Foldable Robotics

Class 2: Origami & Kinematics I

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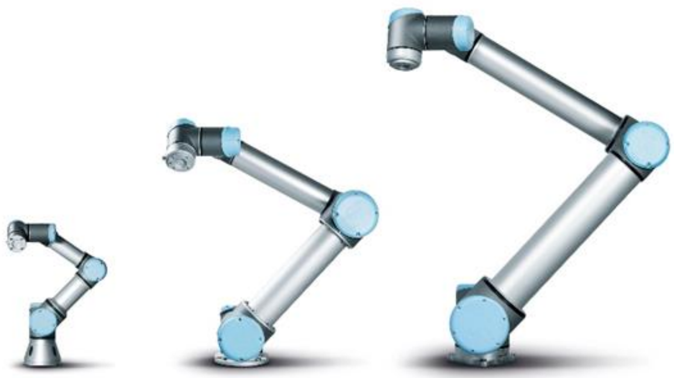
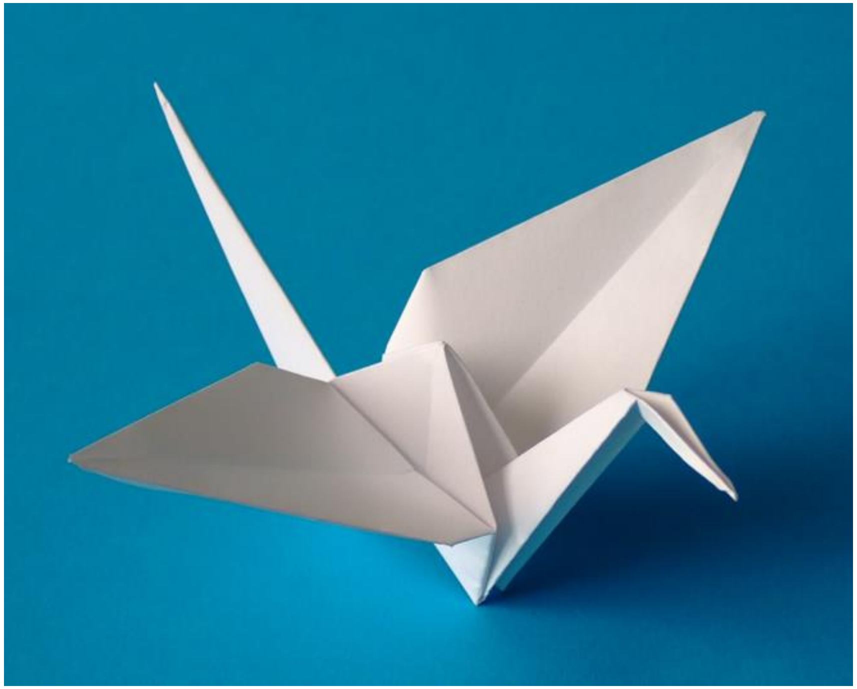
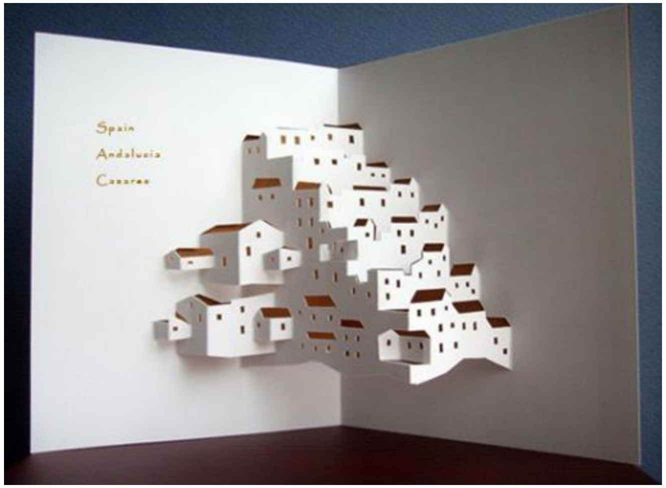
Robotics



- Kinematics: DH Parameters, Jacobians
- Dynamics: Motion, Mass
- Control: Inverse Kinematics, Dynamics
- Traditional Link-joint-link-joint construction
  - Typical materials: metal, gears, belts, drives

Origami/Kirigami/Popups



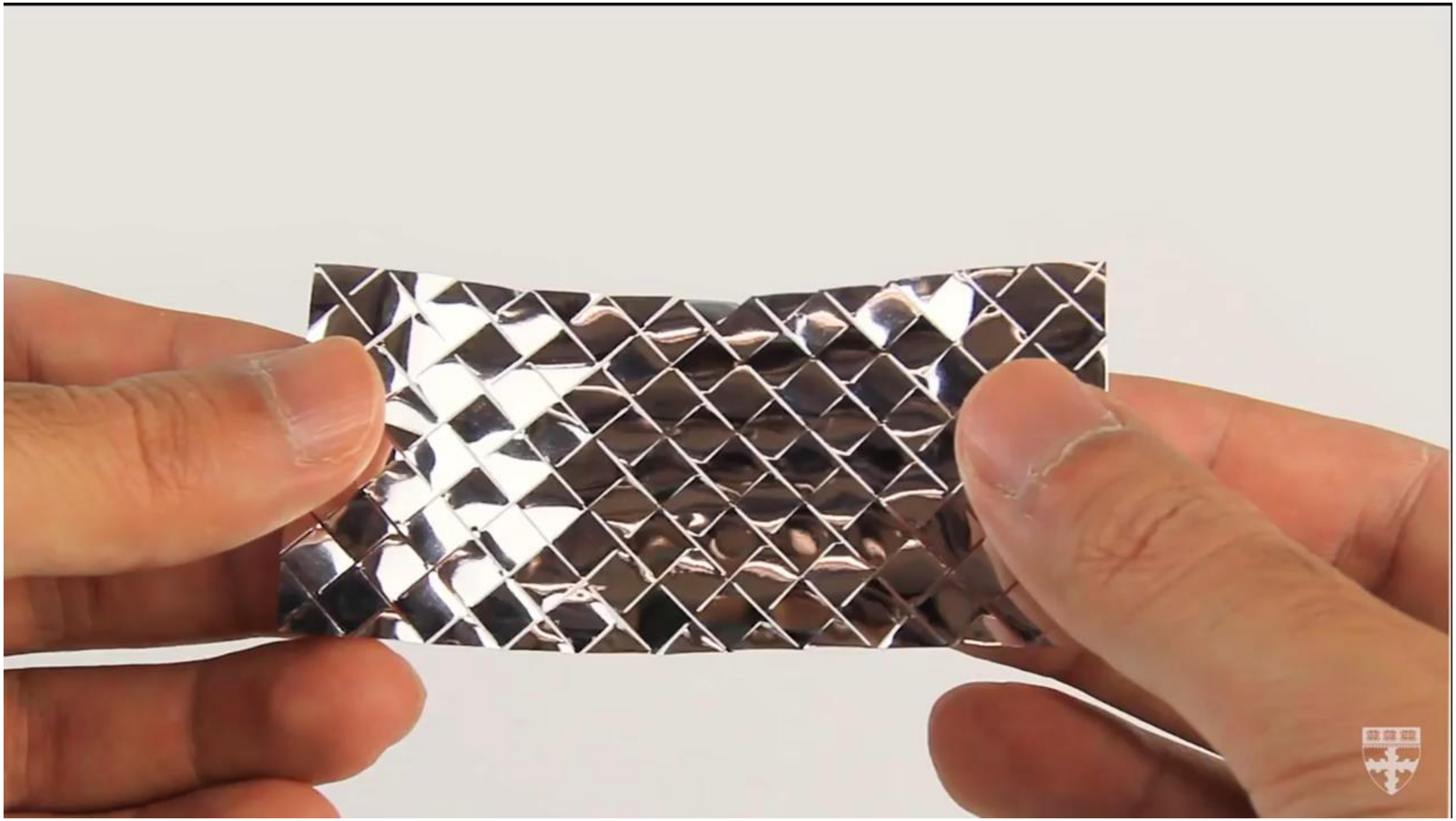


Miura Fold

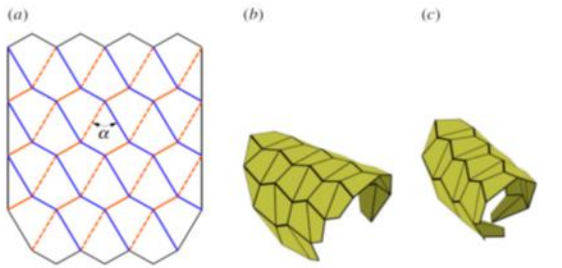
Twist Fold

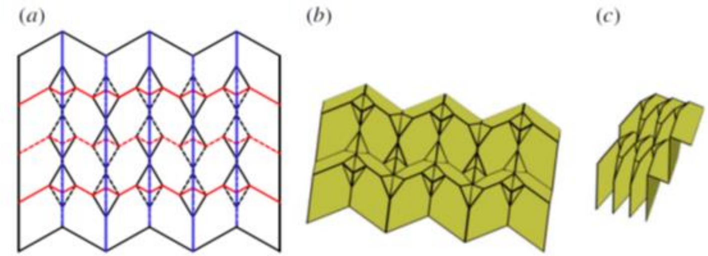
Twist Fold

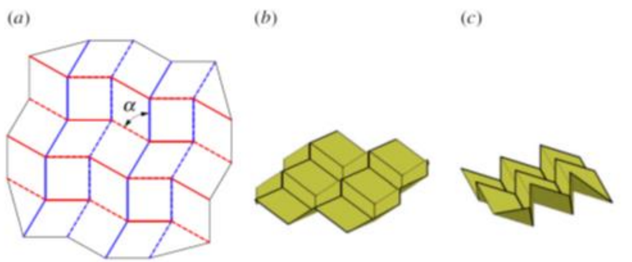
Kirigame+Buckling

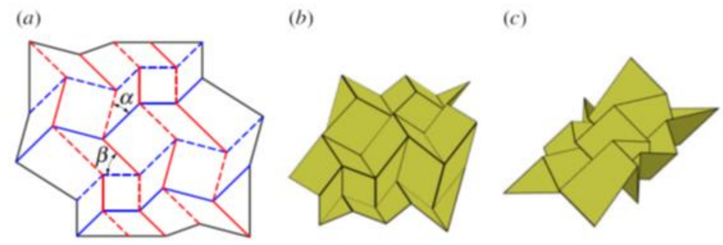


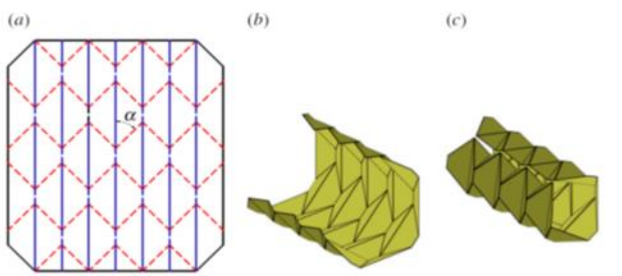
Tesselations

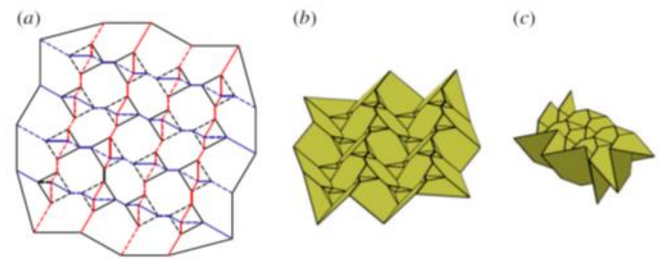






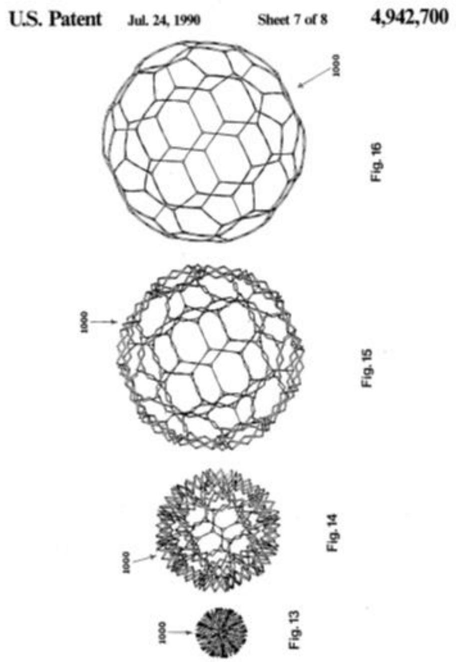






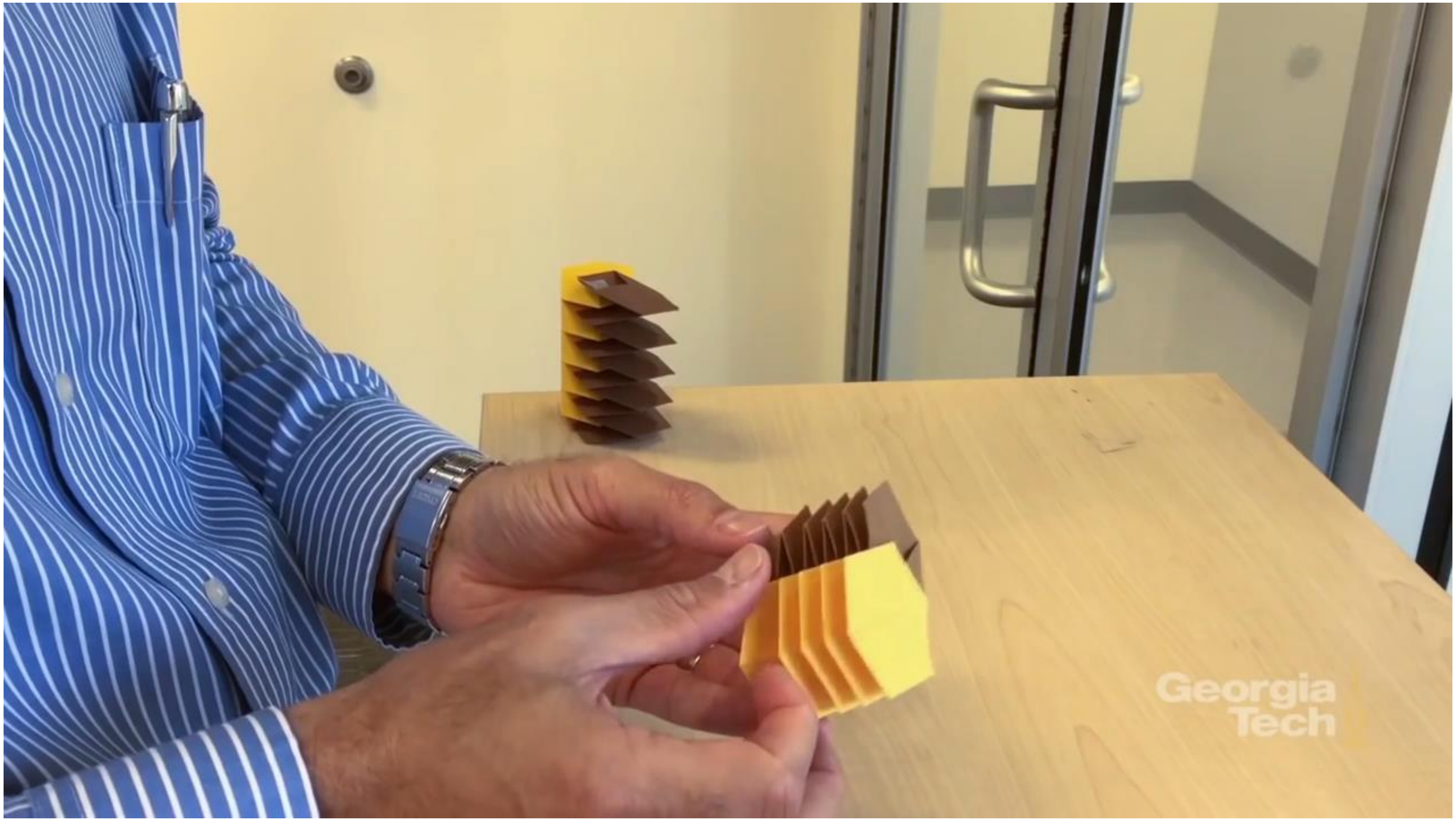
[1] T. A. Evans, R. J. Lang, S. P. Magleby, and L. L. Howell, “Rigidly foldable origami gadgets and tessellations Subject Category : Subject Areas : Author for correspondence :,” 2015.

