vapor Pressure /Temperature Relationship.

The vapor pressure exerted by the liquid varies with the temperature, as shown in Fig.2-2. Vaporization can be considered negligible until the temperature has reached the boiling point. At this temperature the vapor pressure P tends to exceed the actual pressure π, that is, P > π. This results in the stream of vapor issue from the boiling liquid. Because there is no resistance to departure of this vapor, a sufficient flow is emitted that (through the heat balance) automatically prevents the temperature from rising beyond the boiling point.

The vapor pressure corresponding to the boiling temperature is infinitesimally grater than the total pressure, but this minute difference is sufficient to provide the vapor flow that maintains the status quo.

The boiling heat balance is presented in Fig.2-3. The equilibrium equation that computes the vapor rate, v contains a gain factor G that is large enough to keep the (P - π) difference very small.

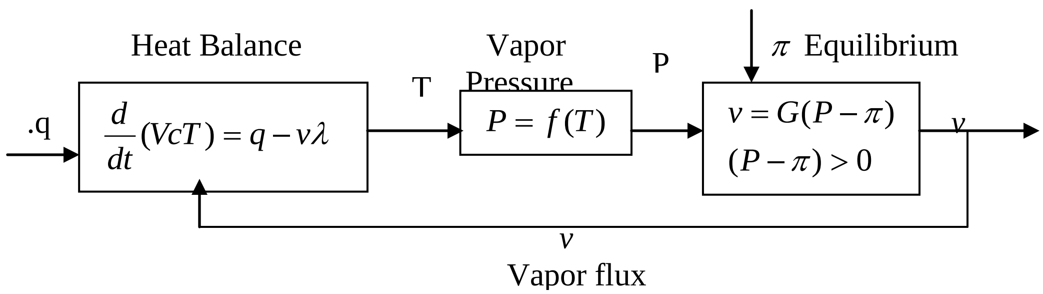


Fig.2-3. Model for Equilibrium Balance.

This model is the "natural" definition of the system. It can be regrouped as shown in Fig.2-4. The overall diagram presented in Fig.2-5 shows that the system has two inputs, π, and q, and two outputs, T, and v.

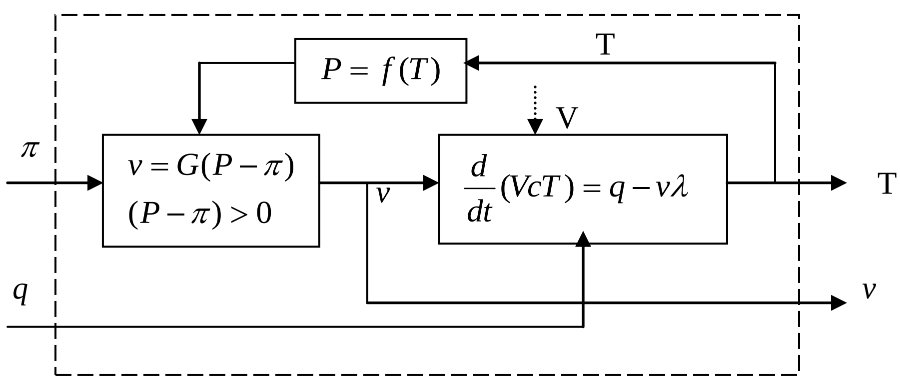


Fig.2-4. Input-Output Relationship for Microscopic Boiling Model.

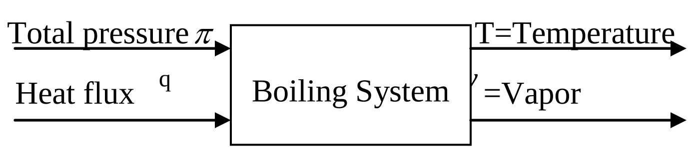


Fig.2-5. Microscopic Input-Output Relationship.

2.1.2 THE PHENOMENON OF BOILING SYSTEM [1]

The phenomenon of boiling system is such that the temperature responds only to the total pressure π and the vapor flow only to the heat flux q. This leads to the more convenient mode shown in Fig.2-6. The heat balance is used to establish the vapor flux, whereas the system pressure π indicates the temperature. In most cases the differential term $d/dt\,(VcT)$ is very small in comperation with q and can be neglected.

In summary, the only way to change the temperature in case of boiling a single component liquid, is to change the total pressure. Changing the heating rate, this changes only the rate of evolution of vapor. The cause-and-effect relationships for single boiling fluid can be status as:

- Pressure (P) establishes the boiling temperature (T);
- Heat flux (q) establishes the vapor rate (v).

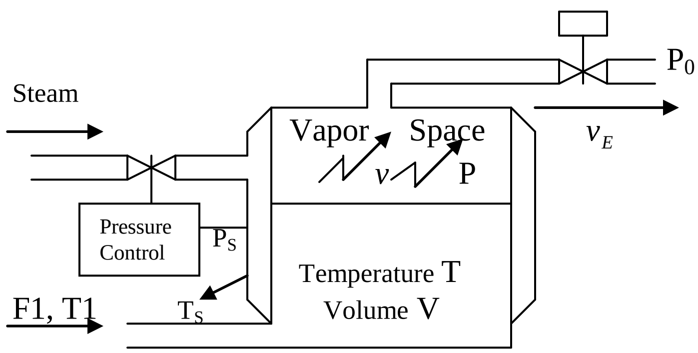


Fig.2-6. Continuous Flow Boiling system.

The complexity of boiling will be assumed for the jacket vessel. Fig.2-6.shows the feed flow supplied in liquid form and the exit flow withdrawn as vapor. The mathematical model for this boiler consists of simultaneous mass and energy balance.

The mass balance equation of the liquid is:

$$\frac{dV}{dt} = F1 - v \quad , \tag{2-2}$$

where F1 is the feed rate and v is the boil up rate. The mass balance equation on the vapor is:

$$\frac{d}{dt}(m_G) = v - v_E \quad , \tag{2-3}$$

where v_E is the flow of vapor through exit valve.

Because equilibrium is assumed to exist at all times between liquid and vapor, an energy balance for the vapor is not necessary, and the vapor temperature is assumed to be the same as the liquid temperature. The energy balance in the liquid is:

$$\frac{d}{dt}(VcT) = F_1cT_1 + q - v(cT + \lambda) \quad , \tag{2-4}$$

where $(cT + \lambda)$ is an approximation of the vapor enthalpy. The pressure in the vapor space is obtained from the gas-law relationship:

$$PV_G = mRT \qquad \text{and} \qquad V_G = V_0 - \frac{V}{\phi} \quad , \tag{2-5}$$

where ϕ represents the density (mass/unit Vol) and V_0 the total volume of vessel. The temperature is obtained from the pressure/boiling temperature relationship:

$$T = f(p) = \frac{c_2}{(\ln p - c_1)} \quad . \tag{2-6}$$

If the effluent valve is fixed, the flow rate of vapor through the valve v_E will be:

$$v_E = k\sqrt{P(P - P_0)} \quad . \tag{2-7}$$

The heat flux q is:

$$q = UA(T_S - T) \quad , \tag{2-8}$$

where T_S is the steam temperature.

1.1.3 PROCEDURE FOR ASSEMBLING THE NON-LINEAR MODEL

With the equations for each part of the system defined, the procedure for assembling the model will now be as follows:

A. Boundary values:

1. Inlet flow: F_1;
2. Inlet temperature: T_1;
3. Jacket steam pressure: P_S;
4. Exit pressure: P_0;

B. Equations:

1. Valve: $v_E = K\sqrt{P(P-P_0)} \to v_E$ (2-9)
2. Gas law: $PV = m_G RT \to P$ (2-10)
3. Vapor mass balance: $\dfrac{dm_G}{dt} = v - v_E \to m_G$ (2-11)
4. Boiling point: $T = f(P) \to T$ (2-12)
5. Jacket heat: $q = UA(T_S - T) \to q$ (2-13)
6. Heat balance:

$$\frac{d}{dt}(VcT) = F_1 c T_1 + q - (cT + \lambda)v \to v \qquad (2\text{-}14)$$

7. Mass balance on liquid: $\dfrac{dV}{dt} = F_1 - v \to V$ (2-15)
8. Gas volume: $V_G = V_0 - \dfrac{V}{\phi} \to V_G$ (2-16)

The equations described above are assembled in Fig.2-7.

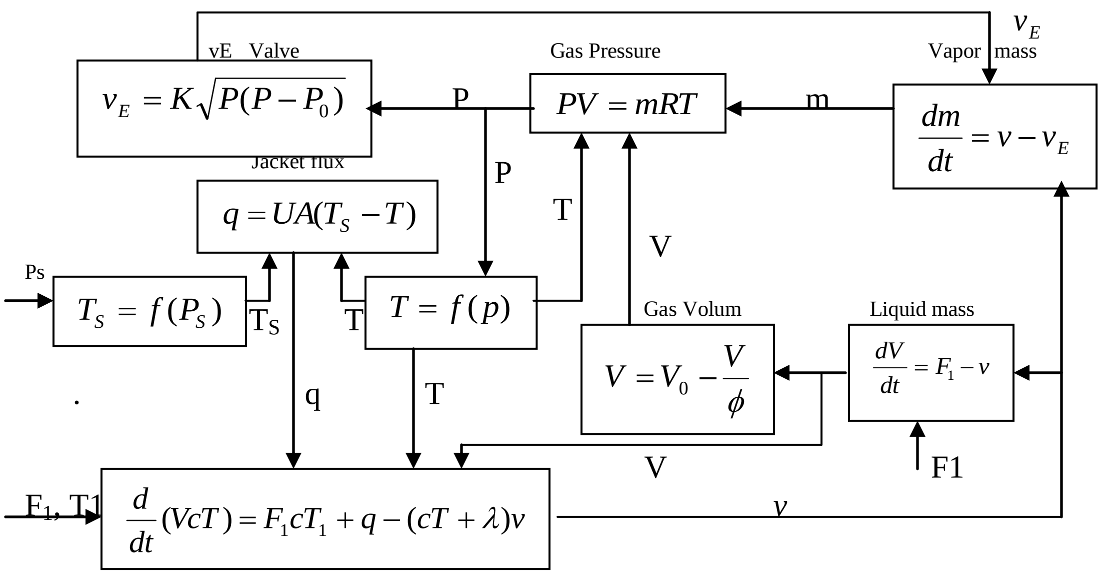


Fig.2-7. Model for continuous flow Boiling Jacketed Vessel.

2.1.4 BUILDING THE NON-LINEAR MODEL

With the equations for each part of the system defined in *Section 2.1.3*, the system equations can be determined as:

$$eq1 = P = \frac{m_G R(T+273)}{V_G} \quad \text{, (Pressure from gas law)} \tag{2-17}$$

$$eq2 = T = \frac{c_2}{\ln P - c_1} - 273 \quad , \tag{2-18}$$

(Boiling point temperature employing Antoine equation)

$$eq3 = q = UA(T_S - T) \quad \text{, (Jacket heat)} \tag{2-19}$$

$$eq4 = v = \frac{q}{T - T_1 + \lambda} \quad \text{, (Boiling rate)} \tag{2-20}$$

$$eq5 = v_E = k\sqrt{P(P - P_0)} \quad \text{, (Out vapor flow rate)} \tag{2-21}$$

$$deq1 = \frac{dm_G}{dt} = v - v_E \quad \text{, (Vapor mass balance)} \tag{2-22}$$

Hence, the mathematical model (in MIMO case) could be built up considering the following variables:

- Input variables:
  - Inlet temperature: $u_1=T_1$;
  - Jacket temperature: $u_2=T_{S;}$
  - Exit pressure: $u_3= P_0$;
- State Variables:
  - Vapor pressure: $X_1=P$;
  - Boiling temperature: $X_2=T$;
  - Vapor mass: $X_3= m_G$;
- Output variables:
  - Outlet vapor flow rate: $Y_1 = v_E$;
  - Temperature: $Y_2 = X_2 = T$;
  - Vapor mass: $Y_3 = X_3 = m_G$

- Constants:

R=1.98moles/ ft^3 ,c1=13.96 $KJoule/(Kmol.^\circ C)$,c2=-5210.6 $KJoule/(Kmol.^\circ C)$

,V_G=30000 ft^3 , λ=9717 PCU/mole, UA=1700, K=5.7 $ft^3/(Kmol.\text{sec.})$

While the general form of a state space description of a mathematical model is:

$$\begin{aligned}\dot{\overline{X}} &= A\overline{X} + B\overline{u} \\ \overline{Y} &= C\overline{X} + D\overline{u}\end{aligned} \quad , \tag{2-23}$$

where: A, B, C, D : are matrixes (A is the state matrix, B is the control matrix, C is output matrix, D is the direct transformation matrix), the boiling system can be written in this form:

$$\overline{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} P \\ T \\ m_G \end{bmatrix} \quad , \quad \overline{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_S \\ P_0 \end{bmatrix} \quad \text{and} \, \overline{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} T \\ v_E \\ P \end{bmatrix}. \tag{2-24}$$

To obtain the corresponding system equations for the state space description of the non-linear model, I started form (2-17)-(2-22). Taking the derivative of the equations (2-17), (2-18), (2-22):

$$\dot{P} = \frac{R}{V}(m_G.\dot{T} + (T + 273).\dot{m}_G) \quad , \tag{2-25}$$

$$\dot{T} = \frac{-c_2}{P(\ln p - c_1)^2}.\dot{P} \quad , \tag{2-26}$$

$$\dot{m}_G = v - v_E \quad , \tag{2-27}$$

and substituting the equations (1-20), (1-21) on the last equation:

$$\dot{m}_G = \frac{q}{T - T_1 + \lambda} - k\sqrt{P(P - P_0)} \Rightarrow \dot{m}_G \quad , \tag{2-28}$$

The following result is obtained (substituting the equation (2-26) on the equation (2-25) :

$$\dot{P}(1 + \frac{R.m_G}{V}.\frac{-c_2}{P(\ln P - c_1)^2}) = \frac{R(T + 273)}{V}.\dot{m}_G \Rightarrow \dot{P} \ . \tag{2-29}$$

Since:

$$\dot{T} = \frac{-c_2}{P(\ln p - c_1)^2} . \dot{P} \Rightarrow \dot{T} \qquad , \tag{2-30}$$

with the equations (2-28), (2-29), (2-30) substituting the input, output and state variables as:

$u_1=T_1$ $\quad X_1=P$ $\quad$,

$u_2=T_S$ $\quad X_2=T$ $\quad$;

$u_3= P_0$ $\quad X_3= m_G$ $\quad$;

$$Y_1 = v_E$$

$$Y_2 = X_2 = T;$$

$$Y_3 = X_3 = m_G, \tag{2-31}$$

the state space representation of the non-linear model (in MIMO case) will be :

$$\dot{X}_1 = \left(\frac{R(X_2 + 273).X_1.(\ln X_1 - c_1)^2}{V.X_1.(\ln X_1 - c1)^2 + c_2 R X_3} \right) \left(\frac{UA(u_2 - X_2)}{X_2 - u_1 + \lambda} - K\sqrt{X_1(X_1 - u_3)} \right)$$

$$\dot{X}_2 = \left(\frac{-c_2 R(X_2 + 273)}{V.X_1.(\ln X_1 - c1)^2 + c_2 R X_3} \right) \left(\frac{UA(u_2 - X_2)}{X_2 - u_1 + \lambda} - K\sqrt{X_1(X_1 - u_3)} \right)$$

$$\dot{X}_3 = \frac{UA(u_2 - X_2)}{X_2 - u_1 + \lambda} - K\sqrt{X_1(X_1 - u_3)}$$

$$Y_1 = K\sqrt{X_1(X_1 - u_3)}$$

$$Y_2 = X_2$$

$$Y_3 = X_3 \, . \tag{2-32}$$

2.2 LINERAZATION OF THE NON-LINEAR MODEL

2.2.1 CALCULATION OF THE OPERATING POINT

Considering the importance of the operating point [1], the following conditions will be applied to the system:

- The liquid level is maintained at a fixed position by a level controller. This makes V and V_G constants and also the feed flow $F_1 = v_E$;
- The liquid is initially cold and heated up to its boiling point. After boiling starts, the pressure rises to its equilibrium level, raising the temperature to a higher value.

At equilibrium point (operating point) the vapor mass will be constant, therefore:

$$\frac{d}{dt}(m_G) = v - v_E = 0 \quad , \tag{2-33}$$

under this condition I can solve the system equations (2-17)-(2-22) and compute the steady state values for the input, state variables. I will denote the steady state values of these variables by using character (o):

$$X_{1o} \equiv Steady.State.value.of.X_1$$
$$u_{1o} \equiv Steady.State.value.of.u_1$$

As a result, the following numerical results are obtained using the Mathematica Program , and Matlab Simulink (A.1):

u_{1o}=T_1=15 ºC X_{1o}=P=1.68301 atom

u_{2o}=T_S=150 ºC X_{2o}=T=114.71 ºC (2-34)

u_{3o}=P_0=1 atom X_{3o}=m_G=65.7711 (unit of mass),

I obtained the same results using both software.

2.2.2 LINEARIZATION USING TAYLOR SERIES [2]

Mathematically, a linear differential equation is one for which the following two properties hold:

1. If x (t) is a solution, then c·x (t) is also a solution, where c is a constant;

2. If x_1 is a solution and x_2 is also a solution, then $x_1 + x_2$ is a solution.

As a result, linearization is realized taking the nonlinear functions, expanding them in Taylor series around the steady state operating point level, and neglecting all terms after the first partial derivatives.

Let us assume I have a nonlinear function $f(x_1, x_2, t)$ of a process, where in our case x_1 is the pressure, x_2 is the temperature.

