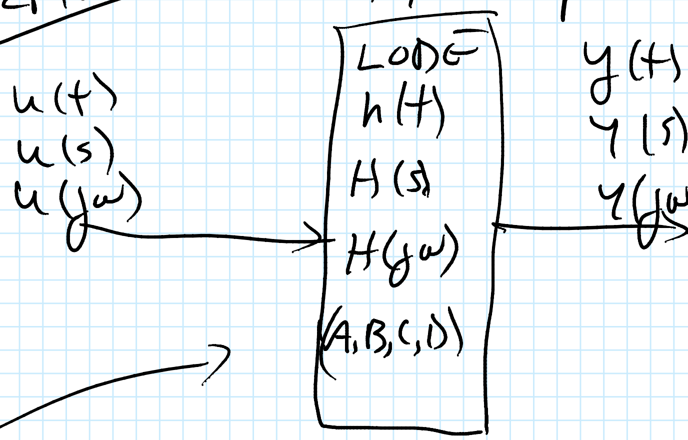
2-11-16 Lecture 7

Thursday, February 11, 2016
2:03 PM

Interlude



$y(t) = h(t) * u(t)$ LTI causal

$Y(s) = H(s)U(s)$

$Y(j\omega) = H(j\omega)U(j\omega)$

$H(s) = \mathcal{L}\{h(t)\}$

transfer matrix

impulse response

frequency response

$s = j\omega$

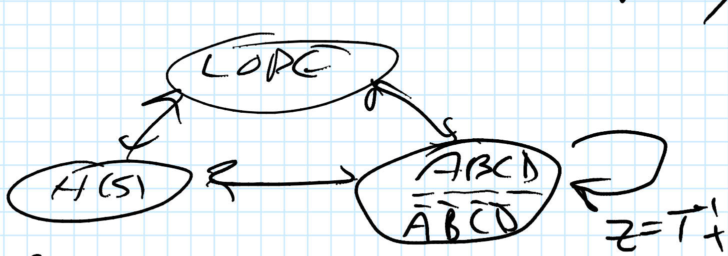
gain at frequency ω

state space

$$y(t) = Ce^{At}x(0) + C\int_0^t e^{A(t-\tau)}Bu(\tau)d\tau + Du(t)$$

n^{th}-order LODE in $\mathbb{R}^1$

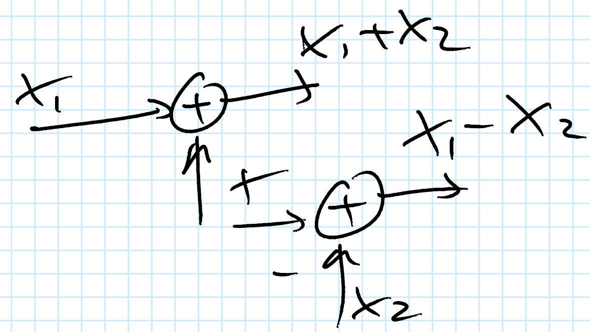
1^{st}-order LODE in $\mathbb{R}^n$



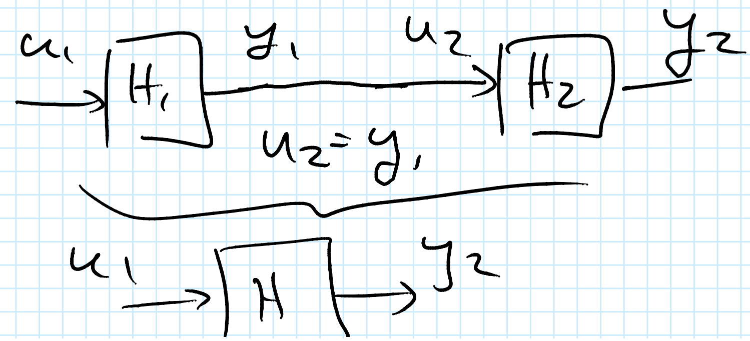
Interconnected Systems

Block diagrams:

adders/summer



cascade



$H = H_1 \cdot H_2$?