Deployable Structures

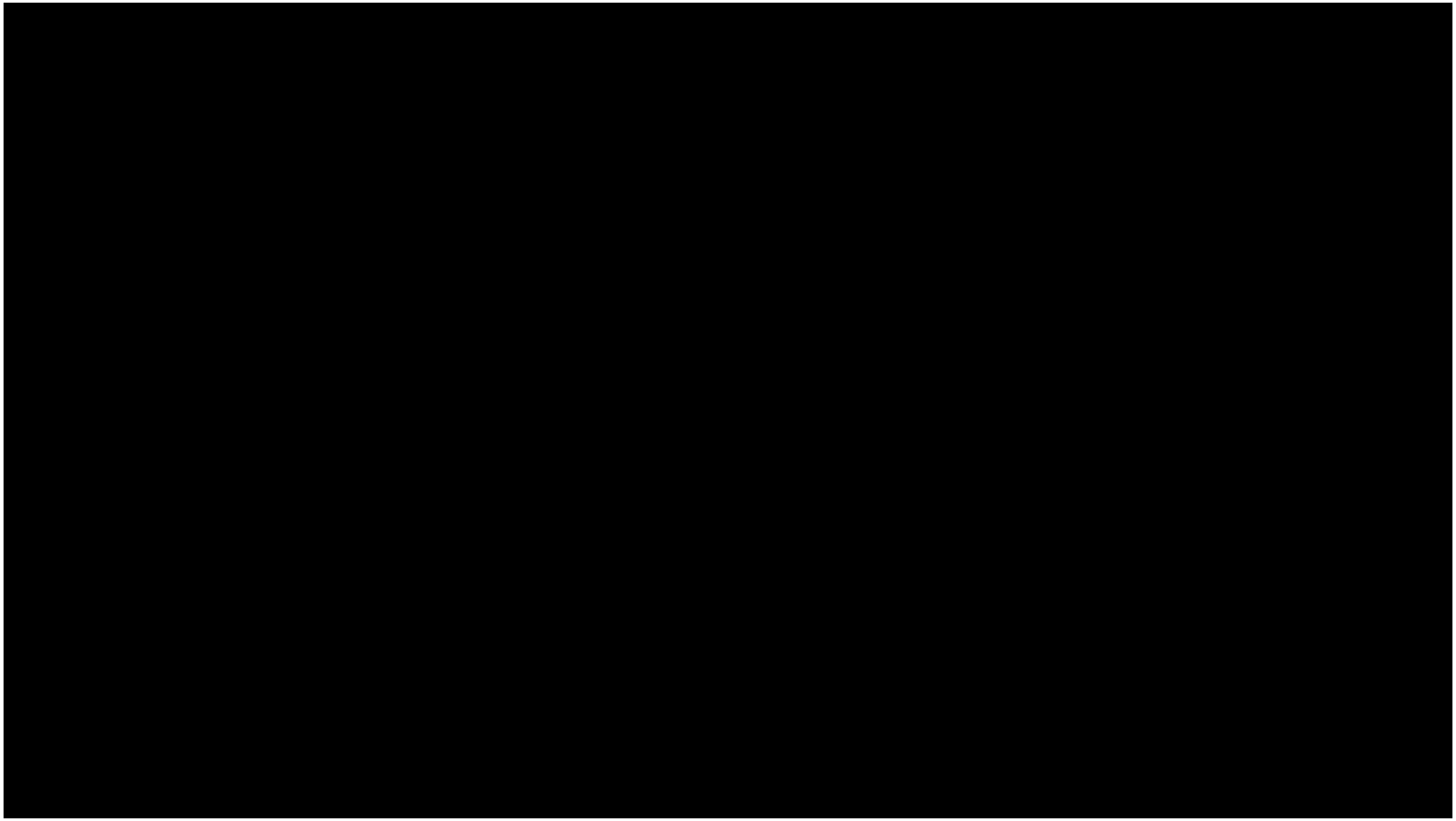


more on this later…

Deployable Structure

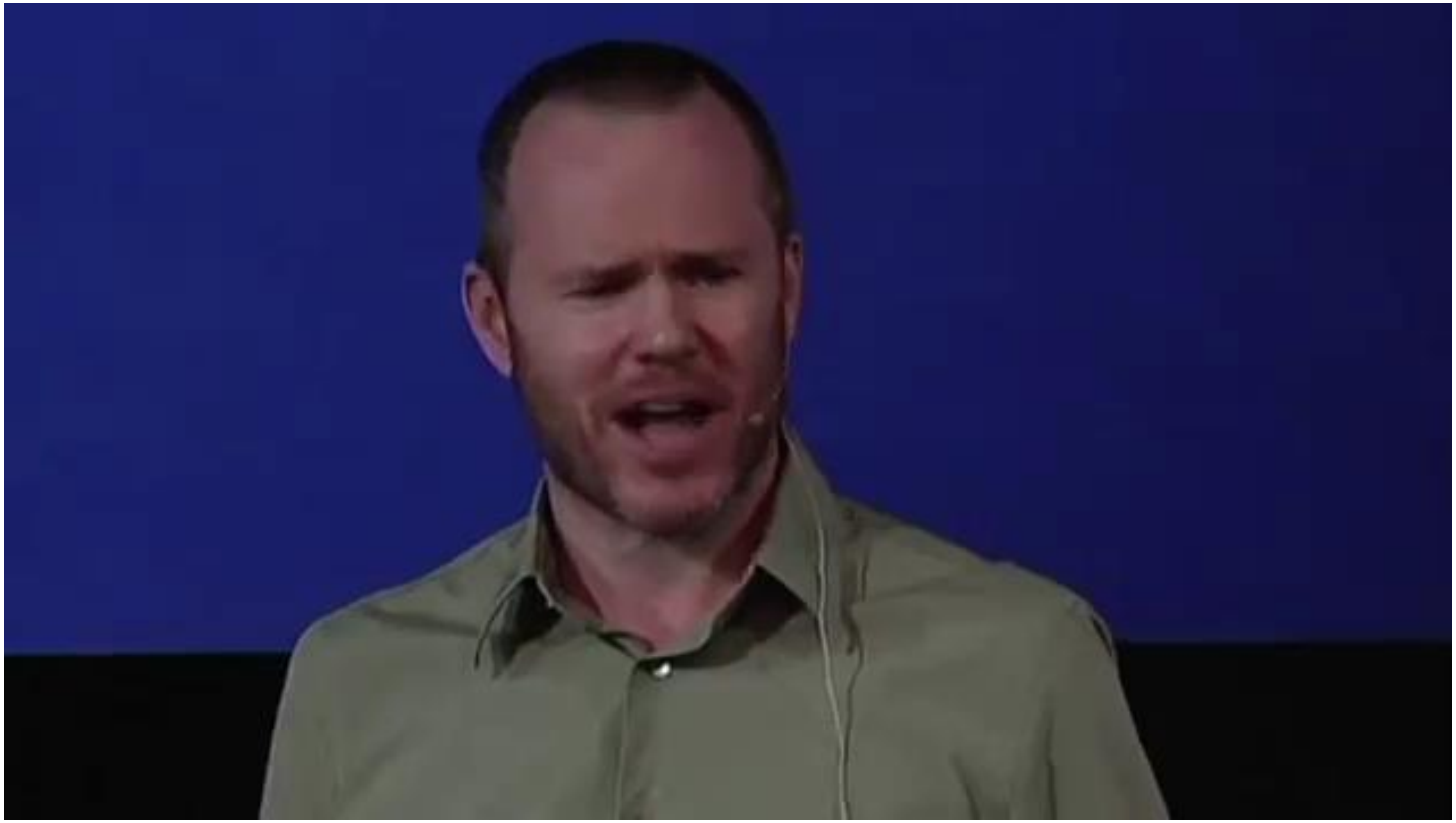


Reconfigurable Materials



Deployable Structure

Popup Books



Kinematics

- Language used to describe this…to a point
- Then you have to start “bending” the rules
- But first, what are the rules?

Kinematics

- What are the different kinds of joints?

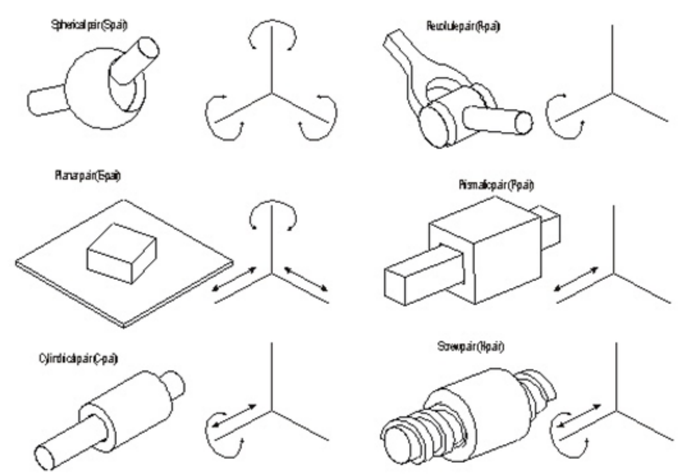
Lower Order Pairs

Kinematics: Joints

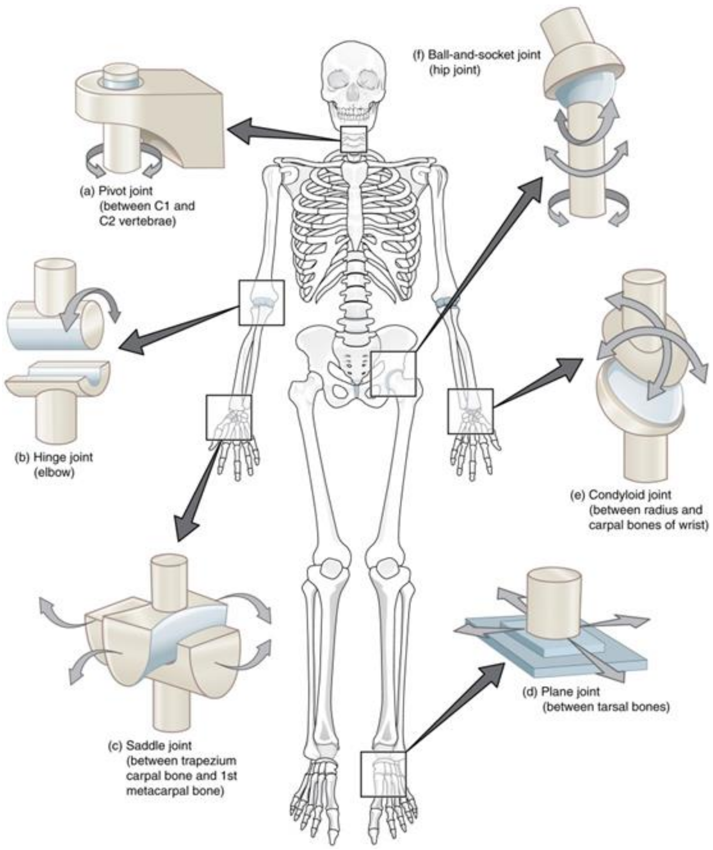
- Rigid

- Revolute
- Prismatic
- Helical
- Cylindrical
- Spherical
- Planar

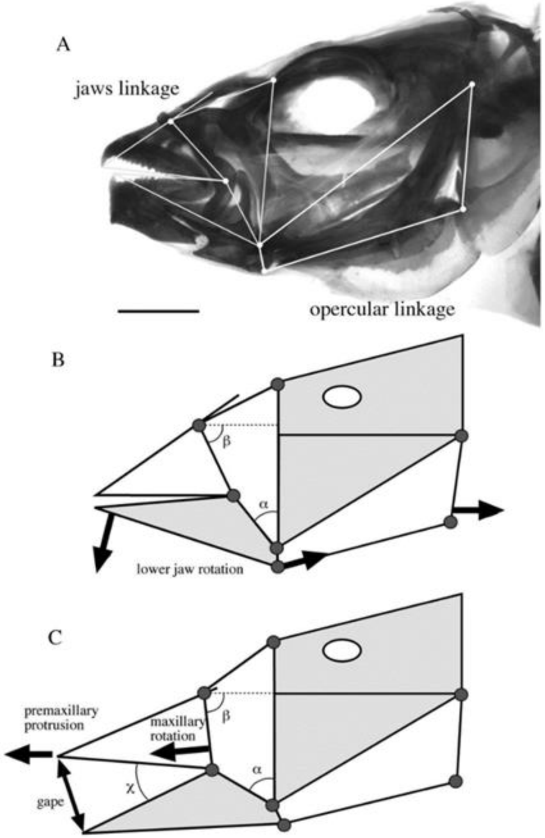
FIGURE 7. There are six lower order kinematic pairs -, spherical, planar, cylindrical, revolute, prismatic and screw can be represented easily in the Physical Markup Language.



Biological Connection



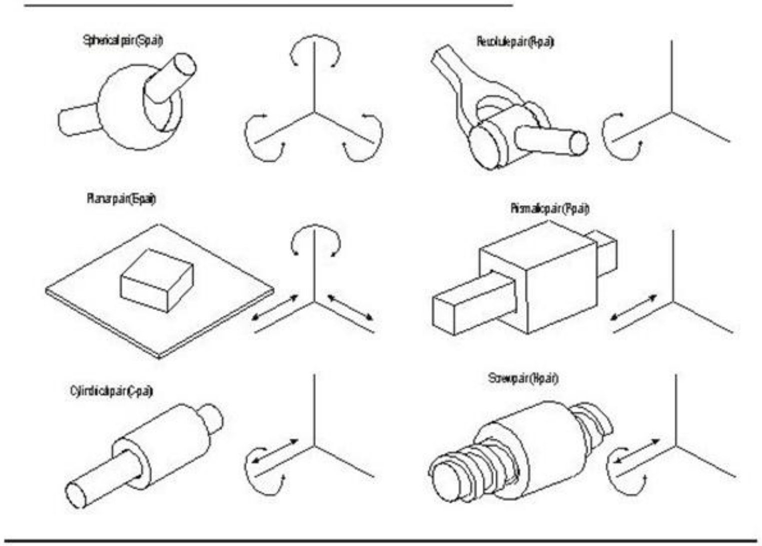
Linkage Inspiration



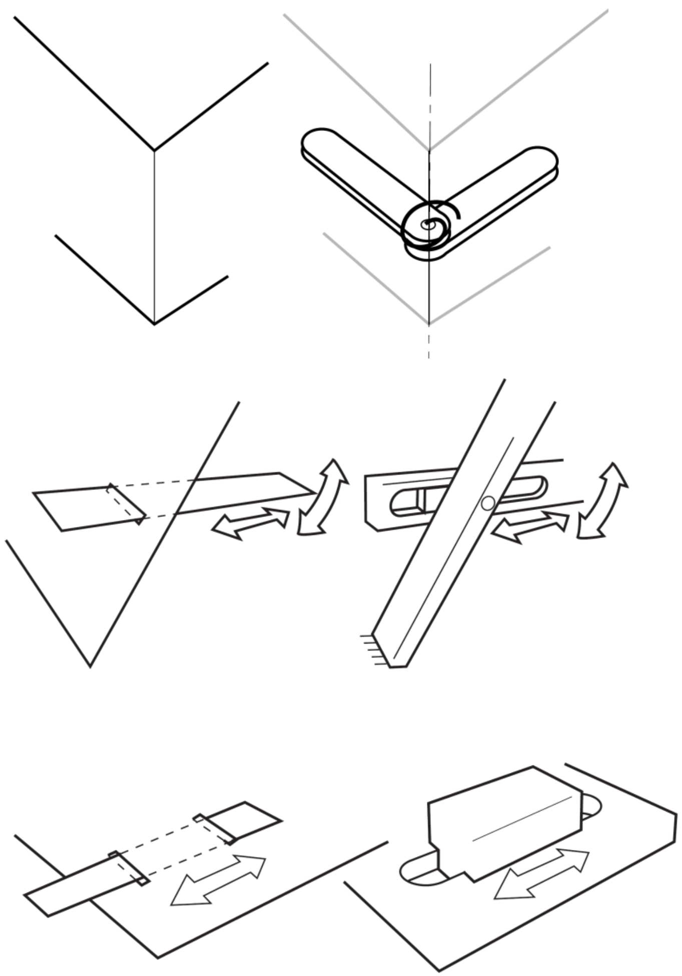
Linkages & Kinematics

- Easy to fabricate
- Easy to connect
- Low friction, even with bushings
    - Linear not as good, jamming

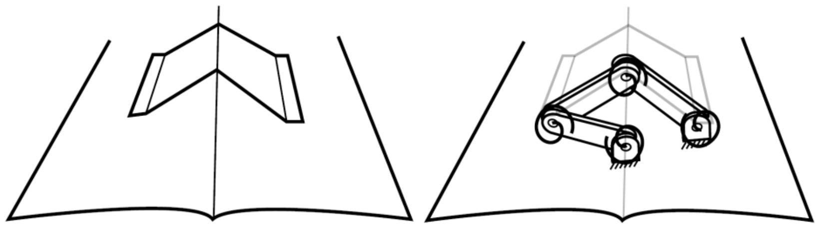
Traditional Robotic Degrees of Freedom

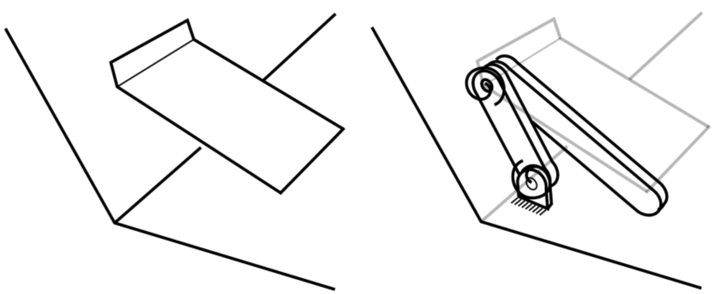


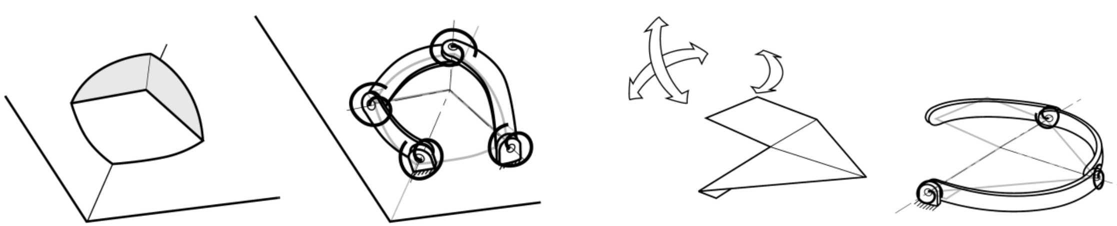
Origami & Popups:



