Rotational Motion

When we consider things that rotate, we unconsciously place them within one of two categories:

- Experiencing the rotation from **within** the rotating frame of reference. (riding a merry-go-round, going around a corner in a car, the spinning tube of a washing machine)
- Experiencing the rotation from **outside** the rotating frame of reference. (a ball on a string, a spinning wheel, the hands of a clock)

When experienced from within a rotating frame of reference, there appears to be a force "pulling" things outward, away from the center of motion. This "force" IS NOT REAL, but it has mistakenly been given a name anyway -- centrifugal.

This is wrong! This centrifugal force does not exist!

For an object to move in a circle, a net centripetal force, directed inward, is necessary. If you have circular motion, you must have a net force directed inward, and this net force is called the CENTRIPETAL FORCE.

Note: the centripetal force is not necessarily a force in and of itself -- it is just the name we give the net force -- the sum of all real forces acting on an object that are directed into the center of motion.
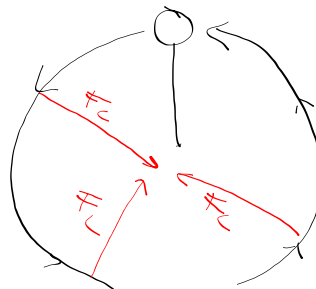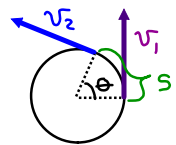
Deriving an expression for the centripetal acceleration and centripetal force required for uniform circular motion.

Uniform circular motion (UCM): motion in a circle at constant speed and at a constant radius.

Assume mass *m* travels at speed *v* with a radius *r*.
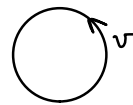
IF WE WISH TO FIND THE CENTRIPETAL ACCELERATION $\vec{a_c}$, WE NEED TO DETERMINE $\Delta v/\Delta t$
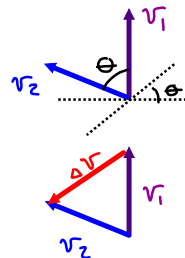
IN Δt, THE MASS WILL HAVE TRAVELLED THE DISTANCE $v\Delta t$

$$v\Delta t = s = \text{ARC LENGTH}$$

$\theta = s/r$ (FROM EARLIER THIS YEAR)

$$\theta = \frac{(v\Delta t)}{r}$$

APPLY $\theta = s/r$ TO THIS AS WELL:

$$\theta = \frac{s}{r} = \frac{\Delta v}{v}$$

$$\theta = \frac{\Delta v}{v} \implies \Delta v = \theta v$$

$$\Delta v = \left(\frac{v\Delta t}{r}\right) v$$

DIVIDE BOTH SIDES BY Δt: $\quad \frac{\Delta v}{\Delta t} = \frac{v^2}{r} = \vec{a_c}$

FOR UCM:

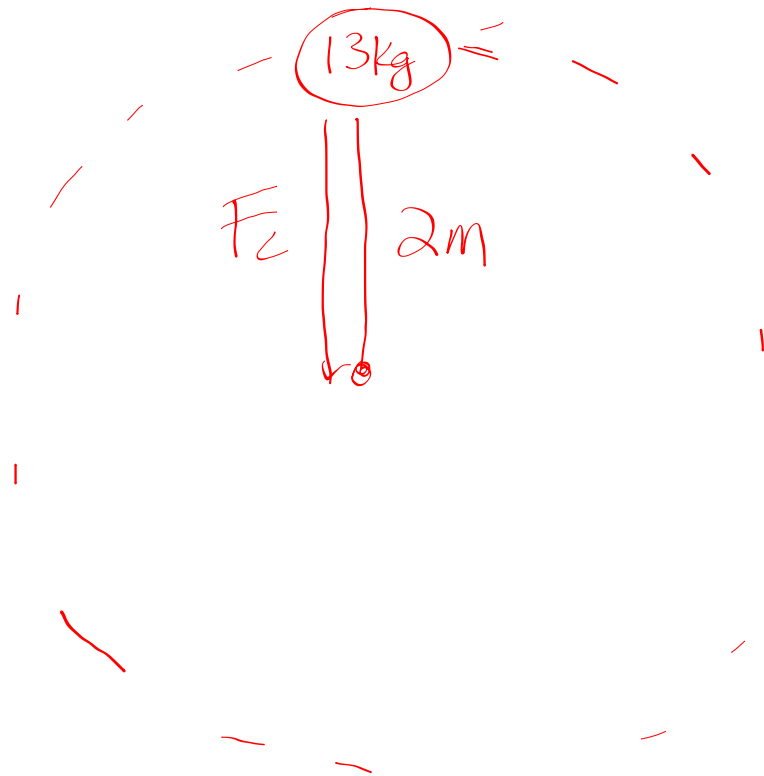$$\boxed{\begin{array}{l} |\vec{a_c}| = \text{CENTRIPETAL ACCELERATION} \\ \vec{a_c} = \frac{v^2}{r} \quad \text{(ALWAYS DIRECTED INWARD)} \end{array}}$$

