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Generic Methodology for Control System Development

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The twentieth century has been an era of dramatic technological development. Arguably, automation has been at the forefront of spurring on the technical advances of this century. Central to automation is feedback control. Applications of automation and control range from household appliances, to advanced aerospace and defense technology, to industrial process control, to robotics and automation in manufacturing. In this paper we give an overview of control systems and the perspective of the “MAD” control theorist on the control system design. We also present a systematic methodology for control systems engineering, including two subsections, one about defining objectives and one giving a specific approach called the integrated control design procedure, developed by Thomas Marlin of McMaster University¹. We conclude with a section that summarizes key aspects of modelling, with a focus on process control models. As a disclaimer, note that this write-up is still under development and should not be viewed as complete or conclusive. Also, all ideas presented here have been developed a personal perspective of the author, with the exception of the integrated control design procedure, control design form, and checklists taken from Marlin.

1 Control Systems

To begin it is useful to define several terms. First, let us note that

a system is a collection of interconnected components, working together towards some common objective.

From this context we define

a control system is a system whose components have been deliberately configured to collectively achieve a desired objective.

¹*Process Control: Designing Processes and Control Systems for Dynamic Performance*, Thomas E. Marlin, McGraw-Hill, New York, 1995.

These definitions are generic and apply to a variety of systems. The field of study called *general systems theory* is concerned with applying such definitions to biological systems, socio-economic systems and organizations, and others. Here, however, we restrict our discussion to engineering applications. Specifically, we are most interested in the case where we are given a physical system and our goal is to find a way to force a desired behavior or response. We call this *control system engineering* (or controller design). Fundamentally, the goal of control systems engineering is

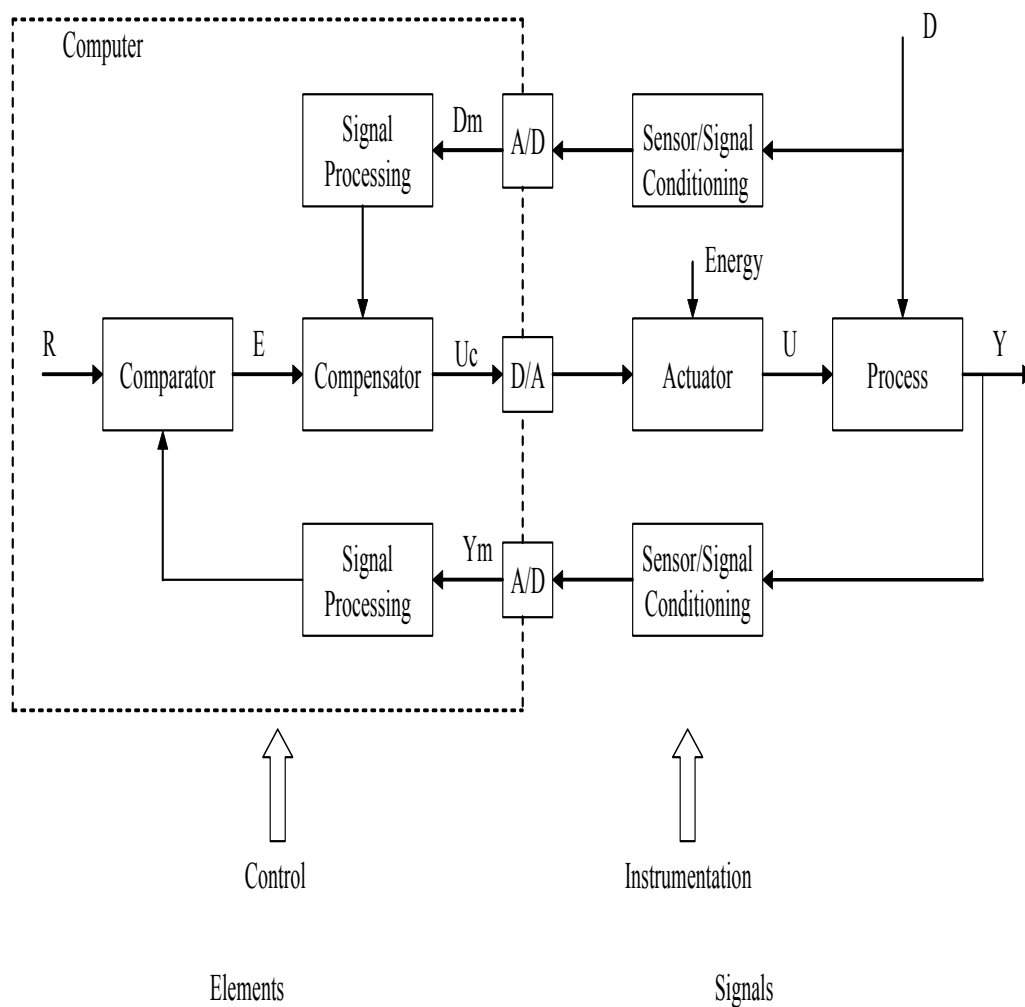
to modify a given physical system by adding a controller so that the resulting new system (physical system plus controller) will have a desired behavior.