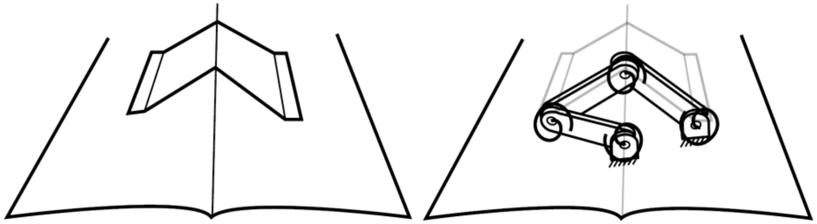
Loop Mechanisms

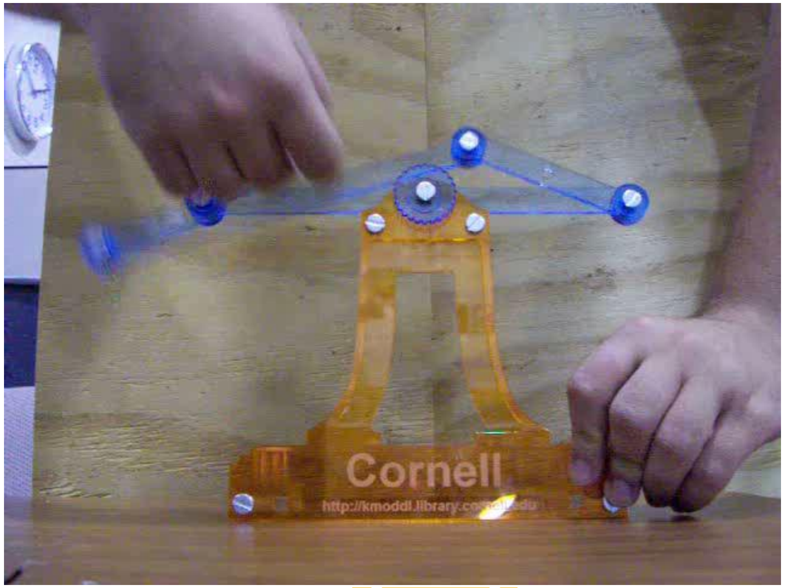




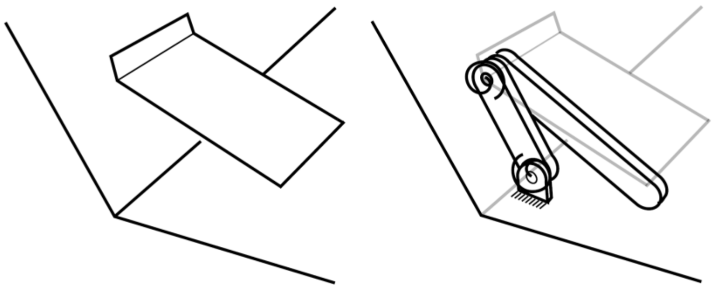


Loop Mechanisms

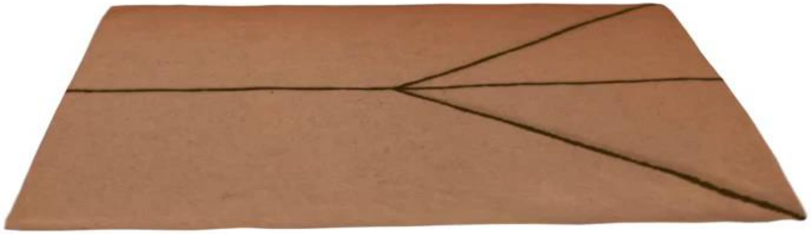




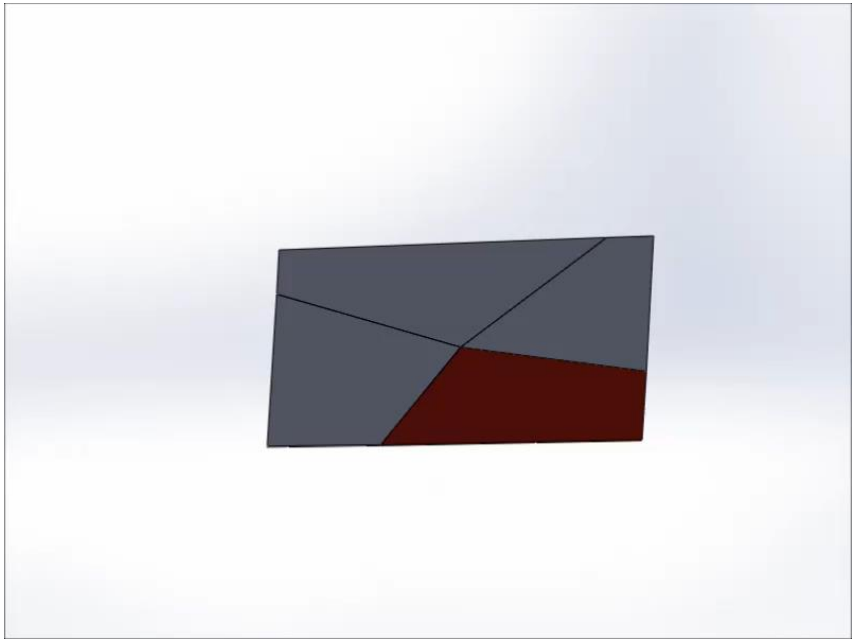
Loop Mechanisms

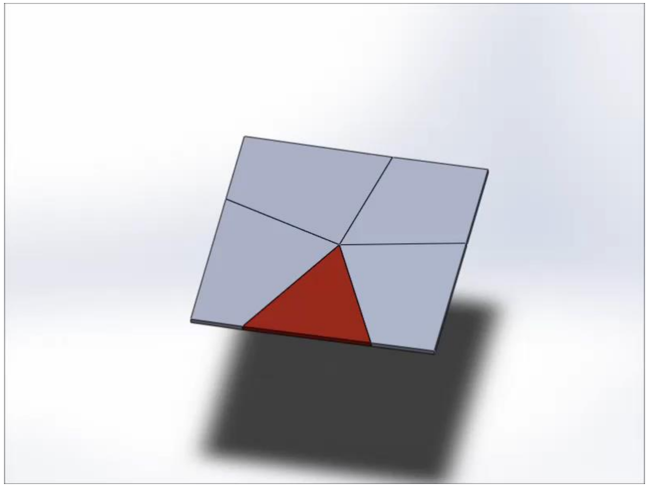


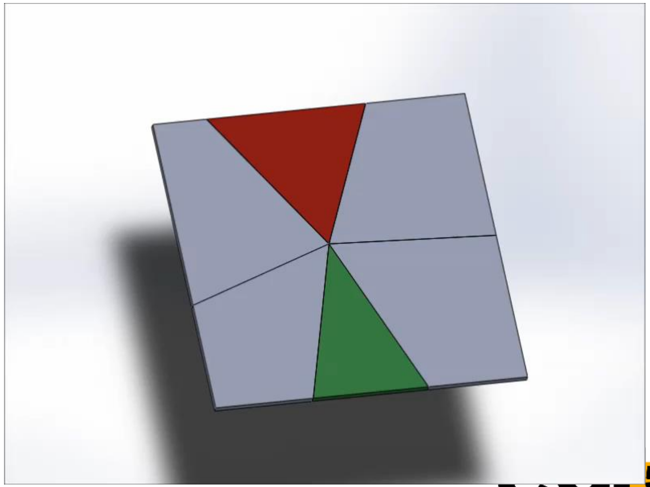
Folded Paper – Kinematics



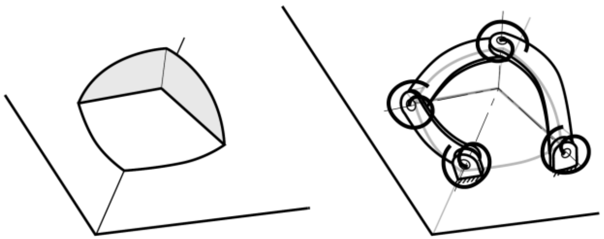
Hinges arranged about a vertex

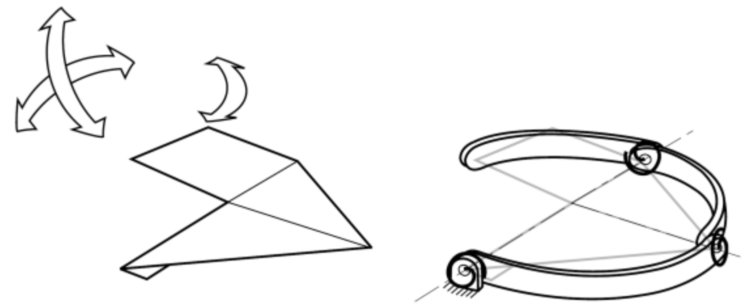


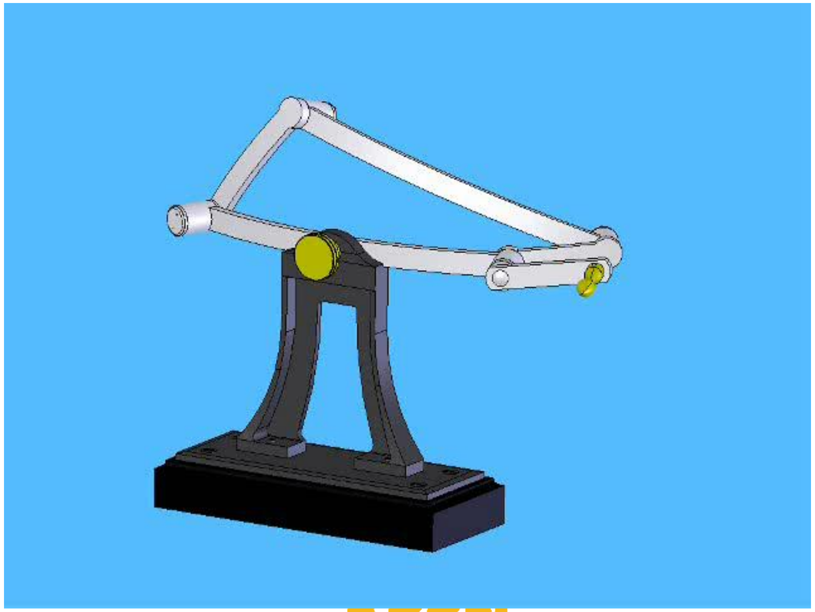




Loop Mechanisms



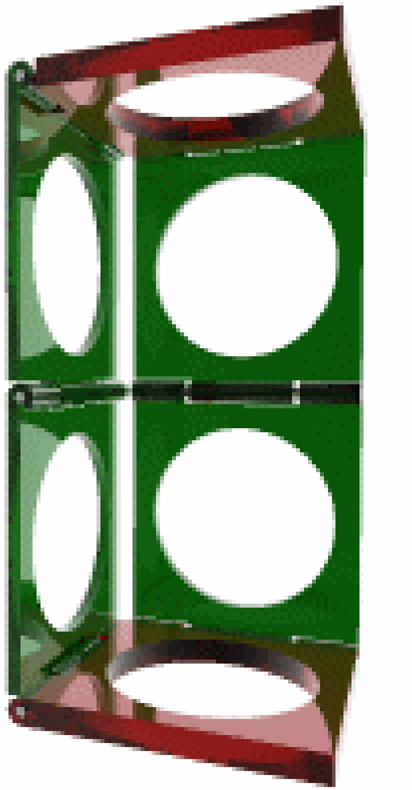




Spherical Mechanisms

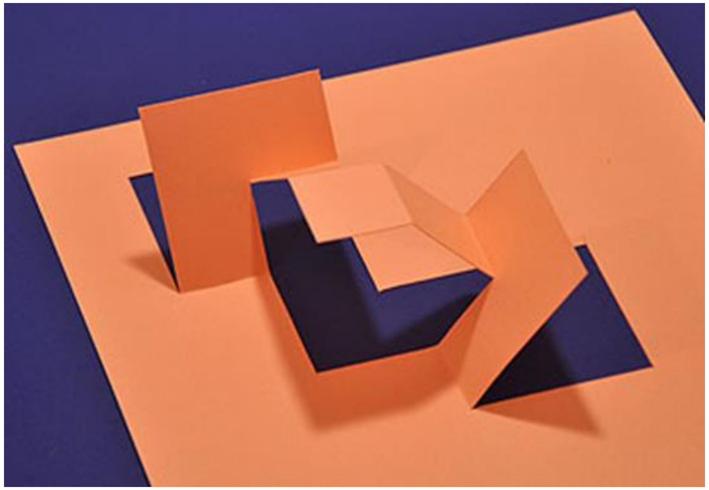
- Similar to four-bar linkages.
- Same as the v-fold we saw earlier
- Rotations about a point
- Will study more…

Sarrus Linkage



How do you make a sarrus linkage in a flat sheet?

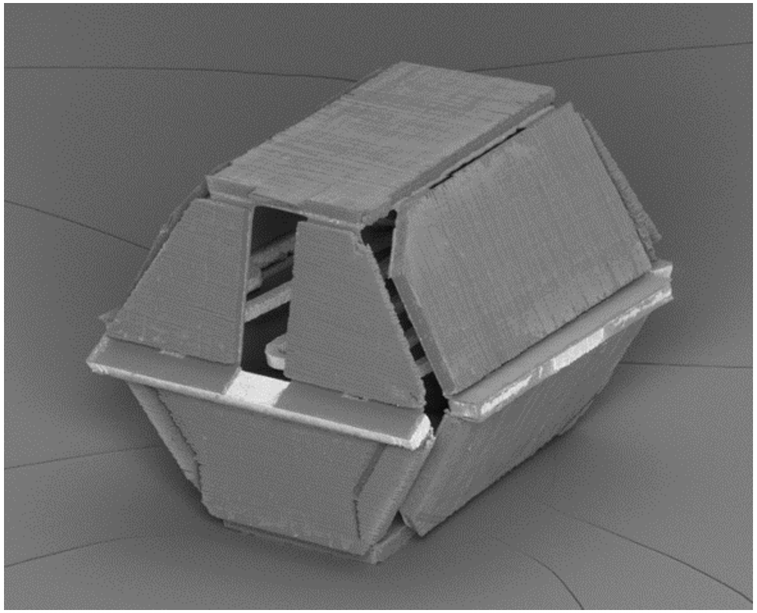
Sarrus Linkage in one Layer



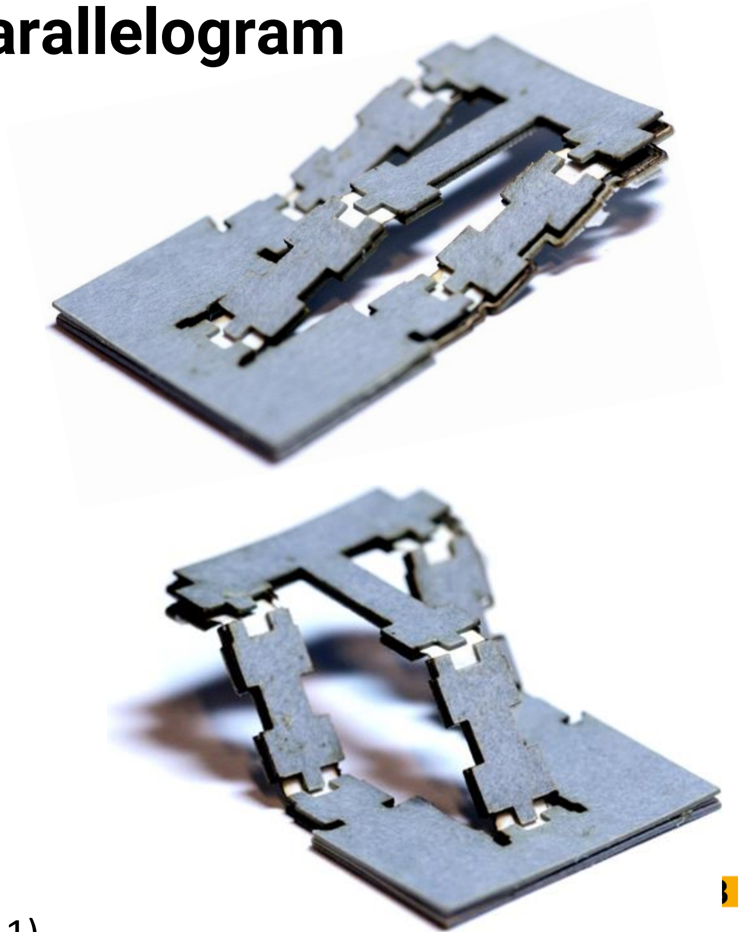
http://www.cutoutfoldup.com/1703-sarrus-straight-line-polyhedral-linkage.php

What devices can a hinge make?

Sarrus Linkage

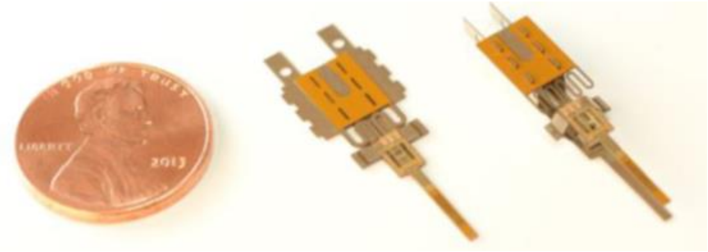


Parallelogram



Whitney, J. P., Sreetharan, P. S., Ma, K. Y., & Wood, R. J. (2011).

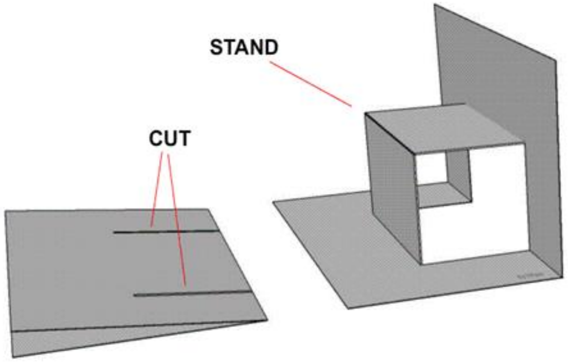
Sarrus-based microrobots

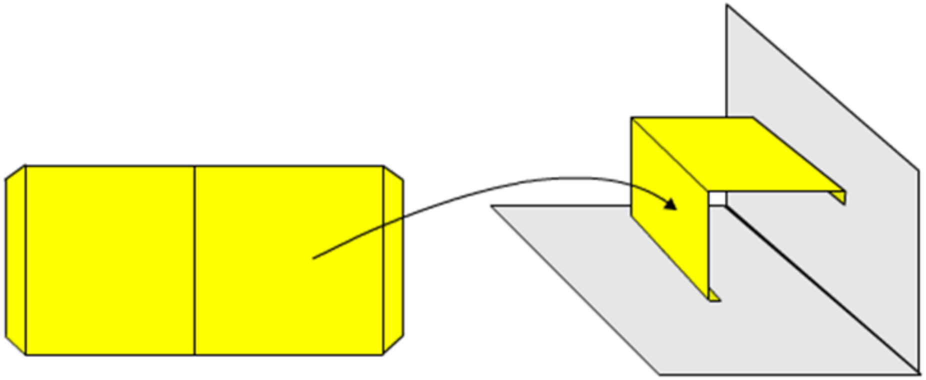


[1] J. B. Gafford, S. B. Kesner, R. J. Wood, and C. J. Walsh, “Force-sensing surgical grasper enabled by pop-up book MEMS,” in *2013 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2013, pp. 2552–2558.

[1] P. S. Sreetharan, J. P. Whitney, M. D. Strauss, and R. J. Wood, “Monolithic fabrication of millimeter-scale machines,” *J. Micromechanics Microengineering*, vol. 22, no. 5, p. 055027, May 2012.

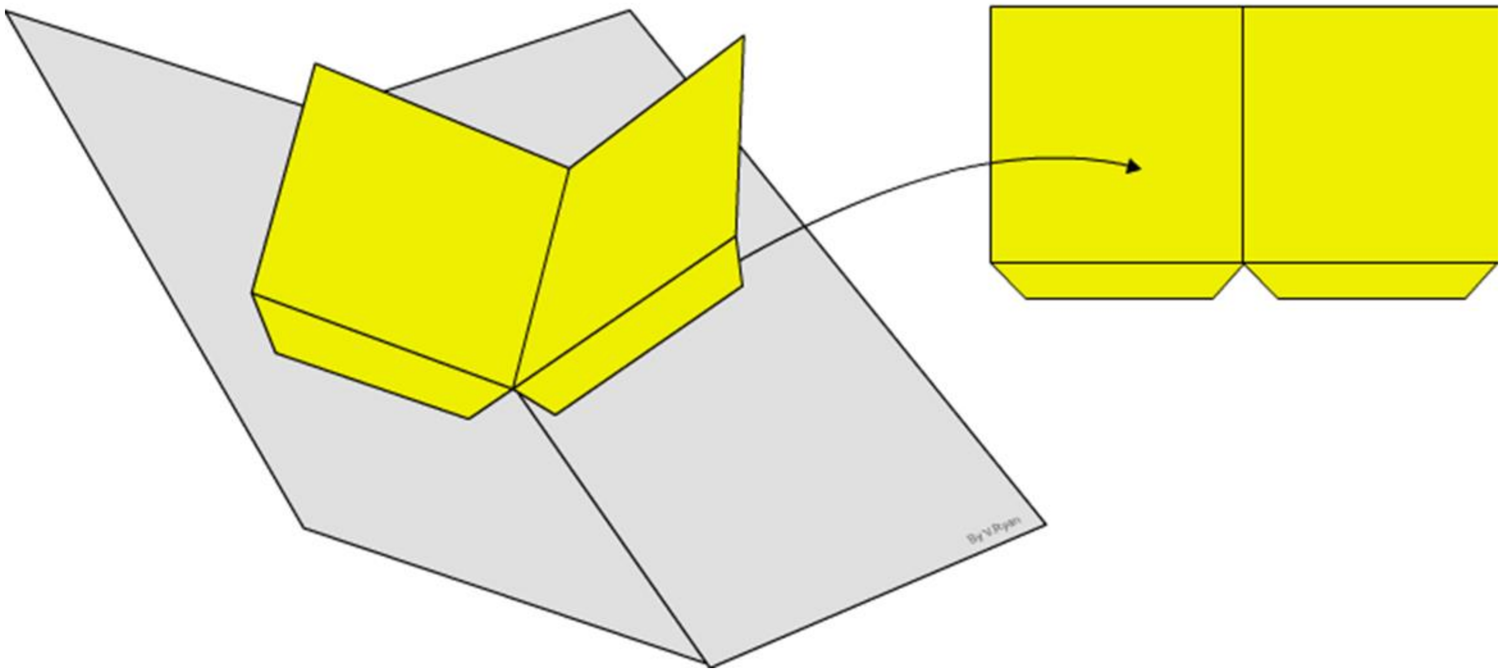
Parallelogram





http://www.technologystudent.com/designpro/popup1.htm

V-Fold(Spherical)



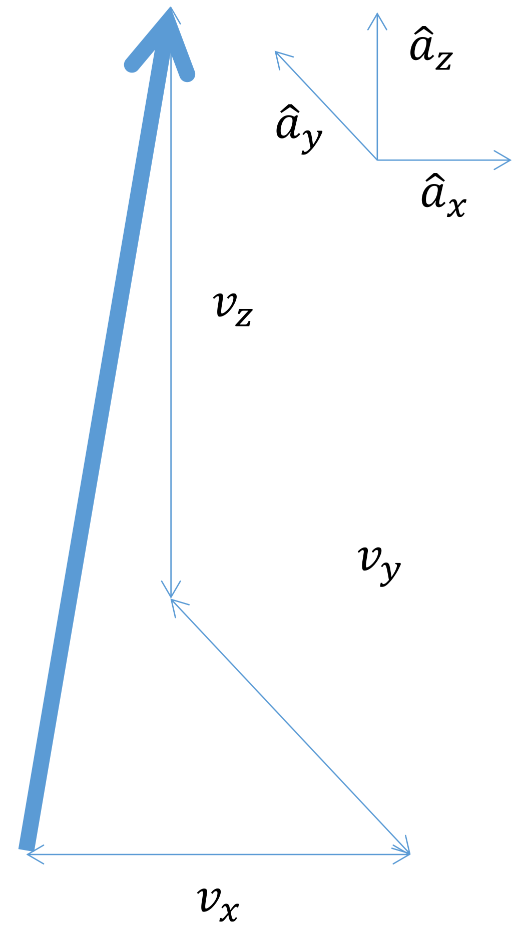
http://www.technologystudent.com/designpro/popup1.htm

Vectors

- Magnitude and Direction
- Often split into components:

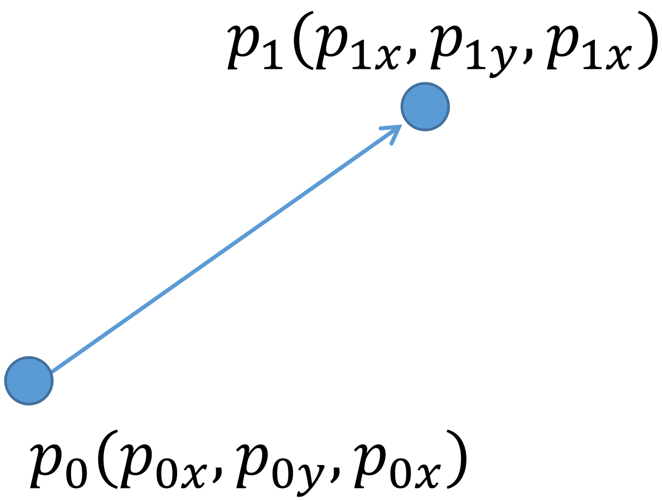
$$\vec{v} = v_x\hat{a}_x + v_y\hat{a}_y + v_z\hat{a}_z$$

- Regular vectors use an $\overrightarrow{arrow}$.
- Unit vectors use a $\widehat{hat}$.
- Zero vector looks like this: $\vec{0}$

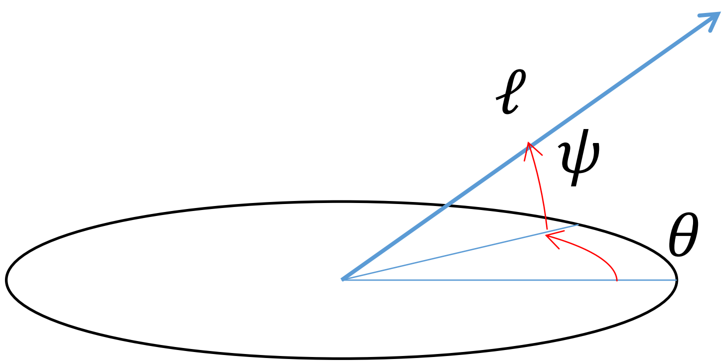


How many numbers in a vector?