Remark: I will denote the steady state values with (o) symbol:

$$X_{1o} \equiv Steady.State.value.of.X_1$$
$$X_{2o} \equiv Steady.State.value.of.X_2$$

Now I expand the function $f(x_1, x_2, t)$ around its steady state value:

$$f(x_1, x_2, t) = f(x_{1o}, x_{2o}) + \left(\frac{\partial f}{\partial x_1}\right)\bigg|_{(x1o,x2o)} (x_1 - x_{1o}) + \left(\frac{\partial f}{\partial x_2}\right)\bigg|_{(x1o,x2o)} (x_2 - x_{2o})$$

$$+ \left(\frac{\partial^2 f}{\partial x_1{}^2}\right)\bigg|_{(x1o.x2o)} \frac{(x_1 - x_{1o})^2}{2!} + \ldots \qquad . \tag{2-35}$$

The linearization was carried out in our case by truncating the series after the first partial derivatives.

$$f(x_1, x_2, t) = f(x_{1o}, x_{2o}) + \left(\frac{\partial f}{\partial x_1}\right)\bigg|_{(x1o,x2o)} (x_{1-} x_{1o}) + \left(\frac{\partial f}{\partial x_2}\right)\bigg|_{(x1o,x2o)} (x_{2-} x_{2o}) . \tag{2-36}$$

2.2.3 LINEARIZATION OF THE CONTINUOUS FLOW BOILING SYSTEM

Using the above described procedure, I took the non-linear functions of the boiling system, (2-32), and expanding them in Taylor series expansions, around the steady state operating point, (2-34), the linearized mthematical model can be obtained:

$$\begin{aligned}
\dot{X}_1 &= f_1(X_1, X_2, X_3, u_1, u_2, u_3) \\
\dot{X}_2 &= f_2(X_1, X_2, X_3, u_1, u_2, u_3) \\
\dot{X}_3 &= f_3(X_1, X_2, X_3, u_1, u_2, u_3) \\
Y_1 &= g_1(X_1, X_2, X_3, u_1, u_2, u_3) \\
Y_2 &= g_2(X_1, X_2, X_3, u_1, u_2, u_3) \\
Y_3 &= g_3(X_1, X_2, X_3, u_1, u_2, u_3)
\end{aligned} \tag{2-37}$$

The steady state operating point is:

O.P. = (X_{1o}, X_{2o}, X_{3o}, u_{1o}, u_{20}, u_{30})

= (1.68301, 114.71, 65.7711, 15, 150, 1) . (2-38)

If now I expand the functions f1, f2, f3, f4 around the O.P and knowing that:

$$\dot{x}_i(t) = f_i(x_0, u_0) + \sum_{j=1}^{n} \left(\frac{\partial f_i(x,u)}{\partial x_j} \right)_{x_0, u_0} (x_j - x_{0j}) + \sum_{j=1}^{n} \left(\frac{\partial f_i(x,u)}{\partial u_j} \right)_{x_0, u_0} (u_j - u_{0j}) \quad , \tag{2-39}$$

introducing:

$$\begin{aligned}
\Delta X_1 &= X_1 - X_{1o}; \Delta u_1 = u_1 - u_{1o} \\
\Delta X_2 &= X_2 - X_{2o}; \Delta u_2 - u_2 - u_{2o} \quad , \\
\Delta X3 &= X_3 - X_{3o}; \Delta u_3 = u_3 - u_{3o}
\end{aligned} \tag{2-40}$$

and considering:

$$\dot{X}_i(t) - (f_i)_{o.p} = \Delta \dot{X}_i(t) \quad , \tag{2-41}$$

the formulas for computing the linearized model will be:

$$\Delta \dot{X}_i(t) = \sum_{j=1}^{3} \left(\frac{\partial f_i(x,u)}{\partial x_j} \right)_{(xo,uo)} \Delta x_j + \sum_{j=1}^{n=3} \left(\frac{\partial f_i(x,u)}{\partial u_j} \right)_{(xo,uo)} \Delta u_j$$

$$\Delta Y_i = \sum_{j=1}^{3}\left(\frac{\partial g_i}{\partial x_j}\right)_{(x_0,u_0)} \Delta x_j + \sum_{j=1}^{n=3}\left(\frac{\partial g_i}{\partial u_j}\right)_{(x_0,u_0)} \Delta u_j \quad . \tag{2-42}$$

As result, the state space representation of the linearized model is obtained by Mathematica Program and Matlab Simulink (A.2) in the following form:

$$\dot{\bar{X}} = \begin{bmatrix} -0.17387527237 & -0.00480476838 & 0.00000000137 \\ -2.98043376844 & -0.08235930868 & 0.00000002349 \\ -6.289359099594509 & -0.1737966 & 0 \end{bmatrix} X + \begin{bmatrix} 0.00001721075 & 0.00478756052 & 0.12368185620 \\ 0.00029501235 & 0.08206434591 & 2.12005061487 \\ 0.00062254200 & 0.17317410822 & 4.47378055054 \end{bmatrix} u$$

$$\bar{Y} = \begin{bmatrix} 6.2894 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} X + \begin{bmatrix} 0 & 0 & -4.4738 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} u \text{..} (2-43)$$

Where: $\bar{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} P \\ T \\ m_G \end{bmatrix}$, $\bar{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_S \\ P_0 \end{bmatrix}$, and $\bar{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} v_E \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} v_E \\ T \\ m_G \end{bmatrix}$. (2-44)

Again, I obtained the same results using both software programs.

2.3 SIMULATION OF THE LINEAR MODEL

The Simulink Model for the continuous flow boiling system was realized in Matlab (Fig.2-8) and involves three inputs and three outputs. Considering the signals u_1, u_2 and u_3 as inputs, the individual step-response of the three outputs Y_1 , Y_2, and Y3 can be plotted (Fig.2-9a, Fig.2-9b, and Fig. 2-9c) using the Matlab Program (A.3).

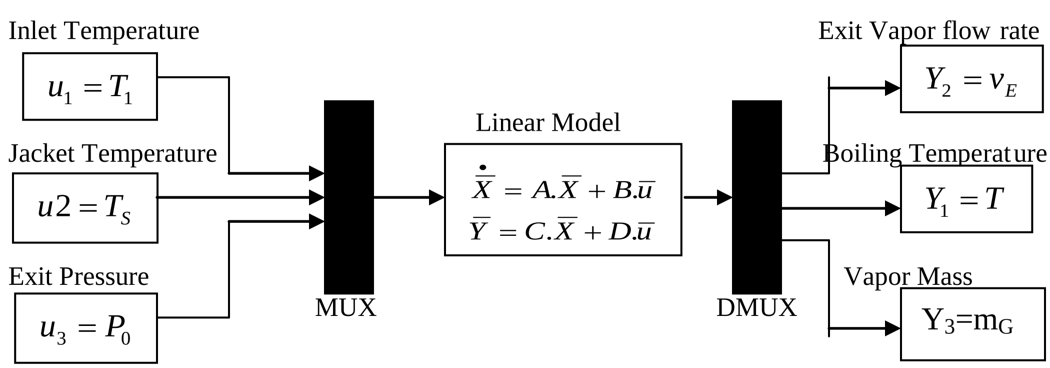


Fig. 2-8. The Simulink Model of the Linear Model.

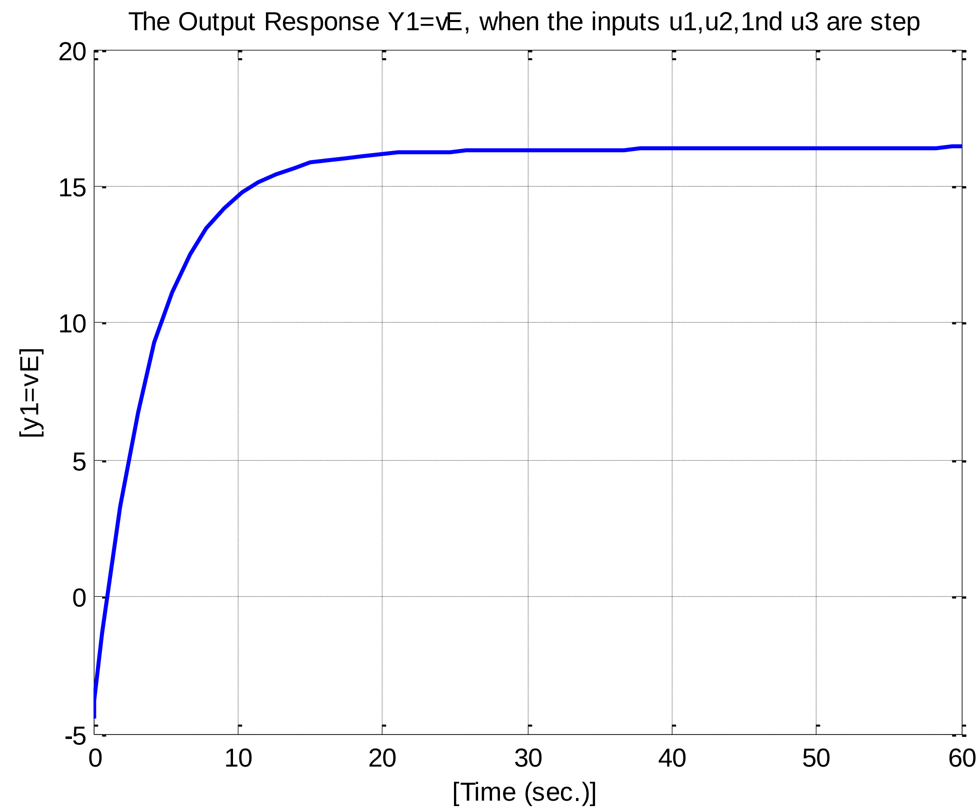


Fig. 2-9.a. The Output Step-Response Y_1=vE, when the Inputs are u_1, u2 and u_3.

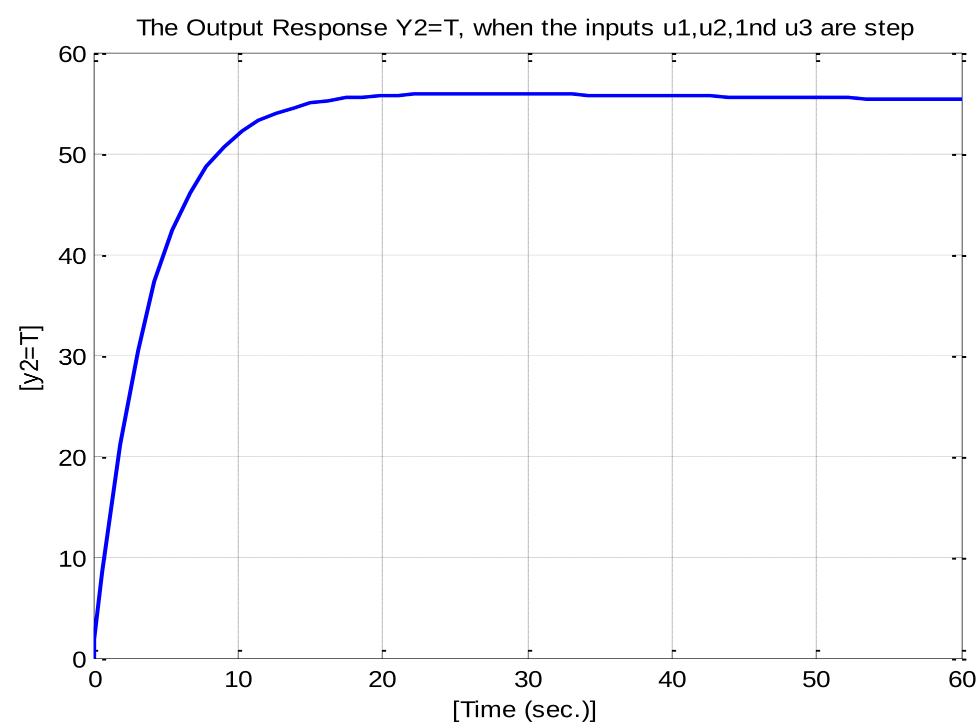


Fig. 2-9.b. The Output Step-Response $Y_2 = T$ when the Inputs are u_1, u2 and u_3.

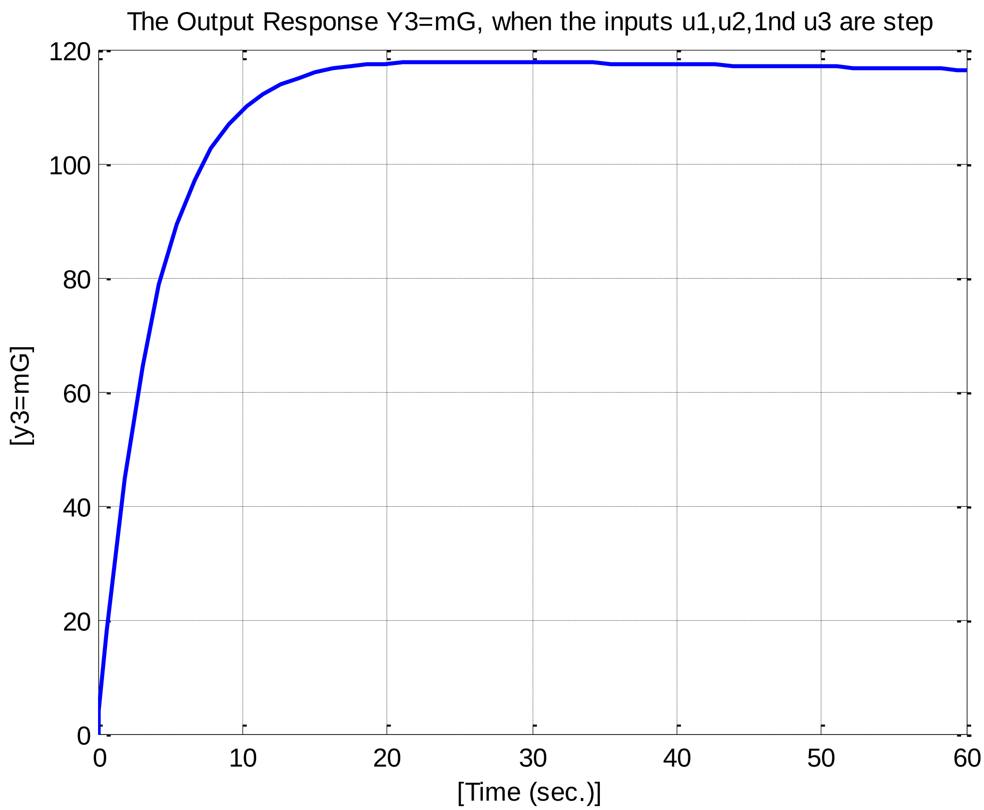


Fig. 2-9.c. The Output Step-Response $Y_3 = m_G$ when the Inputs are u_1, u2 and u_3.

2.4.1 MATRIX TRANSFER FUNCTION OF THE MIMO BOILING SYSTEM

The P-canonical form can be applied for every multivariable system (Fig.2-10). The matrix transfer function in this case (number of the inputs=number of the outputs) is given by:

$$\bar{Y}(s) = H(s)\bar{u}(s) \quad \text{, where: } H(s) = [H_{ij}], \quad \text{where: i, j=1… n} \qquad (2\text{-}45)$$

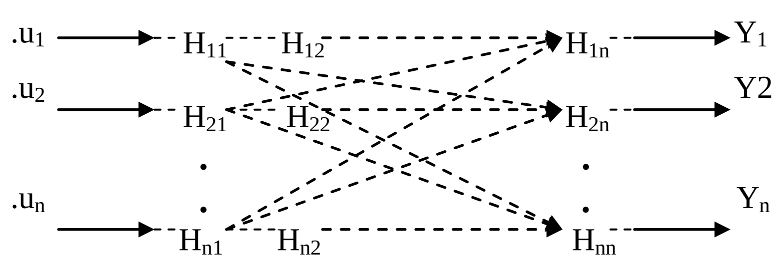


Fig. 2-10. Canonical form of a general multivariable system.

In my case the dependency between the outputs and the inputs of the boiling system is described by relation (2-46) and Fig.2-11.:

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \\ Y_3(s) \end{bmatrix} = \begin{bmatrix} H_{11}(s) & H_{12}(s) & H_{13}(s) \\ H_{21}(s) & H_{22}(s) & H_{23}(s) \\ H_{31}(s) & H_{32}(s) & H_{33}(s) \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \\ u_3(s) \end{bmatrix} \tag{2-46}$$

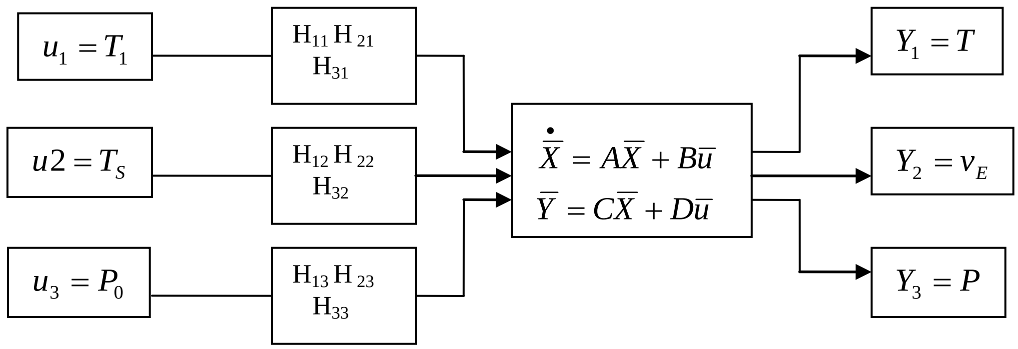


Fig. 2-11. The MIMO Boiling system.

I can compute the transfer function matrix H(s) using the Matlab Command as:

$$[num, den] = ss2tf(A, B, C, D, 1);$$

$$H11 = \frac{y1}{u1} = \frac{0.1082 * 10^{-3}}{s + 0.2562}; \quad H21 = \frac{y2}{u1} = \frac{0.295 * 10^{-3}}{s + 0.2562}; \quad H31 = \frac{y3}{u1} = \frac{0.6225 * 10^{-3}}{s + 0.2562};$$

$$[num, den] = ss2tf(A, B, C, D, 2);$$

$$H12 = \frac{y1}{u2} = \frac{0.0301}{s + 0.2562}; \quad H22 = \frac{y2}{u2} = \frac{0.0821}{s + 0.2562}; \quad H32 = \frac{y3}{u2} = \frac{0.1732}{s + 0.2562};$$

$$[num, den] = ss2tf(A, B, C, D, 3);$$

$$H13 = \frac{y1}{u3} = \frac{-4.4738s - 0.3685}{s + 0.2562}; \quad H23 = \frac{y2}{u3} = \frac{2.1201}{s + 0.2562}; \quad H33 = \frac{y3}{u3} = \frac{4.4738}{s + 0.2562};$$

(2-47)

As result, I obtained the following transfer function matrix:

$$H(s) = \begin{bmatrix} \dfrac{0.1082*10^{-3}}{s+0.2562} & \dfrac{0.0301}{s+0.2562} & \dfrac{-4.4738s-0.3685}{s+0.2562} \\ \dfrac{0.295*10^{-3}}{s+0.2562} & \dfrac{0.0821}{s+0.2562} & \dfrac{2.1201}{s+0.2562} \\ \dfrac{0.6225*10^{-3}}{s+0.2562} & \dfrac{0.1732}{s+0.2562} & \dfrac{4.4738}{s+0.2562} \end{bmatrix} \tag{2-48}$$

SECTION 3

ANALYSIS OF THE CONTINUOUS FLOW BOILING SYSTEM

3.1 MIMO CONTROLLER, OBSERVER, AND GILBERT REALIZATIONS

The transfer function matrix of the MIMO Boiling system in physically motivated form is obtained from (2-48) as follow:

$$H(s) = \begin{bmatrix} \frac{0.1082*10^{-3}}{s+0.2562} & \frac{0.0301}{s+0.2562} & \frac{-4.4738s-0.3685}{s+0.2562} \\ \frac{0.295*10^{-3}}{s+0.2562} & \frac{0.0821}{s+0.2562} & \frac{2.1201}{s+0.2562} \\ \frac{0.6225*10^{-3}}{s+0.2562} & \frac{0.1732}{s+0.2562} & \frac{4.4738}{s+0.2562} \end{bmatrix} \quad (3\text{-}1)$$

3.1.1 THE CONTROLLER FORM (CCF)

The steps to obtain the controller form are:

- The denominators of the columns are already the same
- Rewrite the transfer matrix function as:

$$H(s) = \begin{bmatrix} \frac{0.1082*10^{-3}}{s+0.2562} & \frac{0.0301}{s+0.2562} & \frac{0.7768}{s+0.2562} \\ \frac{0.295*10^{-3}}{s+0.2562} & \frac{0.0821}{s+0.2562} & \frac{2.1201}{s+0.2562} \\ \frac{0.6225*10^{-3}}{s+0.2562} & \frac{0.1732}{s+0.2562} & \frac{4.4738}{s+0.2562} \end{bmatrix} + \begin{bmatrix} 0 & 0 & -4.4738 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (3\text{-}2)$$

- By inspection, the controller form:

$$\dot{X} = \begin{bmatrix} -0.2562 & 0 & 0 \\ 0 & -0.2562 & 0 \\ 0 & 0 & -0.2562 \end{bmatrix} X + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0.1082*10^{-3} & 0.0301 & 0.7768 \\ 0.295*10^{-3} & 0.0821 & 2.1201 \\ 0.6225*10^{-3} & 0.1732 & 4.4738 \end{bmatrix} X + \begin{bmatrix} 0 & 0 & -4.4738 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} u \qquad (3\text{-}3)$$

I proved this correct by working backwards to find the original transfer matrix

$$H(s) = C(sI - A)^{-1} B + D$$

*Matlab command: H=C*inv (s*eye (3, 3)-A)*B+D;* I got the same original T.F.M.