Figure 1 shows a block diagram of a generic control system. Beginning on the right-hand side of the figure we see the *process* that is to be controlled. This is also called the *system* or the *plant*. This is usually what we begin with in the control design process. There is an assumed *output* from the process. This signal, Y , is what we want to control to behave in a desired way.

The process shown in Figure 1 is acted on by two *inputs*, U and D . The input U comes from an *actuator*, which is a device that “acts” on the process to affect the system behavior. Usually an actuator is an energy conversion device. The actuator signal is also called the *manipulated variable* and the actuator may be called the *final control element*. The actuator signal is what we adjust or manipulate to force changes in the process behavior. The other input, D , is called a *disturbance*. It is understood to be an independent energy source that affects the process behavior, but which we cannot manipulate. Moving to the left of the process we see two measurement blocks labeled *sensor/signal conditioning*. Like the actuator block, these denote physical hardware. The measurement blocks will typically include some type of *transducer* that responds to a physical variable as well as an *energy conversion*. These are collectively called a *sensor*. Often the sensor signal is also acted on by some type of filtering or other *signal conditioning* to make the signal suitable for processing at the next stage. The figure indicates two sensor/signal conditioning blocks. While we always assume we have some type of measurement of the system behavior (Y), many times we do not have a measurement of the disturbance D . In industrial environments, the term *instrumentation* is often used to describe the physical actuator and measurement hardware.

Almost all control systems today use some type of digital computations to implement their algorithms. Thus it is necessary to convert between analog and digital signals in the overall system. In Figure 1 these conversions are indicated by the blocks labeled A/D and D/A , representing analog-to-digital and digital-to-analog, respectively. These blocks define the interface between the instrumentation (actuators and measurement devices) and the decisionmaking and control logic executed in a computer or other digital device.

Moving inside the control block in Figure 1 we see two blocks labeled *signal processing*. These blocks may carry out various filtering, scaling, or biasing operations on the incoming signal, often for the purpose of converting the signals to engineering units. Some of these operations, such as *anti-aliasing filtering* may also be integrated into the A/D blocks. Finally,



Comparater
 Compensator
 Software Signal Processing
 A/D and D/A Signal Conversions
 Hardware Signal Conditioning
 Sensor/Transducer
 Actuator (Final Control Element)
 Process, Plant, or System

R: reference, set point,
 E: error; often $E=R-Y_m$
 U_c : commanded actuator signal
 U: plant input
 D: disturbance
 Y: plant output
 Y_m : measured output
 D_m : measured disturbance

Figure 1: Block diagram of a generic control system.

the measured, conditioned, and processed value of the measured process output, Y_m , is passed to the *comparator* block, where it is compared to the reference R , to form an *error* signal, E . Most often the comparator block simply performs a subtraction, so that the error is just $E = R - Y_m$. The final block to discuss is the *compensator*. The compensator is the heart of the control system. The compensator uses the error signal as well as any disturbance measurements, D_m , that may be available and then decides on appropriate adjustments to the commanded actuator signal, U_c . These decisions are usually embodied in algorithms called the *control law*. The combination of the comparator, the compensator, and any associated signal processing is called the *controller* or simply the *control*. Finally, the system is said to be *closed-loop* if the controller is connected to the plant via the actuator and measurement blocks. If these blocks are disconnected, then we say the system is operating in an *open-loop* mode.

Referring again to Figure 1, we would point out several features. First, in general all the signals shown could be vectors. If they are scalars we refer to the system as *single-input, single-output (SISO)*. If they are vectors, then the system is said to be *multivariable* or *multi-input, multi-output (MIMO)*. Also, not all the blocks or the signals shown in the figure are necessarily present in every application. In particular, in many discussions we often assume that the disturbance is not present, the actuator and measurement blocks are lumped in with the plant, the A/D and D/A conversions are transparent, and the comparator is simply a subtraction. Thus, Figure 1 effectively reduces to Figure 2. This is a simple *unity feedback* configuration and is the standard block diagram used to illustrate the basic concepts of control theory in undergraduate courses in feedback control. This figure allows us to illustrate the distinction between two different, but related problems:

1. Control System Engineering: *Given a process, determine all the other blocks in Figure 1 so that the closed-loop system has desired properties.*

This is to be distinguished from

2. Control System Design: *Given P , determine C in Figure 2 so that the closed-loop system desired properties.*

Although Problem 1. is what must be solved in applications, Problem 2. is what is typically covered in most introductory course control systems.