Two Points

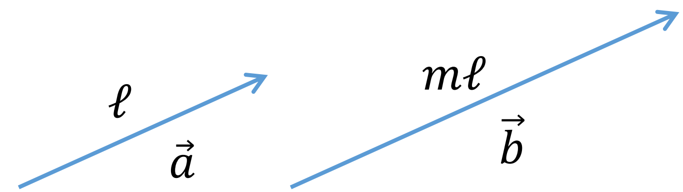


Spherical Coordinates

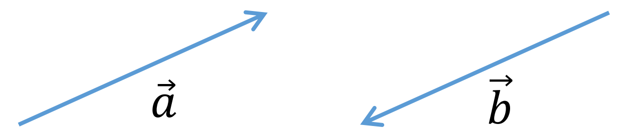


Vector Math

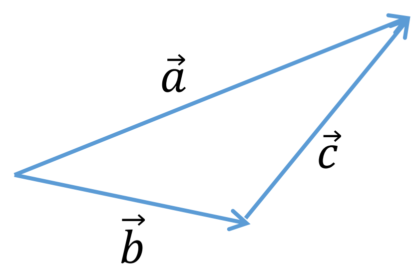
- Multiplication: $\vec{a} = m\vec{b}$



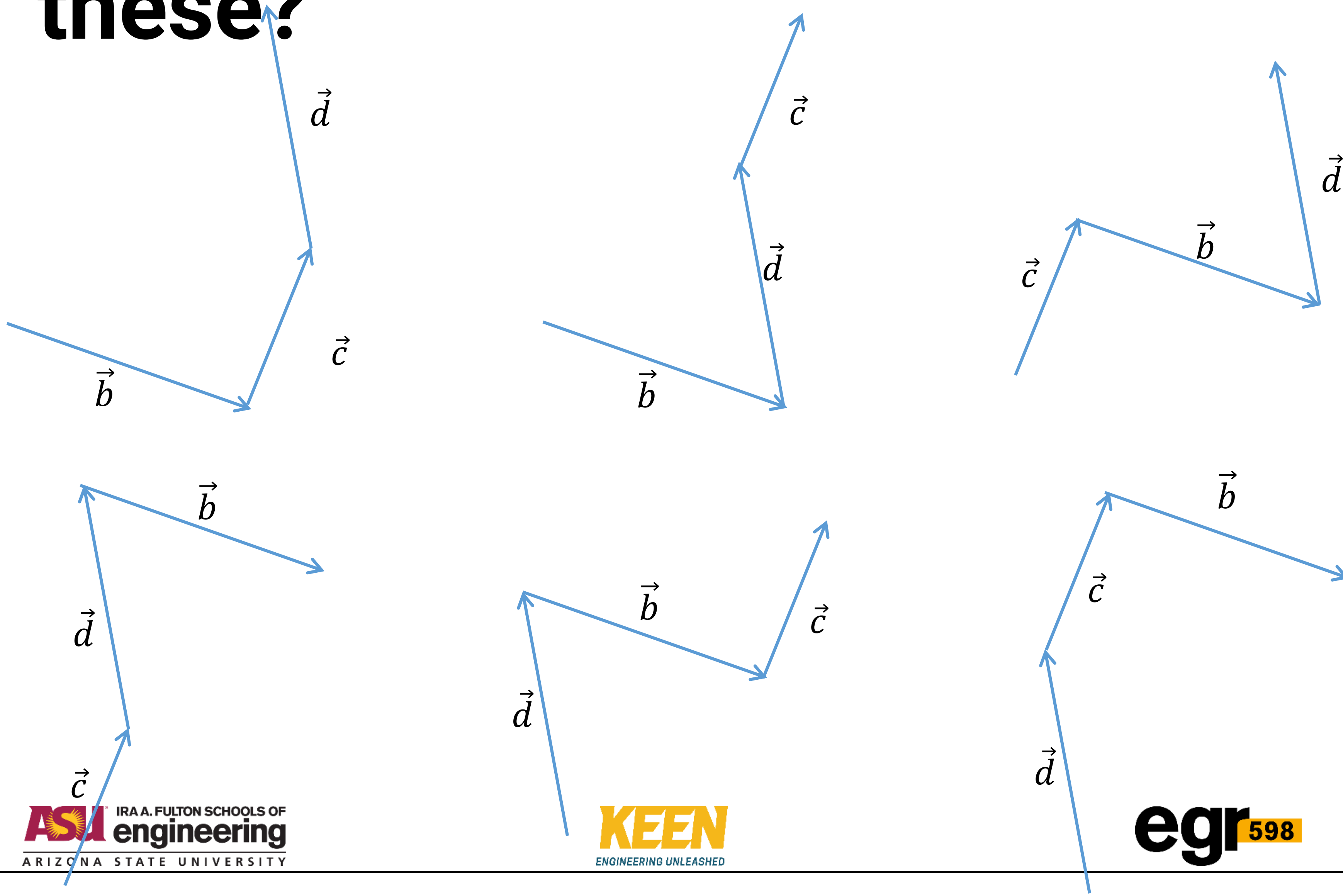
- Negativity: $\vec{a} = -\vec{b}$



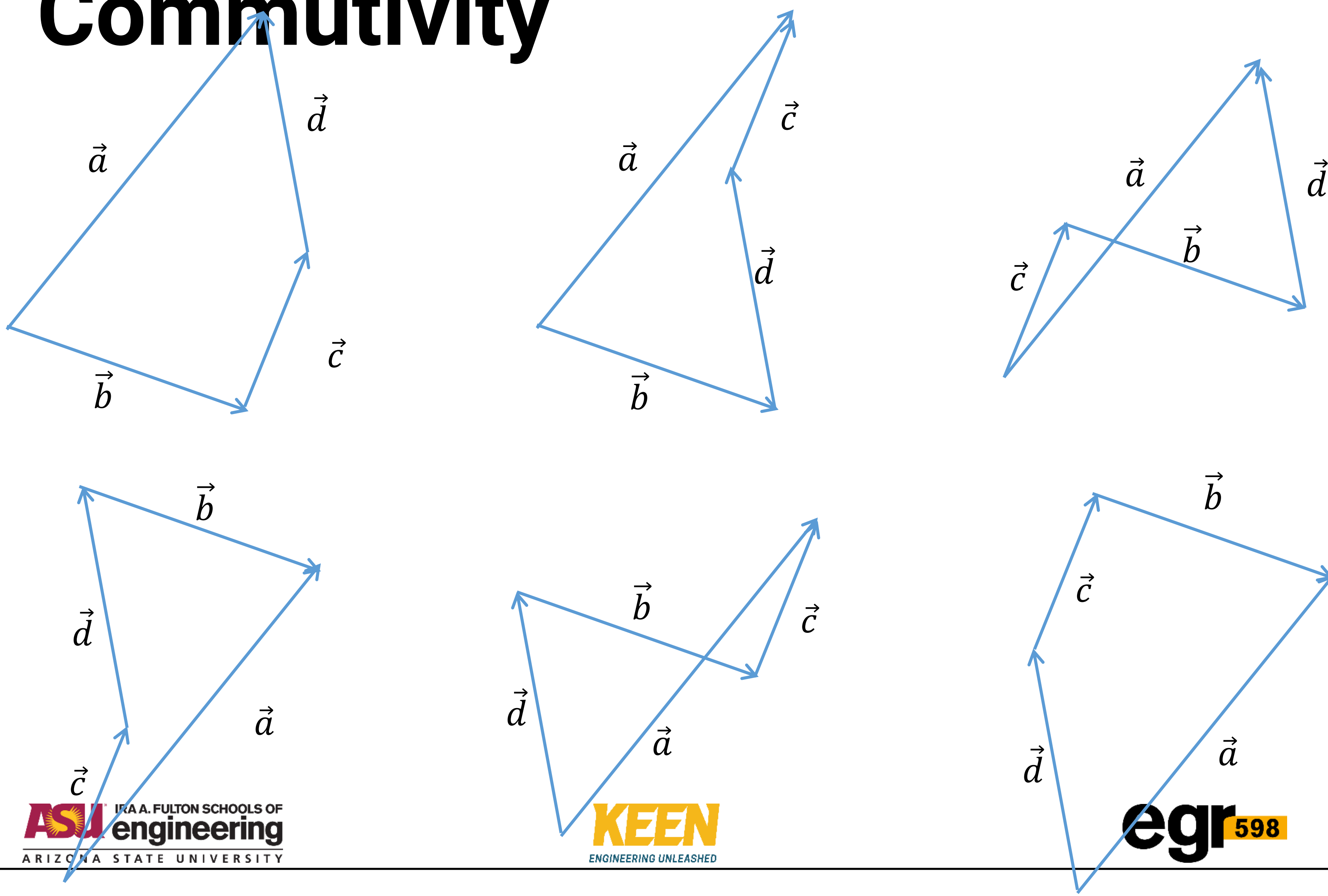
- Addition: $\vec{a} = \vec{b} + \vec{c}$



What is similar about all these?



Equivalent Systems from Commutivity



Mathematical Properties

- Commutivity:

$$\vec{u} + \vec{v} = \vec{v} + \vec{u}$$

- $$\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$$

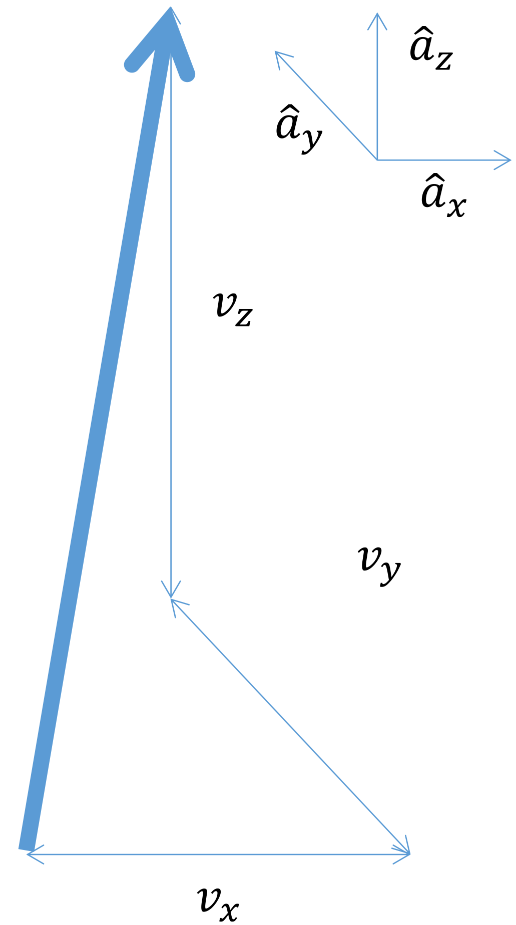
- $$a(\vec{v} + \vec{w}) = a\vec{v} + a\vec{w}$$

Vectors

- Magnitude and Direction
- Often split into components

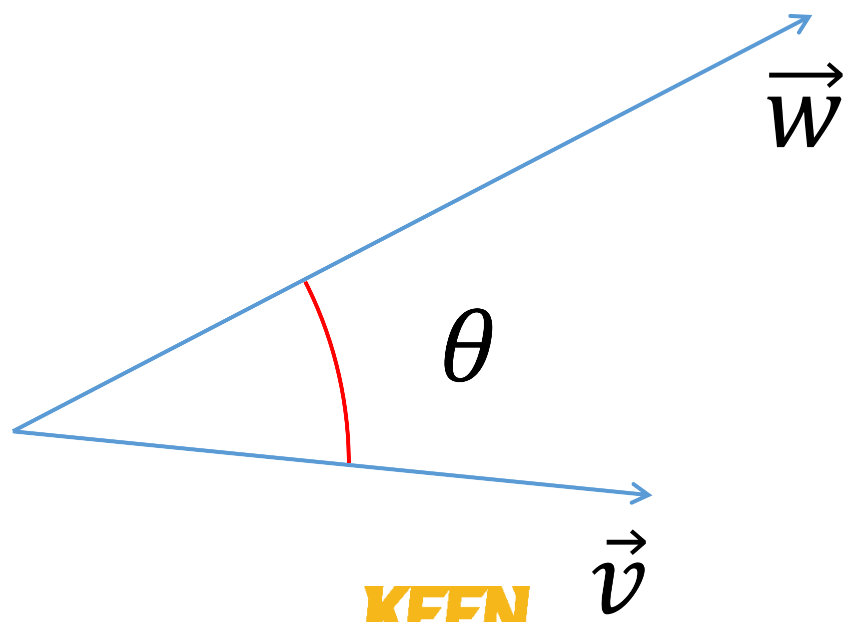
$$\vec{v} = v_x\hat{a}_x + v_y\hat{a}_y + v_z\hat{a}_z$$

- Regular vectors use an $\overrightarrow{arrow}$.
- Unit vectors use a $\widehat{hat}$.



Dot product

$$\vec{v} \cdot \vec{w} \triangleq |\vec{v}|\,|\vec{w}| \cos\theta$$



Magnitude of a Vector

$$|v| = \sqrt{\vec{v} \cdot \vec{v}}$$

Properties of Dot Product

- Commutative

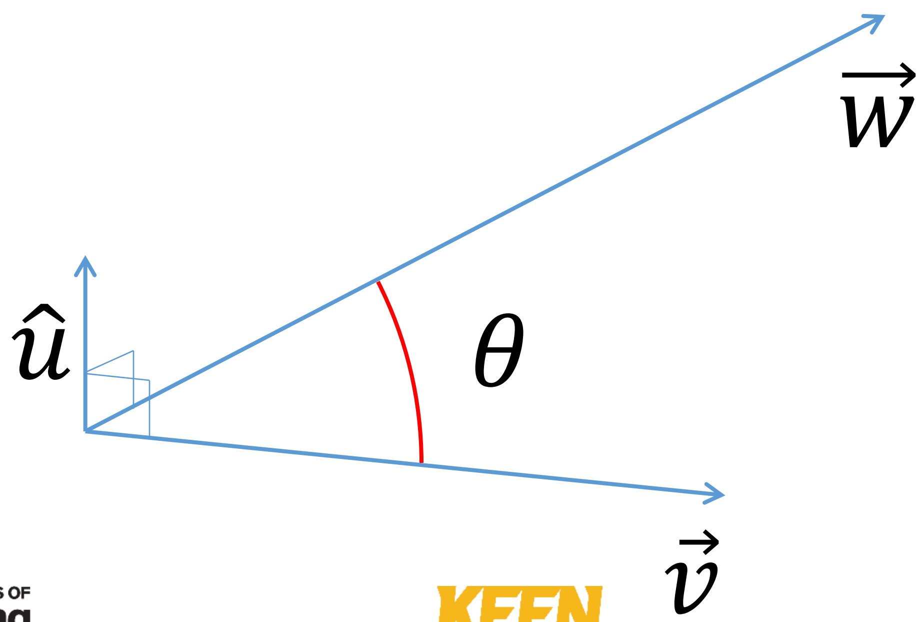
$$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$$

- Di

$$\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$$

Cross Product

$$\vec{v} \times \vec{w} \triangleq |\vec{v}|\,|\vec{w}|\sin\theta\,\hat{u}$$



Cross Product Properties

- $(\vec{u} \times \vec{v}) = |u||v| \sin(\theta)\, \hat{w}, \vec{u} \perp \hat{w}, \vec{v} \perp \hat{w}$
- $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$
- $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$
- $c\vec{u} \times \vec{v} = \vec{u} \times c\vec{v} = c(\vec{u} \times \vec{v})$
- $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \cdot \vec{w} = (\vec{w} \times \vec{u}) \cdot v$
- $\vec{u} \times (\vec{v} \times \vec{w}) \neq (\vec{u} \times \vec{v}) \times \vec{w}$
- $\vec{u} \times (\vec{v} \times \vec{w}) = \vec{v}(\vec{u} \cdot \vec{w}) - \vec{w}(\vec{u} \cdot \vec{v})$

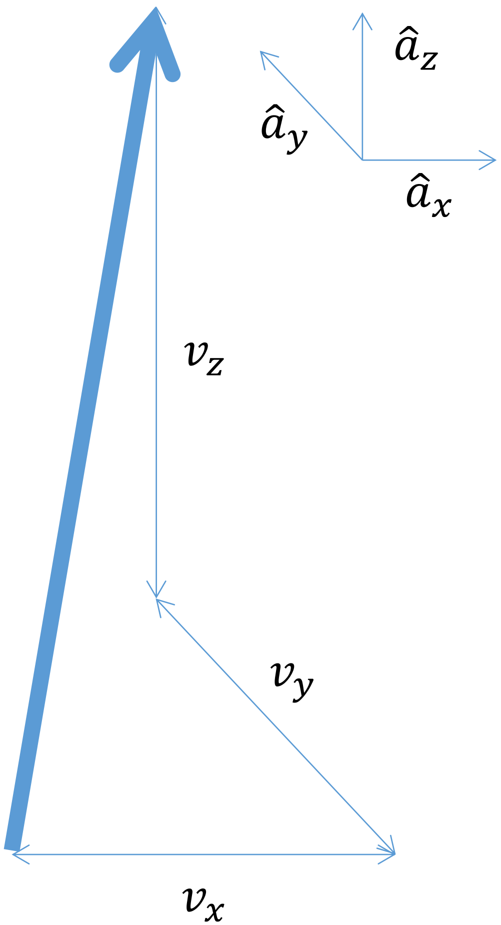
No-no's

- Multiply a vector by another vector
- Equate scalars to vectors
- Use a cross product on a scalar
- …lots more

