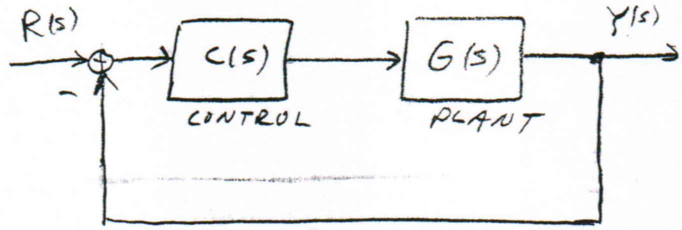
ARBITRARY POLE ASSIGNMENT

CONSIDER UNITY FEEDBACK SYSTEM:



WRITE $C(s)$ AND $G(s)$ IN TERMS OF THEIR NUMERATORS AND DENOMINATORS. I.E., LET

$$G(s) = \frac{n_p(s)}{d_p(s)} \quad \text{AND} \quad C(s) = \frac{n_c(s)}{d_c(s)}$$

THEN THE CLOSED-LOOP SYSTEM IS

$$\boxed{\frac{Y(s)}{R(s)} = \frac{CG}{1+CG} = \frac{n_c n_p}{d_c d_p + n_c n_p}}$$

FACT MOST IMPORTANT FEATURE IS THE CHARACTERISTIC EQTN OF CLOSED-LOOP SYSTEM.

$$\Delta(s) = 1 + CG = d_c d_p + n_c n_p = 0$$

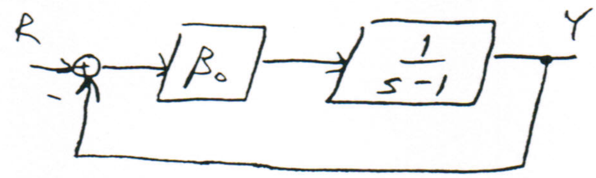
POLES OF CLOSED-LOOP DETERMINE:

(1) STABILITY

(2) TRANSIENT RESPONSE

QUESTION GIVEN G (i.e., n_p and d_p) WHEN AND HOW CAN WE PICK C (i.e., n_c and d_c) SO $\Delta(s)$ HAS DESIRED POLES?

1. Let $G(s) = \frac{1}{s-1}$ (unstable) and $C(s) = \beta_0$



(a) $\frac{Y(s)}{R(s)} =$

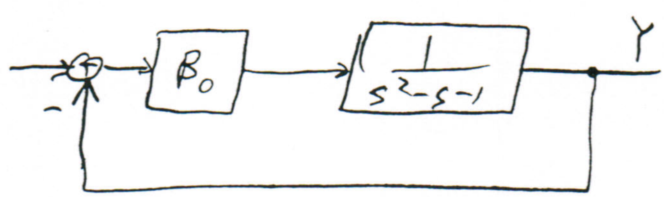
(b) What are the poles of the closed-loop as a function of β_0?

poles =

(c) Can you make the poles anything you want by picking β_0 properly?

Circle: YES / NO

2. Let $G(s) = \frac{1}{s^2-s-1}$ (unstable) and $C(s) = \beta_0$



(a) $\frac{Y(s)}{R(s)} =$

(b) Write $\Delta(s)$ as a polynomial whose coefficients depend on β_0.

$\Delta(s) =$

(c) $\Delta(s)$ in (2b) is a second-order monic polynomial of the form

$$\Delta(s) = s^2 + \delta_1 s + \delta_0 .$$

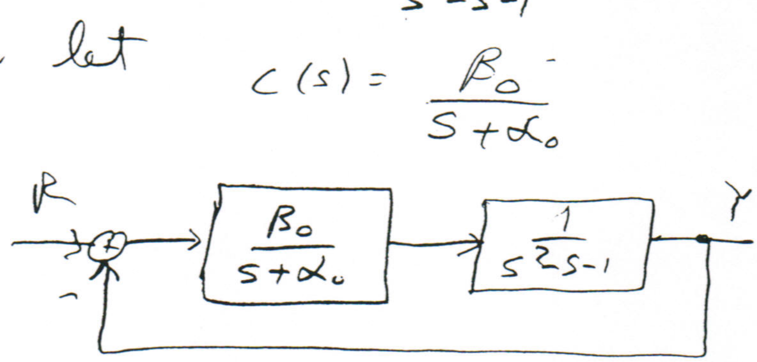
Notice that the two poles of $\Delta(s)$ can be completely specified by specifying the values of δ_1 and δ_2

For $\Delta(s)$ as given in (2b), is it possible to make the roots of $\Delta(s)$ any thing you want by properly choosing β_0? Circle : YES/NO

Is it possible to stabilize this system by proper choice of β_0? Circle : YES/NO

(3) Now let $G(s) = \frac{1}{s^2 - s - 1}$ as above, but now let

$$C(s) = \frac{\beta_0}{s + \alpha_0}$$



(a) Write $\Delta(s)$ as a polynomial whose coefficients depend on α_0 and β_0

$\Delta(s) =$

(b) Notice that now

$$\Delta(s) = s^3 + \delta_2 s^2 + \delta_1 s + \delta_0$$

How many coefficients must be assigned to specify the three poles of $\Delta(s)$?