We conclude this section by referring again to Figure 2. We may think of the control system as fundamentally consisting of three parts. First, the plant produces an output in response to its input. This output is the behavior to be controlled. Second this behavior is compared to the reference input, which we can think of as the desired behavior or system objective. Third, the error between the actual behavior and the desired behavior is then used by the compensator, which produces a signal to the system that acts to correct the behavior. Notice again that this is a generic description. In engineering we are primarily concerned with the control of physical systems, in which the outputs to be controlled are physical

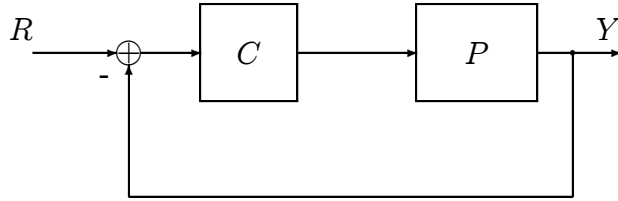


Figure 2: Simplified control configuration.

quantities such as voltage, velocity, chemical concentration, etc. However, in general, these ideas are applicable to a wide variety of systems, such as economic systems, natural resource management, decision support systems, etc. In these more abstract settings the inputs and outputs may be described as objectives and performance, respectively. Thus, control system engineering is a true interdisciplinary field, although applications have been in the engineering arena.

2 Control System Design: The “MAD” Control Theorist

We continue by considering Problem 2. defined above. Our particular perspective is that there are three essential activities required to design a control system. Referring to Figure 2, given a physical system P that we want to control, along with a desired behavior or performance for the controlled system, we determine a control law, C , that will cause the closed-loop system to exhibit the desired behavior by:

1. **Modelling** (mathematically) the system, based on measurement of essential system characteristics.
2. **Analysis** of the model to determine the properties of the system.
3. **Design** of the controller which, when coupled with the model of the system, produces the desired closed-loop behavior. This will involve development of
 - (a) Control law algorithms.
 - (b) Measurement and testing techniques for the specific physical system.
 - (c) signal processing and Signal conditioning algorithms necessary for interfacing the sensor and controller to the physical system and to each other.

- (d) Simulation studies of the individual components of the control system as well as simulation of the closed-loop system in which all the components are interconnected. Simulation studies are an essential part of the design and development process and are highly dependent on the models obtained from the measurement process.

Note that in addition to these three activities (modelling, analysis, and design), there are two other key activities in the controller development process, although these are associated more with Problem 1. (the control engineering process) than with Problem 2. These are:

1. **Development of performance specifications** that define the objective of the control design.
2. **Implementation of the controller** through software and hardware realizations of the control law, including complete specification of the sensor, signal processing, and control elements, and final assembly, testing and validation, delivery, and operation of the control system.

The five activities described above are summarized in Figure 3, which shows an overall conceptual flowchart of the control system design process. As shown in the figure, starting with a system we wish to control (defined as including the plant, sensors, and actuators), we proceed with two tasks in parallel: defining the required performance specifications and developing a model of the process. The modelling activity will often include some form of measurement to determine key system properties. Note that mathematical modelling is a particularly important part of the process of control system development. By having a framework for describing the system in a precise way, it is possible to develop rigorous techniques for analyzing and designing systems. Once a math model is available and we have decided the goal of the design, it is possible to proceed with the analysis of the model and design of the control law. As shown in the figure, simulation cannot be separated from analysis and design, and the process of arrive at a math model of the controller is itself a feedback process. Once a controller model is defined it is necessary to evaluate its effectiveness in combination with the math model of the process (via simulation of the complete control system) before proceeding to implementation.

3 The Control System Engineering Process: A Systematic Design Methodology

In this section we consider the control system engineering process, with a focus toward industrial practice. Consider that in order to develop a feasible control strategy for a given problem it is important to establish a framework for the project that makes sense from the perspective of users who will ultimately benefit from the control system. The following comments identify some important issues that should be addressed in order to develop a