3.1.2 THE OBSERVER FORM (OCF)

Similarly, the observer form of the Boiling system:

$$\dot{X} = \begin{bmatrix} -0.2562 & 0 & 0 \\ 0 & -0.2562 & 0 \\ 0 & 0 & -0.2562 \end{bmatrix} X + \begin{bmatrix} 0.1082*10^{-3} & 0.0301 & 0.7768 \\ 0.295*10^{-3} & 0.0821 & 2.1201 \\ 0.6225*10^{-3} & 0.1732 & 4.4738 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} X + \begin{bmatrix} 0 & 0 & -4.4738 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} u \qquad 3\text{-}4)$$

*Matlab command: H=C*inv (s*eye (3, 3)-A)*B+D;* I got the same original T.F.M.

3.1.3 THE GILBERT FORM (DCF)

In this case the diagonal form is the same as the controller form.

3.2 CONTROLLABILITY AND OBSERVABILITY

3.2.1 CONTROLLABILITY

Theorem: The following are equivalent [3]:

1) System (A, B) is controllable.
2) W_c^{-1} Exists.
3) Rank $(B, AB, A^2B, ..., A^{n-1}B) = n$
4) Rank $[A - \lambda I), B] = n$ (3-5)

For the MIMO Boiling system:

$$C(A,B) = [B, AB, A^2B] = \begin{bmatrix} 0 & 0.004 & 0.123 & 0 & -0.001 & -0.031 & 0 & 0.000 & 0.008 \\ 0.0003 & 0.082 & 2.120 & 0 & -0.021 & -0.543 & 0 & 0.005 & 0.139 \\ 0.0006 & 0.173 & 4.473 & 0.0001 & -0.044 & -1.146 & 0 & 0.011 & 0.293 \end{bmatrix}$$

$Rank(C(A,B)) = 3$, therefore the system is controllable.

3.2.2 OBSERVABILITY

Theorem: The following are equivalent [3]:

1) System (A, C) is observable.
2) W_O^{-1} Exists.
3) $O(A,C) = [C; CA; C^2A; \ldots\ldots\ldots\ldots C^{n-1}A]$ is invertible.
4) Rank $\begin{bmatrix} \lambda I - A \\ B \end{bmatrix} = n$ (3-6)

For the MIMO Boiling system:

$$O(A,C) = \begin{bmatrix} 6.2894 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1.0937 & -0.0302 & 0 \\ -2.9804 & -0.0824 & 0 \\ -6.2894 & -0.1738 & 0 \\ -6.8789 & -0.1899 & 0 \\ -2.9804 & -0.0824 & 0 \\ -6.2894 & -0.1738 & 0 \end{bmatrix}; Rank(A,C) = 3 \Rightarrow \text{ The system is observable.}$$

Since the Diagonal form is controllable and observable, then the diagonal form is minimal.

3.3 STABILITY

Theorem: The system is stable if all eigenvalues of A matrix have non-positive real parts [5]. In case of boiling system:
$EigenvaluesOf(A) = eig(A) = 0, -0.001, -0.2562 \Rightarrow$ All eigenvalues have non-positive real parts $\Rightarrow$ the system is stable.

3.3.1 BIBO STABILITY

The system is BIBO stable if each entry of H(s) has its poles in Re(s) <0 (e.g. open left half plane).In case of Boiling system, the poles of each entry of H(s) are at P=-0.2562 then, the system is BIBO stable.

3.3.2 INTERNAL STABILITY

The system is internally stable iff eig (A) are in (O.L.H.P) ($\lambda_i < 0$).Apply this theorem to the original system (physical motivated form), the eigenvalues of A matrix: $EigenvaluesOf(A) = eig(A) = 0, -0.001, -0.2562 \Rightarrow \lambda 1 = 0 \Rightarrow$ the physical form is not internally stable. In the other hand, since the diagonal form is controllable and observable then $BIBO \Leftrightarrow Internall$ stable .The diagonal form is BIBO stable $\Rightarrow$ the diagonal form is internally stable.

3.4 FREQUENCY RESPONSE

The figure 3-1 and figure 3-2 show the properties of frequency response of the system using Bode plot and Nyquist plot:

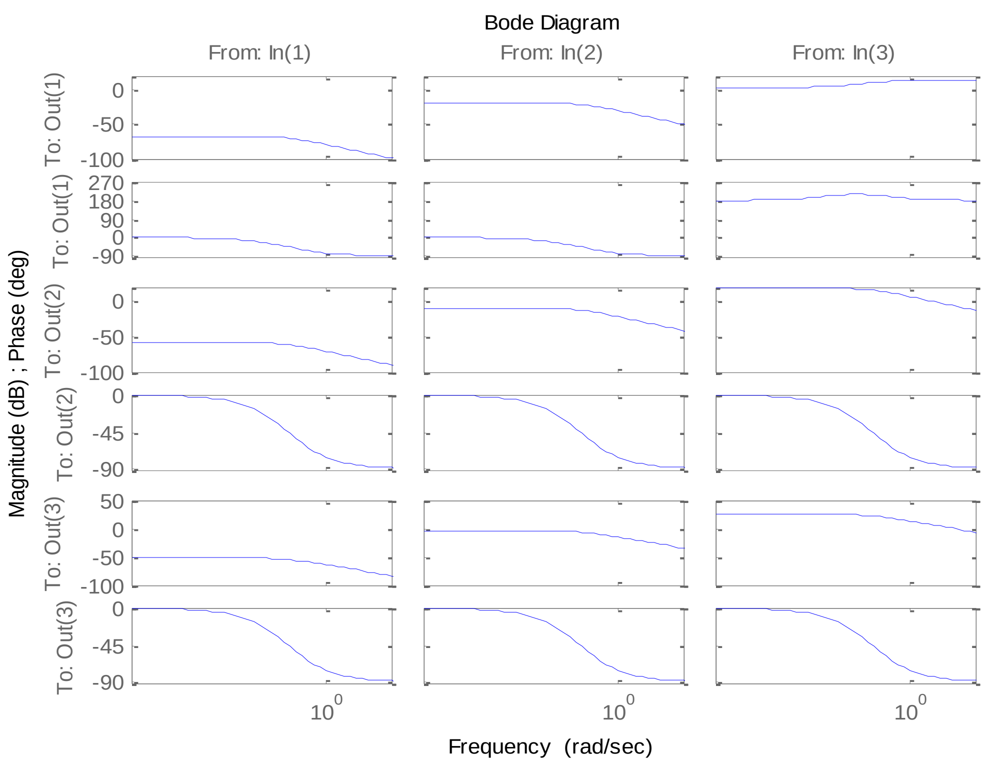


Fig. 3-1 Bode diagram of MIMO Boiling system

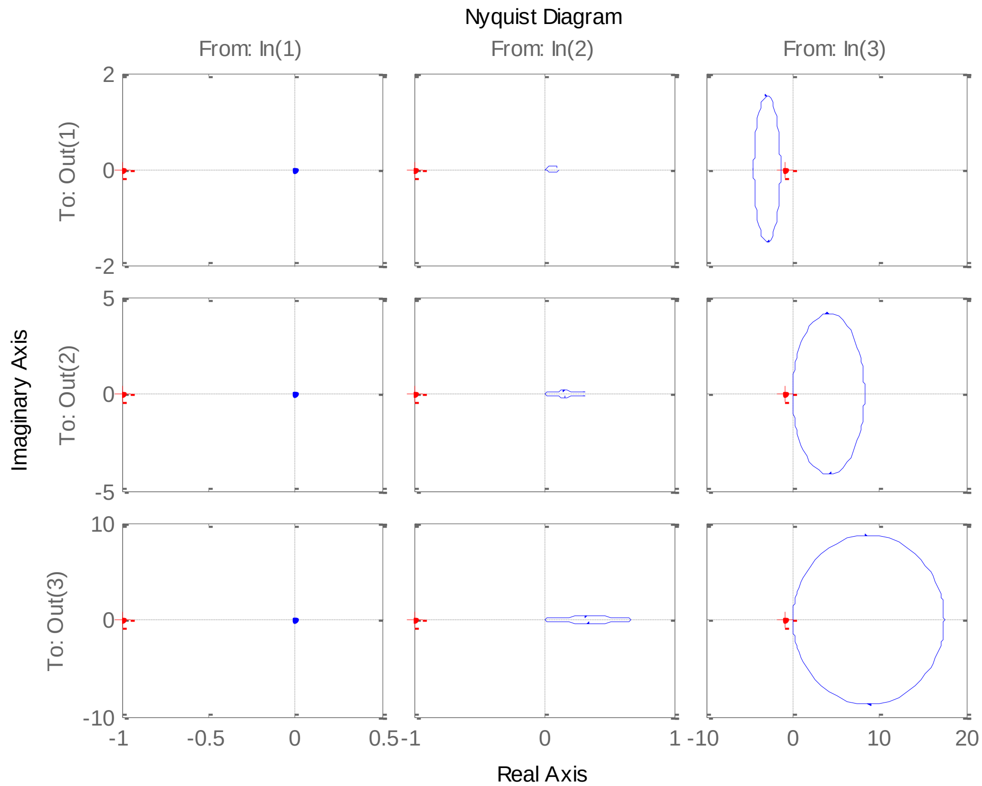


Fig. 3-1 Nyquist diagram of MIMO Boiling system

SECTION 4

CONTROL OF THE CONTINUOUS FLOW BOILING SYSTEM

4.1 CONTROL OF A CONTINUOUS FLOW BOILING SYSTEM IN SINGLE INPUT SINGLE OUTPUT (SISO) APPROACH

For the SISO system approach, I considered as input variable the inlet temperature (u_1=T_1); and as output variable the boiling temperature ($Y = X_2 = T$) (Fig. 4-1).

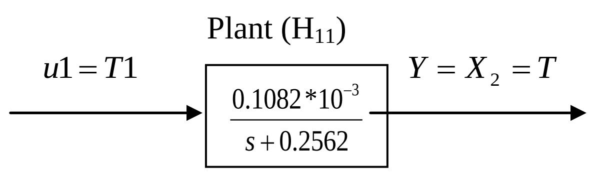


Fig. 4-1. SISO system approach of the continuous flow boiling system and its transfer function.

The problem of the measurement unit and the actuator is very important, because they make the connection and the analog-digital conversion between the plant and the controller (Fig. 4-2).

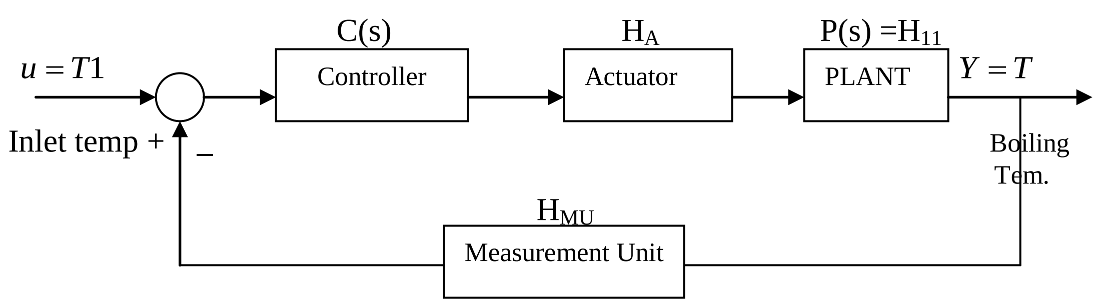


Fig. 4-2. Block Diagram of the SISO system approach.

I considered for the Measurement Unit and Actuator the following transfer functions:

$H_{MU} = 1$ (unity feedback) ,

$H_A = A$ (constant(I have chosen A=1)) . (4-1)

4.1.1 CONTROLLER DESIGN USING ERROR FEEDBACK

The steps of design a controller for the SISO Boiling system using error feedback are:

- I have chosen H_{11} transfer function for the SISO plant :

$$P(s) = H_{11}(s) = \frac{0.1082 * 10^{-3}}{s + 0.2562} = \frac{n_p}{d_p} \tag{4-2}$$

- I considered the following transfer function for a controller:

$$C(s) = \frac{\beta_1 s + \beta_0}{\alpha_1 s + \alpha_0} = \frac{n_c}{d_c} \tag{4-3}$$

- Determine the closed loop characteristic equation as:

$$\Delta(s) = nc.np + dc.dp = \alpha_1 s^2 + (0.2562\alpha_1 + \alpha_0 + 0.1082 * 10^{-3}\beta_1)s + (0.2562\alpha_0 + 0.1082 * 10^{-3}\beta_0)$$

$$\Delta_d(s) = \delta_2 s^2 + \delta s_1 + \delta_0 = \alpha_1 s^2 + (0.2562\alpha_1 + \alpha_0 + 0.1082 * 10^{-3}\beta_1)s + (0.2562\alpha_0 + 0.1082 * 10^{-3}\beta_0) \tag{4-4}$$

- Equating the both sides in the last equation I got :

$$\begin{bmatrix} \delta_2 \\ \delta_1 \\ \delta_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.2562 & 1 & 0.1082 * 10^{-3} & 0 \\ 0 & 0.2562 & 0 & 0.1082 * 10^{-3} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_0 \\ \beta_1 \\ \beta_0 \end{bmatrix} \Rightarrow \underline{\delta} = A\underline{X} \tag{4-5}$$

- Suppose I want all the poles at P_d= -1 $\Rightarrow \Delta_d(s) = (s+1)^2 = s^2 + 2s + 1$(the desired characteristic equation).

- Solving this equation ($\underline{\delta} = A\underline{X}$) $\Rightarrow \underline{X} = pinv(A).\underline{\delta} = 10^3 * \begin{bmatrix} 0.0010 \\ 0.0019 \\ -1.2293 \\ 4.7982 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_0 \\ \beta_1 \\ \beta_0 \end{bmatrix}$. (4-6)

- Then, the resulting controller transfer function is:

$$C(s) = \frac{\beta_1 s + \beta_0}{\alpha_1 s + \alpha_0} = \frac{-1.2293s + 4.7982}{0.001s + 0.0019}. \tag{4-7}$$

  - I repeated the same steps for different pole locations. As result , I obtained the following:

- For P_d= -15 $\Rightarrow \Delta_d(s) = (s+15)^2 = s^2 + 30s + 225 \Rightarrow$ the resulting controller: $C(s) = \dfrac{\beta_1 s + \beta_0}{\alpha_1 s + \alpha_0} = \dfrac{s - 3.9032}{-2.03*10^{-3} s - 1.38*10^{-4}}$ (4-8)

- For P_d= -20 $\Rightarrow \Delta_d(s) = (s+20)^2 = s^2 + 40s + 400$

  $\Rightarrow$ the resulting controller:

$$C2(s) = \frac{\beta_1 s + \beta_0}{\alpha_1 s + \alpha_0} = \frac{s - 3.9032}{-1.15*10^{-6} s - 1.54*10^{-4}} \quad (4\text{-}9)$$

➢ Applying the transfer functions of the resulting controllers, and Plant, as in Fig.4-3.