note $y_2 = H_2 u_2$

$y_1 = H_1 u_1$

$u_2 = y_1$

$$y_2 = H_2 u_2 = \underline{H_2 H_1} u_1$$

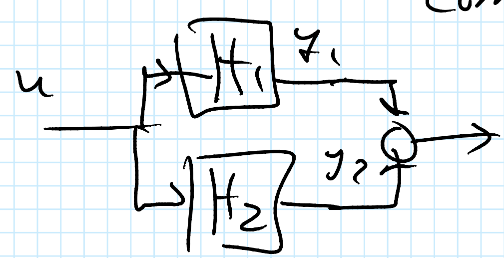
$\neq$

$H_1 H_2$

unless scalar

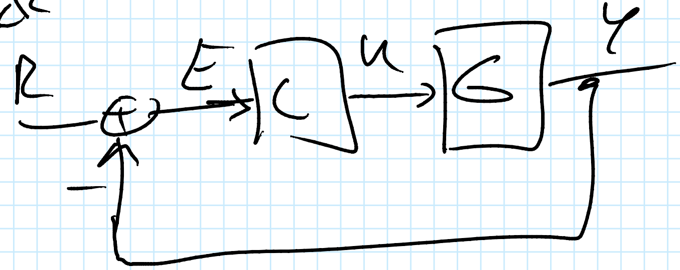
(matrices don't commute)

parallel



$y = (H_1 + H_2) u$

feedback



$$Y = Gu = G \cdot C\, E = G \cdot C \cdot (R - Y)$$

$$= G \cdot C \cdot R - G \cdot C \cdot Y$$

$$Y + G \cdot C\, Y = G \cdot C \cdot R$$

$$(I+GC)Y = G \cdot C \cdot R$$

$$Y = \underbrace{(I+G \cdot C)^{-1} G \cdot C}_{\text{transfer Matrix}} R$$

for SISO

$$\frac{Y}{R} = \frac{GC}{1+GC}$$

using matrix fractions

<u>SISO</u> $\quad G(s) = \frac{n_g(s)}{d_g(s)} \quad C(s) = \frac{n_c(s)}{d_c(s)}$