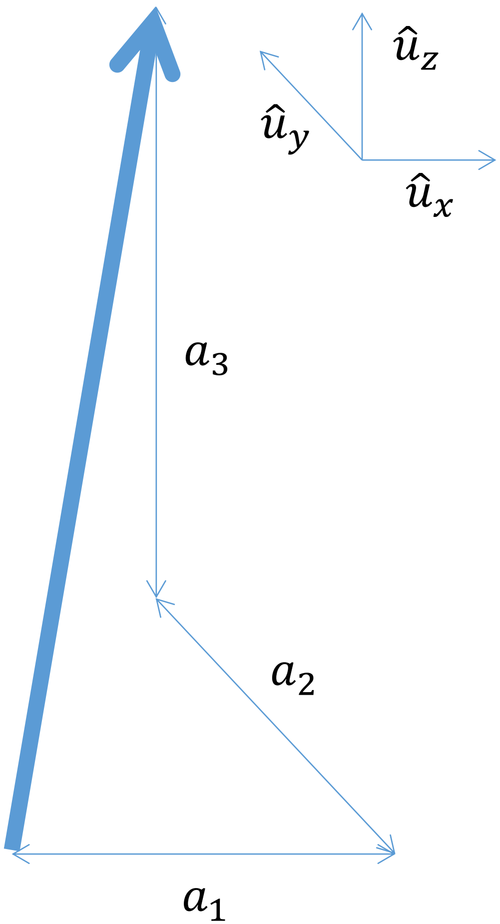
Basis Vectors

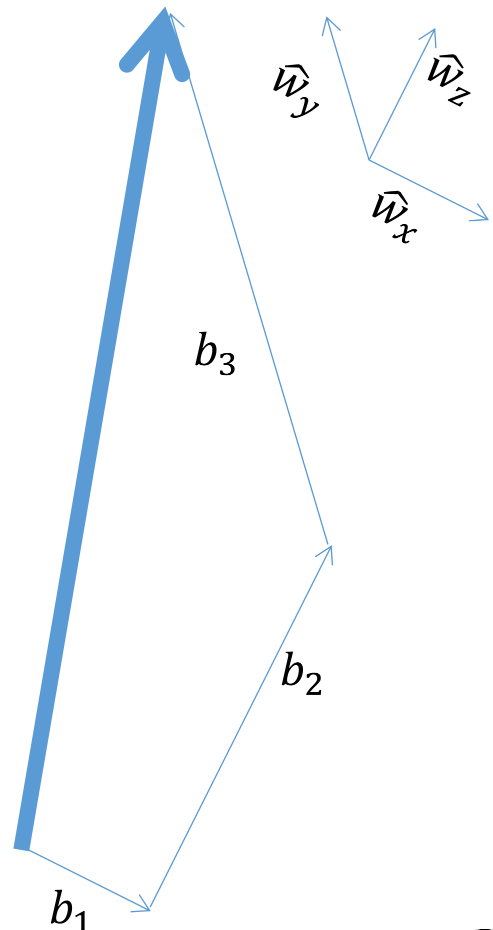
- Orthogonality
- Unit length
- Not unique, just handy
- Expand vector into components

$$\vec{v} = v_x \hat{a}_x + v_y \hat{a}_y + v_z \hat{a}_z$$



Alternate Basis Vectors





$$\vec{v} = a_1\hat{u}_x + a_2\hat{u}_y + a_3\hat{u}_z$$

$$\vec{v} = b_1\hat{w}_x + b_2\hat{w}_y + b_3\hat{w}_z$$

Angle between basis vectors

- $a_x \angle a_x = ?$
- $a_y \angle a_y = ?$
- $a_z \angle a_z = ?$
- $a_x \angle a_y = ?$
- $a_y \angle a_z = ?$
- $a_z \angle a_x = ?$
- $\cos 0 = ?$
- $\cos \frac{\pi}{2} = ?$
- $\sin 0 = ?$
- $\sin \frac{\pi}{2} = ?$

Angle between basis vectors

- $a_x \angle a_x = 0$
- $a_y \angle a_y = 0$
- $a_z \angle a_z = 0$
- $a_x \angle a_y = \frac{\pi}{2}$
- $a_y \angle a_z = \frac{\pi}{2}$
- $a_z \angle a_x = \frac{\pi}{2}$

- $\cos 0 = 1$
- $\cos \frac{\pi}{2} = 0$
- $\sin 0 = 0$
- $\sin \frac{\pi}{2} = 1$

Extended Expressions: Dot Product

- How do I get from:

$$\vec{v} \cdot \vec{w} = (u_x \hat{a}_x + u_y \hat{a}_y + u_z \hat{a}_z) \cdot (v_x \hat{a}_x + v_y \hat{a}_y + v_z \hat{a}_z)$$

- To:

$$\vec{v} \cdot \vec{w} = u_x v_x + u_y v_y + u_z v_z$$

Extended Expressions: Cross Product

- How do I get from:

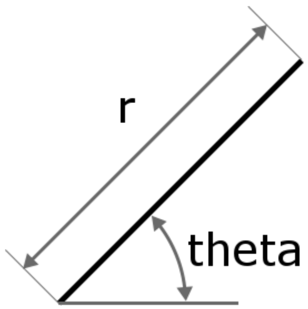
$$\vec{v} \times \vec{w} = (u_x\hat{a}_x + u_y\hat{a}_y + u_z\hat{a}_z) \times (v_x\hat{a}_x + v_y\hat{a}_y + v_z\hat{a}_z)$$

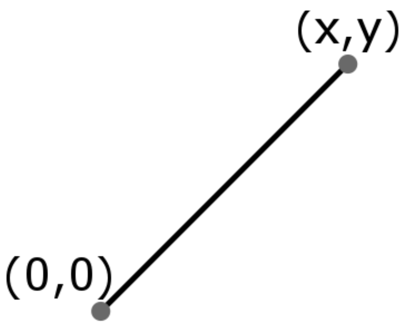
- To:

$$\vec{v} \times \vec{w} = u_x v_y \hat{a}_z - u_x v_z \hat{a}_y - u_y v_x \hat{a}_z + u_y v_z \hat{a}_x + u_z v_x \hat{a}_y - u_z v_y \hat{a}_x$$

Constraint Equations

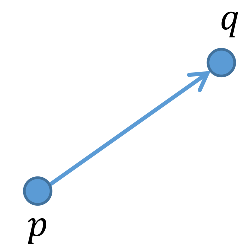
- Many ways to represent the same constraint
- Variables don't necessarily map to DOF

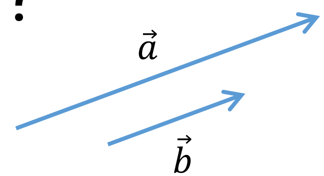


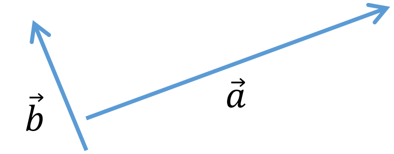


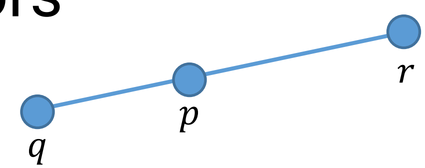
Constraints, Degrees of Freedom

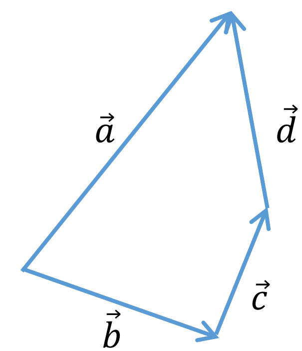
- What are degrees of freedom?
- What is a constraint?
- Describe geometric relationships
  - Distances
  - Perpendicular, Parallel Vectors
  - Point on a line
  - Loops
  - Point a distance from a line
- Resolve Vector Equations

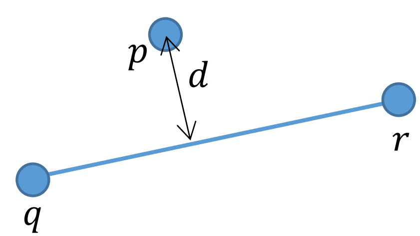






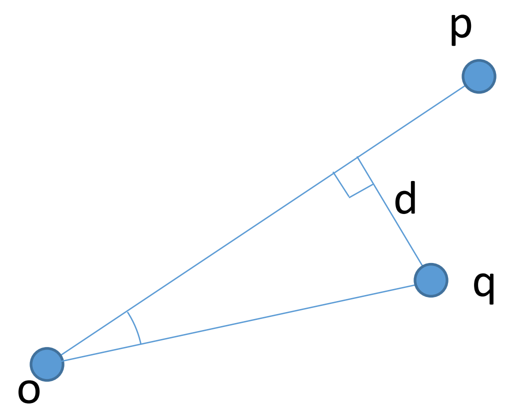






Distance from a Line

- $(\vec{r}^{oq}) \times (\vec{r}^{op}) = |\vec{r}^{oq}||\vec{r}^{op}| \sin(\theta)\, \hat{u}$
- $|(\vec{r}^{oq}) \times (\vec{r}^{op})| = |\vec{r}^{oq}||\vec{r}^{op}| \sin(\theta)$
- $\frac{|(\vec{r}^{oq})\times(\vec{r}^{op})|}{|\vec{r}^{op}|} = |\vec{r}^{oq}| \sin(\theta) = d$
- $\frac{|(\vec{r}^{oq})\times(\vec{r}^{op})|}{|\vec{r}^{op}|} - d = 0$



Degrees of Freedom

$$M = (6-d)(m-1) - \sum_{i=1}^{p}(6-d-f_i)$$

d: mechanism family

m: number of links

p: number of joints

f_i:degree of freedom of each joint

Note: does not account for redundancy

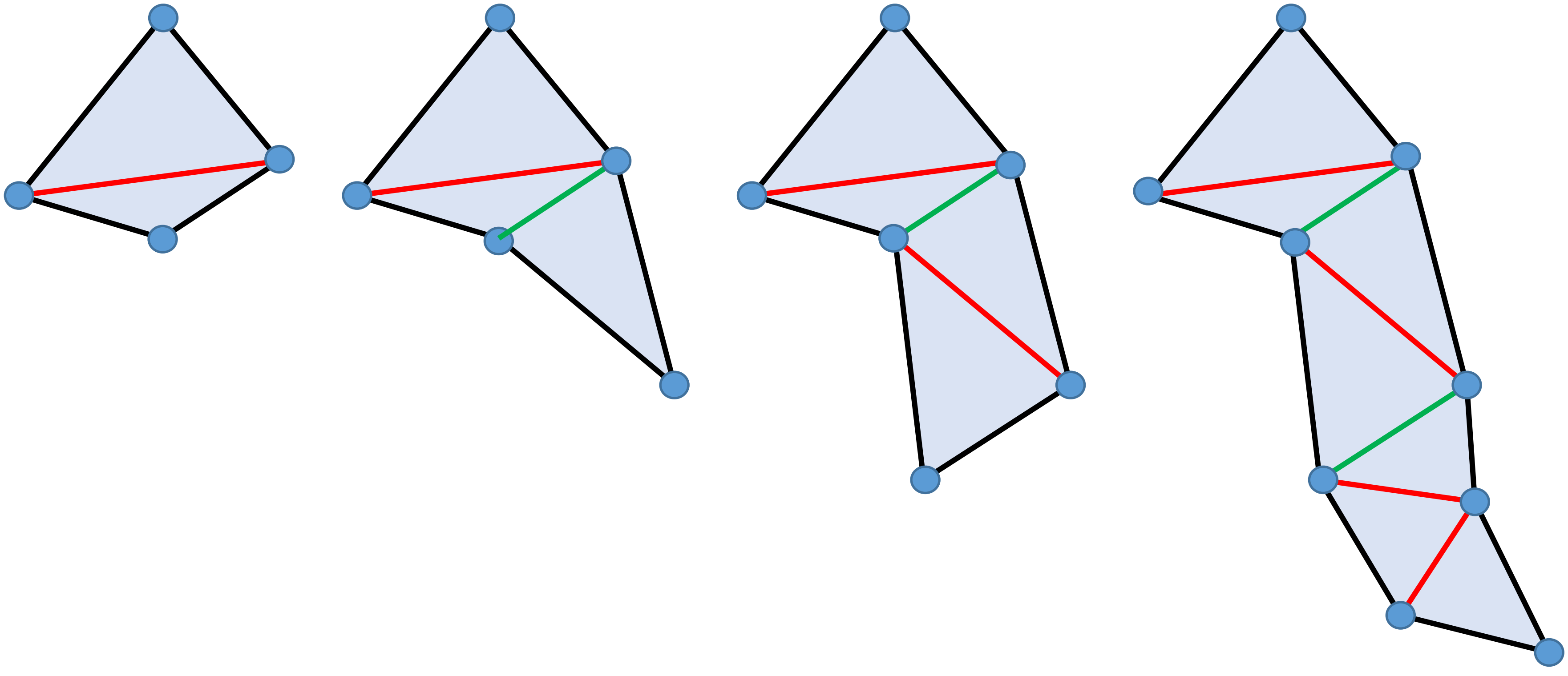
Chebychev–Grübler–Kutzbach criterion

[1] K. J. Waldron, “The constraint analysis of mechanisms,” *J. Mech.*, vol. 1, no. 2, pp. 101–114, Jan. 1966.

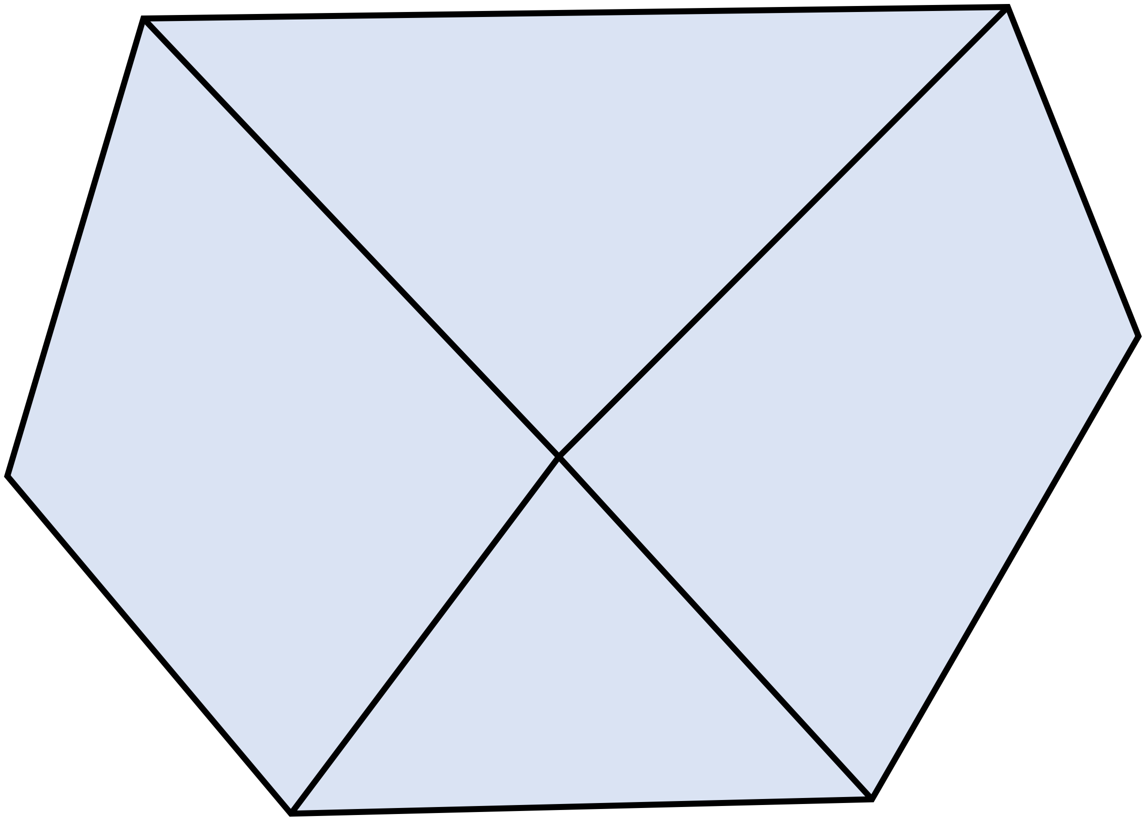
[2] C. Gosselin and J. Angeles, “Singularity analysis of closed-loop kinematic chains,” *IEEE Trans. Robot. Autom.*, vol. 6, no. 3, pp. 281–290, Jun. 1990.

[3] G. Gogu, “Mobility of mechanisms: A critical review,” *Mech. Mach. Theory*, vol. 40, no. 9, pp. 1068–1097, 2005.