$$\Sigma F_{\text{INTO/OUT OF CENTER}} = m\vec{a_c} = m\left(\frac{v^2}{r}\right) = \frac{mv^2}{r}$$

$$\boxed{\Sigma F_{\text{INTO/OUT OF CENTER}} = \frac{mv^2}{r} = \begin{array}{l}\text{CENTRIPETAL} \\ \text{FORCE}\end{array}}$$

$$F_C = \frac{m v^2}{r}$$

EXAMPLE 1: If a 13.0 kg ball on a 2.0-meter string is in uniform circular motion (UCM) with a speed of 8.0 m/sec, what acceleration must the ball need to experience?

$$a_c = \frac{v^2}{r}$$

$$F_c = \frac{mv^2}{r}$$

$$a_c = \frac{v^2}{r}$$

$$= \frac{8^2}{2} = 32 \, \text{m/}_{s^2}$$

13 kg

F_c 2m

EXAMPLE 1: If a 13.0 kg ball on a 2.0-meter string is in uniform circular motion (UCM) with a speed of 8.0 m/sec, what acceleration must the ball need to experience?

$$m = 13.0 \text{ kg}$$
$$r = 2.0 \text{ m}$$
$$v = 8.0 \text{ m/s}$$

$$\vec{a}_c = \frac{v^2}{r} = \frac{8^2}{2} = \boxed{32 \text{ m/s}^2}$$

This acceleration is directed inward, toward the center of motion and is provided by the string. The REAL tensile force in the string provides the centripetal motion in this case.

THE FORCE IN THE STRING:

$$\Sigma F_{IN/OUT} = ma$$

$$F = m\left(\frac{v^2}{r}\right)$$

$$F = \frac{(13)(8)^2}{2} = \boxed{416 \text{ N } \text{ OF TENSION}}$$

EXAMPLE 2: A 350 gram model plane travels at 7.0 m/sec on the end of a 3.1 meter wire. Assume the path of the plane and the wire make a horizontal orbit. What is the tension in the string?

TOP

3.1 m   T

$$T = F_c = \frac{mv^2}{r}$$

$$= \frac{(0.35)49}{3.1}$$

$$= 5.53 \, N$$

EXAMPLE 3: How fast can a car round a corner having a radius of 74.0 meters? Assume the car's mass is 1620 kg, and assume the coefficient of friction between the car's tires and the road is 0.3.
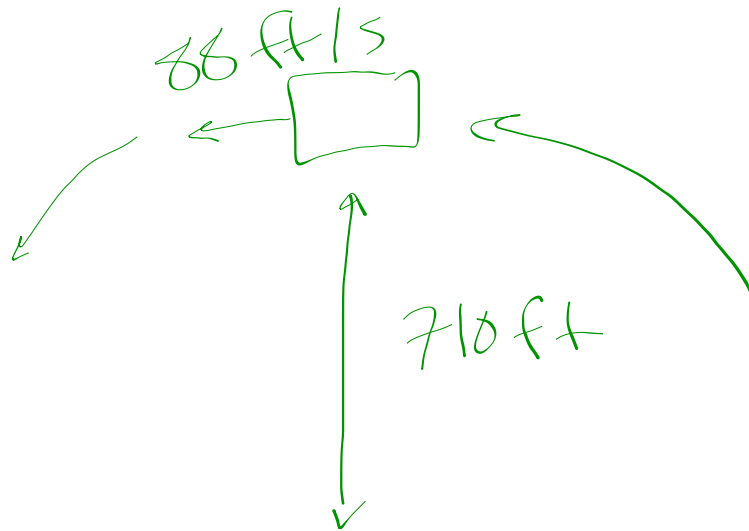
TOP