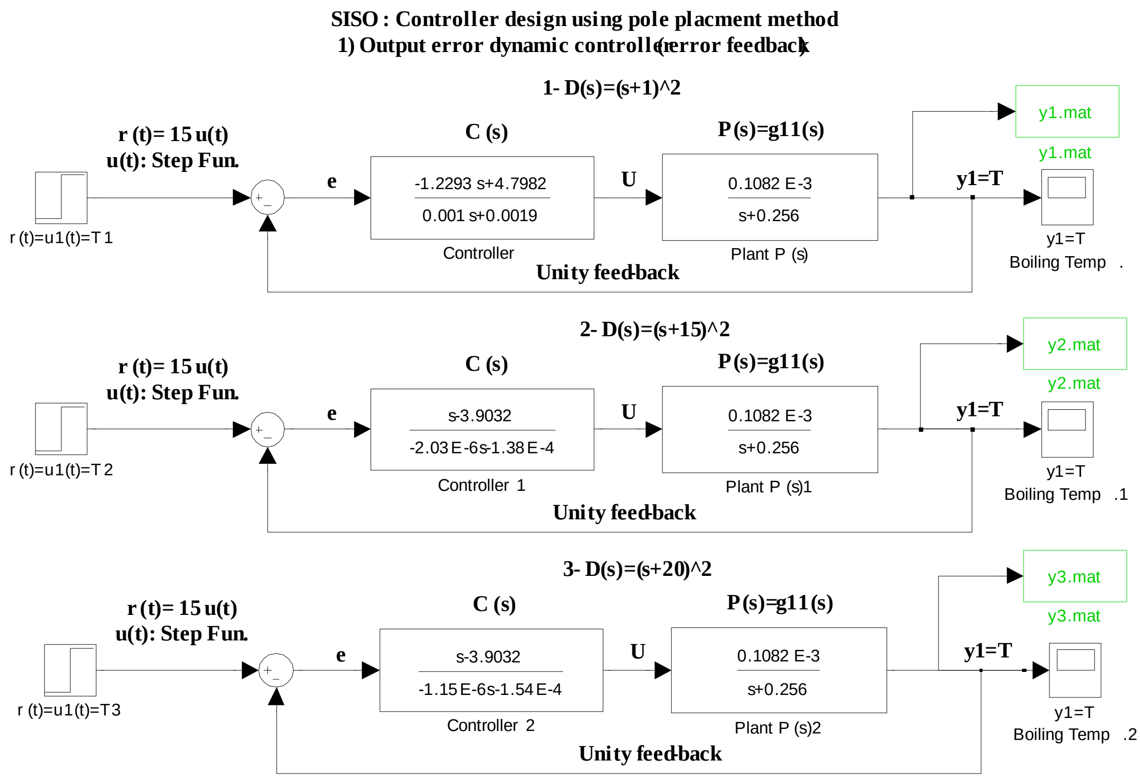
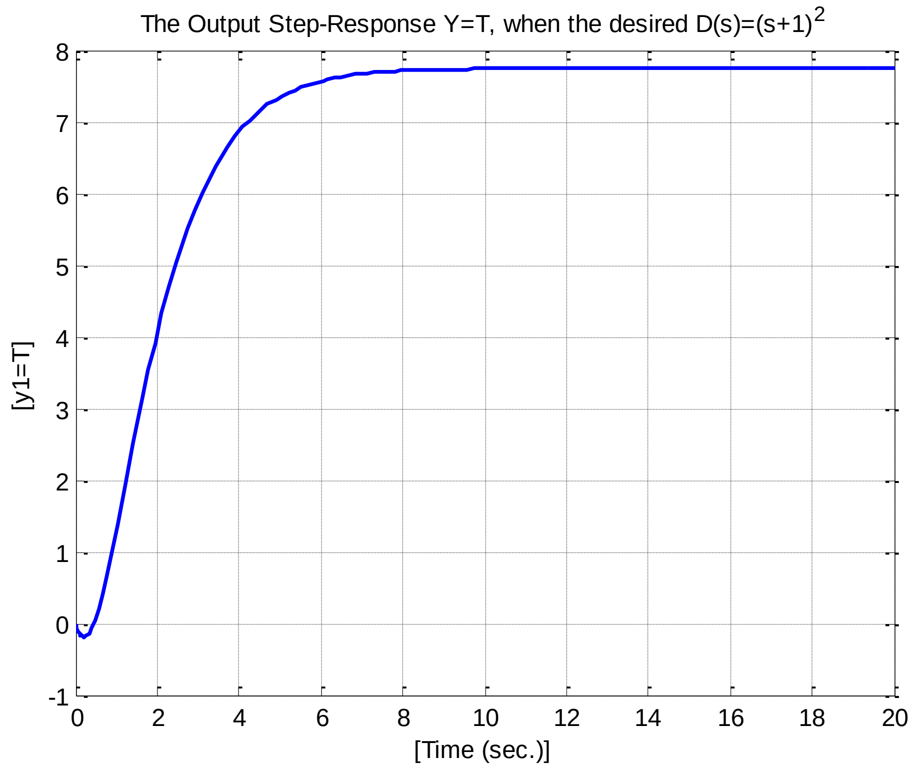
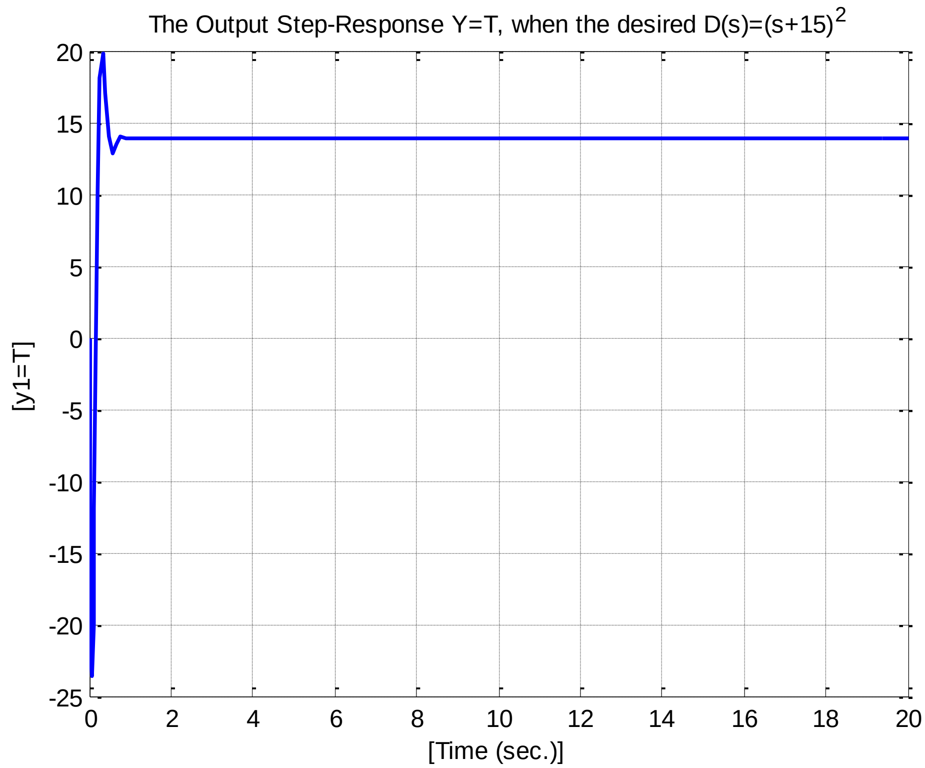


Fig.4-3.the Simulink diagram for a controller design using error feedback pole placement

➢ The output step response for the Boiling temperature (T), when I considered a desired temperature (T_1 is step function from 0 to15 C^o) is shown in the figure4-4 a,b,c.





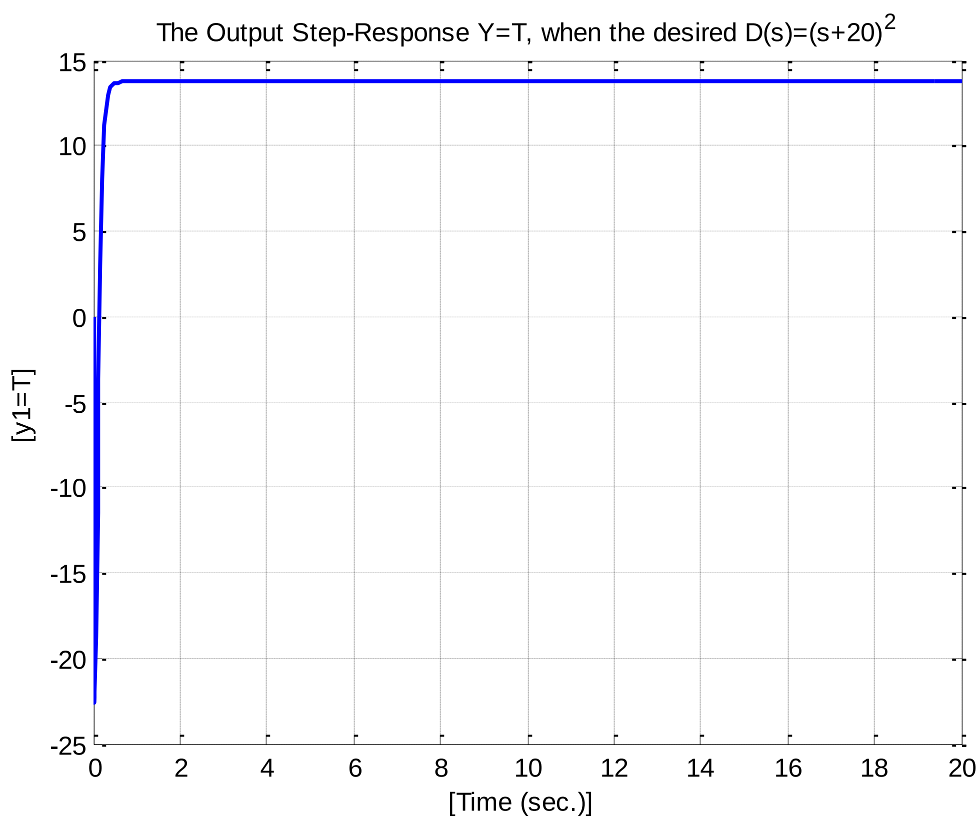


Fig. 4-4 a, b, and c the output response for different desired D(s)

- As it can seen the Boiling Temperature (Y=T) near to reach the settling point ($T_= 15\ ^oC$) when I place the poles far away to the left from the plant poles (last figure, the desired poles where at P=-20).

4.1.2 CONTROLLER DESIGN USING STATE FEEDBACK

The steps of design a controller for the SISO Boiling system using state feedback are:

- Transfer the SISO plant transfer function to the state space description as following:

I have chosen P(s): $P(s) = H_{11}(s) = \dfrac{0.1082*10^{-3}}{s+0.2562}$ .By inspection, the state space representation in controller form is:

$$\dot{X} = -0.256X + u$$
$$y = 0.1082*10^{-3}X \qquad (4\text{-}10)$$

- Let $u = r - KX \Rightarrow \dot{X} = (A - BK)X + B.r$ (4-11)

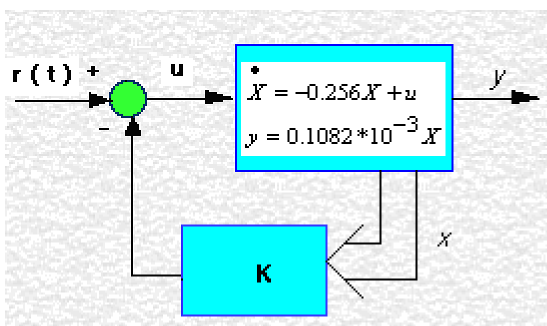


Fig.4-5 State feedback for SISO Boiling system

- By using state feedback I can place the poles anywhere I want. In general, the farther you move the poles, the more control effort it takes.

- Recall that the characteristic polynomial for this closed-loop system is the determinant of (sI-(A-BK)).Suppose I want my desired poles at P_d= -5.12 (twenty times of the plant pole),

$$\Rightarrow (sI - (A - BK)) = s - (-0.256 - K) = s + (0.256 + K) = s + 5.12$$
$$\Rightarrow 0.256 + K = 5.12 \Rightarrow K = 4.864 \qquad (4\text{-}12)$$

I got the same result using the Matlab function (place or acker).

$$K = \text{place}\ (A, B, P_d); \qquad (4\text{-}13)$$

➢ Applying the S.S. equations, and K as in this Fig.

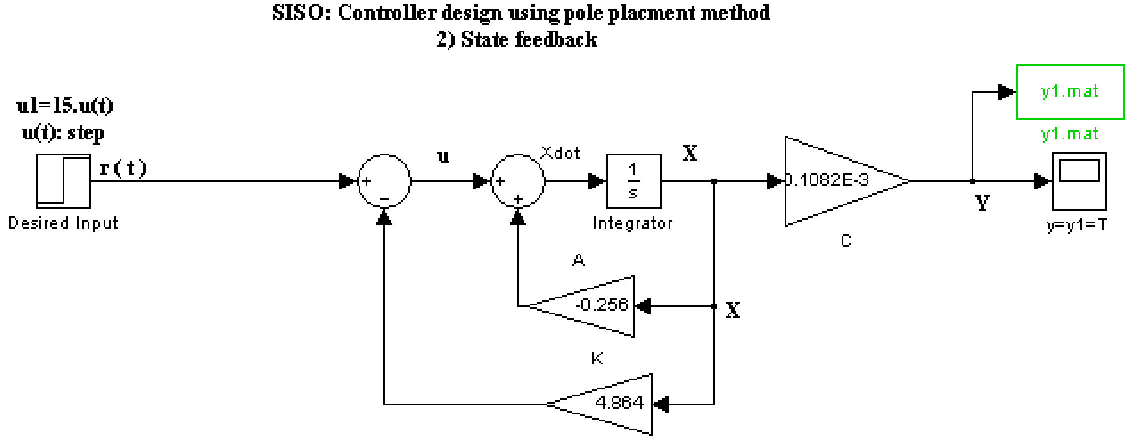


Fig. 4-6 Simulink model of the state feedback for SISO Boiling system without tracking

➢ The output step response for the Boiling temperature (T), when I considered a desired temperature (T_1 is step function from 0 to 15 C^o) is shown in the figure 4-7.

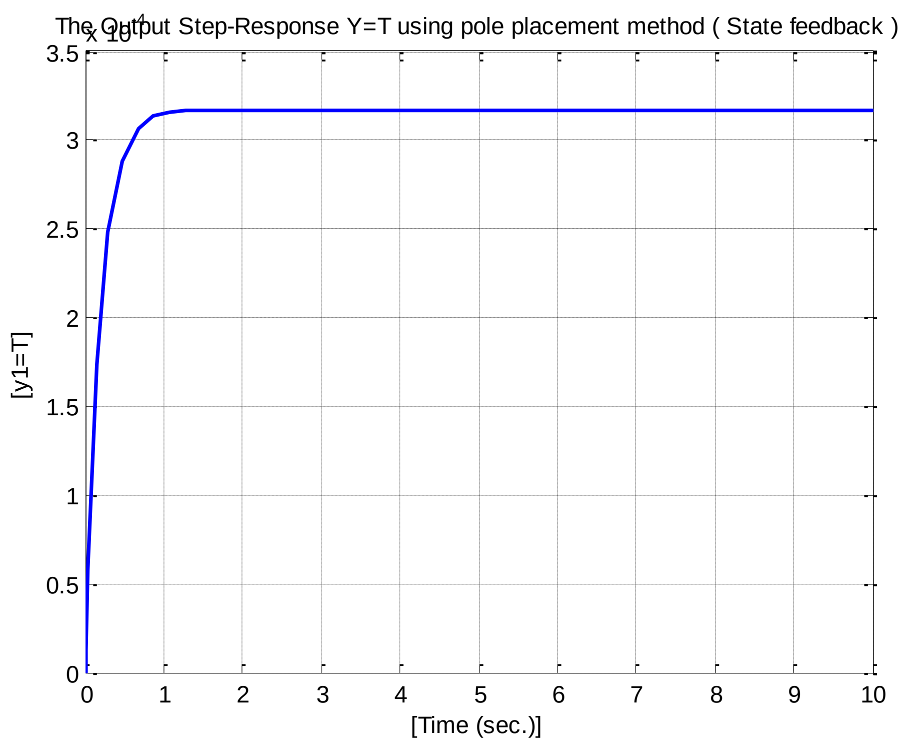


Fig. 4-7 the output response using state feedback without tracking

- As it can seen the system does not track the step well at all; the magnitude not 15, the Boiling Temperature (Y=T) has not reached the settling point ($T_{=}15\ ^{o}C$), that is why I need to track the reference to get the desired value.
- Recall the schematic above, I don't compare the output to the reference; There is no reason to expect that K*x will be equal to the desired output. To eliminate this problem, I scaled the reference input to make it equal to K*x_steadystate. I called this scale factor Nbar as shown in figure 4-8.

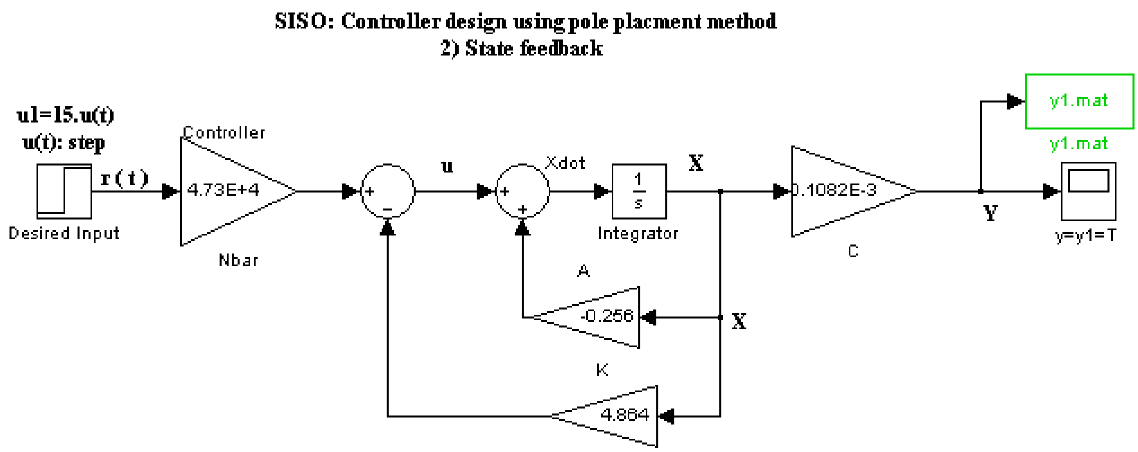


Fig. 4-8 Simulink model of the state feedback for SISO Boiling system with tracking

I got Nbar from Matlab by using the function rscale (A.4),

Nbar=rscale (A, B, C, 0, K).As result, Nbar=4.73E+4. (4-14)

- The output step response for the Boiling temperature (T), when I considered a desired temperature (T_1 is step function from 0 to 15 C^o) is shown in the figure 4-9.

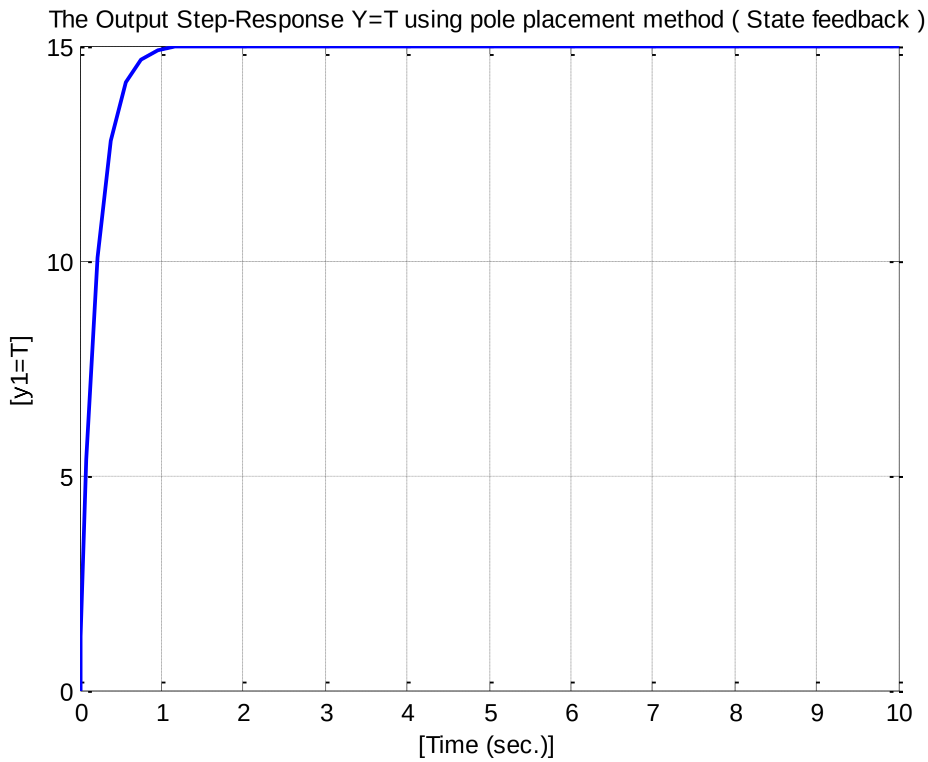


Fig. 4-9 The output response using state feedback with tracking

- As it can seen the Boiling Temperature (Y=T) has reached the settling point (T=15 ^{o}C). Now a step can be tracked reasonably well.

4.1.3 CONTROLLER DESIGN USING OUTPUT FEEDBACK

The steps of design a controller for the SISO Boiling system using output feedback are:

- The state space representation in controller form is:

$$\dot{X} = -0.256X + u$$
$$y = 0.1082 * 10^{-3} X \tag{4-15}$$

- Let $u = r - Ky \Rightarrow \dot{X} = (A - BKC)X + B.r$ (4-16)

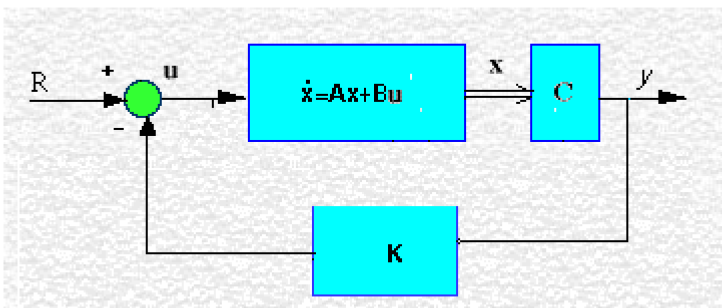


Fig.4-10 Output feedback for SISO Boiling system

- By using output feedback figure 4-10, I can place the poles anywhere I want. In general, the farther you move the poles, the more control effort it takes.