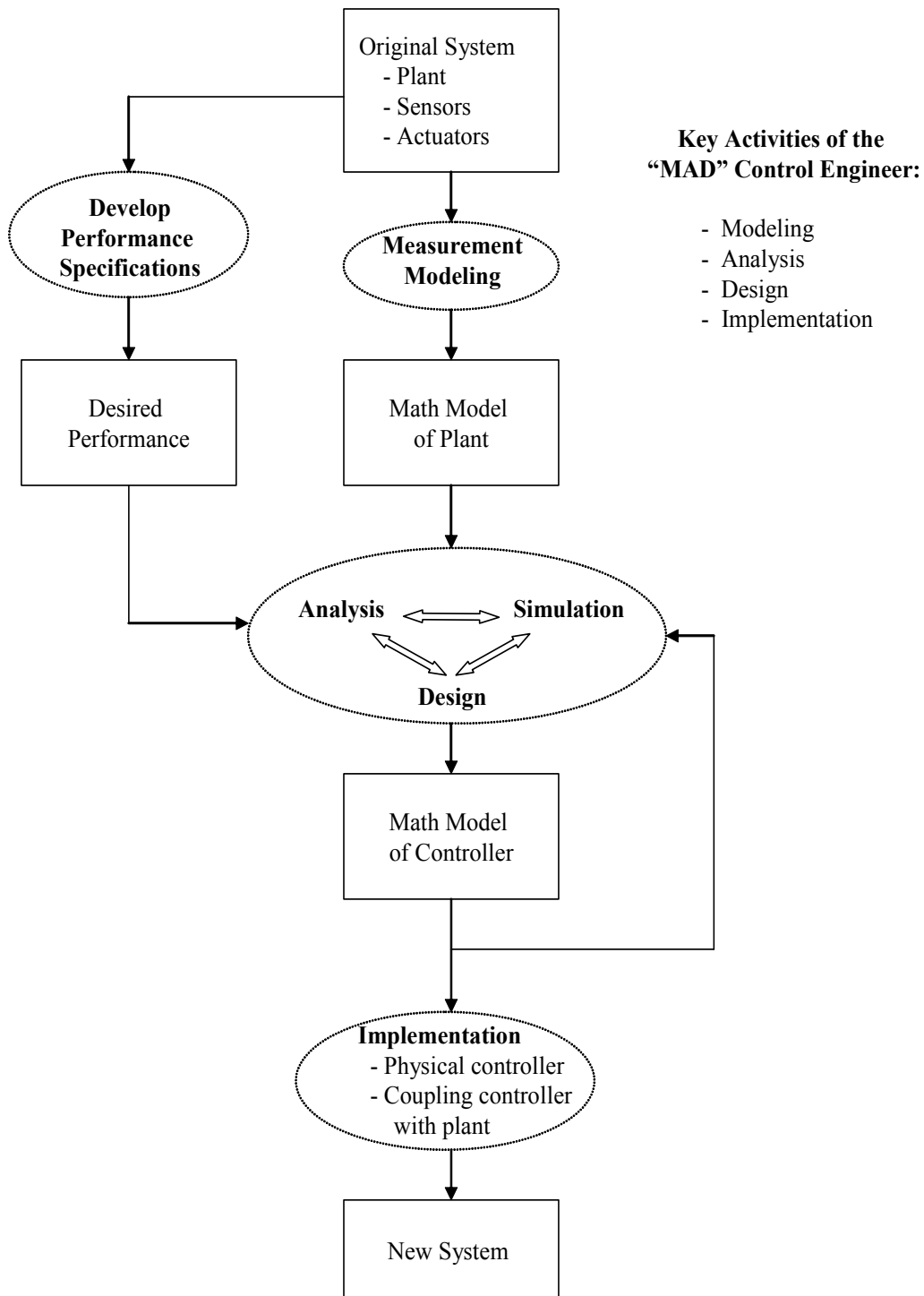


Figure 3: Flowchart of the control system design process.

control scheme that can be implemented and that will be effective in an operational setting. Much of this material is based on and, in some cases, taken from Chapters 24-26 of the excellent text by Marlin¹. We begin with a brief discussion about defining objectives. We then present Marlin's integrated control design procedure.

3.1 Objectives and Functional Specifications

Before any control system development project is started, it is important determine what it is that is to be accomplished. Some questions to consider include:

1. What is the primary objective of the project? That is, what is the purpose or goal of the control scheme:
 - (a) Economic improvement (long-term or short-term goal)?
 - (b) Improved consistency of system outputs or operation?
 - (c) Reduced variability in process outputs?
 - (d) Improved response to input changes?
 - (e) Ability to automatically correct for irregularities in inputs?
 - (f) Ability to reject disturbances?
 - (g) Ability to track time-varying inputs or reject time-varying disturbances?
 - (h) Simply to stabilize or to also optimize?
 - (i) Is robustness an issue?
 - (j) For a given class of systems that you want to control, is the objective always the same? I.e., are there different objectives for different systems in the same class?
 - (k) For a specific system in the class, is the objective always the same. I.e., are there different objectives at different operating ranges (is there an optimal operating regime for a given system)?
2. Given a succinct statement of the objective of the project, how does this translate into well-defined functional specifications that can be used to develop control system performance requirements:
 - (a) What aspects of the system do you want to be able to control in order to meet the primary objective?
 - (b) What system variables must be controlled to meet the primary objective?
 - (c) What criteria defines that the objective has been met (what kind of constraints do you specify for the variables you wish to control)?
 - (d) Can you quantify a cost or performance index?
 - (e) Does the cost index lead to a solvable problem?
 - (f) Can you express the objective in terms of signal norms?

3.2 A Systematic Approach

Answering questions such as those listed above is ultimately more important than the actual control system algorithm design. Consequently, it is helpful to have a procedure that can lead the designer to systematically develop a statement of the control system objectives and then through the remainder of the control system development process. One such procedure, taken from Marlin, emphasizes the following steps:

1. Form a **definition** of the design problem.
2. Determine the **feasibility** of the project.
3. Obtain an **overview** of the problem.
4. Specify an appropriate **control structure** and algorithms.
5. Determine **optimization** strategies.
6. Develop approaches to **monitoring and diagnosis**.