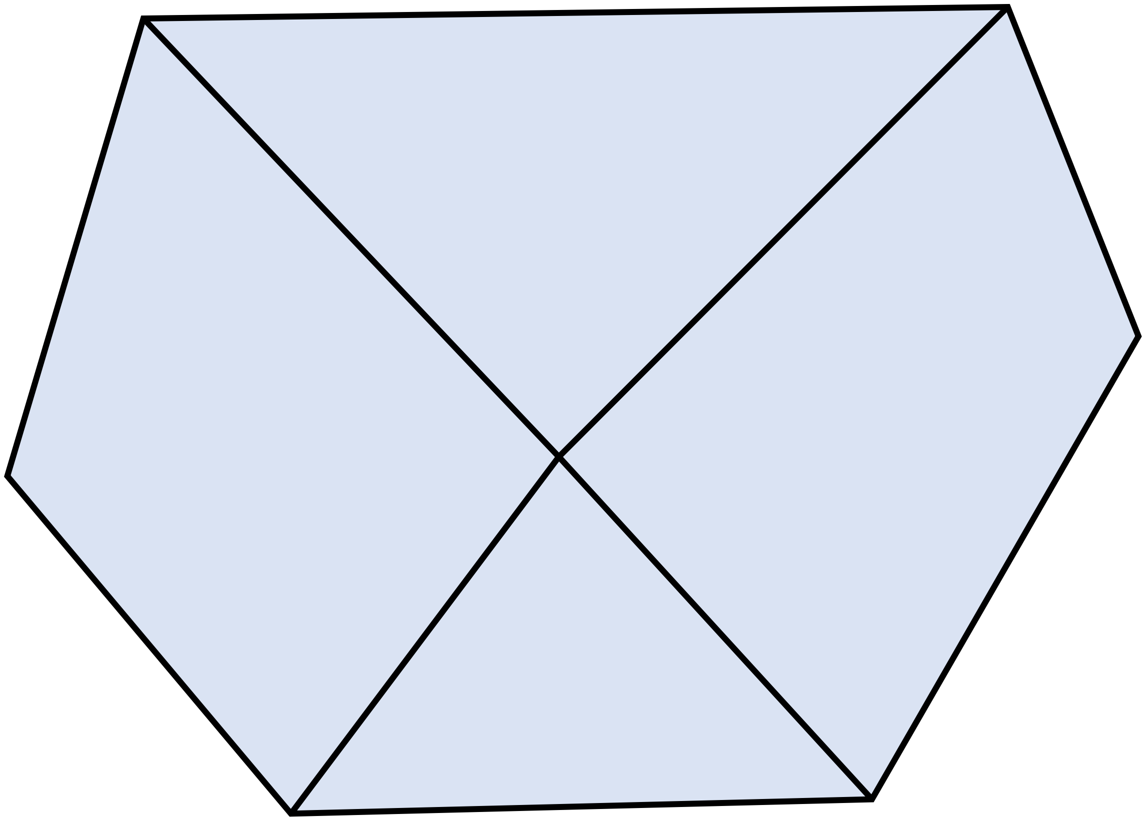
Origami Folds are Joints: Open Chains



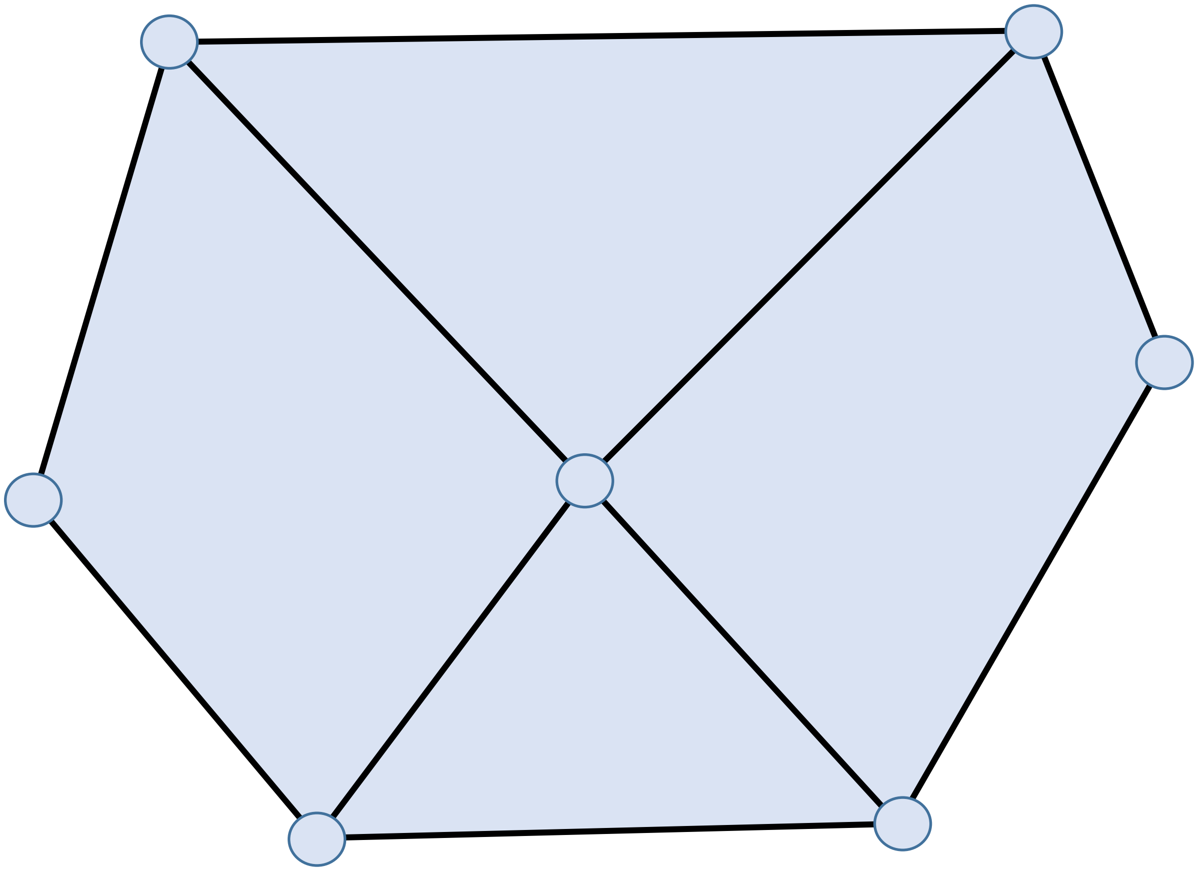
A Spherical Mechanism is a Closed Chain



Number of Degrees of Freedom



Number of Degrees of Freedom



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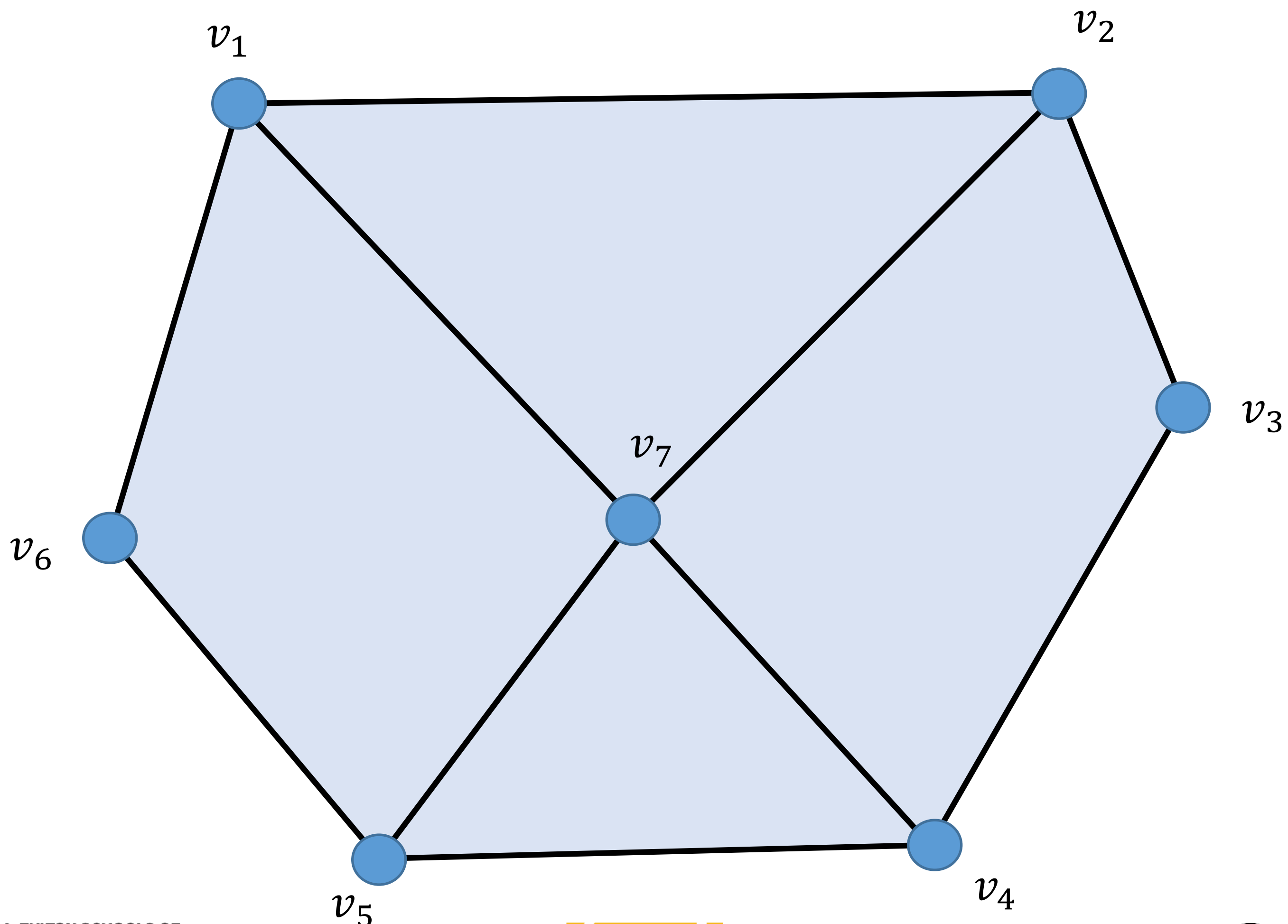
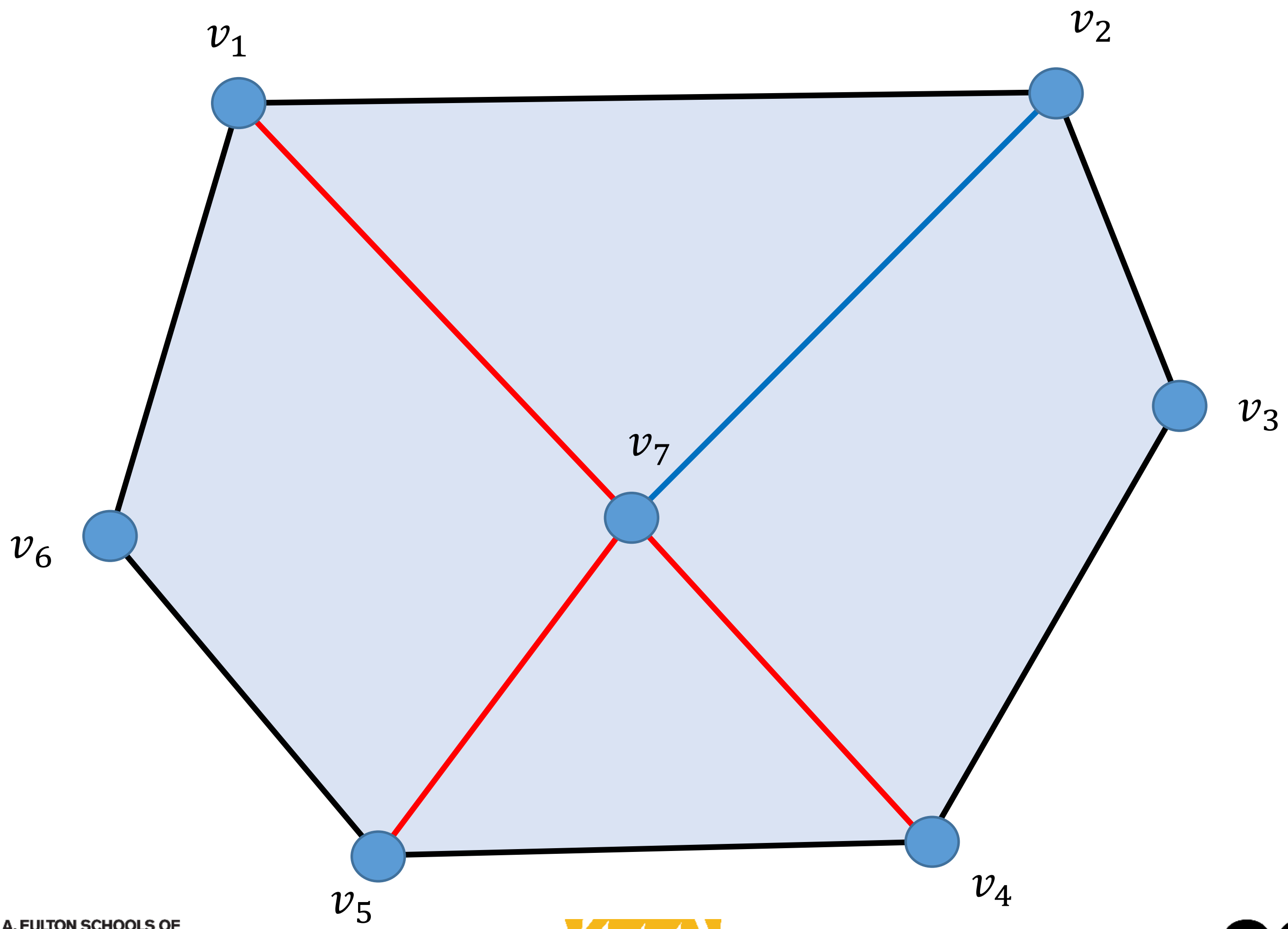
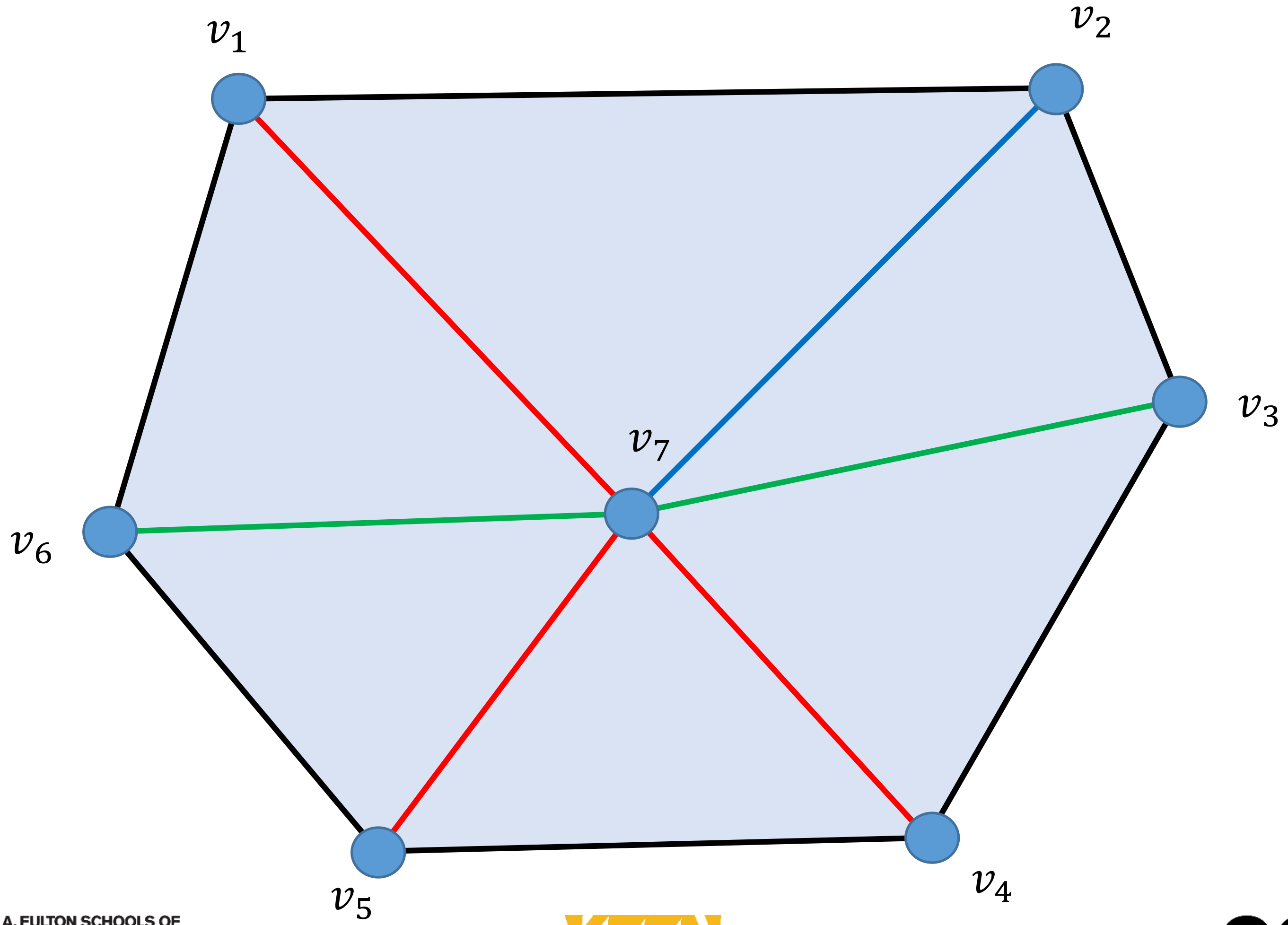


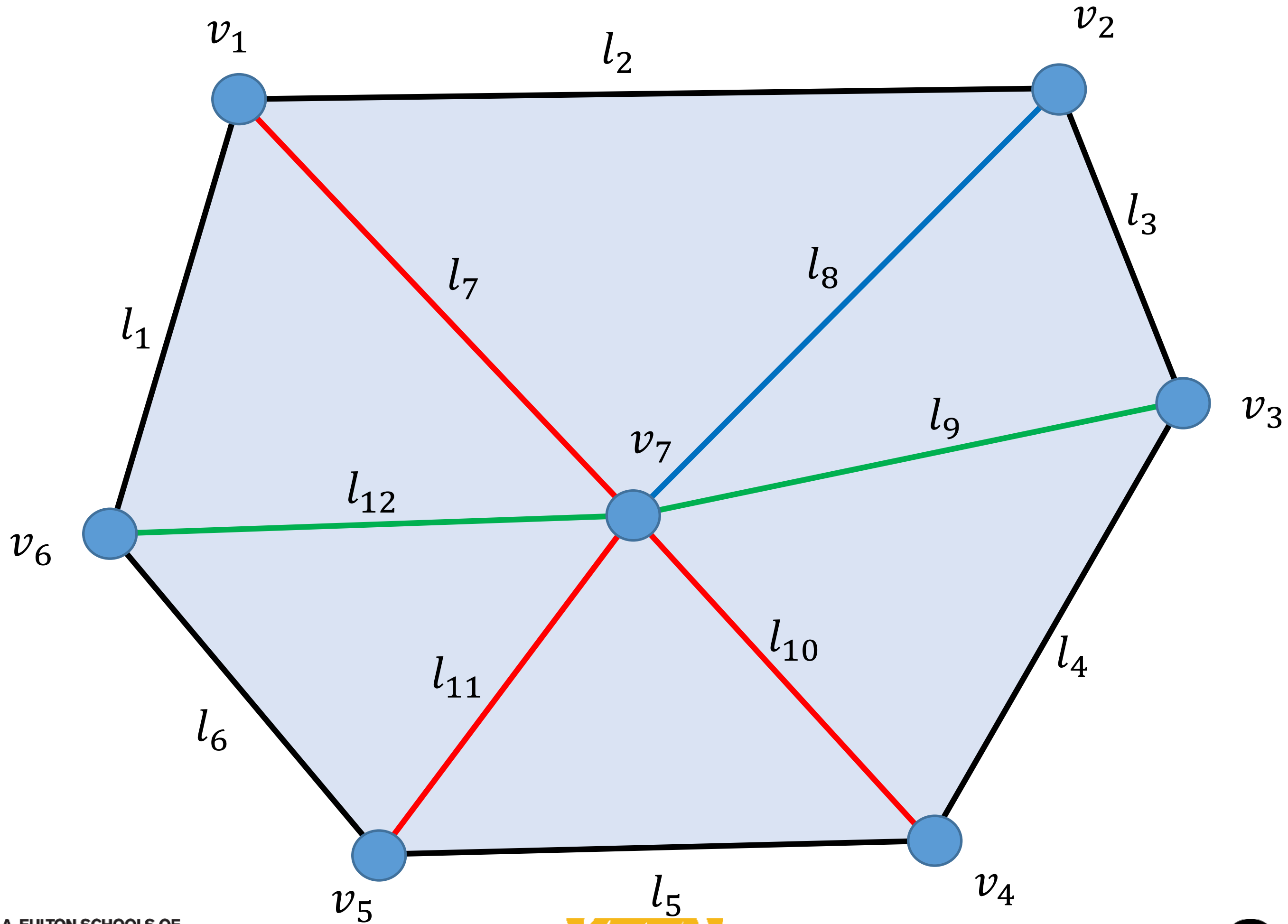
Number of Degrees of Freedom



Number of Degrees of Freedom

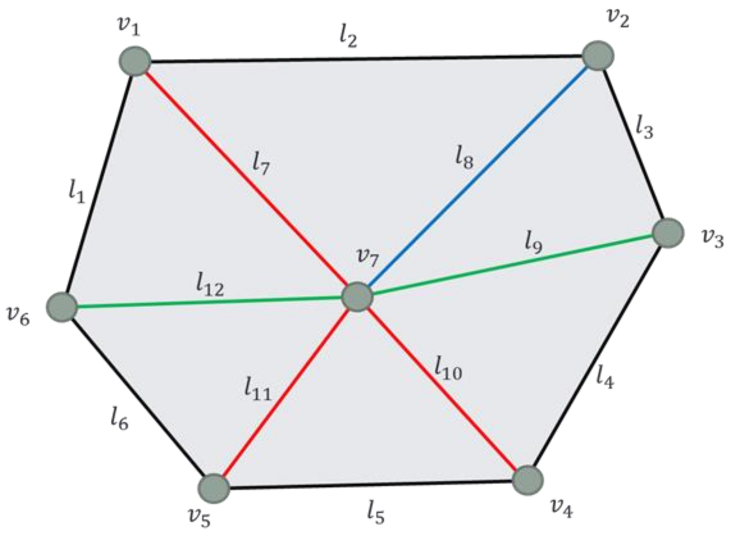


Number of Degrees of Freedom

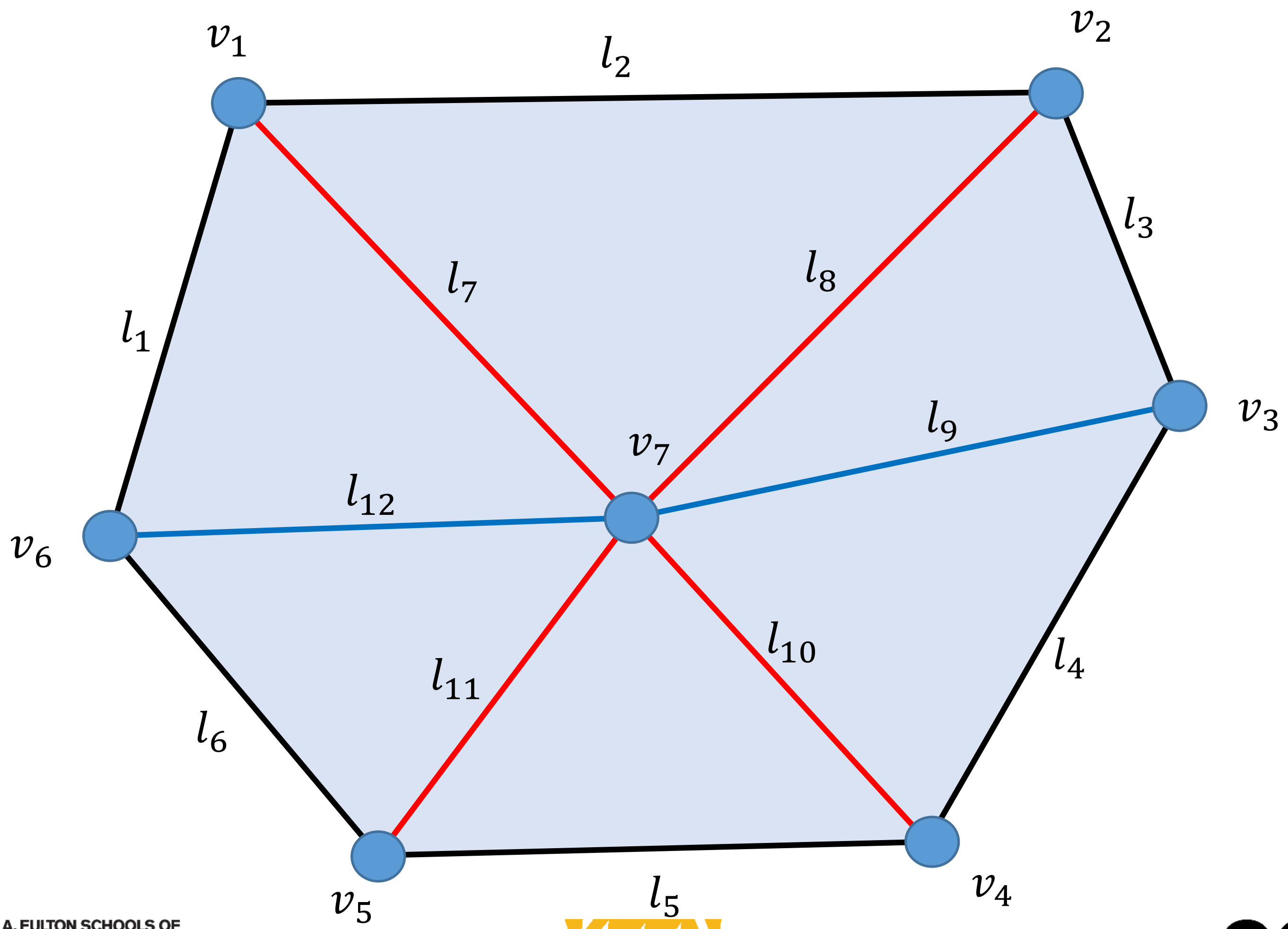


Number of Degrees of Freedom

- n: number of vertices
  - $n = 7$
- m: number of lines
  - $m = 12$
- r: number of green lines
  - $r = 2$
- f : degrees of freedom
  - $f = n * 3 - m - r - 6$
  - $f = 1$

