Computational Physics Homework 5

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1 Link to Github

https://github.com/danbateman01/phys-ga2000/tree/main

2 Problem 1

This problem was quite straight forward, I just used previous techniques to create the necessary functions. I also cleaned up the code a bit with ChatGPT.

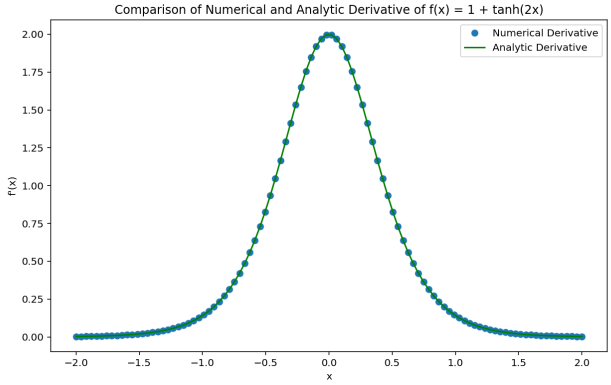

Figure 1: Numerical and Analytical Derivatives

3 Problem 2

3.1 2a

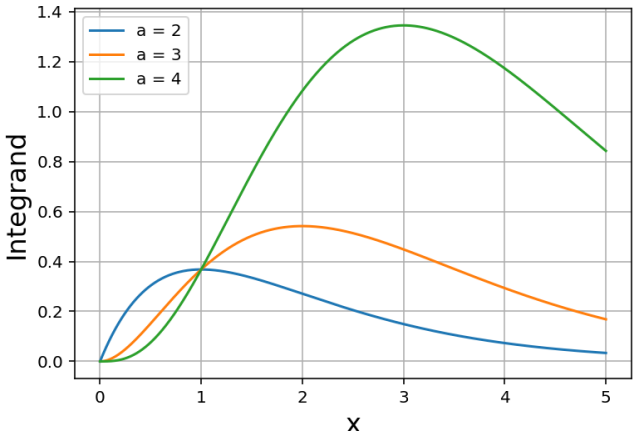

Figure 2: Integrands

To see that the maximum falls at x= a-1 :

3.2 2b

$$f = x^{a-1}e^{-x}$$

$$\frac{df}{dx} = (a-1)x^{a-2}e^{-x} - x^{a-1}e^{-x}$$

$$0 = (a-1)x^{a-2}e^{-x} - x^{a-1}e^{-x}$$

$$\frac{x^{a-1}}{x^{a-2}} = a - 1$$

$$x = a - 1$$

3.3 2c

By plugging in x max as a-1 for$z = 1/2$ you can see the appropriate choice of the parameter c that puts the peak of the integrand for the gamma function at $z = 1/2$ is $c = a - 1$

3.4 2d

One reason this new expression is better is because it has smaller round off errors

3.5 2e and 2f

$$\Gamma(\frac{3}{2}) = 0.88622696$$

$$\Gamma(3) = 2.00000$$

$$\Gamma(6) = 120.00000$$

$$\Gamma(10) = 362880.00000$$

4 Problem 3

4.1 3a

For this problem I just unpacked the data set and plotted it using matplotlib.

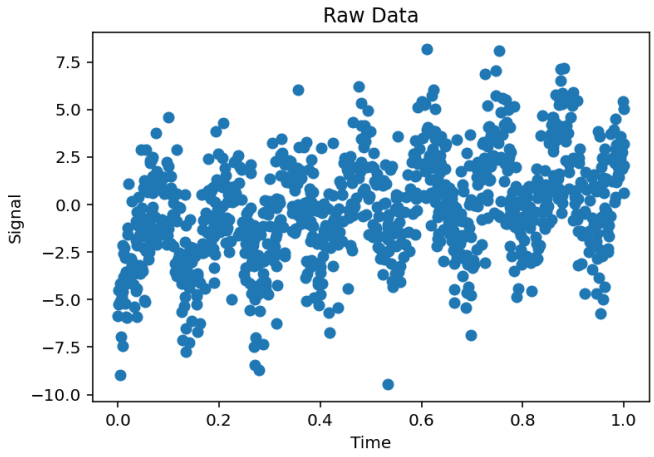

Figure 3: Data Plot

4.2 3b

For this problem I used a for loop and then the SVD code given in class to plot a 3rd order polynomial. You can see that the plot does follow a general trend but does not fit the data set well at all.

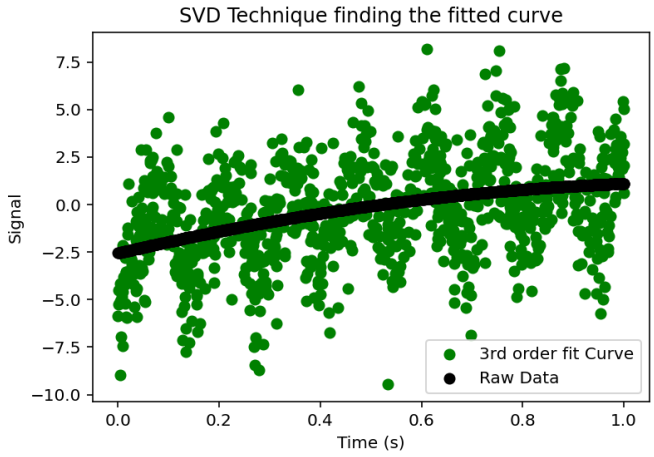

Figure 4: Enter Caption

4.3 c

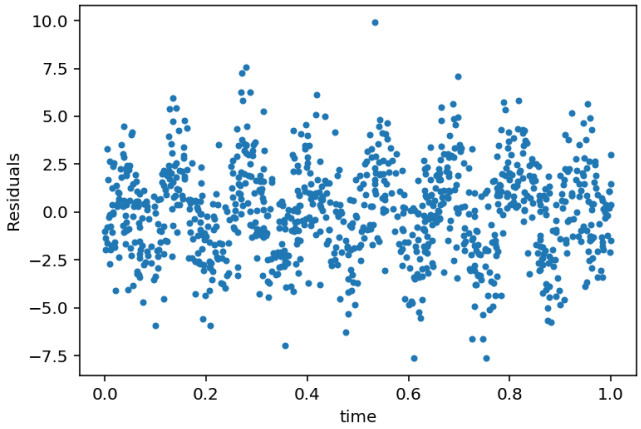

Figure 5: Residuals

4.4 d

Through trial and error of different order polynomials I found that the 36th order polynomial curve looked the best fit. I just changed the order value in my code for part b so there is no extra code for this part.

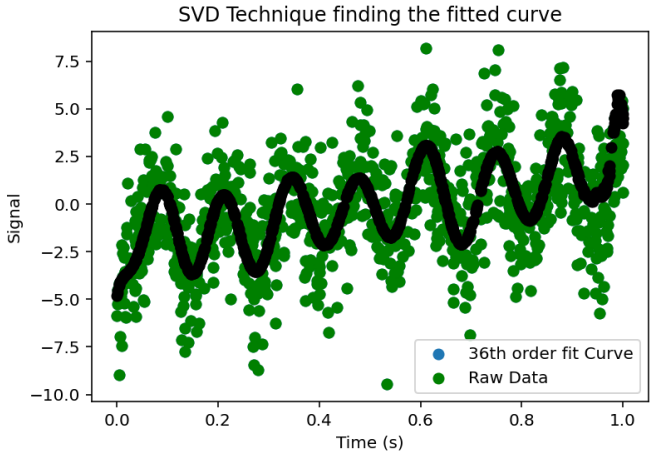

Figure 6: SVD with 36 Order Polynomial