$\Rightarrow$ can write

$$\frac{Y}{R} = \frac{n_c n_g}{d_c d_g + n_c n_g} \quad *$$

consider <u>MIMO</u>

$$C = D_c^{-1} N_c$$

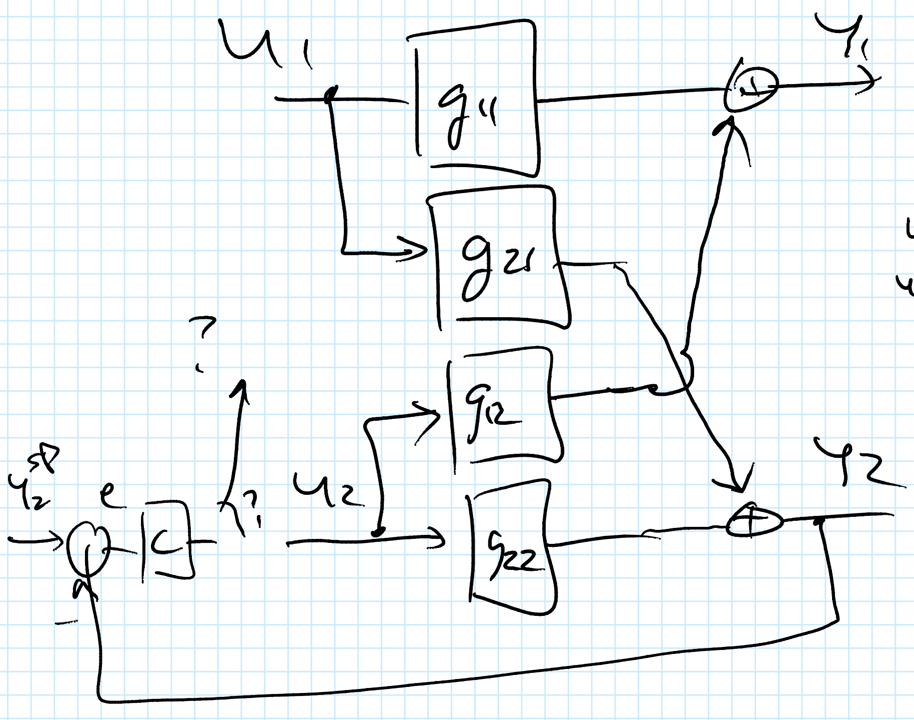
$$G = D_g^{-1} N_g$$

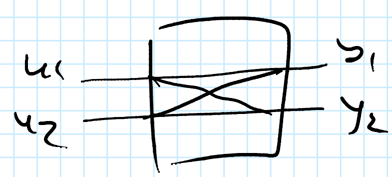
<u>Assignment</u> (Thurs 18th) Derive *

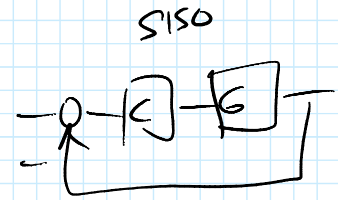
for MIMO case

consider MIMO system can be seen as interconnected SISO blocks

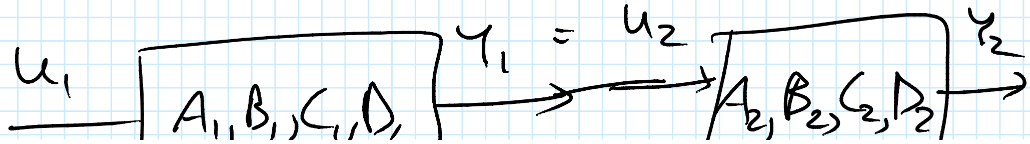
Ex:
$$\begin{pmatrix} Y_1(s) \\ Y_2(s) \end{pmatrix} = \left[\begin{array}{c:c} g_{11}(s) & g_{12}(s) \\ \hdashline g_{21}(s) & g_{22}(s) \end{array}\right] \begin{pmatrix} U_1(s) \\ U_2(s) \end{pmatrix}$$







Interconnected state-space



$$u_1 \longrightarrow \boxed{A_1, B_1, C_1, D_1} \xrightarrow{y_1} \boxed{A_2, B_2, C_2, D_2} \xrightarrow{y_2}$$

$$\underbrace{\qquad\qquad\qquad\qquad}$$

$$u_1 \longrightarrow \boxed{\bar{A}, \bar{B}, \bar{C}, \bar{D}} \longrightarrow y_2$$

$$\dot{x}_1 = A_1 x_1 + B_1 u_1$$

$$y_1 = C_1 x_1 + D_1 u_1$$

$$\dot{x}_2 = A_2 x_2 + B_2 u_2$$

$$y_2 = C_2 x_2 + D_2 u_2$$

$$y_1 = u_2$$

$$\Rightarrow \dot{x}_2 = A_2 x_2 + B_2 (C_1 x_1 + D_1 u_1)$$

$$y_2 = C_2 x_2 + D_2 (C_1 x_1 + D_1 u_1)$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} A_1 & 0 \\ B_2 C_1 & A_2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 D_1 \end{pmatrix} u_1$$

$$y_2 = \begin{bmatrix} D_2 C_1 & C_2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + (D_2 D_1) u_1$$

Aside

disturbances

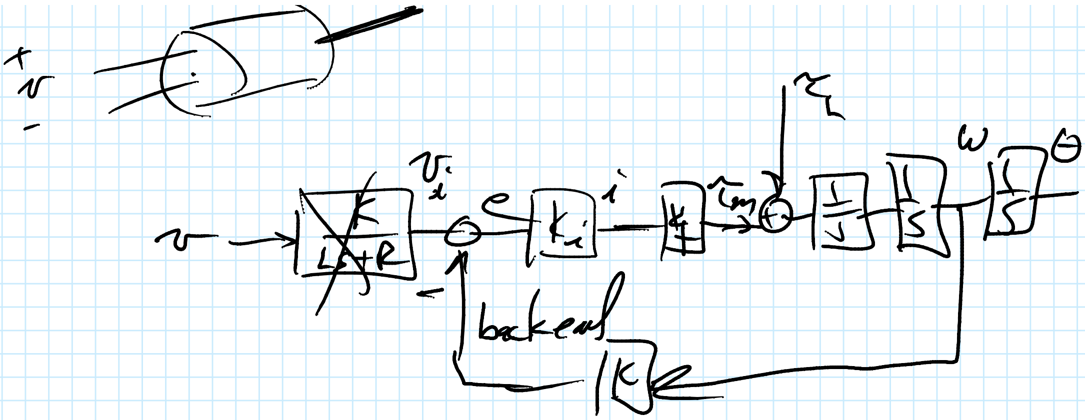
$$\dot{x} = Ax + Bu + Ed$$

↑ disturbance

$$y = Cx + Fn(t)$$

↑ noise

motor



$$J\dot{\omega} = \tau_L + \underbrace{\tau_m}_{K_t K_i (v_i - K\omega)}$$

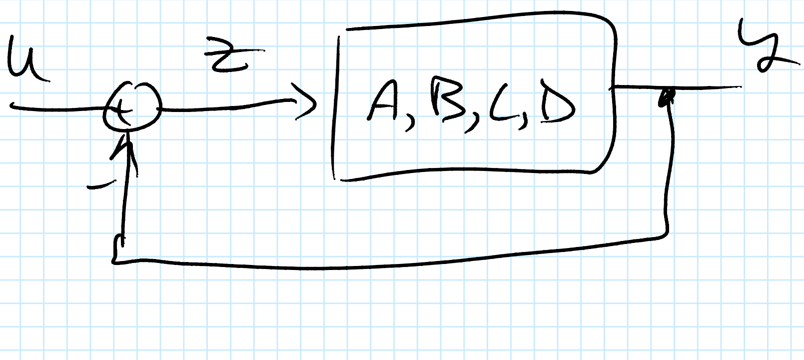
$$J\dot{\omega} = -K_t K_i K \omega + K_t K_i v_i + \tau_L$$