These steps are shown in more detail in Figure 4. Although this procedure may not be completely appropriate for every situation, it does provide an indication of important issues that the designer should consider.

Echoing the comments of the previous section, notice that the first step in the procedure presented in Figure 4 is problem definition. To assist in this process, Marlin has developed the concept of the Control Design Form (CDF). A sample CDF is shown in Figure 5. The CDF prompts the control designer to identify the following items in defining the problem:

1. Control objectives.
 - (a) Safety of personnel.
 - (b) Environmental protection.
 - (c) Equipment protection.
 - (d) Smooth, easy operation.
 - (e) Product quality.
 - (f) Efficiency and optimization.
 - (g) Monitoring and diagnosis.
2. Measurements.
3. Manipulated variables.
4. Constraints.
5. Disturbances.

6. Dynamic responses.

7. Additional considerations.

Of particular interest is the correct identification of control objectives. Marlin has provided some checklists that help prompt the design to consider each objective listed above. These are shown in Figure 6. Note that the other parts of the CDF are related to process modelling. The next section provides information on process modelling for control purposes and can be used as an aid in completing the remainder of the CDF after the objectives have been defined.

TABLE 25.6
Integrated control design procedure

START: Acquire information about the process

- (a) Process equipment and flow structure
- (b) Operating conditions
- (c) Product quality and economics
- (d) Preliminary location of sensors and final elements

[Modify process and instrumentation] ←

1. DEFINITION: Complete the Control Design Form

- (a) Use checklists
- (b) Sample questions
- (c) Prepare a preliminary set of controlled variables

[Modify objectives] ←

2. FEASIBILITY: Determine whether objectives are possible

- (a) Degrees of freedom
- (b) Controllability
- (c) Operating window for key operating conditions

[Iterate] →

3. OVERVIEW: Develop understanding of entire process to enable "look-ahead" in decisions

- | | |
|--|---|
| (a) Key production rate variables | (e) Key product qualities |
| (b) Inventories for potential control | (f) Key constraints |
| (c) Open-loop unstable processes | (g) Key disturbances |
| (d) Complex dynamics (long delays, inverse response, recycle, strong interactions) | (h) Useful manner for decomposing the analysis (and control design), if necessary and appropriate |

4. CONTROL STRUCTURE: Selection of controlled and manipulated variables, interconnections (pairings in decentralized control), and relevant tuning guidelines

- (a) Preliminary decisions on overall process flows and inventories
- (b) Process segment (Unit) 1
- (c) Process segment (Unit) 2

Control Hierarchy (temporal decomposition) for every unit

- | | |
|------------------------|--------------------|
| 1. Flow and inventory | 3. Product quality |
| 2. Process environment | 4. Safety |

[Iterate] →

[Modify control] ←

- (d) Integrate control designs as needed for good overall performance

5. OPTIMIZATION: Strategy for excess manipulated variables

- (a) Clear strategy for improved operation, or
- (b) Measure of profit using real-time data
- (c) Sensors and final elements
- (d) Minimize unfavorable interaction with product quality

[Iterate] →

6. MONITORING AND DIAGNOSIS

- (a) Real-time operations monitoring
 - 1. Alarms
 - 2. Graphic displays and trends
- (b) Process performance monitoring
 - 1. Variability of key variables (histogram and frequency range)
 - 2. Calculated process performances (efficiencies, recoveries, etc.)

[Iterate] →

FINISH: Completed specification, meeting objectives in step 1

- | | |
|--|--------------------------------|
| (a) Process equipment and operating conditions | (e) Safety controls and alarms |
| (b) Control equipment, sensors, and final elements | (f) Optimization |
| (c) Control structure and algorithms | (g) Monitoring calculations |
| (d) Tuning guidelines as needed, e.g., level control and interacting loops | |

Figure 4: A systematic control system design procedure (from Marlin, 1995).

