Chapter 62

Assignment 2: Kinematics – Part 1

62.1 Introduction

The purpose of this assignment is to demonstrate your knowledge of kinematics through exemplar exercises. Part 1 of the assignment looks specifically at the following content:

- Vectors and vector operations
- Basic properties of the quaternion
- The use of vector expressions for defining constraint equations

62.2 Instructions

62.2.1 Part 1: Vector Math

$$\vec{v}_1 = c_1\hat{a}_x + c_2\hat{a}_y + c_3\hat{a}_z \tag{62.1}$$
$$\vec{v}_2 = c_4\hat{a}_x + c_5\hat{a}_y + c_6\hat{a}_z \tag{62.2}$$
$$\vec{v}_3 = c_7\hat{b}_x + c_8\hat{b}_y + c_9\hat{b}_z \tag{62.3}$$

Expand each expression below into basis vector components, and then simplify where possible. Do not just show your answers or skip steps.

1. $|\vec{v}_2|$
2. $\vec{v}_1 \times \vec{v}_3$
3. Now assume you are given more information about the relationship between the A and B reference frames. Simplify your expression for the previous question using $\theta = \frac{\pi}{3}$ and

$$
{}^{a}R^{b}(\theta) = \begin{array}{c|ccc} & b_x & b_y & b_z \\ \hline a_x & \cos\theta & 0 & \sin\theta \\ a_y & 0 & 1 & 0 \\ a_z & -\sin\theta & 0 & \cos\theta \end{array}
$$

62.2.2 Part 2: Quaternion Math

You are given two expressions for quaternions,

$$q_1 = 1 + 2i + 2j \tag{62.4}$$
$$q_2 = 2 + 3j + k \tag{62.5}$$

And a vector defined in the A frame as:

$$\vec{r} = 3\hat{a}_x + 2\hat{a}_y + 4\hat{a}_z$$

1. What is $q_1 + q_2$? Show the expanded form of this expression.
2. What is $q_1 q_2$? Show the expanded form of this expression.
3. Find $\hat{q}_1$, the unit quaternion of q_1
4. Identify the axis of rotation and equivalent rotation angle that $\hat{q}_1$ represents.
5. Using quaternion math, rotate $\vec{r}$ by $\hat{q}_1$.

62.2.3 Part 3: Constraint Equations

1. How many degrees of freedom does a vector have?

2. Say you have three vectors in 3D space, $\vec{a}$, $\vec{b}$, and $\vec{c}$.

    1. Write the necessary constraint equation(s) to constrain $\vec{c}$ such that $\vec{c}$ is parallel to the plane that $\vec{a}$ and $\vec{b}$ belong to. Justify your results with a discussion on degrees of freedom.
    2. Write the necessary constraint equation(s) to constrain $\vec{c}$ such that $\vec{c}$ is perpendicular to the plane that $\vec{a}$ and $\vec{b}$ belong to. Justify your results with a discussion on degrees of freedom.

3. Consider the following figure

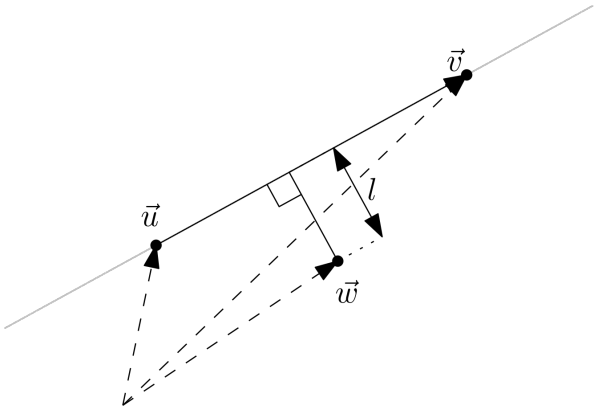

Figure 62.1: caption

1. Write an equation that constrains $\vec{w}$ to be l units *away from* the line that goes through points $\vec{u}$ and $\vec{v}$.
2. If $\vec{u}$ and $\vec{v}$ are fully defined and fixed in space, how many degrees of freedom does vector $\vec{w}$ have?

3. What joint type is this constraint equivalent to?

62.3 Submission

Please include a Jupyter notebook and pdf of the following:

1. Detailed description of the completed steps above
   - Include any chunks of related code as accompanying markdown blocks.
   - Images of handwritten work, parts you've made, and screenshots should be included inline within the ipython notebook. use the following notation:
     `![alt text](assignments/assignment-02/path/to/your/image.jpg)`
     or, if you want to access other html attributes:
     `<img src="path/to/your/image.jpg" width="300px" alt="alt text here">`
2. Code used in solving the assignment (inline in the report in code blocks), along with descriptions detailing your approach.
   - requested plots and figures should be included inline
3. Detailed answers to the discussion questions asked alongside all steps

62.3.1 Other Notes

- Please follow the "Submission Best Practices" document posted on the course website.

- This assignment should be submitted to Canvas as a jupyter notebook (.ipynb) file and pdf.
  - Please ensure that the .ipynb code is fully compiled and *runs without error* in its entirety. This can be done by opening the file in your browser / editor of choice, and by selecting "Restart and Run All" (or similar).
  - Ensure the pdf is legible and makes appropriate use of page scaling to ensure answers are

not cut off, truncated, or spread across multiple pages

