SUMMARY: ARBITRARY POLE PLACEMENT

Transfer Function Given $P(s) = \frac{n_p}{d_p} = \frac{b_n s^n + b_n s^{n-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$

n-th order

Regulator (for stability)

$$\Rightarrow \quad C(s) = \frac{n_c}{d_c} = \frac{\beta_p s^p + \cdots + \beta_1 s + \beta_0}{\alpha_p s^p + \cdots + \alpha_1 s + \alpha_0}$$

Servo (for tracking)

let $R(s) = \frac{n_r}{d_r}$, order r

$$\Rightarrow \quad C(s) = \frac{n_c}{d_r d_1} = \frac{n_c}{d_c} = \frac{\beta_{r+p} s^{r+p} + \cdots + \beta_1 s + \beta_0}{d_r \cdot (\alpha_p s^p + \cdots + \alpha_1 s + \alpha_0)}$$

In each case, use $\boxed{p = n-1}$ and solve

$$\Delta_d(s) = n_p n_c + d_c d_p$$

This gives $(n+p+1)$ equations in $(n+p+1)$ unknowns by equating coefficients of like powers of s ($(n+p+r+1)$ for servo).

State Space

Given $\dot{x} = A\underline{x} + Bu$

Let $u = K\underline{x} + r$

$\Rightarrow \quad \dot{\underline{x}} = (A + BK)\underline{x} + Br = A_{cl}\underline{x} + Br$

Ackerman's formula

(1) Want $\alpha_d(s) = s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0$

(2) Form $\mathcal{C}(A, B) = [B \;\; AB \;\; A^2B \cdots A^{n-1}B]$

(3) Form $\alpha_d(A) = A^n + a_{n-1} A^{n-1} + \cdots + a_1 A + a_0 I$

(4) $K = -[0 \cdots 0 \;\; 1]\, \mathcal{C}^{-1}(A, B)\, \alpha_d(s)$