TABLE 24.1
Preliminary Control Design Form for the flash process in Figure 24.1

TITLE: Flash drum PROCESS UNIT: Hamilton chemical plant DRAWING: Figure 24.1		ORGANIZATION: McMaster Chemical Engineering DESIGNER: I. M. Learning ORIGINAL DATE: January 1, 1994 REVISION No. 1	
Control Objectives			
1.	Safety of personnel (a) The maximum pressure of 1200 kPa must not be exceeded under any (conceivable) circumstances.		
2.	Environmental protection (a) Material must not be vented to the atmosphere under any circumstances.		
3.	Equipment protection (a) The flow through the pump should always be greater than or equal to a minimum.		
4.	Smooth, easy operation (a) The feed flow should have small variability.		
5.	Product quality (a) The steady-state value of the ethane in the liquid product should be maintained at its target of 10 mole% for steady-state operating condition changes of (i) +20 to -25% feed flow (ii) 5 mole% changes in the ethane and propane in the feed (iii) -10 to +50°C in the feed temperature (b) The ethane in the liquid product should not deviate more than ± 1 mole% from its set point during transient responses for the following disturbances: (i) The feed temperature experiences a step from 0 to 30°C. (ii) The feed composition experiences steps of +5 mole% ethane and -5 mole% or propane. (iii) The feed flow set point changes 5% in a step.		
6.	Efficiency and optimization (a) The heat transferred should be maximized from the process integration exchanger before using the more expensive steam utility exchanger.		
7.	Monitoring and diagnosis (a) Sensors and displays needed to monitor the normal and upset conditions of the unit must be provided to the plant operator. (b) Sensors and calculated variables required to monitor the product quality and thermal efficiency of the unit should be provided for longer-term monitoring.		
Measurements			
Variable	Sensor principle	Nominal value and sensor range	Special information
A1	Chromatograph	10, 0-15 mole%	Update every 2 minutes
F1	Orifice	100, 0-200	
F2	Orifice	120, 0-150	
F3	Orifice	100, 0-200	
F4	Orifice	45, 0-90	
F5	Orifice	55, 0-110	
L1	Δ pressure	Range is lower half of drum	
P1	Piezoelectric	5000-15000 kPa	

TABLE 24.1 (CONTINUED)

Measurements			
Variable	Sensor principle	Nominal value and sensor range	Special Inform.
T1	Thermocouple	0, (-50-100 °C	
T2	Thermocouple	25, 0-100 °C	
T3	Thermocouple	90, 0-200 °C	
T4	Thermocouple	45, 0-200 °C	
T5	Thermocouple	25, 0-100 °C	
T6	Thermocouple	25, 0-100 °C	
Manipulated variables			
I.D.	Maximum capacity (at design pressures)		
	(%open, maximum flow)		
v1	100%, 100		
v2	53%, 189		
v3	50%, 200		
v4	14%, 340		
v5	52%, 106		
Constraints			
Variable	Limit values	Measured/Inferred	Hard/soft Penalty for violat
Drum pressure	1200 kPa, high	P1, measured	Hard Personnel injury
Drum level	15%, low	L1, measured	Hard Pump damage
Ethane in F5 product	±1 mole%, (max deviation)	A1, measured, and T6, inferred	Soft Reduced selectivity downstream reactor
Disturbances			
Source	Magnitude	Dynamics	
Feed temperature (T ₁)	-10 to 55°C	Infrequent step changes of 20 °C magni	
Feed rate (F ₁)	70 to 180	Set point changes of 5% at one time	
Feed composition	±5 mole% feed ethane	Frequent step changes (every 1 to 3 hr	
Dynamic responses (Input = all manipulated variables and disturbances) (Output = all controlled and constraint variables)			
Input	Output	Gain	Dynamic mot
v1	{see Example 24.7}		
v2			
v3			
v4			
v5			
Additional considerations			
Liquid should not exit the drum via the vapor line.			

Figure 5: Sample control design form (from Marlin, 1995).

TABLE 25.2
Checklist for smooth operation

Unstable processes (do not reach steady state without control)
• Levels
• Chemical reactors
Processes that are very sensitive to disturbances
• Maximum rate of change of disturbance
Process integration that either propagates or attenuates disturbance
Manipulated variables that are easily interpreted by operating personnel
Disturbance sources

TABLE 25.3
Checklist for product quality

Target average value and variability
• One or multiple specifications
• Average value
• Variability
• \pm deviation from target at which product is unacceptable
Variability in a property that affects future use by customer
• Standard deviation or other measure
• Nonlinearity between measurement and quality in future use
Disturbances that affect quality
• Magnitude
• Frequency
Factors affecting control performance
• Availability of on-stream measurement
• Degrees of freedom
• Controllability
• Feedback dynamics
• Modelling errors

TABLE 25.1
Checklist for safety, equipment, and environment

Limitations on operating conditions due to equipment, material, e.g.,
• Composition
• Flow
• pH
• Pressure
• Temperature
Explosion
• Fuel source
• Oxidizing source
• Energy source
Release of hazardous material
Failure of process equipment
Failure of control equipment
Human mistakes and their consequences

TABLE 25.4
Checklist for efficiency and optimization

External manipulated variables not used for control, potentially for optimization
Changes in targets or inputs (disturbances)
• Frequency
• Need for optimization
• Complexity of optimizing strategy
Parallel units
• Different product quality
• Different yields
• Energy consumption
Recycle flows
• Composition of recycle
Separation units
• Energy-yield tradeoff
Chemical reactors
• Conversion
• Yield
Operating condition
• Internal optimum
• Operation at a constraint

TABLE 25.5 Checklist for monitoring and diagnosis
Performance that changes rapidly
• Alarms
• Emergency shutdowns
• Constraint violations
• Product quality
• Inventories
Performance that changes slowly
• Heat transfer coefficient
• Catalyst activity
• Corrosion
• Coking or fouling
Performance requiring complex calculations
• Fired heater efficiency
• Turbine and compressor efficiency
Utilization of control
• Percent of time in automatic
Temporal correlation of good or poor operation with external disturbances (feed type, equipment operation, and so forth)

Figure 6: Control objectives checklist (from Marlin, 1995).