# of coefficients =

(c) How many free parameters do we have available in our controller (i.e., α_0 and β_0)?

free parameters =

(d) Can you arbitrarily specify the 3 coefficients δ_2, δ_1, and δ_0 by choosing the free parameters α_0 and β_0?

Circle: YES / NO

(e) Why or Why not?

(4) Repeat Problem 3 using

$$C(s) = \frac{\beta_1 s + \beta_0}{s + \alpha_0}$$

(a) $\Delta(s) =$

(b) # of coefficients =

(c) # of free parameters =

(d) Arbitrary assignment of 3 coefficients using 3 free parameters?

Circle: YES/NO

(e) Why?

5 Now let $G(s) = \dfrac{1}{s^3 - s^2 - 2s - 3}$

(a) $C(s) = \dfrac{\beta_1 s + \beta_0}{s + \alpha_0}$

(i) What is order of $\Delta(s)$?

order =

(ii) How many coefficients must be prescribed in $\Delta(s)$ to specify its 4 poles?

$\boxed{\#\text{ of Coefficients} = \qquad\qquad}$

(iii) Can all the coefficients in $\Delta(s)$ be adjusted by proper choice of the 3 free parameters in $C(s)$? Circle: YES/NO

(iv) Why or Why NOT?

(b) $C(s) = \dfrac{\beta_2 s^2 + \beta_1 s + \beta_0}{s^2 + \alpha_1 s + \alpha_0}$. Repeat (i) – (iii)

(i) order $\Delta(s) =$

(ii) # of coefficients in $\Delta(s) =$

(iii) Can set coefficients of $\Delta(s)$ using 5 free parameters in $C(s)$?

Circle: YES/NO

6 Let $G(s) = \dfrac{s^2+2s+4}{s^3-2s^2-s+3}$

$$C(s) = \frac{\beta_2 s^2+\beta_1 s+\beta_0}{s^2+\alpha_1 s+\alpha_0}$$

Compute

$$\Delta(s) = d_c d_p + n_c n_p$$
$$= s^5+\delta_4 s^4+\delta_3 s^3+\delta_2 s^2+\delta_1 s+\delta_0$$

Expand $\Delta(s)$ out and equate like powers of s to show that

ATTACH YOUR WORK

$$\begin{pmatrix}\delta_4\\ \delta_3\\ \delta_2\\ \delta_1\\ \delta_0\end{pmatrix} = \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & 0\\ -2 & 1 & 2 & 1 & 0\\ -1 & -2 & 4 & 2 & 1\\ 3 & -1 & 0 & 4 & 2\\ 0 & 3 & 0 & 0 & 4\end{array}\right]\begin{pmatrix}\alpha_1\\ \alpha_0\\ \beta_2\\ \beta_1\\ \beta_0\end{pmatrix} + \begin{pmatrix}-2\\ -1\\ 3\\ 0\\ 0\end{pmatrix}$$

Use MATLAB to find the correct values of $\alpha_1, \alpha_0, \beta_2, \beta_1,$ and β_0 to give all the roots of $\Delta(s)$ at $s=-1$.

$C(s) =$

(8) Now let's generalize our results.

(a) Go back to part (2) through part (4). What was the order of the plant G?

$\boxed{\text{order } G(s) = \quad}$

(b) What was the order of the controller that finally worked?

$\boxed{\text{order } C(s) = \quad}$

(c) Repeat questions (8a) and (8b) for the problem of part (5)

(i) $\boxed{\text{order } G(s) = \quad}$

(ii) $\boxed{\text{order of } C(s) \text{ that worked } = \quad}$

(d) Suppose
$$G(s) = \frac{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

let
$$C(s) = \frac{\beta_p s^p + \beta_{p-1} s^{p-1} + \cdots + \beta_1 s + \beta_0}{s^p + \alpha_{p-1} s^{p-1} + \cdots + \alpha_1 s + \alpha_0}$$

What is the minimum value of p that will allow us to set the $n+p$ coefficients of $\Delta(s)$?

$\boxed{p \geq \quad}$