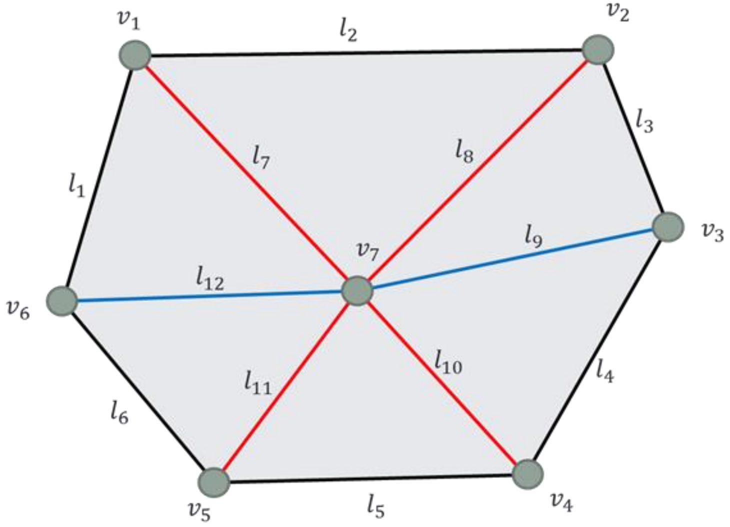


Number of Degrees of Freedom

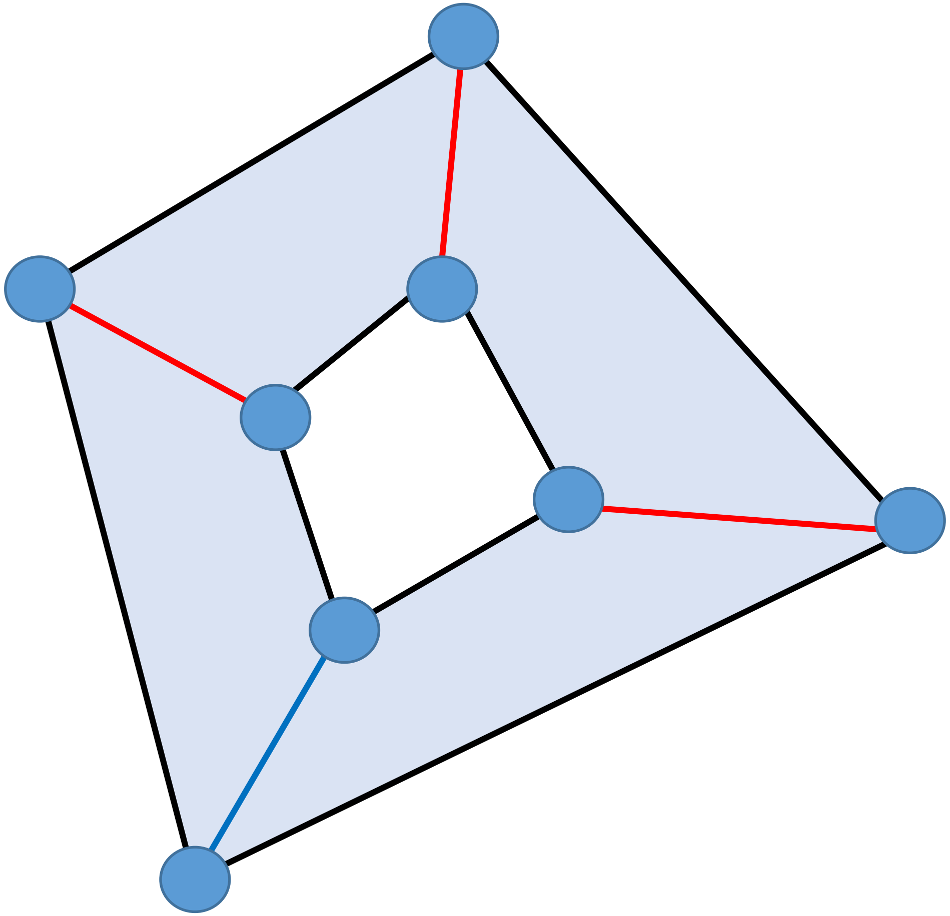


Number of Degrees of Freedom

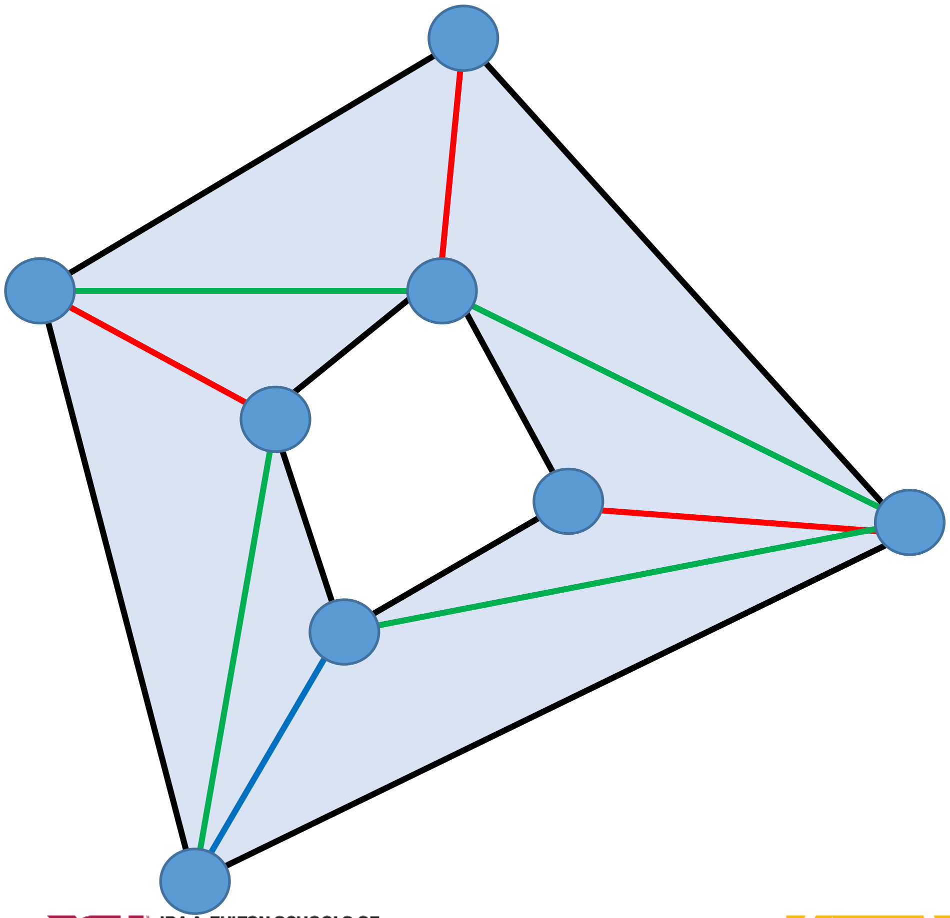
- n: number of vertices
  - $n = 7$
- m: number of lines
  - $m = 12$
- r: number of green lines
  - $r = 0$
- f : degrees of freedom
  - $f = n * 3 - m - r - 6$
  - $f = 3$



Failures: What's the problem?

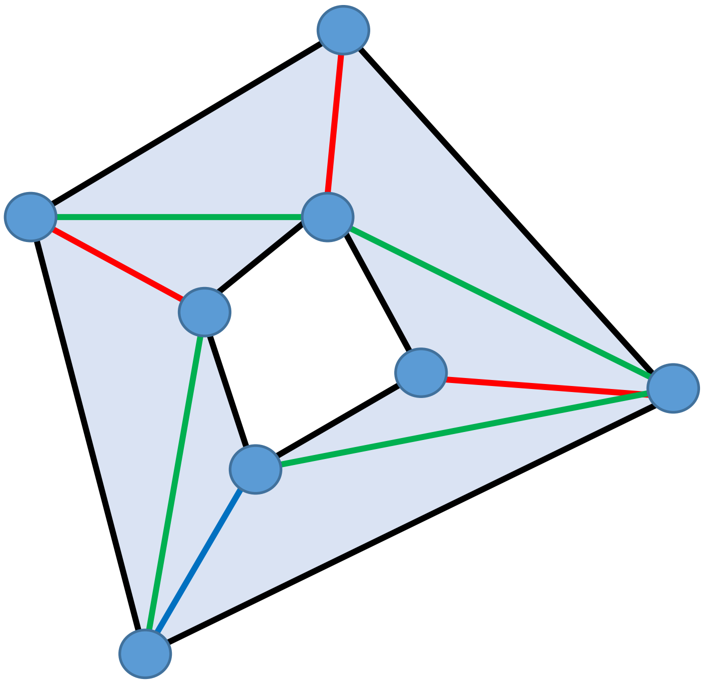


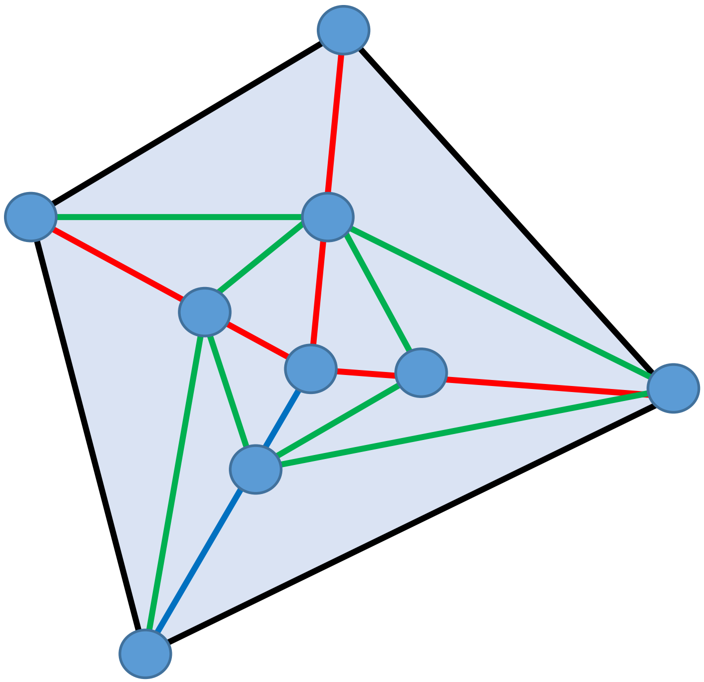
Failures: What's the problem with holes?



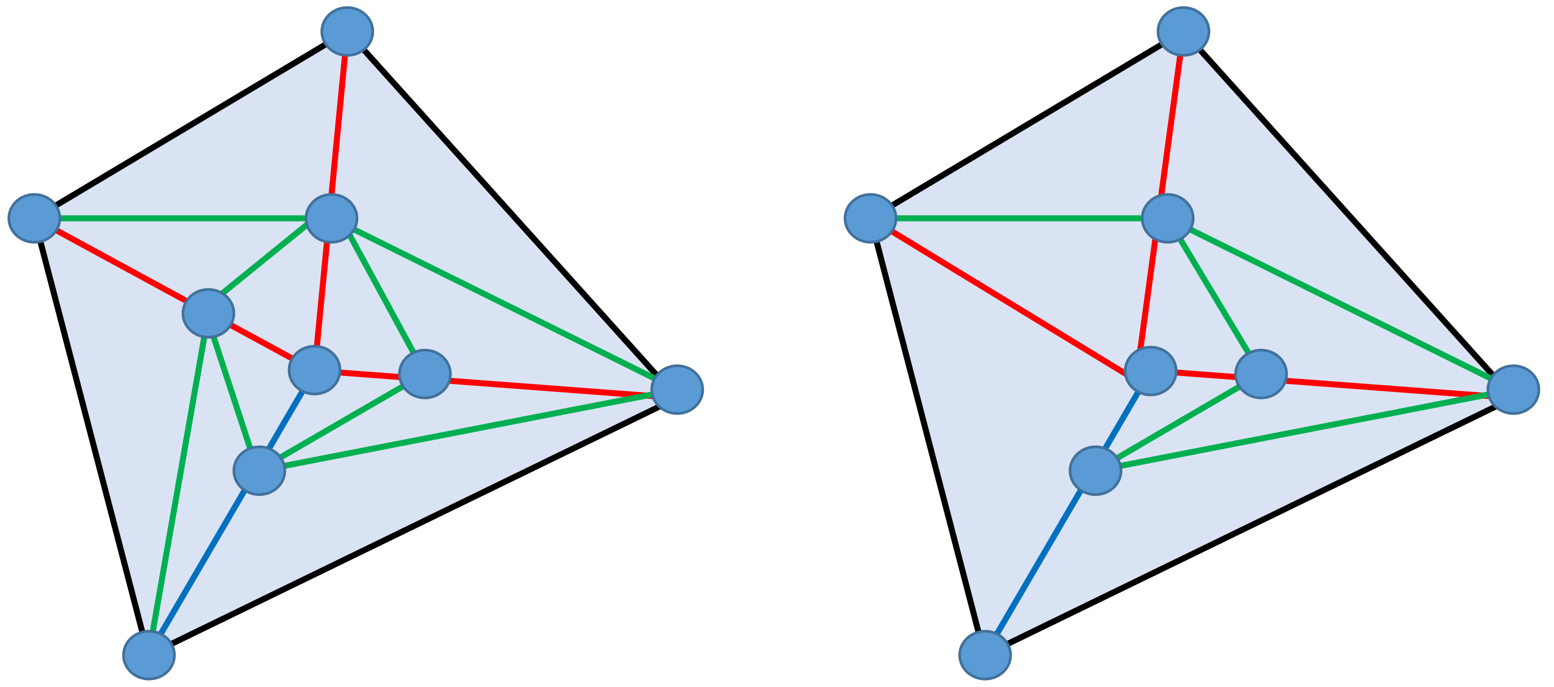
$$f = n * 3 - m - r - 6$$
$$f = 8 * 3 - 12 - 4 - 6$$
$$f = -2$$

Failures: What's the difference?

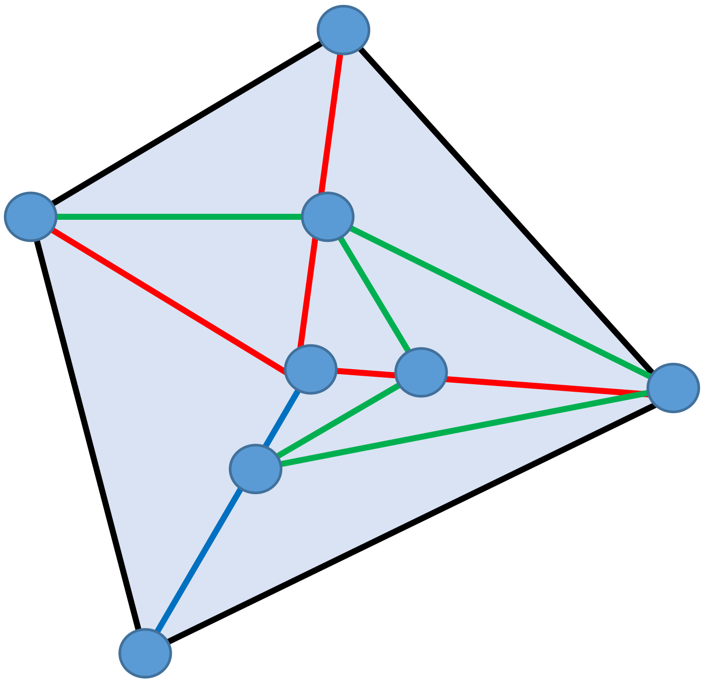


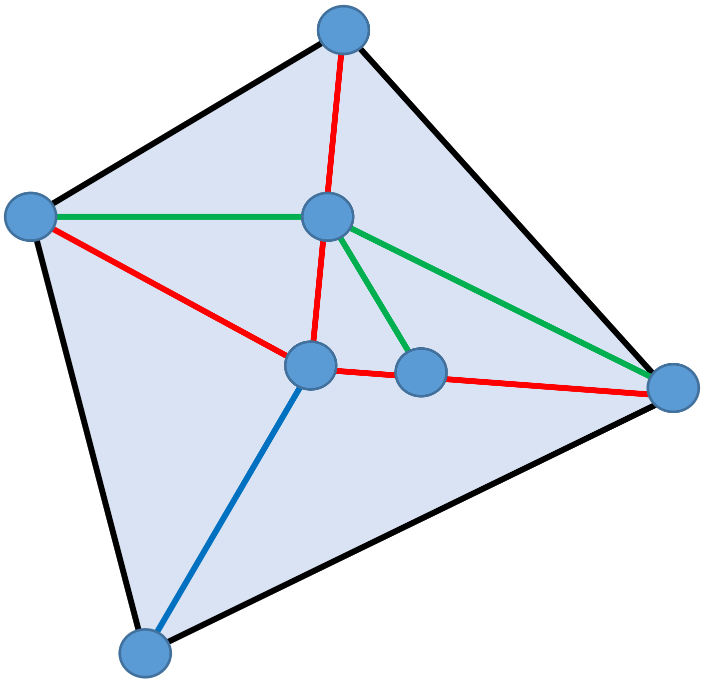


Failures: What's the problem with holes?

