Computational Physics Homework 4

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1 Link to Github

https://github.com/danbateman01/phys-ga2000/tree/main

2 Abstract

This problem set explores the uses of Gaussian quadrature for problems such as the variation of heat capacity with temperature and harmonic oscillators.

3 Problem 1

Here I used Debye's theory of solids to model how the heat capacity of aluminium varies with temperature. Using Gaussian quadrature to evaluate the integral, I first attempted to import gaussxw but i couldn't get that to work so I instead created a function. In figure 1 you can see a good representation of the function. For the convergence test in part c you can clearly see the function converges after N=5.

4 Problem 2

$$V(a) = \frac{1}{2}m(\frac{dx}{dt})^2 + V(x)$$

$$\frac{dx}{dt} = \sqrt{\frac{2}{m}(V(a) - V(x))}$$

$$\int_0^{\frac{T}{4}} dt = \sqrt{\frac{m}{2}} \int_0^a \frac{dx}{\sqrt{V(a) - V(x)}}$$

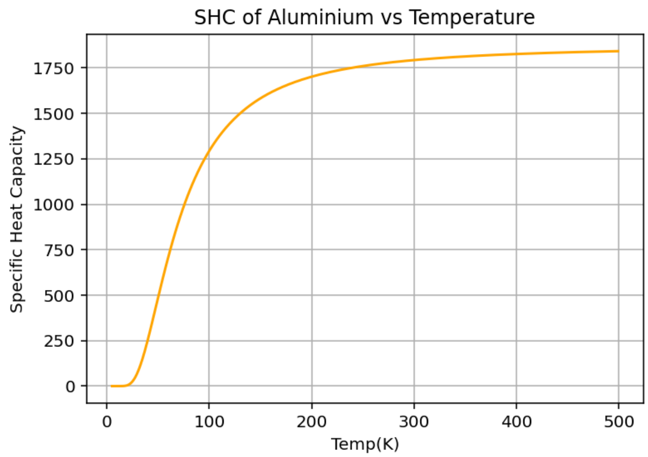

Figure 1: Specific Heat Capacity of Aluminum Vs Temperature

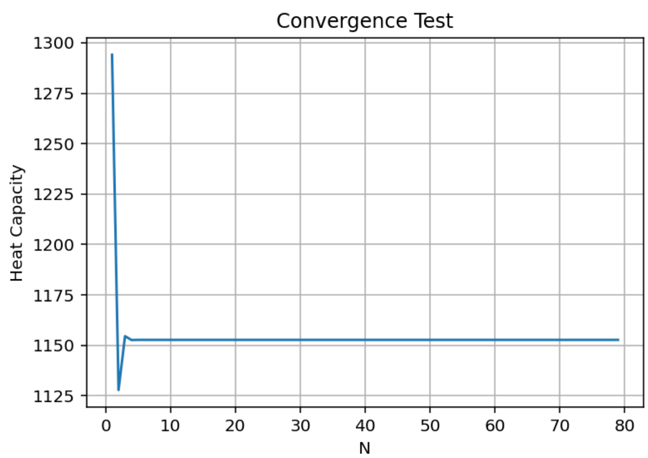

Figure 2: Convergence Test

$$T = \sqrt{8m} \int_0^a \frac{dx}{\sqrt{V(a) - V(x)}}$$

The reason that the oscillator gets faster as the amplitude increases and the period diverges as the amplitude goes to zero is because, by the equation, when you increase the amplitude linearly you increase the potential by the power of 4 and the speed by a power of 2 while the amplitude only increases to the power of 1. The opposite is true also causing the period to diverge as the amplitude reaches 0.

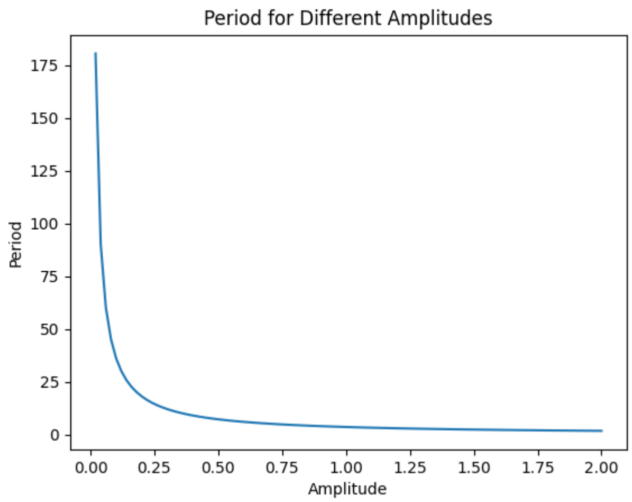

Figure 3: Period Vs Amplitude

5 Problem 3

This problem looked at uncertainty in the quantum harmonic oscillator, I used the gaussxw function from the book and created a plot of the wave-functions. For part c I found the uncertainty to be 2.345 which agrees with the given value in the book.

$$Uncertainty = 2.34521$$

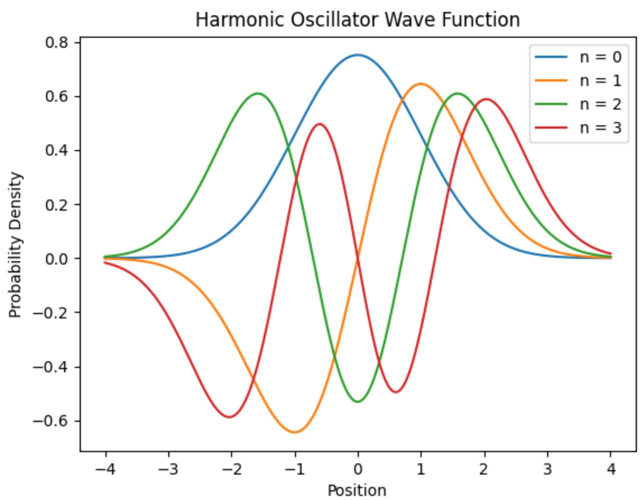

Figure 4: Harmonic Oscilator For N = 0,1,2,3

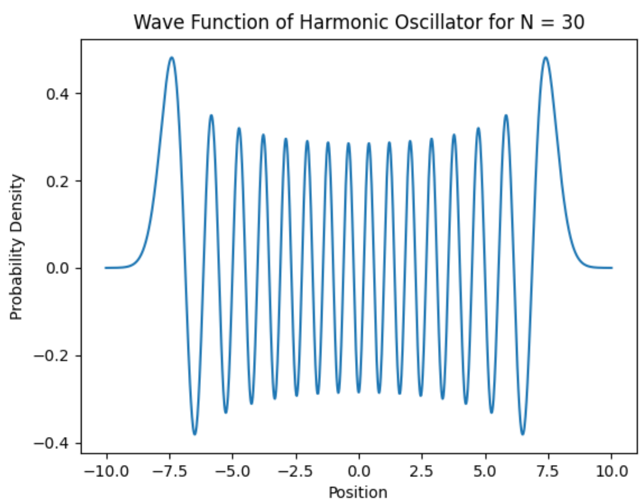

Figure 5: Oscillator at N=30