- Recall that the characteristic polynomial for this closed-loop system is the determinant of (sI-(A-BKC)).Suppose I want my desired poles at P_d= 25,

$$\Rightarrow (sI-(A-BKC)) = s-(-0.2562-0.1082*10^{-3}K) = s+(0.2562+0.1082*10^{-3}K) = s+5.12$$
$$\Rightarrow 0.2562+0.1082*10^{-3}K = 5.12 \Rightarrow K = 2.2869*10^{5}$$

I got the same result using the Matlab function (place or acker).

$$K = \text{place}\ (A, C, P_d); \tag{4-17}$$

- Applying the S.S. equations, and K as in figure 4-11.

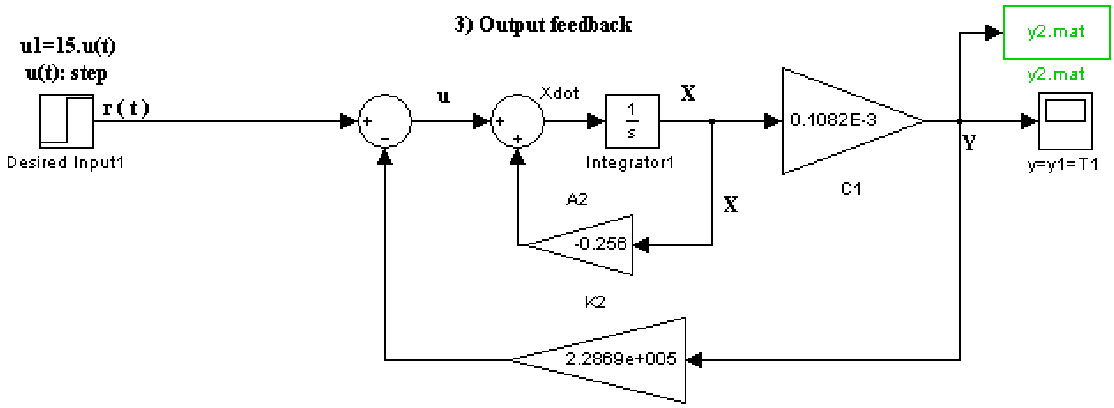


Fig. 4-11 Simulink model of the state feedback for SISO Boiling system without tracking

- The output step response for the Boiling temperature (T), when I considered a desired temperature (T_1 is step function from 0 to 15 C^o) is shown in the following Fig.

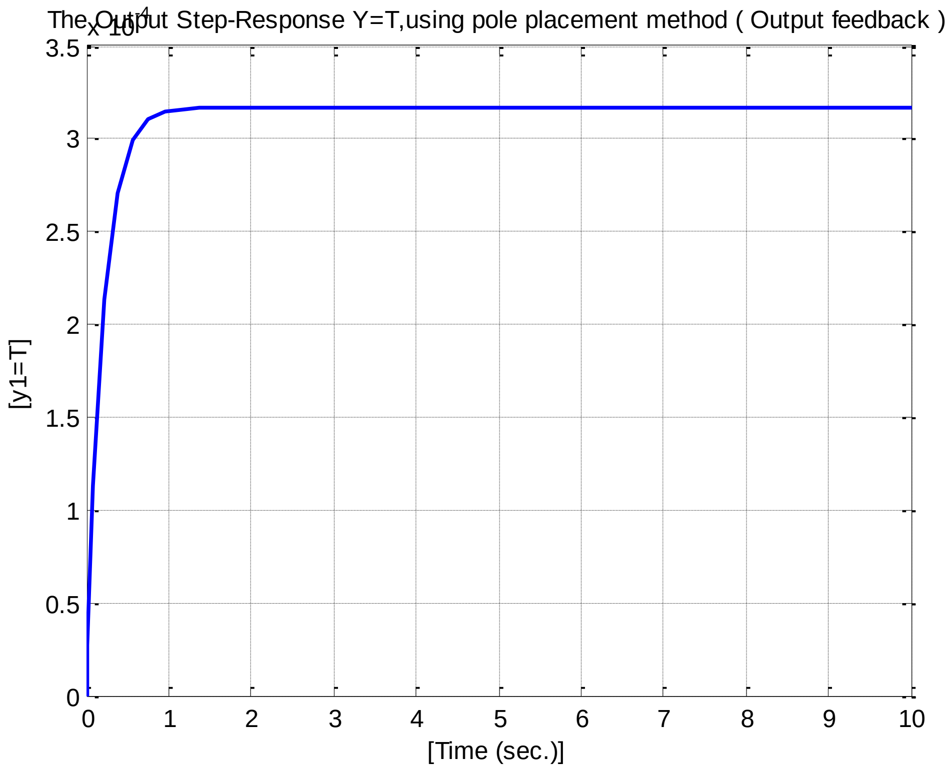


Fig. 4-12 The output response using output feedback without tracking

- As it can seen the system does not track the step well at all; the magnitude not 15, the Boiling Temperature (Y=T) has not reached the settling point (T=15 °C), that is why we need to track the reference to get the desired value.
- As before I scaled the reference input to make it equal to steady state figure 4-13. I called this scale factor Nbar. Nbar=rscale (A, B, C, 0, K).As result, Nbar = $2.11*10^9$

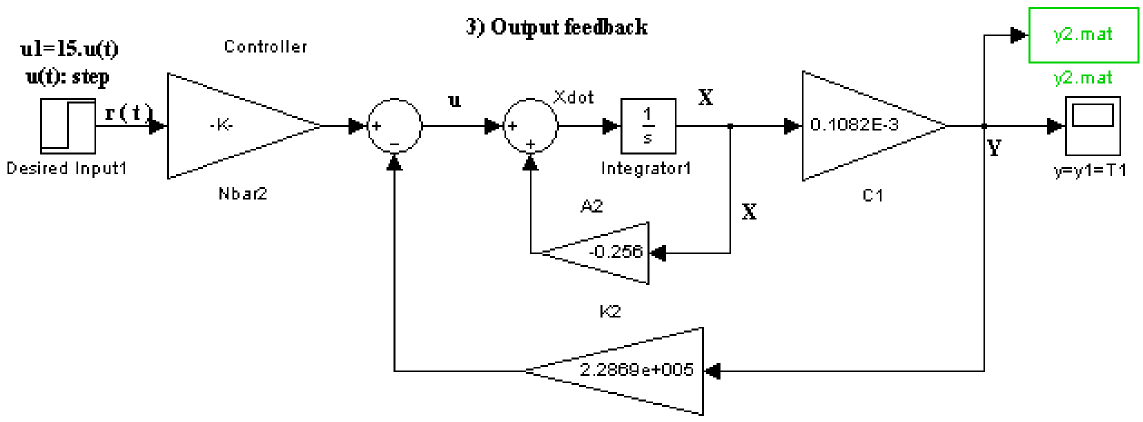


Fig. 4-13 Simulink model of the state feedback for SISO Boiling system with tracking

- The output step response for the Boiling temperature (T), when I considered a desired temperature (T_1 is step function from 0 to 15 C^o) is shown in the following Fig.

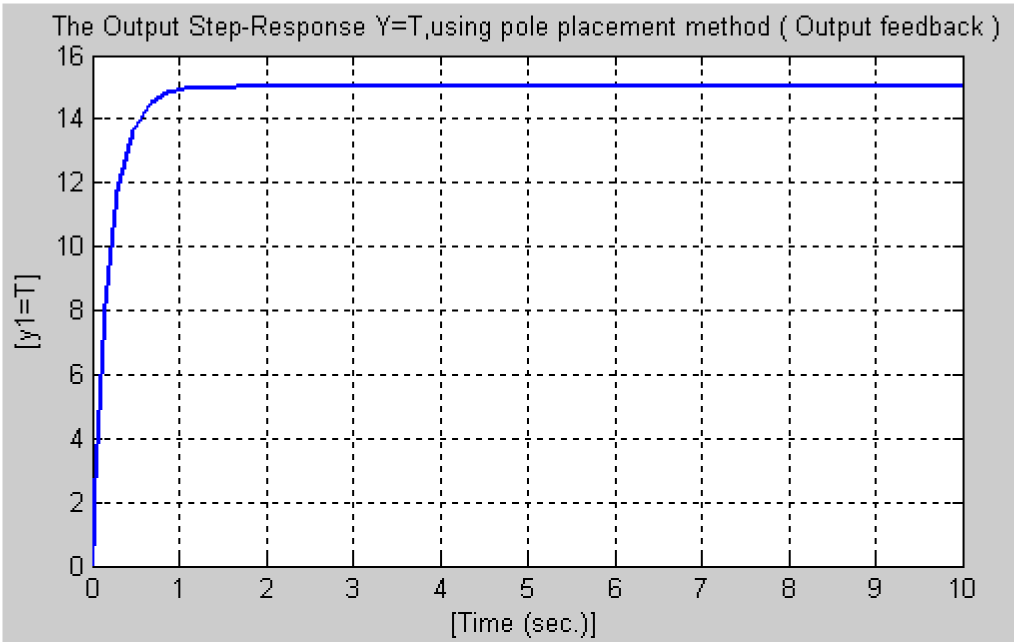


Fig. 4-14 The output response using output feedback with tracking

- As it can seen the Boiling Temperature (Y=T) has reached the settling point ($T_{=}15$ ^oC). Now a step can be tracked reasonably well.

4.1.4 USING THE SYLVESTER MATRIX TECHNIQUE TO DESIGN AN ARBITRARY POLE PLACMENT CONTROLLER THAT CAN ALSO TRACK A SINUSOID

In this part, I will use the Sylvester matrix technique to design an arbitrary pole placement controller that can also track a sinusoid with zero steady-state error

- I have chosen P(s): $P(s) = H_{11}(s) = \dfrac{0.1082 * 10^{-3}}{s + 0.2562} = \dfrac{np}{dp}$, plant order (n=1). (4-18)

$$R = \frac{nr}{dr} = \frac{\omega^2}{s^2 + \omega^2} \qquad (4\text{-}19)$$

- $P \geq n - 1$, where P is the controller order. $P \geq 0 \Rightarrow dc = \alpha 0$

$$C(s) = \frac{nc}{drdc} = \frac{\beta_2 s^2 + \beta_1 s + \beta_0}{(s^2 + \omega^2)\alpha_0} \qquad (4\text{-}20)$$

- The characteristic equation becomes :

$$\Delta(s) = ncnp + dcdpdr$$
$$\Delta(s) = 0.1082 * 10^{-3}(\beta_2 s^2 + \beta_1 s + \beta_0) + \alpha_0 (s^2 + \omega^2)^2 (s + 0.2562)$$

$$\Delta(s) = \alpha_0 s^5 + 0.2562\alpha_0 s^4 + 2\omega^2 \alpha_0 s^3 + (0.5124\omega^2 \alpha_0 + 0.1082 * 10^{-3} \beta_2) s^2 +$$
$$+ (\omega^4 \alpha_0 + 0.1082 * 10^{-3} \beta_1) s + (0.1082 * 10^{-3} \beta_0 + 0.2562\omega^4 \alpha_0)$$
$$\Delta(s) = \delta_5 s^5 + \delta_4 s^4 + \delta_3 s^3 + \delta_2 s^2 + \delta_1 s + \delta_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4-21)$$

$$\begin{bmatrix} \delta_5 \\ \delta_4 \\ \delta_3 \\ \delta_2 \\ \delta_1 \\ \delta_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.2562 & 0 & 0 & 0 \\ 2\omega^2 & 0 & 0 & 0 \\ 0.5124\omega^2 & 0.1082 * 10^{-3} & 0 & 0 \\ \omega^4 & 0 & 0.1802 * 10^{-3} & 0 \\ 0.2562\omega^4 & 0 & 0 & 0.1802 * 10^{-3} \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \beta_2 \\ \beta_1 \\ \beta_0 \end{bmatrix} = \underset{-}{\delta} = A \underset{-}{X}$$

(4-22)

- Suppose I want all the poles at P_d= - 1 , and $\omega = 1 rad/\sec$

$\Rightarrow \Delta_d(s) = (s+1)^5 = s^5 + 5s^4 + 10s^3 + 10s^2 + 5s + 1$(the desired characteristic equation).

➢ Solving this equation ($\underline{\delta} = A\,\underline{X}$) $\Rightarrow$ $\underline{X} = pinv(A).\underline{\delta} = 10^4 * \begin{bmatrix} 0.0004 \\ 7.1592 \\ 0.5560 \\ -0.1173 \end{bmatrix} = \begin{bmatrix} \alpha_0 \\ \beta_2 \\ \beta_1 \\ \beta_0 \end{bmatrix}$. (4-23)

➢ Then, the resulting controller transfer function is:

$$C(s) = \frac{\beta_2 s^2 + \beta_1 s + \beta_0}{(s^2 + \omega^2)\alpha_0} = \frac{7.1592s^2 + 0.5560s - 0.1173}{0.0004(s^2 + 1)}. \quad (4\text{-}24)$$

➢ Applying the C(s), P(s), and the input transfer functions as shown in this figure 4-15.

Note: The Controller is designed at $\omega = 1 rad/\sec$ in both cases (the same controller is applied in both cases)

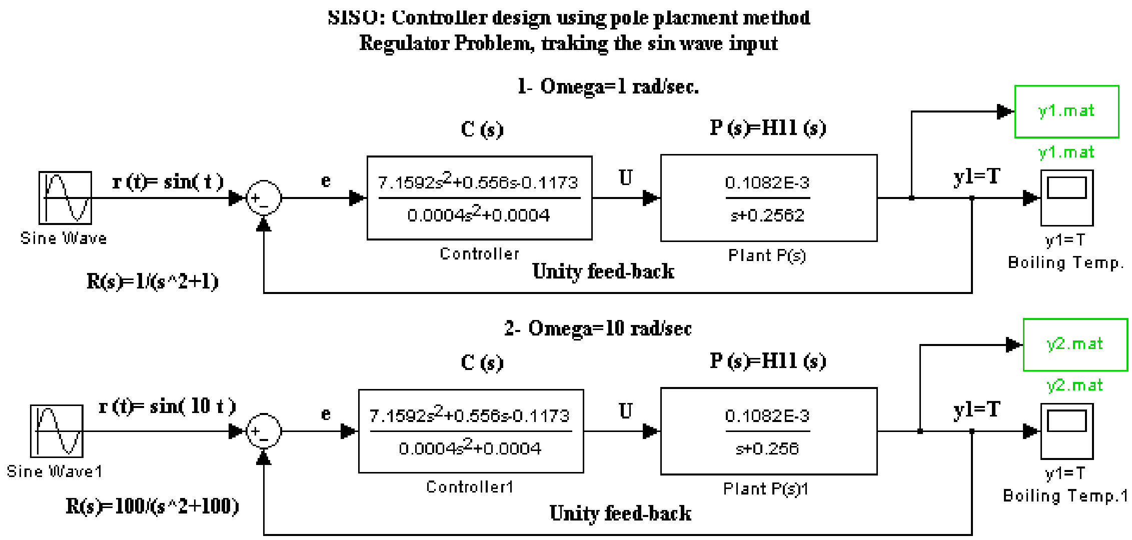


Fig. 4-15 Controller design when the input is sin wave

➢ The output response for the boiling temperature, when I considered a desired input (R (t) =sin (t)) is shown in the figure 4-16.

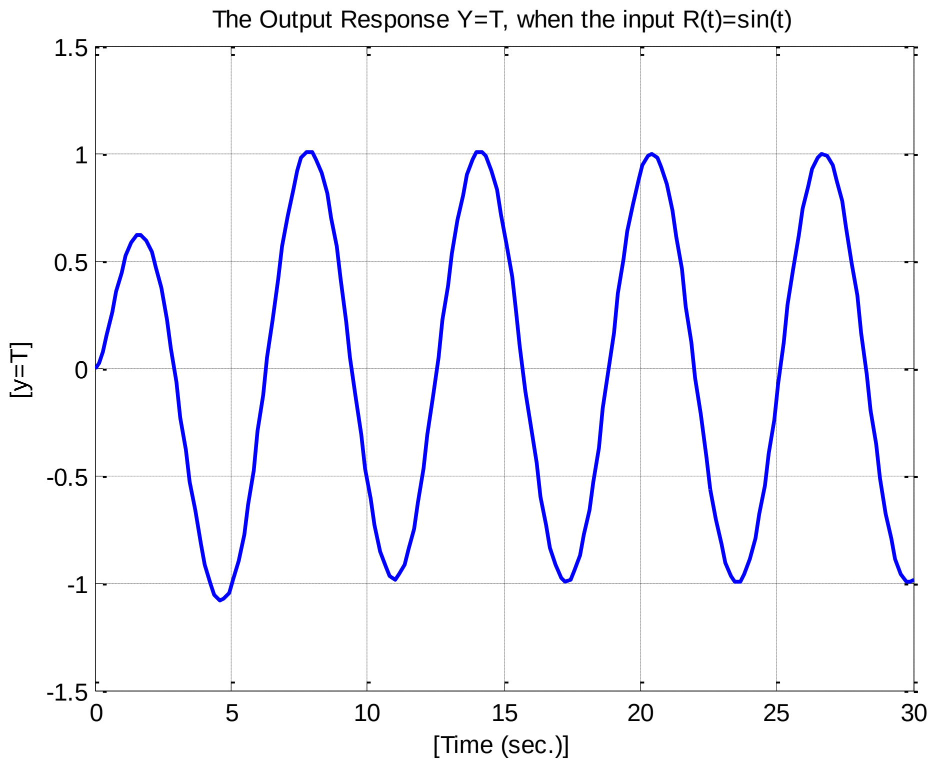


Fig 4-16 The output response, when the input R(t)=sin(t)

- As it can seen the Boiling Temperature (Y=T) has reached the desired input (R (t) =sin (t)), when I consider $\omega = 1 rad/\sec$. Now a sin is tracked reasonably well.

- The output response for the boiling temperature in the second case, when I considered the desired input (R (t) =sin (10 t), $\omega = 10 rad/\sec$) is shown in the figure 4-17.