speed voltage (in) disturbance

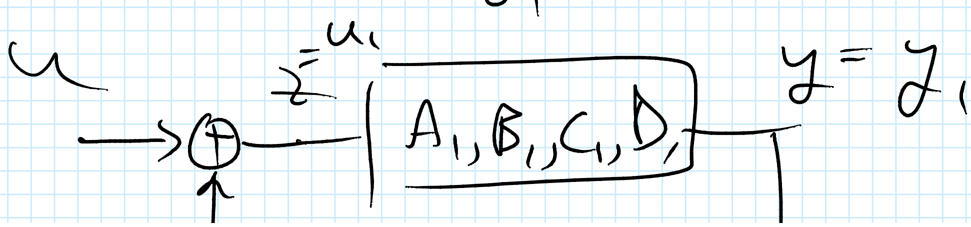
Assignment (Thurs 18th)

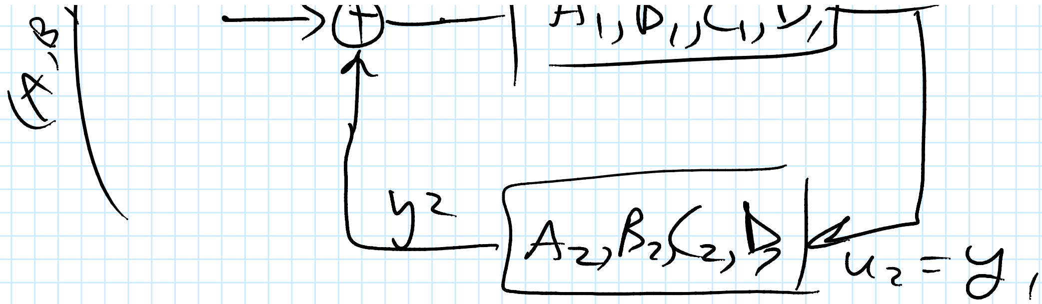
Exercise Fig 1.1d in text

For one of these find composite (A,B,C,D)



or





4.0 Analysis : Stability

2 kinds:
- Input-Output (BIBO) (zero state)
- or
- Internal (zero input)

<u>Def.</u> A relaxed systems (all I.C. = 0) is bounded-input / bounded output (BIBO) stable if and only if for any bounded input the output is bounded

Let

$$G(t) = \begin{bmatrix} g_{11}(t) & \cdots & g_{1m}(t) \\ \vdots & & \\ g_{q1}(t) & \cdots & g_{qm}(t) \end{bmatrix}$$

$q \times m$ impulse response of system

$$G(s) = \mathcal{L}\{G(t)\}$$ transfer matrix

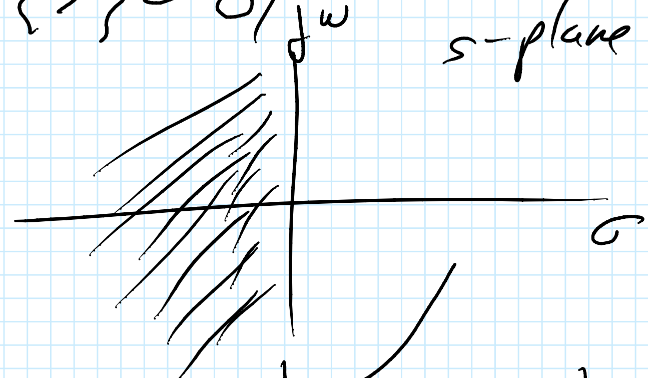
Theorem: The following are equivalent:

1) System is BIBO stable

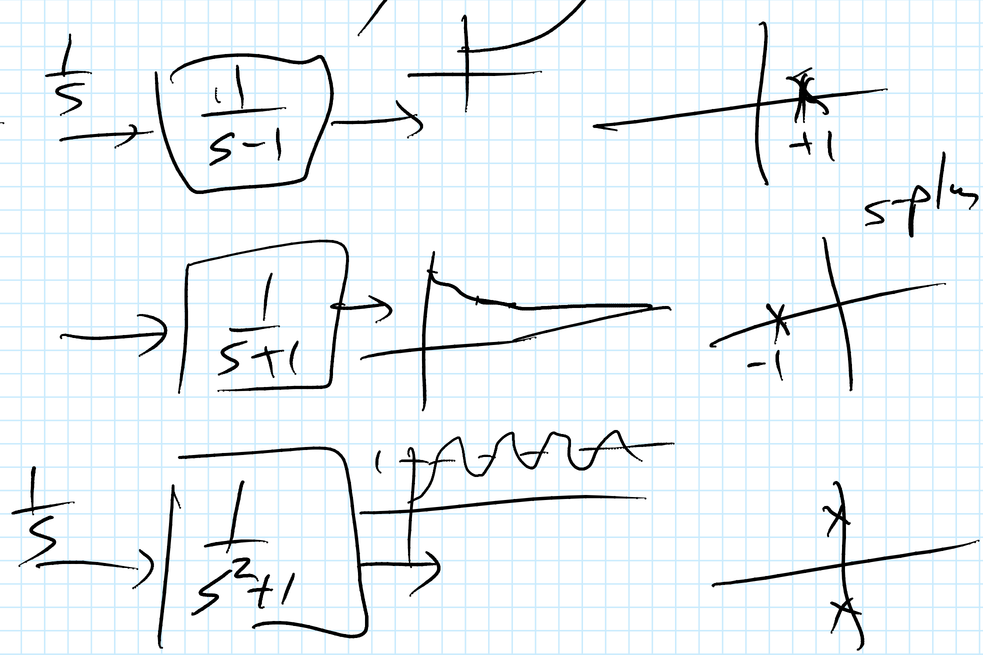
2) $\int_{-\infty}^{t} |g_{ij}(t)|\, dt < 0 \quad \forall t$ (for all)

i.e. $\|g_{ij}\|_{L_1}$ bounded

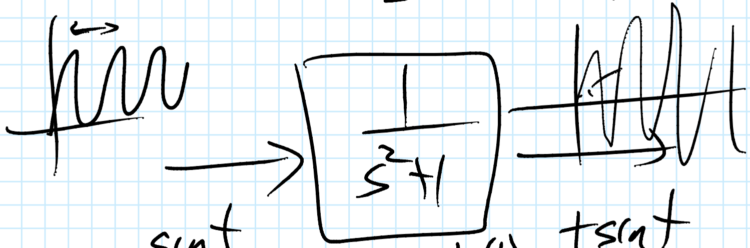
3) Each entry in $G(s)$ has poles in open left-half plane i.e. $\{s : \mathrm{Re}\{s\} < 0\}$



Ex.



by definition
seems O.K.



$\sin t$

$\to \frac{1}{s^2+1}$

$y(t) = t\sin t$

$y(t)$
$Y(s) = \left(\frac{1}{s^2+1}\right)^2$

• Internal stability

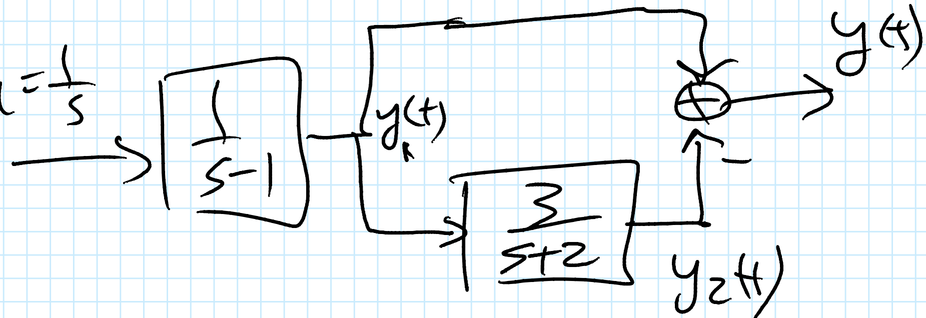
An unforced system $\dot{x} = Ax$
is ~~zero-input~~ stable if
$x(t) \to 0$ for all possible IC

Theorem $\dot{x} = Ax$ is zero-input
stable iff all the eigenvalues of
A are in open left-half plane

Q. Does BIBO stable $\overset{?}{\Longleftrightarrow}$ zero-input stable

A. No

Exercise $u = \frac{1}{s}$



Compute composite $H(s)$ ask: BIBO stab?

compute (A, B, C, D) ask: internally stab?

compute expressions in time domain for $y_1(t)$, $y_2(t)$, $y(t)$