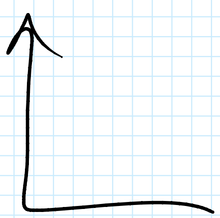
3-1-16 Lecture 12

Tuesday, March 1, 2016
2:11 PM

* Change: - Take-home exam give out on this Thurs 3-3

- Project Part 2 push to Thurs 3-10 (complete)



<u>Example</u>: Getting minimal realizations

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{2}{s+1} \\ \frac{-1}{(s+1)(s+2)} & \frac{1}{s+2} \end{bmatrix}$$

get ccf (use common denominator for columns)

$$G(s) = \begin{bmatrix} \frac{s+2}{s^2+3s+2} & \frac{2s+4}{s^2+3s+2} \\ \frac{-1}{s^2+3s+2} & \frac{s+1}{s^2+3s+2} \end{bmatrix}$$

$$\Rightarrow \quad A = \left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & -3 \end{array}\right] \quad B = \left[\begin{array}{c|c} 0 & 0 \\ 1 & 0 \\ \hline 0 & 0 \\ 0 & 1 \end{array}\right]$$

$$\Rightarrow A = \left[\begin{array}{cc:cc} - & - & - & - \\ 0 & 6 & 0 & 1 \\ 0 & 0 & -2 & -3 \end{array}\right] \qquad \left[\begin{array}{c:c} 0 & 0 \\ 0 & 1 \end{array}\right]$$

$$C = \left[\begin{array}{cc:cc} 2 & 1 & 4 & 2 \\ \hdashline -1 & 0 & 1 & 1 \end{array}\right]$$

check cont. ✓ (by construction)

obs ✗

$$\mathcal{O}(A,C) = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix} = \left[\begin{array}{cccc} 2 & 1 & 4 & 2 \\ -1 & 0 & 1 & 1 \\ \hline -2 & -1 & -4 & -2 \\ 0 & -1 & -2 & -2 \\ \hline 2 & 1 & 4 & 2 \\ 2 & 3 & 4 & 4 \\ \hline \end{array}\right] \begin{array}{l} \checkmark \longrightarrow v_1^T \\ \checkmark \longrightarrow v_2^T \\ \times \\ \checkmark \longrightarrow v_3^T \\ \times \\ \times \\ \times \\ \times \end{array}$$

rank = 3 ✓

$\Rightarrow$ not obs.

Find the obs. part

$$\Rightarrow \text{Define } V^{-1} = \begin{bmatrix} v_1^T \\ v_2^T \\ v_3^T \\ v_4^T \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 & 2 \\ -1 & 0 & 1 & 1 \\ 0 & -1 & -2 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

v_4 is arbitrary

$$\bar{A} = V^{-1}AV = \overset{A_{on}}{\left[\begin{array}{ccc:c} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -2 & -3 & 0 \\ \hdashline -1/2 & -1 & -1/2 & 2 \end{array}\right]} \qquad \bar{B} = V^{-1}B = \left[\begin{array}{cc} 1 & 2 \\ 0 & 1 \\ -1 & -3 \\ \hdashline 0 & 1 \end{array}\right] B_{on}$$

$$\bar{C} = CV = \left[\begin{array}{ccc:c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$

C_m

claim (A_m, C_m) is obs. and

$$G(s) = C_m (sI - A_m)^{-1} B_m$$
$$= C(sI - A)^{-1} B$$

Exercise: Try to get an $(A\,B\,C\,D)$ using Gilbert Diagonal form

6.2 Transfer Matrix Perspective on MIMO Systems

Aside

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$b_1 \quad b_2$

(A, B) cont
(A, b_1) not cont
(A, b_2) not cont

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \;\&$$

$A = \begin{bmatrix} & & \\ 0 & 0 & 3 \end{bmatrix}$

(A, b_1) cont $\qquad b_1$

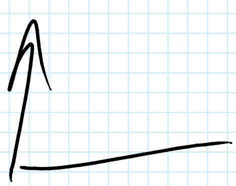
$u = Kx$

$$= \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

consider $\quad K = \begin{bmatrix} 0 & k_1 & 0 \\ k_2 & 0 & k_3 \end{bmatrix}$

x_2-related control action comes through b but other comes through k_2

all control action comes through b_1 $\longrightarrow$ $K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ 0 & 0 & 0 \end{bmatrix}$



Ex:
$$\tau_{11} \dot{y}_1 + y_1 = K_{11} u_1 + K_{12} u_2$$
$$\tau_{12} \dot{y}_1 + \tau_{22} \dot{y}_2 + y_2 = K_{21} u_1$$

$\mathcal{L}$

$$D_L(s) \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = N_L(s) \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$D_L(s)\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = N_L(s)\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\left[\begin{array}{c|c} \tau_{11}s+1 & 0 \\ \hline \tau_{12}s & \tau_{22}s+1 \end{array}\right]\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & 0 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

matrix fraction $\quad Y(s) = D_L^{-1}(s)\, N_L(s)\, U(s)$

$$Y(s) = G(s)\, U(s)$$

transfer matrix

- state-space to matrix fraction

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

y, u are vectors

$\Rightarrow$ can find

$$Y(s) = G(s)\, U(s)$$
$$= D_L^{-1}(s)\, N_L(s)\, U(s) \quad \text{left MF}$$
$$= N_R(s)\, D_R^{-1}(s)\, U(s) \quad \text{right MF}$$

ASIDE

in SISO $g(s) = \frac{n(s)}{d(s)} = d^{-1}(s)\, n(s) = n(s)\, d^{-1}(s)$

Given $G(s) = C(sI-A)^{-1}B + \cancel{D}$ →0 for now

Define

where $A-JC$ stable

$$D_L(s) = I - C(sI-A+JC)^{-1}J$$
$$N_L(s) = C(sI-A+JC)^{-1}B$$
$$\Bigg\} \quad G = D_L^{-1} N_L$$

$D_R =$

$N_R =$