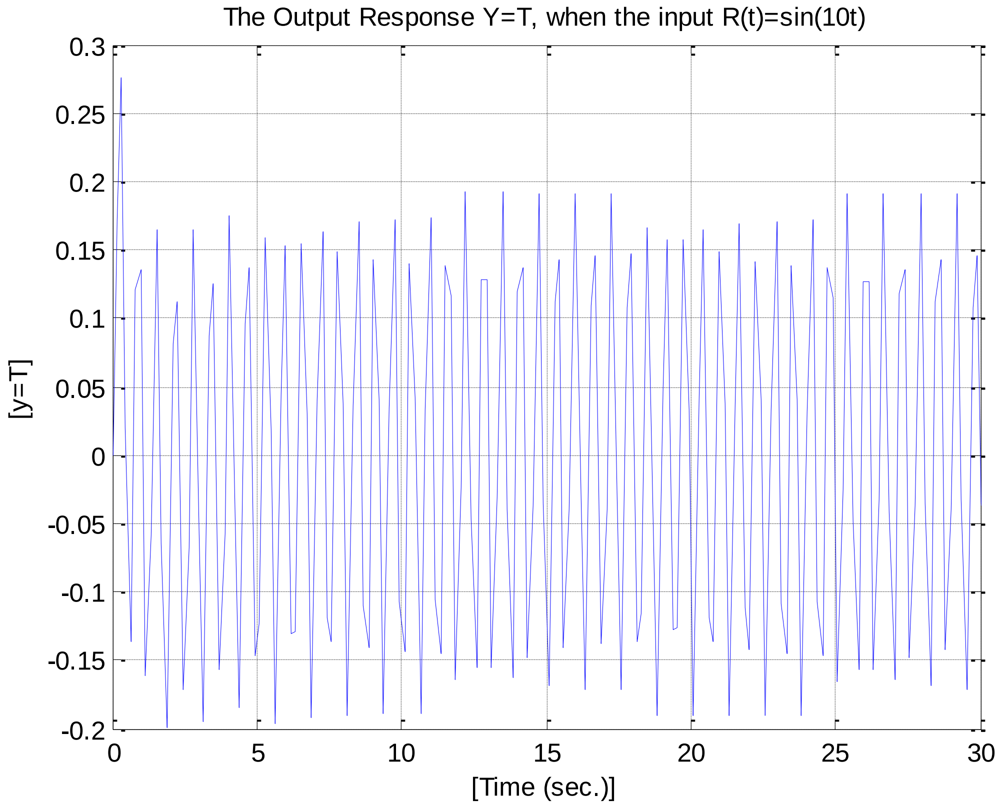


Fig 4-17 The output response, when the input R (t) =sin (10 t)

- As it can seen the Boiling Temperature (Y=T) does not reached the desired input (R (t) =sin (10 t)), when I consider $\omega = 10 rad/\sec$. Now a sin is not tracked well, because I designed the controller at $\omega = 1 rad/\sec$. Then, it is obvious that the controller can track the input only at designed frequency which in this case $\omega = 1 rad/\sec$.

4.1.5 A PARAMETERIZATION OF ALL STABILIZING CONTROLLERS

- Given plant transfer function:

$$P(s)=H_{11}(s)=\frac{0.1082*10^{-3}}{s+0.2562}=\frac{\frac{0.1082*10^{-3}(s-1)}{(s+0.2562)^2}}{\frac{(s-1)}{(s+0.2562)}}=\frac{np}{dp}. \tag{4-25}$$

$$np=\frac{0.1082*10^{-3}(s-1)}{(s+0.2562)^2}\quad,\quad dp=\frac{(s-1)}{(s+0.2562)} \tag{4-26}$$

- Let the transfer function of the controller:

$$C=\frac{X+qdn}{Y-qnp} \tag{4-27}$$

- Solve: $npX(s)+dpY(s)=1$; (4-28)

Where: $X(s)=\dfrac{nx}{(s+0.2562)^P}, Y(s)=\dfrac{ny}{(s+0.2562)^P}$, substitute X, and Y into the equation (4-28), and equate the needs parameters in both sides. I got P=1.so Bezoit equation becomes, $nx=\beta_1 s+\beta_0,\ ny=\alpha_1 s+\alpha_0$

- Substituting nx,ny,X, and Y in the (4-28), I got:

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0.1082*10^{-3} & 0 & -0.7438 & 1 \\ -0.1082*10^{-3} & 0.1082*10^{-3} & -0.2562 & -0.7438 \\ 0 & -0.1082*10^{-3} & 0 & -0.2562 \end{bmatrix}\begin{bmatrix}\beta_1\\ \beta_0\\ \alpha_1\\ \alpha_0\end{bmatrix}=\begin{bmatrix}1\\ 0.7684\\ 019684\\ 0.0168\end{bmatrix} \tag{4-29}$$

- Solving this equation ($\underline{\delta}=A\underline{X}$) $\Rightarrow \underline{X}=pinv(A).\underline{\delta}=10^3*\begin{bmatrix}-0.6947\\ 2.711\\ 0.000504\\ 0.000723\end{bmatrix}=\begin{bmatrix}\beta_1\\ \beta_0\\ \alpha_1\\ \alpha_0\end{bmatrix}$.(4-30)

- Substituting these parameters into the controller transfer function equation, as result I got:

$$C(s)=\frac{10^{-3}(-0.6947s+2.711)(s+0.2562)+q(s+0.2562)(s-1)}{(0.504s+0.723)(s+0.2562)-q(0.1802*10^{-3})(s-1)} \tag{4-31}$$

- Pick several q's, using Matlab program [7], I demonstrate (1+C(s) P(s)) is stable for different q.

When I consider: $q = \frac{1}{s + 0.2562} \Rightarrow Roots = -0.7707 + J0.596, and - 7707 - J0.596$

I got stable roots for different stable q's.

5. CONTROL OF A CONTINUOUS FLOW BOILING SYSTEM IN MULTI INPUT MULTI OUTPUT (MIMO) APPROACH

This chapter describes the procedure of controller design using a state feedback controller with integral error feedback, a state feedback controller with integral error feedback with state feedback implemented via observer system, and decentralized controller.

5.1 DESIGN A STATE FEEDBACK CONTROLLER WITH INTEGRAL ERROR FEEDBACK

The steps of design:

- **PLANT:** The transfer matrix function of MIMO boiling system is defined in equation (2-43) in the form:

$$\dot{\overline{X}} = A\overline{X} + B\overline{u}$$
$$\overline{Y} = C\overline{X} + D\overline{u}$$

Where: $\overline{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} P \\ T \\ m_G \end{bmatrix}$, $\overline{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_S \\ P_0 \end{bmatrix}$, and $\overline{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} v_E \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} v_E \\ T \\ m_G \end{bmatrix}$. (5-1)

- **CONTROL:** The control law in this case:

$$\begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} - \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = r - CX - Du \quad , \quad \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = -KX + K_1\omega \Rightarrow u = -\overline{\overline{K}} \begin{bmatrix} X \\ \omega \end{bmatrix}$$

Where: $\overline{\overline{K}} = [K \quad -K1]$ (5-2)

➢ **CLOSED LOOP:** The closed loop equations will be:

$$\begin{aligned}\dot{\bar{X}} &= A\bar{X} + B\bar{u}\\ \bar{Y} &= C\bar{X} + D\bar{u}\end{aligned} \quad \Rightarrow \quad \begin{aligned}\dot{X} &= AX + B(-\bar{K})\begin{bmatrix} X \\ \omega \end{bmatrix}\\ y &= CX + D(-\bar{K})\begin{bmatrix} X \\ \omega \end{bmatrix}\end{aligned}, \quad \dot{\omega} = r - y \Rightarrow \dot{\omega} = r - CX - D(-\bar{K})\begin{bmatrix} X \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} \dot{X} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}\begin{bmatrix} X \\ \omega \end{bmatrix} - \begin{bmatrix} B \\ D \end{bmatrix}\begin{bmatrix} K & -K_1 \end{bmatrix}\begin{bmatrix} X \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix}[r]_{(3\times1)}$$

$$\begin{bmatrix} \dot{X} \\ \dot{\omega} \end{bmatrix} = (\bar{A} - \bar{B}\bar{K})\begin{bmatrix} X \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix}[r]_{(3\times1)}$$

Where: $\bar{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ D \end{bmatrix}, and\ \bar{K} = \begin{bmatrix} K & -K_1 \end{bmatrix}$ (5-3)

$\Rightarrow$ Need $\bar{K}$ to make $(\bar{A} - \bar{B}\bar{K})$ stable.

$$\bar{A} = \begin{bmatrix} -0.1739 & -0.0048 & 0 & 0 & 0 & 0 \\ -2.9804 & -0.0824 & 0 & 0 & 0 & 0 \\ -6.2894 & -0.1738 & 0 & 0 & 0 & 0 \\ -6.2894 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} 0 & 0.0048 & 0.1237 \\ 0.0003 & 0.0821 & 2.1201 \\ 0.0006 & 0.1732 & 4.4738 \\ 0 & 0 & -4.4738 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{5-4}$$

$$Rank(\bar{A}, \bar{B}) = rank([\bar{B}, \bar{A}\bar{B}, \bar{A}^2\bar{B}, \ldots\ldots, \bar{A}^5\bar{B}]) = 6 \tag{5-5}$$

Therefore it is controllable.

$$\bar{K} = place(\bar{A}, \bar{B}, [-0.5, -1, -1.5, -2, -2.5, -3]) \tag{5-6}$$

$$\bar{\bar{K}} = 1.0e + 010 * \begin{bmatrix} -1.8803 & -2.0056 & 1.0028 & 0.0004 & 0.0052 & -0.0026 \\ 0.0067 & 0.0072 & -0.0036 & -0.0000 & -0.0000 & 0.0000 \\ -0.0000 & -0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0000 \end{bmatrix} \quad (5\text{-}7)$$

Finally, we should check that the poles are right:

$Eigenvalues = eig(\bar{\bar{A}} - \bar{\bar{B}}\,\bar{\bar{K}})$

Eigenvalues =

-0.5005
-3.0000
-0.9995
-1.4998
-2.5002
-2.0000 (5-8)

Almost there, but to simulate we need to remember that $\bar{\bar{K}} = [K \quad -K1]$. So we compute:

$$K = 1.0e + 10 * \begin{bmatrix} -1.8803 & -2.0056 & 1.0028 \\ 0.0067 & 0.0072 & -0.0036 \\ -0.0000 & -0.0000 & 0.0000 \end{bmatrix},$$

$$K_1 = 1.0e + 07 * \begin{bmatrix} -0.3717 & -5.1740 & 2.5857 \\ 0.0013 & 0.0185 & -0.0092 \\ -0.0000 & -0.0000 & 0.0000 \end{bmatrix}. \quad (5\text{-}9)$$

- **SIMULATION RESULT:**

Here is a simulink diagram for this setup, figure 5-1:

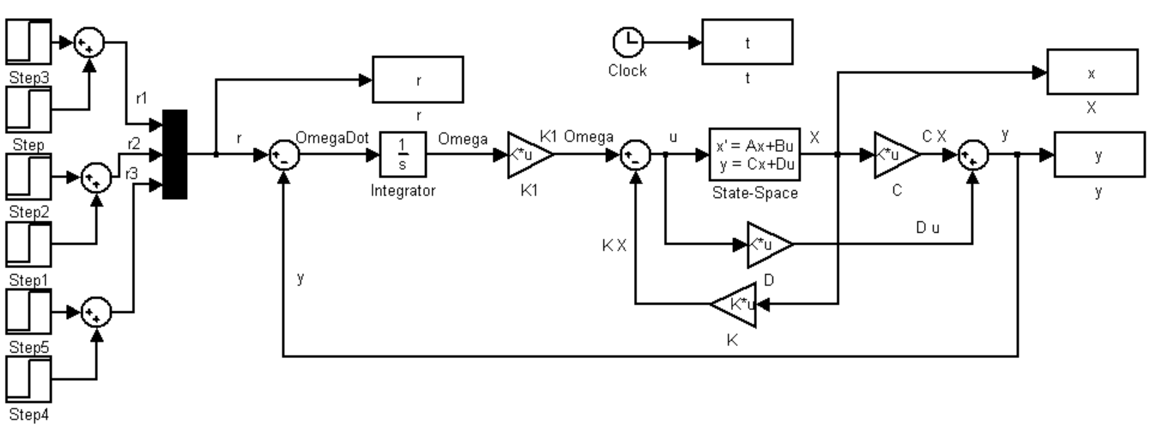


Fig. 5-1 A state feedback controller with integral error feedback

Next, here is the result figure 5-2, looking at r1, r2, and r3 and y1, y2, and y3 versus time. We see the setpoints are maintained.

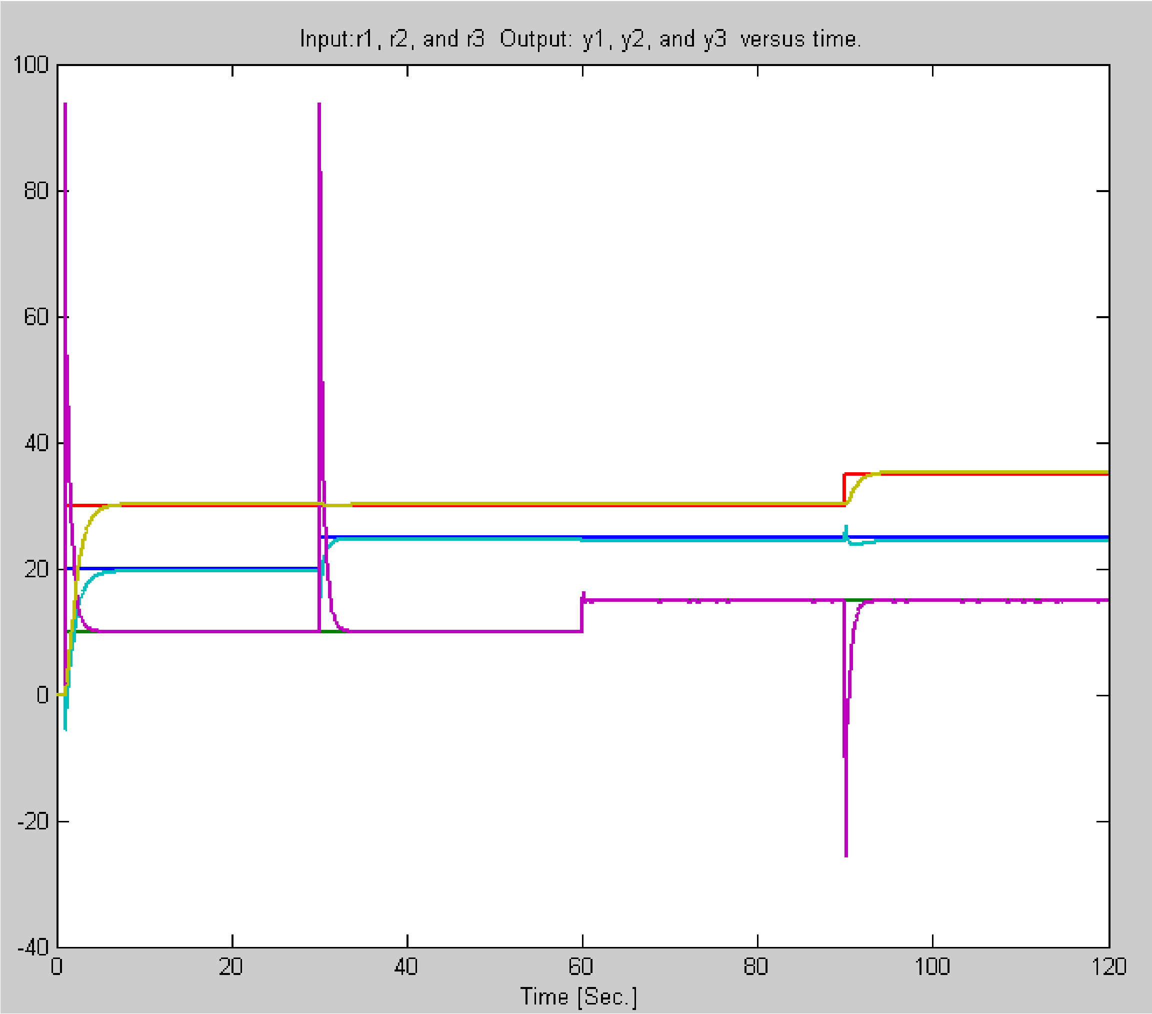


Fig. 5-2 Simulation result y1, y2, y3 and r1, r2, r3 versus time

5.2 DESIGN A STATE FEEDBACK CONTROLLER WITH INTEGRAL ERROR FEEDBACK WITH STATE FEEDBACK IMPLEMENTED VIA OBSERVER

The steps of the design:

- **PLANT:** The transfer matrix function of MIMO boiling system is defined in equation (2-43) in the form:

$$\dot{\overline{X}} = A\overline{X} + B\overline{u}$$
$$\overline{Y} = C\overline{X} + D\overline{u}$$

Where: $\overline{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} P \\ T \\ m_G \end{bmatrix}$, $\overline{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_S \\ P_0 \end{bmatrix}$, and $\overline{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} v_E \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} v_E \\ T \\ m_G \end{bmatrix}$. (5-10)

- **OBSERVER:** The observer equation is:

$$\dot{\hat{X}} = (A - LC)\hat{X} + LY + Bu \tag{5-11}$$

- **CONTROL:** The control low:

$$u = r - K\hat{X}$$

Where: $\overline{\overline{K}} = \begin{bmatrix} K & -K1 \end{bmatrix}$ (5-12)

- **CLOSED LOOP:** I summarized the closed loop equations as:

$$\begin{aligned} \dot{\overline{X}} &= A\overline{X} + B\overline{u} \\ \overline{Y} &= C\overline{X} + D\overline{u} \end{aligned} \Rightarrow \begin{aligned} \dot{X} &= AX + B(r - K\hat{X}) \\ y &= CX + D(r - K\hat{X}) \end{aligned} \Rightarrow \begin{aligned} \dot{X} &= AX - BK\hat{X} + Br \\ y &= CX - DK\hat{X} + Dr \end{aligned}$$

$$\dot{\hat{X}} = (A - LC)\hat{X} + L(CX + Du) + B(r - K\hat{X}) = LCX + (A - LC - BK - LDK)\hat{X} + (LD + B)r$$

$$\begin{bmatrix} \dot{X} \\ \dot{\hat{X}} \end{bmatrix} = \begin{bmatrix} A & -BK \\ LC & A - LC - BK - LDK \end{bmatrix} \begin{bmatrix} X \\ \hat{X} \end{bmatrix} + \begin{bmatrix} B \\ LD + B \end{bmatrix} .r$$

$$A_{C1} = \begin{bmatrix} A & -BK \\ LC & A - LC - BK - LDK \end{bmatrix}, \qquad B_{C1} = \begin{bmatrix} B \\ LD + B \end{bmatrix} \tag{5-13}$$

- To design observer, need L to make $(A-LC)$ stable. $(A-LC)$ stable iff $A^T - C^T L^T$ stable $\Rightarrow$ $L = place(A^T, B^T, [-0.5,-1,-1.5])^T$

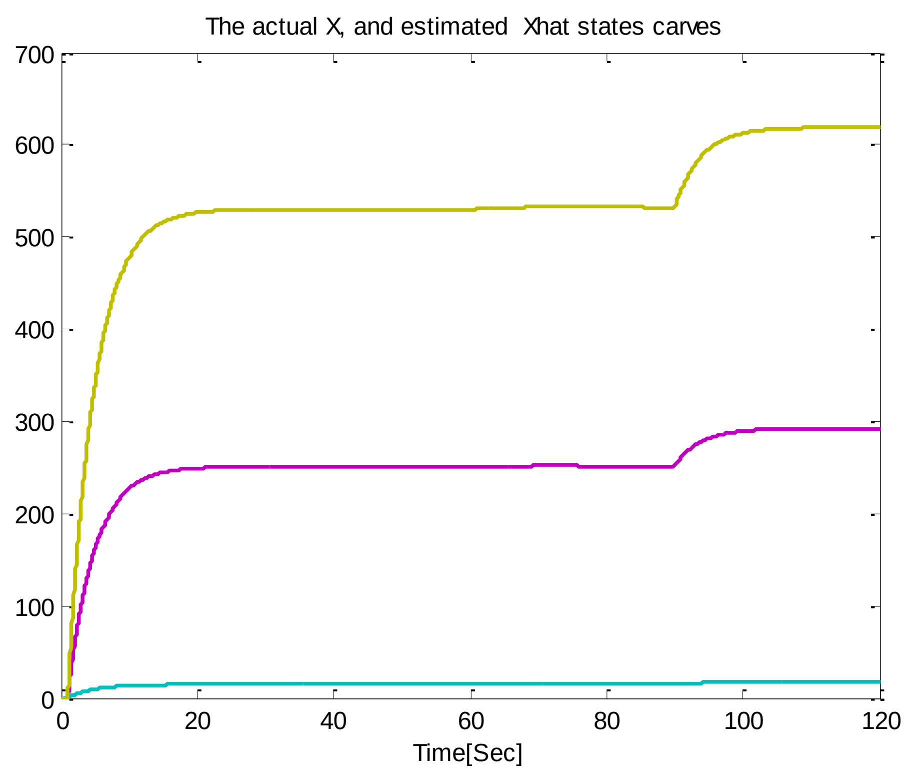
$$L = 1.0\mathrm{e}+004 * \begin{bmatrix} 0.0440 & 0.0440 & 0.0440 \\ -1.5929 & -1.5929 & -1.5929 \\ 1.3165 & 1.3165 & 1.3165 \end{bmatrix} \text{, it's the observer gain.} \quad (5\text{-}14)$$