4 Process Modelling

Figure 7 shows a generalized system configuration for control. In this figure P is the plant (system to be controlled) and C is the controller. We identify the vector signals w, u, z , and y as

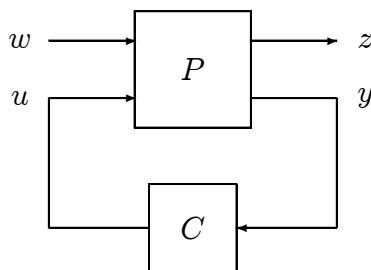


Figure 7: A generalized controller configuration.

- w : exogenous inputs which cannot be changed by the controller (typically disturbances or reference commands).
- u : control inputs which can be influenced by the controller.
- z : controlled outputs which are not directly measured.
- y : measured outputs.

The signals z and y are not necessarily distinct. This is the standard framework used in most of the recent approaches to controller design, particularly in H_∞ control theory.

The typical task in control system development is to pick the controller C , given a model of the process (also called the plant P), so that some type of performance objective is met. In order to carry out such a program, it is important to have as much information as possible about the dynamics of the process to be controlled (sometimes called the system). Of primary importance are (i) the system inputs; (ii) the system outputs; and (iii) process characteristics, including steady-state or DC gain, system time constant, and transportation or time delays.

4.1 Inputs

Relative to *inputs*, the following issues should be addressed:

1. What are the inputs to the system?

2. What variables can be controlled or adjusted to affect the process.
3. Which of these can be controlled automatically?
4. How often can the inputs be changed?
5. How much accuracy do we have in specifying the inputs (i.e., if we specify a particular value, how sure are we that we get that value and how far off might it be)?
6. What type of disturbance inputs do we expect to affect the process (i.e., inputs that we might or might not be able to measure, but which we cannot control)?
7. Are there limits on the values that can be commanded (actuator limits)?
8. What are the correct units to use for these variables?
9. How do these answers vary between a real system as found in industry versus assumptions made in modelling?

4.2 Outputs

Relative to *outputs*, the following issues should be addressed:

1. What are the outputs for the system?
2. What variables would we like to measure?
3. What variables can be measured?
4. What variables cannot be measured, but can be inferred from measurements of other variables?
5. Which of the measurements can be done automatically?
6. How often can we take measurements of each variable?
7. How much accuracy do we have in measuring the variables?
8. What type of noise or disturbances do we expect to affect the measurements?
9. Are there max/min limits on the variables to be measured?
10. What are the correct units to use for these variables?
11. How do these answers vary between a real system as found in industry versus assumptions made in modelling?

Other issues related to inputs and outputs include:

1. Is this a local problem or a global problem?
2. Does the system operate around a setpoint?
3. Is the system subject to load disturbances?
4. Does the system exhibit significant time delay (transportation lag)?

4.3 Process Model

Once inputs and outputs are identified, an initial attempt can be made to develop a model for the system. Some key questions here include:

1. Is the model dynamic or static?
2. Is the model linear or nonlinear?
3. Is the model finite-dimensional?
4. Does the model change or degrade with time?
5. What kind of uncertainty do you have in the model: structured or unstructured or parametric?
6. Is the model stochastic?
7. Is the model qualitative or linguistic?
8. Do human operators perform well controlling the system manually?

For process control system, it is common to assume a first order system with time delay (see the next subsection for a description of these parameters). In this case the we can develop a transient model using data obtained from step response experiments. For a multivariable system the model is summarized in transfer matrix, using Laplace transforms. For a two-input, two-output system, such a transfer matrix is shown below. In this transfer matrix the DC gains are based on percent changes in the inputs from a nominal condition and the outputs are also defined as percent changes based on the nominal output.

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{K_{11}e^{-T_{11}s}}{(\tau_{11}s+1)} & \frac{K_{12}e^{-T_{12}s}}{(\tau_{12}s+1)} \\ \frac{K_{21}e^{-T_{21}s}}{(\tau_{21}s+1)} & \frac{K_{22}e^{-T_{22}s}}{(\tau_{22}s+1)} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

In a process control situation, what we need to begin the controller design process is a complete and correct version of this transfer matrix for a practical operating point that makes sense for the industrial system. That is, we need to know answers to the following questions

1. For each input, if all others are held constant, and the input in question is given a step change from its nominal value (measured in percent change in the input), find:
 - (a) For each output, the percent change in its final steady-state value.
 - (b) For each output, the time it takes before any change is observed.
 - (c) For each output, the time it takes before the output comes to its final value.
2. Do any of the responses exhibit overshoot (indicating that the first-order model assumption is invalid)?
3. What nominal operating points should be used in developing an initial model.
4. Should you concentrate on developing a transient model for a typical industrial system as opposed to a general class of systems that includes the system you are interested in controlling.