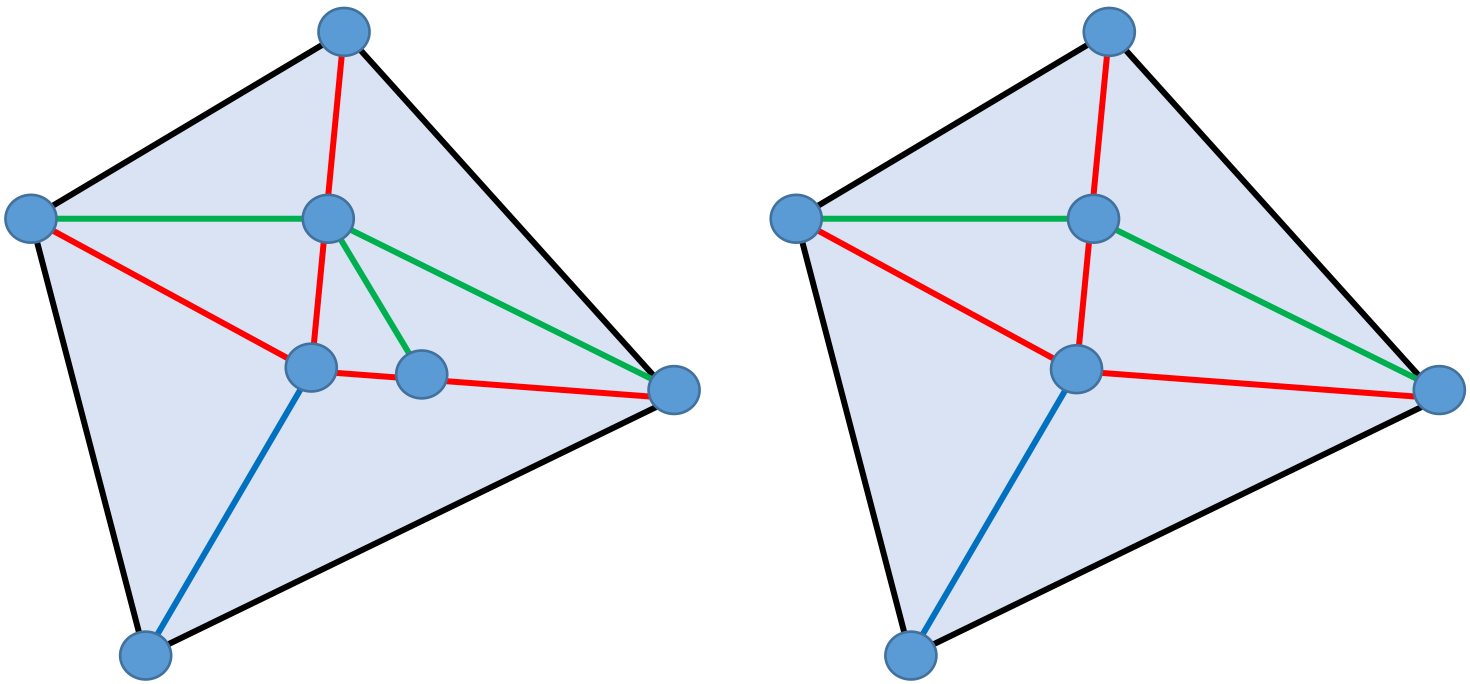


Failures: What's the problem with holes?

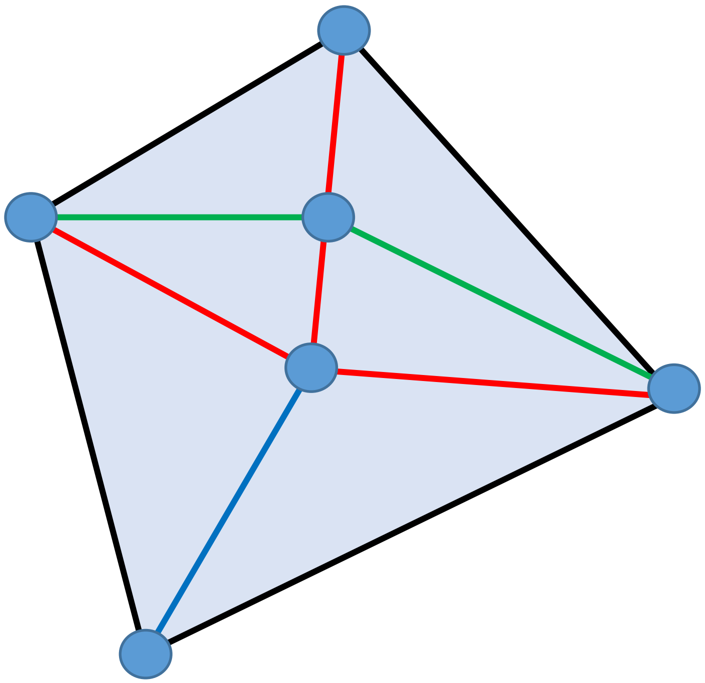


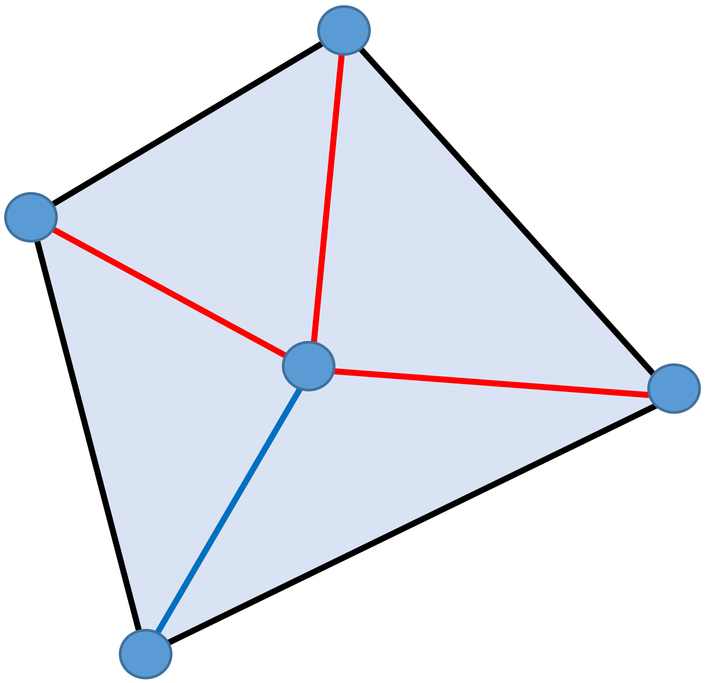


Failures: What's the problem with holes?

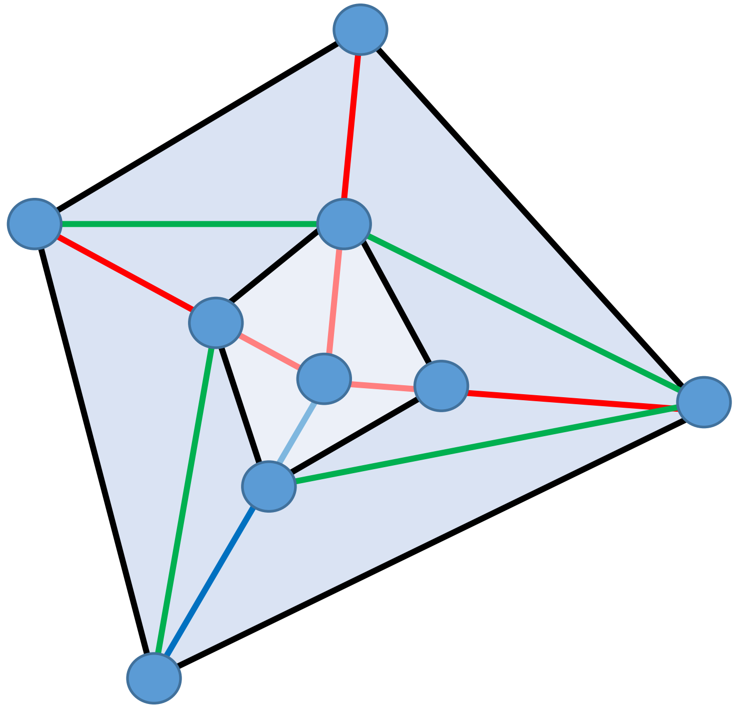


Failures: What's the problem with holes?



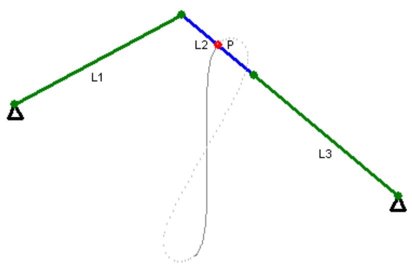


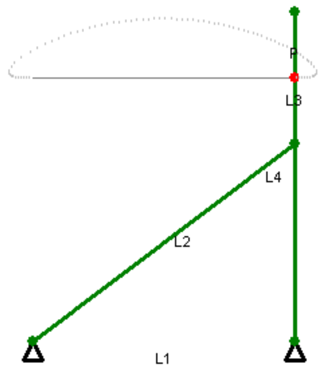
Failures: What's the problem with holes?

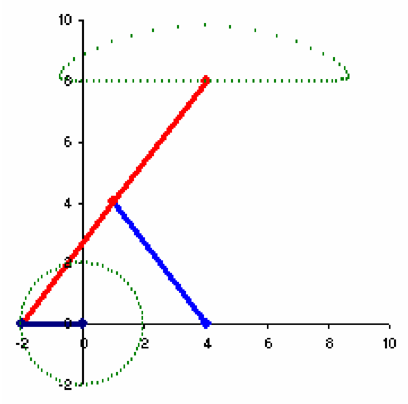


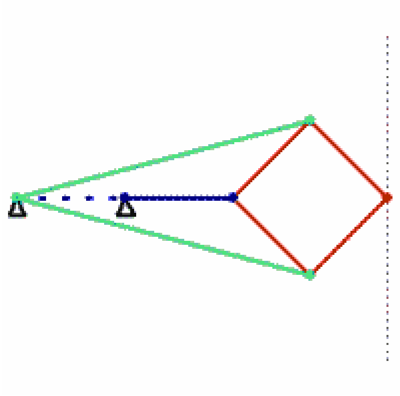
Holes hide geometric constraints

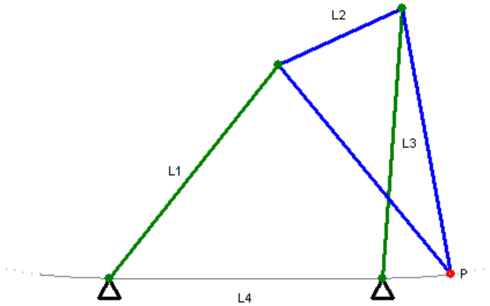
Straight-line mechanisms



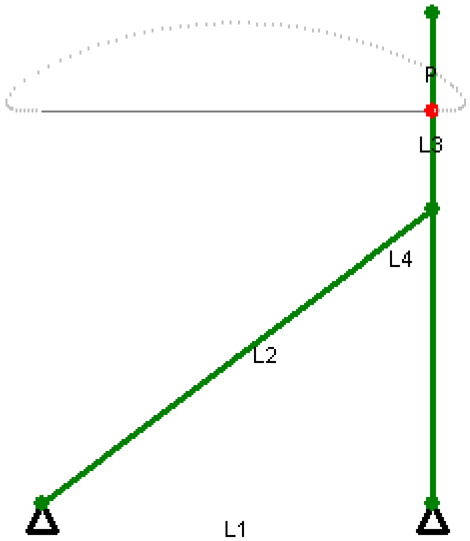


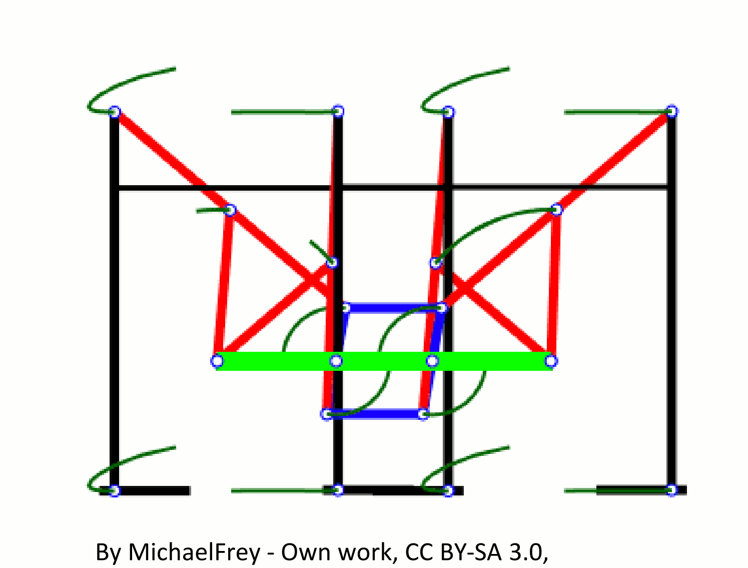






Chebshyev





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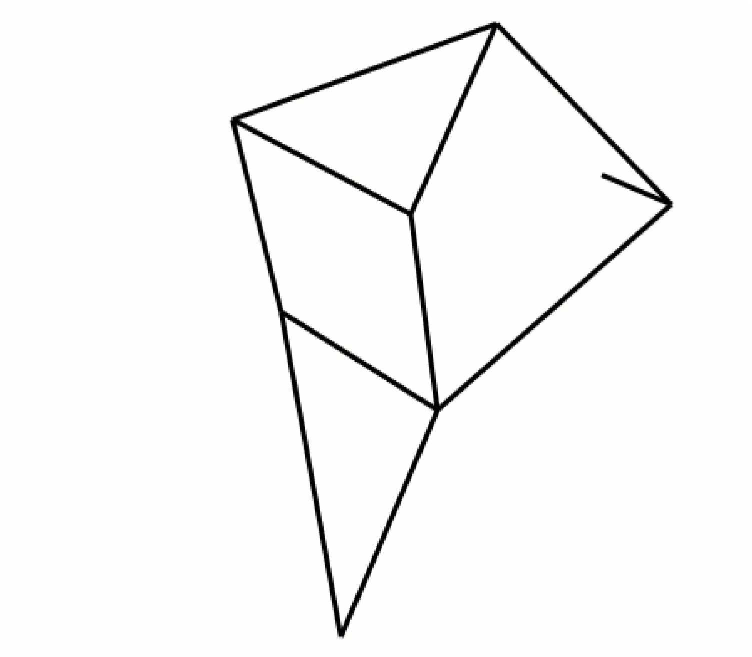
Jansen’s Mechanism



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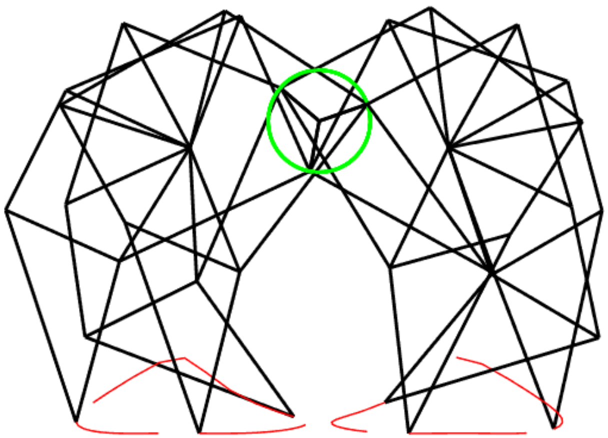

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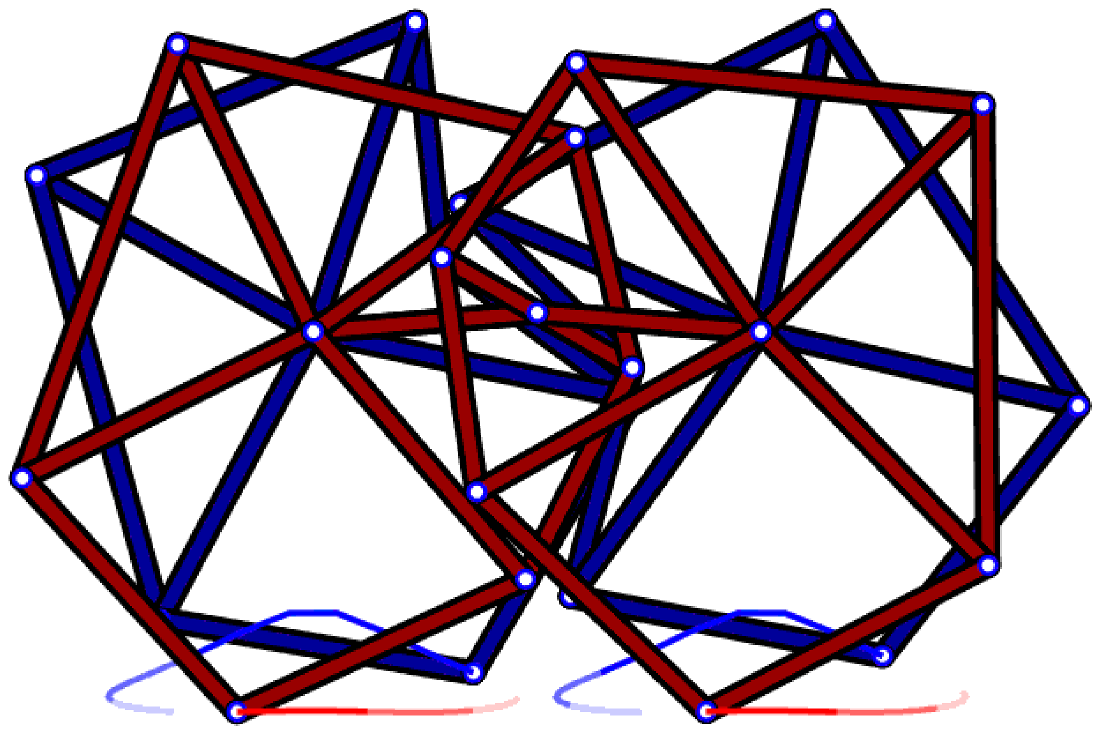
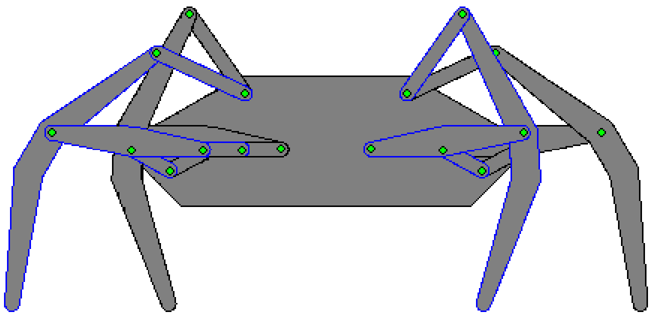
Strandbeest

Similar, but different



Other Mechanisms



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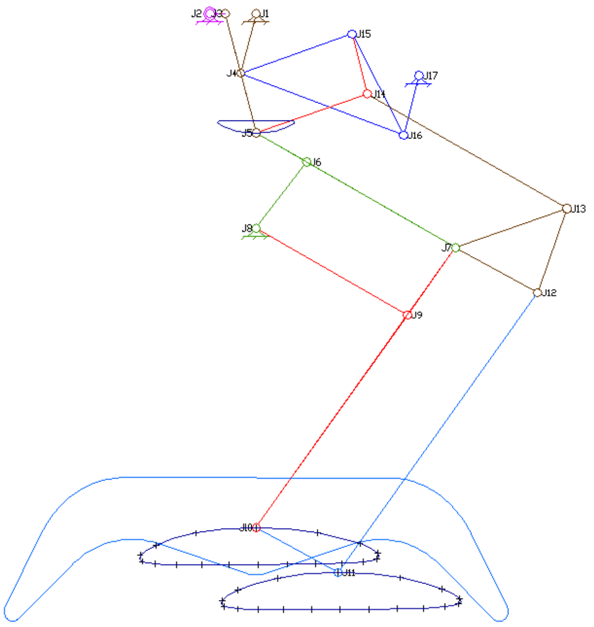

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Other Mechanisms

Jupyter Coding Exercise

Graphical Synthesis Exercise

Other References

- http://4volt.com/projects/jansen/
- http://robotronics09.blogspot.com/2011/01/klann-mechanism.html
- http://sites.uci.edu/markplecnik/projects/leg_mechanisms/

Popup Mechanisms

- [1] Smithsonian Institution, “Paper Engineering: Fold, Pull, Pop and Turn,” 2010.
- [1] B. G. Winder, S. P. Magleby, and L. L. Howell, “Kinematic Representations of Pop-Up Paper Mechanisms,” *J. Mech. Robot.*, vol. 1, no. 2, p. 021009, 2009.

Kinematics Reading

- J. M. McCarthy and G. S. Soh, Geometric Design of Linkages, vol. 11. New York, NY: Springer New York, 2011.
- G. Gogu, “Mobility of mechanisms: A critical review,” Mech. Mach. Theory, vol. 40, no. 9, pp. 1068–1097, 2005.
- K. J. Waldron, “The constraint analysis of mechanisms,” J. Mech., vol. 1, no. 2, pp. 101–114, Jan. 1966.
- C. Gosselin and J. Angeles, “Singularity analysis of closed-loop kinematic chains,” IEEE Trans. Robot. Autom., vol. 6, no. 3, pp. 281–290, Jun. 1990.
- B. G. Winder, S. P. Magleby, and L. L. Howell, “Kinematic Representations of Pop-Up Paper Mechanisms,” *J. Mech. Robot.*, vol. 1, no. 2, p. 021009, 2009.