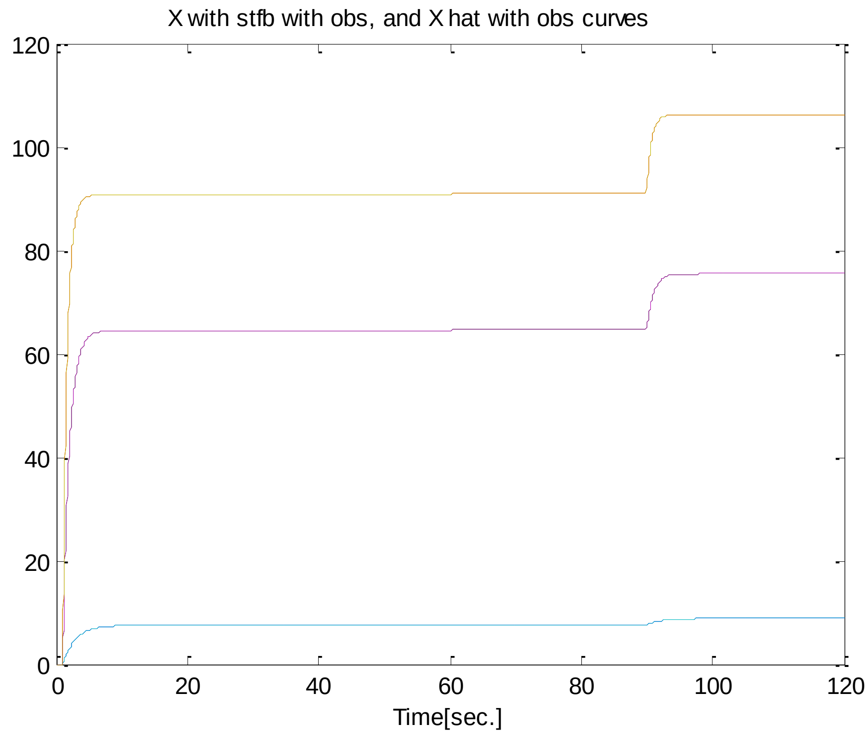
- I obtained K , and K1 as in section 4.1.1 (A.5):

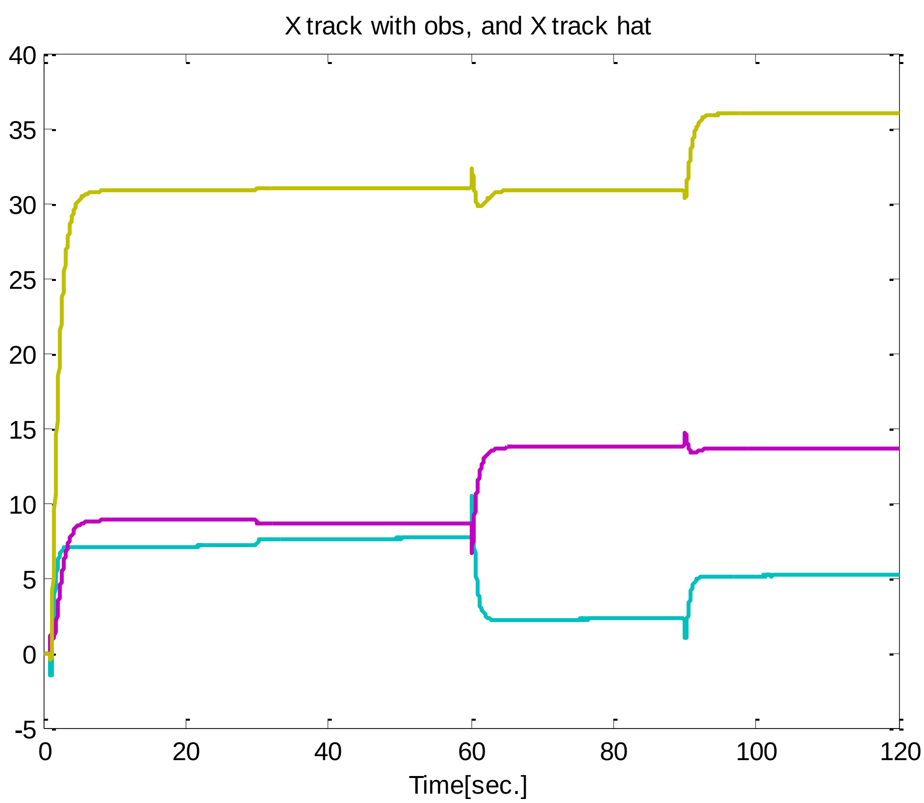
$$K = 1.0\mathrm{e}+10 * \begin{bmatrix} -1.8803 & -2.0056 & 1.0028 \\ 0.0067 & 0.0072 & -0.0036 \\ -0.0000 & -0.0000 & 0.0000 \end{bmatrix}$$

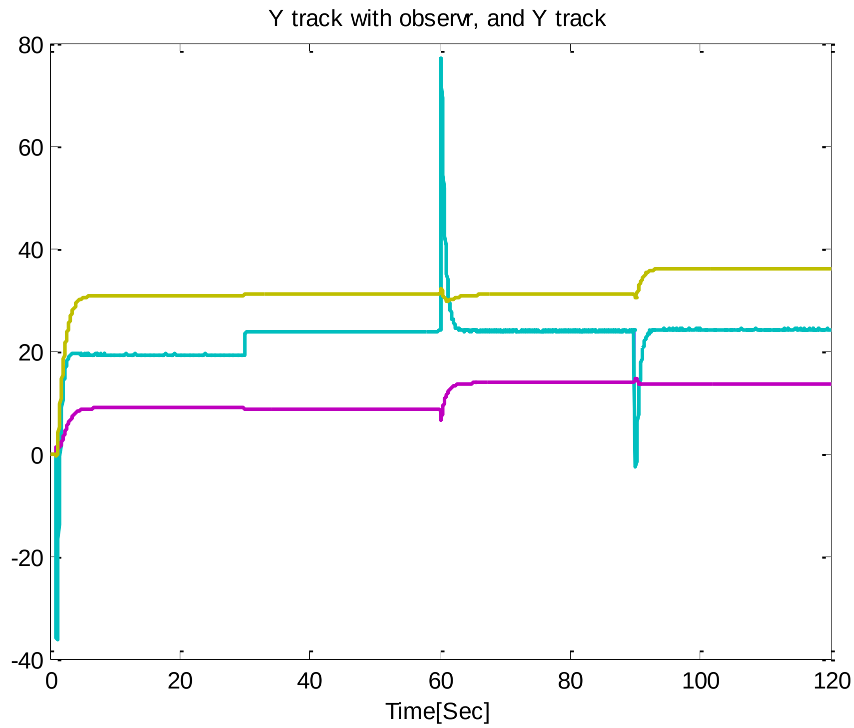
$$K_1 = 1.0\mathrm{e}+07 * \begin{bmatrix} -0.3717 & -5.1740 & 2.5857 \\ 0.0013 & 0.0185 & -0.0092 \\ -0.0000 & -0.0000 & 0.0000 \end{bmatrix} \quad (5\text{-}15)$$

- Finally, state feedback controller with integral error feedback with state feedback implemented via an observer was implemented in Simulink (A.6) and the simulation results for step changes in each input is shown in the figures 5-3 a,b,c,d, and e.




X with stfb with obs, and X hat with obs curves
120
100
80
60
40
20
0
0
20
40
60
80
100
120
Time[sec.]


X track with obs, and X track hat
40
35
30
25
20
15
10
5
0
-5
0
20
40
60
80
100
120
Time[sec.]



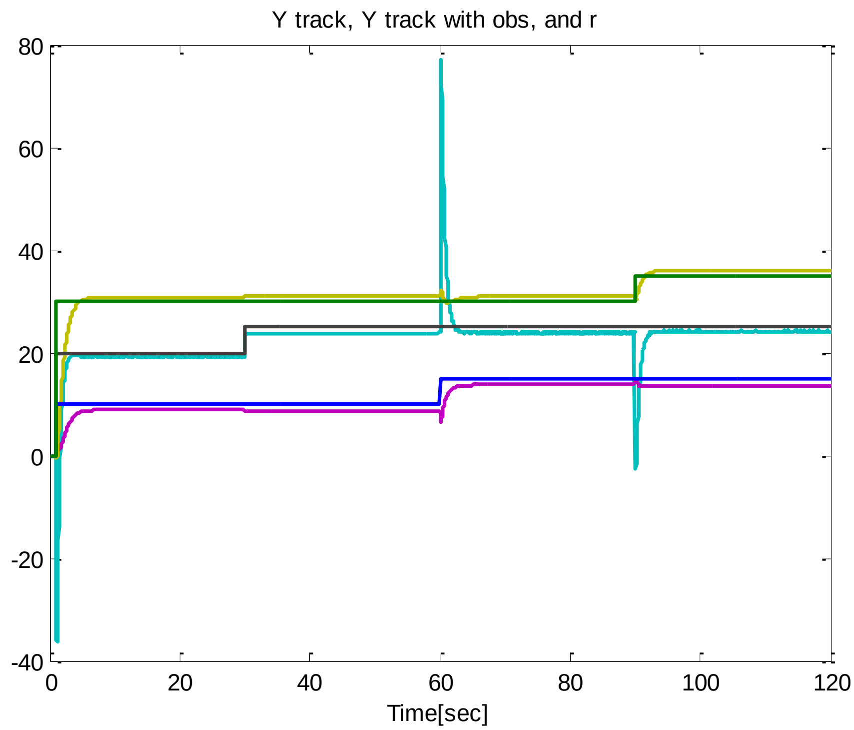


Fig. 5-3 a, b, c, d, and e. the simulation results for step changes in each input.

As it can see, the observer does the job and the state feedback track the desired setpoints for step changes in each input.

5.3 DECENRALIZED CONTROLLER FOR MIMO BOILING SYSTEM

First, the relative gain array matrix will be derived in order to define the proper pairing of inputs and outputs for the use of decentralized controllers. The steps of the design:

- I compute the Dc-gain as: $\begin{matrix} K = \lim H(s) \\ s \to 0 \end{matrix} \quad \Rightarrow \quad K = \begin{bmatrix} 0.0042 & 0.1170 & -1.4380 \\ 0.0012 & 0.3204 & 8.2750 \\ 0.0024 & 0.6760 & 1.74620 \end{bmatrix}$

(5-16)

- Define $H = (K^{-1})^T = 1.0\text{e}+005 * \begin{bmatrix} 0.0023 & 0.0000 & -0.0000 \\ -7.3647 & 0.1887 & -0.0063 \\ 3.4902 & -0.0894 & 0.0030 \end{bmatrix}$ (5-17)

- The relative gain array matrix is computed as: $\Lambda = H \bullet * K$

$$\Lambda = 1.0\text{e}+003 * \begin{bmatrix} 0.000953929631650 & 0.000000591566598 & 0.000045478801752 \\ -0.847971844194889 & 6.044485643490019 & -5.195513799295130 \\ 0.848017914563239 & -6.043486235056618 & 0.5196468320493376 \end{bmatrix}$$

(5-18)

As it can see fro the above matrix, the diagonal elements are the closest elements to 1, then proper pairing of inputs and outputs are1-1, 2-2, and 3-3 pairing.

- I carried out the controller for each channel using the frequency domain with the following steps:
  - I determine the transfer function of the open loop system (with unit value of the controller’s gain);
  - I draw the Bode diagrams;
  - I choose a value for the phase margin (in the [45, 60]˚ domain), namely 60˚;
  - For this value I read from the Bode-diagrams the amplitude and recalculate the gain of the open loop system from what the controller’s gain can be calculated ($k_0=k_pk_r$);
  - The time constant of the controller was chosen in the way to compensate the plant’s big time constant.

As result, the PI-controllers (for $H_{11}(s)$, $H_{22}(s)$ and $H_{33}(s)$):

$$H_{c1}(s) = \frac{2328.676}{3.9s}(3.9s+1);$$
$$H_{c2}(s) = \frac{3.0687}{3.9s}(3.9s+1);$$
$$H_{c3}(s) = \frac{0.0563}{3.9s}(3.9s+1);$$

(5-19)

➢ Finally, I built the model in Simulink figure 5-4, and the simulation results are shown in figure5-5 a, b, and c.