Note that not all systems are easily modelled using first-order approximations with time delay. However, such questions can sometimes make sense for other classes of systems.

4.4 Process Characteristics

There are three specific parameters that are typically used to specify the transient behavior of a process: the steady-state or DC gain, denoted K ; the system time constant, denoted τ , but also characterized by the system settling time, $T_s = 5\tau$; and the system transportation or time delay, T_d . These are briefly described as:

1. *Steady-State Gain*: This parameter specifies the effect the steady-state value of the input variable has on the steady-state value of the output variable.
2. *Time Constant*: This is better described using the concept of settling time, which is the time it takes a process to reach a new steady-state after a change has been made in the input value. The settling time is approximately equal to five time constants. Note that these times are measured after the time delay.
3. *Time Delay*: This is the time it takes for the output to begin to change following a change in the input.

Note that in process control we are usually interested in the effect that changes in inputs have on the outputs. Thus we assume the system is operating at some nominal condition

when it encounters a change in an input value. We then typically model the resulting change in the output value from its nominal value. These concepts are illustrated in Figures 8 and 9. In Figure 8 the input is denoted as $u(t)$ and is shown to consist of a nominal value u_0 and a step change Δu added to the nominal value. Likewise, the output response is the superposition of the nominal response y_0 and the resulting change in the output Δy due to the change in input, Δu . Figure 9 illustrates a typical input/output response for a first-order

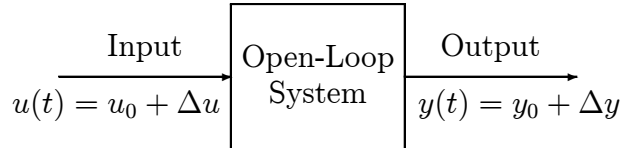


Figure 8: Configuration for transient response measurement.

system with time delay that is subject to a step change in the input at time $t = t_0$. In this figure we see the time delay T_d as well as the settling time T_s . The final value of the change in the input, Δy , is related to the step change in the input Δu by

$$\Delta y = K \Delta u$$

where K is the steady-state gain. Note that at the nominal conditions, before the input changes we also have $y_0 = K u_0$.

As an example, suppose that we wish to model the effect of a unit change in the input, $u(t)$, on the output, $y(t)$. Assuming a first-order linear model with time delay, and assuming the step change occurs at time $t = t_0$ we can develop a relationship of the form:

$$\Delta y(t) = K \left(1 - e^{-\frac{t-t_0-T_d}{\tau}} \right), \quad \text{for } t \geq T_d,$$

where $\Delta y(t)$ denotes the change in the output from its initial value. Note that for the purpose of controller design it is often useful to write this model in a different form, using Laplace transforms. In this case we would write the transfer function relating changes in output due to changes in input as

$$G(s) = \frac{\Delta Y(s)}{\Delta U(s)} = \frac{K e^{-T_d s}}{(\tau s + 1)}$$

The goal of the modelling effort for control purposes is to identify the values of K , T_d , and τ for each output due to each input. Then the total change in any output is considered to be the sum of the change due to all the inputs (this assumes superposition applies, a common assumption in most process control applications). Such information can usually be obtained by measuring the effect of small step changes in each input (starting from some nominal

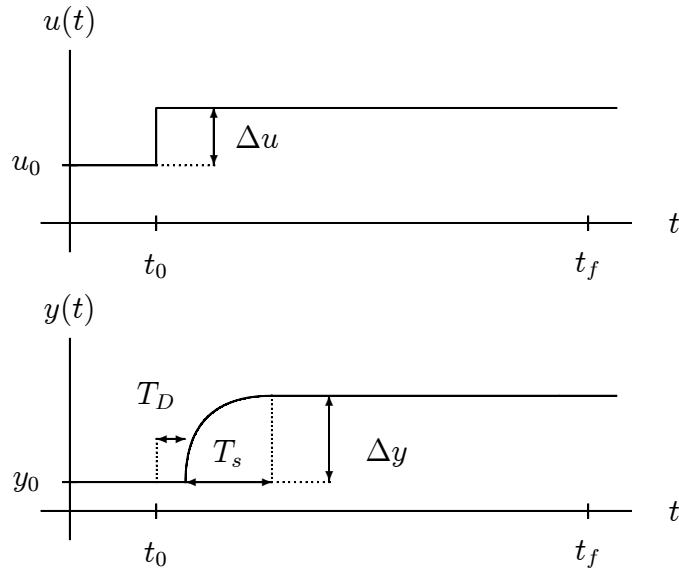


Figure 9: Input and output responses.

operating point) on each output. Thus, one would ideally conduct a series of experiments in which step changes are made to each input, one at a time (with all other parameters held constant), and the effect on each output is measured.