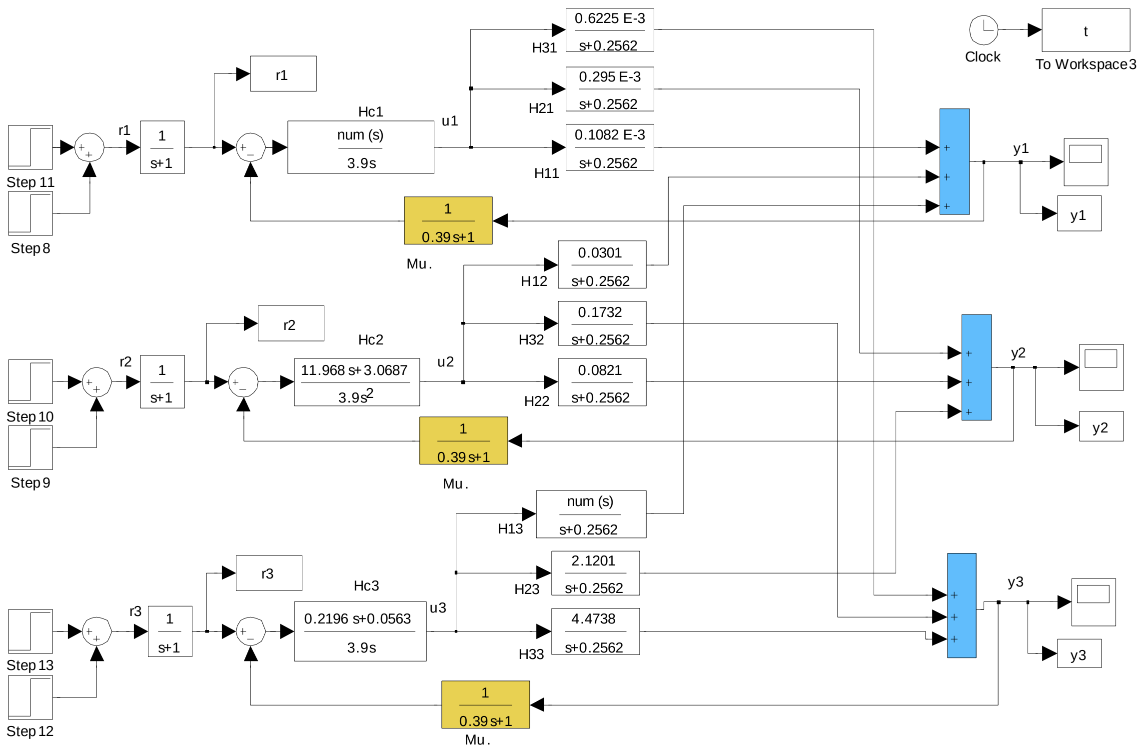


Fig. 5-4 the Simulink model of the multi-loop MIMO Boiling system

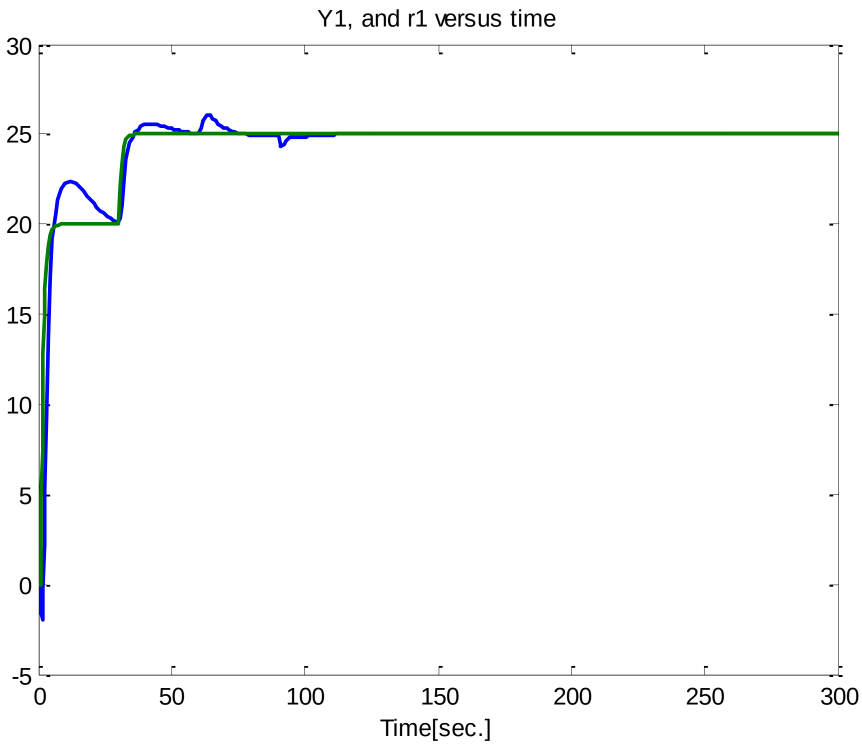


Fig. 5-5. a. the response curves y1, and r1 versus time

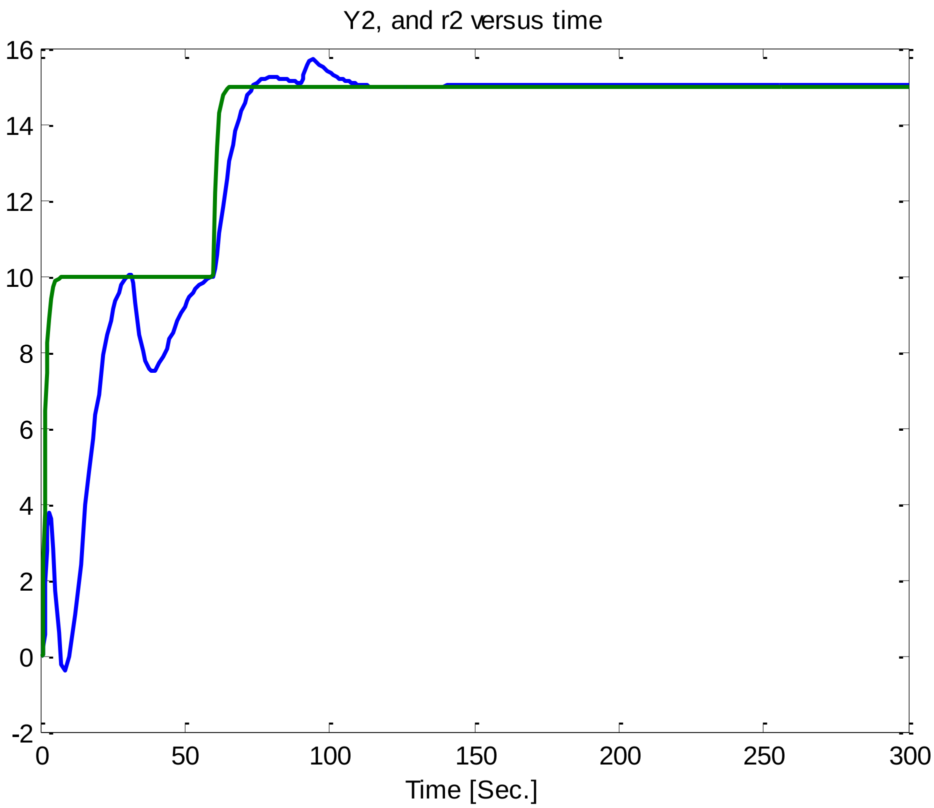


Fig. 5-5. b. the response curves y2, and r2 versus time

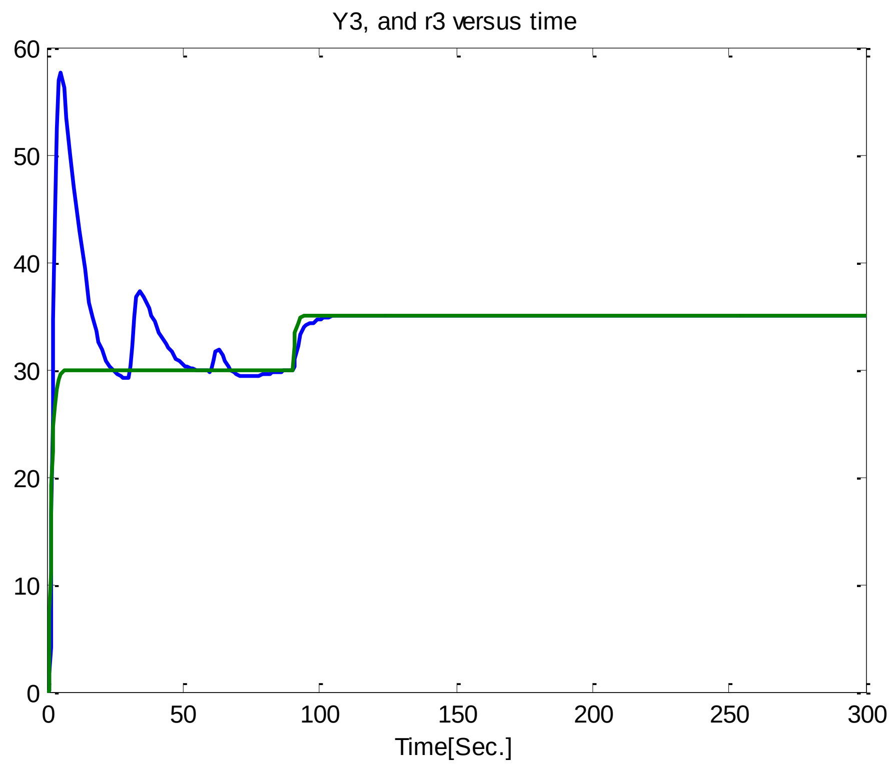


Fig. 5-5. c. the response curves y3, and r3 versus time

As it can see, the set points are maintained and the three attenuations (oscillations) are appears in output curves due to the changes in the inputs at time 1, 30, 60, and 90 sec.

5.4 DESIGN A STATE FEEDBACK CONTROLLER WITH INTEGRAL ERROR FEEDBACK USING OBTIMAL GAIN

Using the Matlab program I computed the optimal gain using the built in function in Matlab:

$$[K, S, E] = lqr(Abar, Bbar, Q, R)$$

Where: Abar, and Bbar are computed in the equation (5-4), and

$$Q = eye(6,6), R = 0.01eye(3,3)$$

As result, the optimal gain:

$$K = 10^3 * \begin{bmatrix} -0.0343 & 1.0325 & -0.4884 \\ 0.0385 & 0.0462 & -0.0140 \\ -0.0235 & -0.0237 & 0.0218 \end{bmatrix} \quad , \quad K1 = \begin{bmatrix} -0.0977 & 9.0407 & -4.2603 \\ 8.5880 & 2.2647 & 4.5810 \\ -5.1048 & 3.6131 & 7.8002 \end{bmatrix}$$

Finally, run the simulink mode which was demonstrated in figure 5-1.The simulation result is shown in figure 5-6

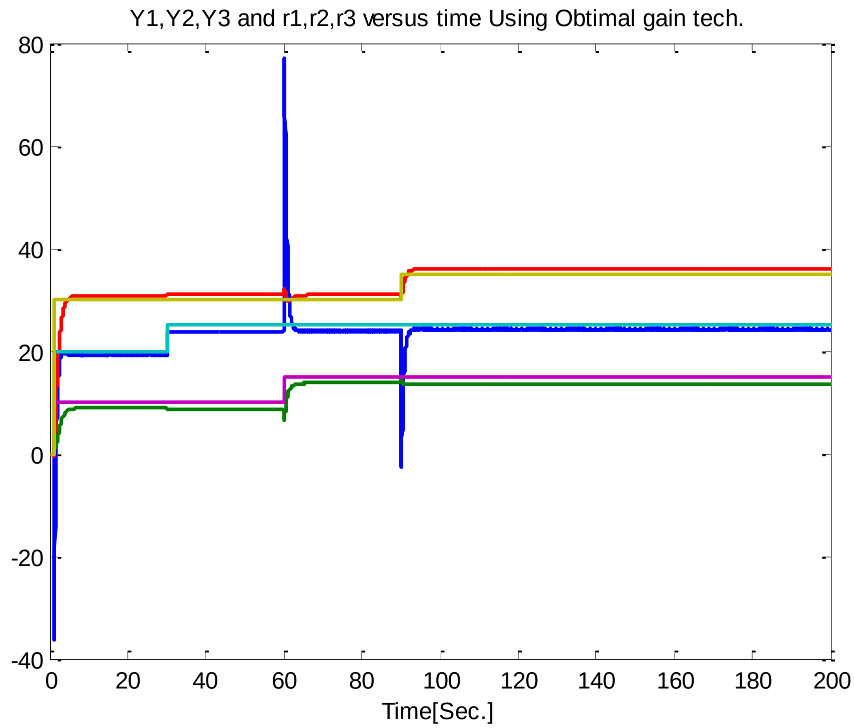


Fig. 5-6 Controller design using LQR tech.

6. Conclusions

My task was, first to study the modeling analysis and control of a continuous flow boiling system (SISO system and MIMO system approach)..

The non-linear mathematical model of a continuous flow boiling system has been developed employing the mass and heat balance equations. I took the non-linear functions of the boiling system, and expanding them in Taylor series expansions around the steady state operating point, the linearized mathematical model was obtained. The results were presented using MATLAB-Simulink simulations. I have demonstrated based on the presented results the necessity of a controller.

The control of the continuous flow boiling system in *SISO approach* was presented, using error feedback; state feedback, output feedback, Sylvester matrix technique, and a parametization of all stabilizing controllers are made.

The control of continuous flow boiling system in *MIMO approach* was presented using a state feedback controller with integral error feedback, a state feedback controller with integral error feedback with state feedback implemented via observer system, and decentralized controller, and finally the optimal gain method.

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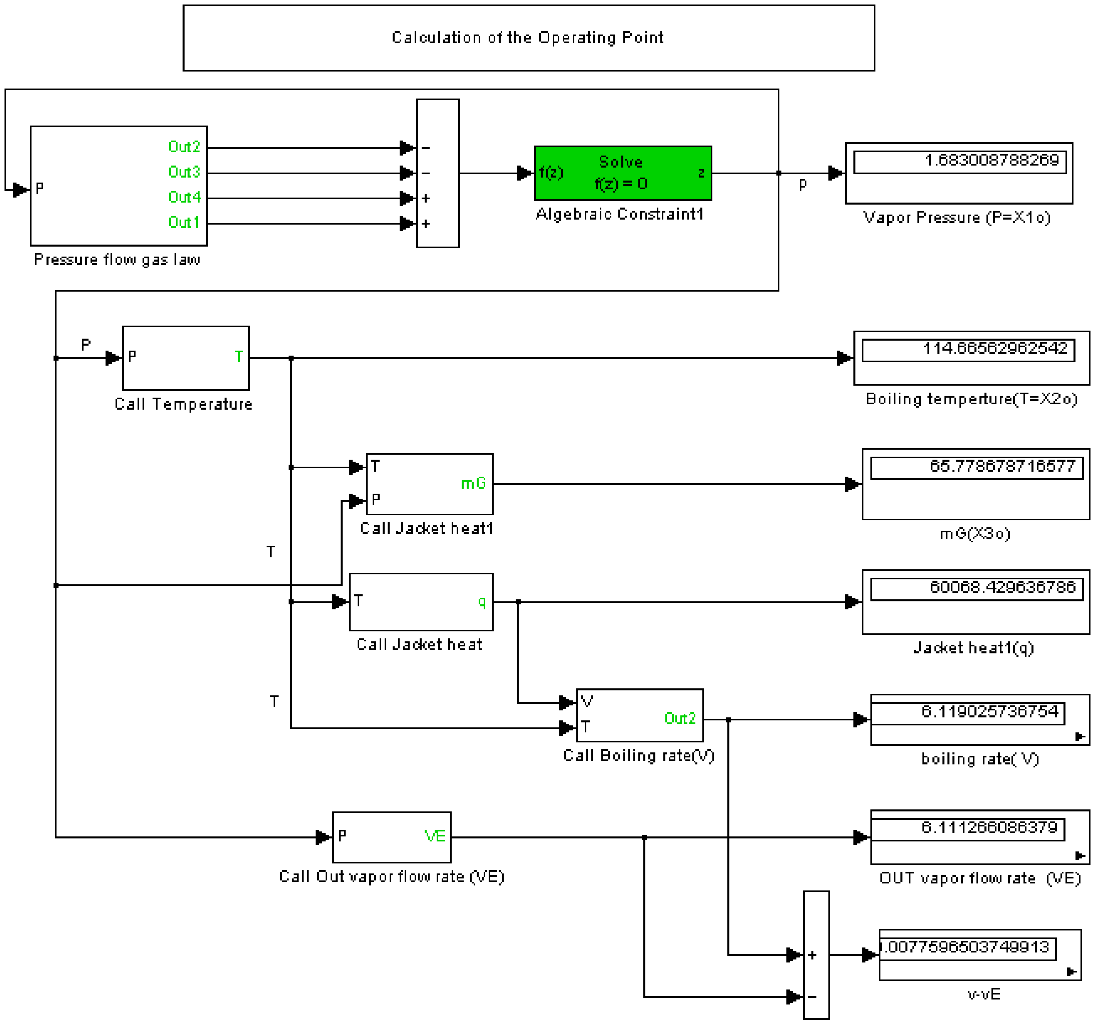
[2] LUYBEN W. L. “Process Modeling, Simulation, and Control for Chemical Engineers”, McGraw-Hill Book Co., New York.

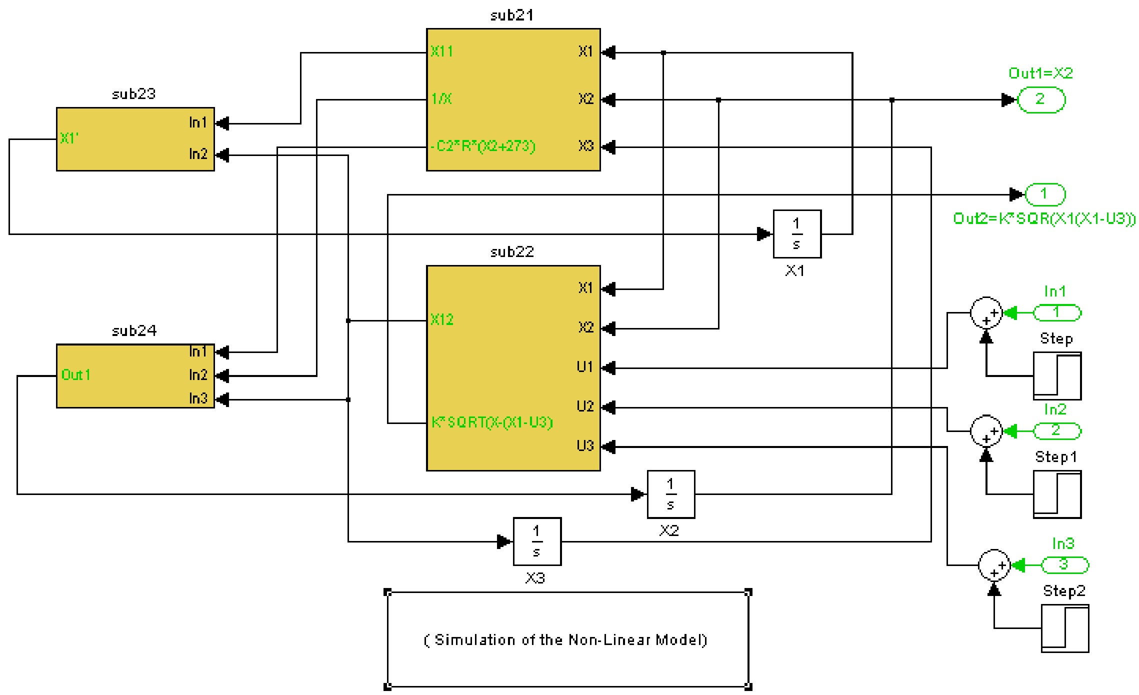
[3] KIVEN MOOR. “EGGN 517 lecture notice”. Spring semester 2009.

APPENDIXES

A.1 Calculation of the Operating Point using Matlab Simulink



A.2 Linearization of the Non-Linear Model using Matlab Simulink



EDU>> [A,B,C,D]=linmod('SYS')

A =

-0.1739	-0.0048	0.0000
-2.9804	-0.0824	0.0000
-6.2894	-0.1738	0

B =

0.0000	0.0048	0.1237
0.0003	0.0821	2.1201
0.0006	0.1732	4.4738

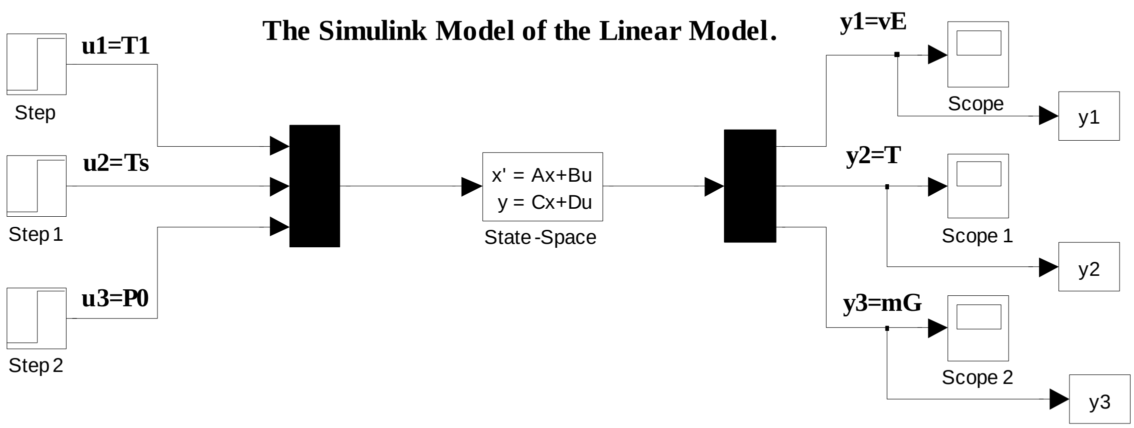
C =

6.2894	1	0
0	1	0
0	0	1

D =

0	0	-4.4738
0	0	0
0	0	0

A.3 Simulation of the Linear Model



A.4 Rascal function to compute Nbar

```
function[Nbar]=rscale(A,B,C,D,K)
% the function rscale(A,B,C,D,K) finds the scale factor N which will
% eliminate the steady-state error to a step reference
% using the schematic below:
s = size(A,1);
Z = [zeros([1,s]) 1];
N = inv([A,B;C,D])*Z';
Nx = N(1:s);
Nu = N(1+s);
Nbar=Nu + K*Nx;
```

A.5 A state feedback controller with integral error feedback with state feedback implemented via observer system.

```
clear all
close all
clc

A=[-0.1739,-0.0048,0.0000
   -2.9804,-0.0824,0.0000
   -6.2894,-0.1738,0]
B=[0.0000,0.0048,0.1237
   0.0003,0.0821,2.1201
   0.0006,0.1732,4.4738]
C=[ 6.2894     ,0    ,0
    0          ,1    ,0
    0          ,0    ,1]
D=[ 0     ,0     ,-4.4738
    0     ,0     ,0
    0     ,0     ,0]

a=A;b=B;c=C;d=D;
display('Obs design - place eig of A-LC at [-0.5;-1;-1.5]')
```

```
%Obs design - place eig of A-LC at [-0.5;-1;-1.5]
g=[1;1;1];
Cbar=g'*C;
Rank_A_Cbar=rank([Cbar;Cbar*A;Cbar*A^2]);
Lbar=place(A',Cbar',[-0.5;-1;-1.5])';

l=Lbar*g'
Eigvalues=eig(A-l*C)

display('state feedback with no integral error poles at [-0.5;-1;-
1.5]')

% state feedback with no integral error poles at [-0.5;-1;-1.5]
k=place(A,B,[-0.5;-1;-1.5])
% k=g*kbar
Eigvalues=eig(A-B*k)

Cont_M=[B A*B A^2*B]
Rank_C=rank([B A*B A^2*B])
Obser_M=[C; C*A; C^2*A]
Rank_O=rank([C; C*A; C^2*A])
Eig_A=eig(A)

display('State feedback with integral error, overall poles of state
feedback at[-0.5;-1;-1.5;-2;-2.5;-3]')

%State feedback with integral error, overall poles of state feedback at
%[-0.5;-1;-1.5;-2;-2.5;-3]
Abar=[A zeros(3,3);-C zeros(3,3)]
Bbar=[B;D]
RankAbar_Bbar=rank([Bbar,Abar*Bbar,Abar^2*Bbar,Abar^3*Bbar,Abar^4*Bbar,
Abar^5*Bbar])
% kbar=place(Abar,Bbar,[-1,-1.5,-2,-2.5,-3,-3.5])
Kbar=place(Abar,Bbar,[-0.5,-1,-1.5,-2,-2.5,-3]);

Eigvalues=eig(Abar-Bbar*Kbar)
K=[Kbar(1,[1:3]);Kbar(2,[1:3]);Kbar(3,[1:3])]

K1=-[Kbar(1,[4:6]);Kbar(2,[4:6]);Kbar(3,[4:6])]
```

A.6 The Simulink model of a state feedback controller with integral error feedback with state feedback implemented via observer system.