74m
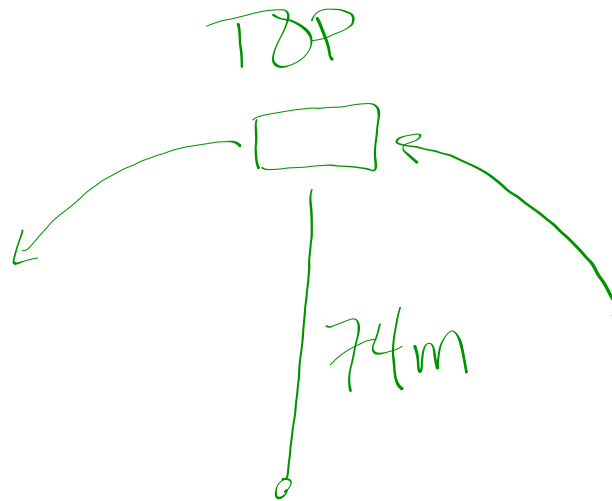
$$F_{Fr} = F_c$$

$$\mu m g = \frac{m v^2}{r}$$

$$F_{Fr} = \mu F_N = \mu m g$$

$$v = \sqrt{\mu \cdot g \cdot r}$$

$$= 14.7 \, \text{m/s}$$

EXAMPLE 4: What minimum coefficient of friction is needed to prevent a car travelling at 60.0 mph from sliding off a curve having a radius of 710.0 feet?

$\rightarrow$ 88 ft/sec

88 ft/s

710 ft

$$F_{fr} = F_c$$

$$\mu mg = \frac{mv^2}{r}$$

$$\mu = \frac{v^2}{r \cdot g} = \frac{88^2}{(710)(32.2)}$$

$$\boxed{\mu = 0.341}$$

EXAMPLE 5: How much should a road with a 110 meter curve be banked (what is the angle) to accommodate cars travelling at 25 m/sec (so that friction is NOT required to get the car around the corner)?

25 m/s

CAR

F_c | 110 m

F_c

F_N

car

θ | θ F_N

mg | F_{Ny}

F_{Nx}

$F_{Ny} = mg$

θ F_N

F_{Nx}

$\tan \theta = \dfrac{o}{a}$

$\tan \theta = \dfrac{F_{Nx}}{mg}$

$F_{Nx} = mg \tan \theta$

$\Sigma F = F_c = \boxed{\dfrac{mv^2}{r}} = F_{Nx} = \boxed{mg \tan \theta}$
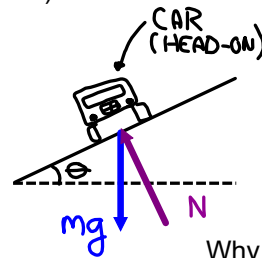
$\dfrac{mv^2}{r} = mg \tan \theta$

$\tan \theta = \dfrac{v^2}{r \cdot g}$

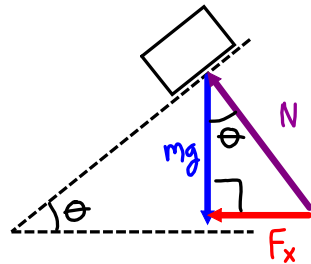$\theta = \tan^{-1} \dfrac{v^2}{r \cdot g} = \dfrac{25^2}{110 \cdot 9.8}$

$\theta = 30.1°$

EXAMPLE 5: How much should a road with a 110 meter curve be banked (what is the angle) to accommodate cars travelling at 25 m/sec (so that friction is NOT required to get the car around the corner)?

CAR (HEAD-ON)

Of the two real forces acting on the car (there is no friction in this problem), only the normal force has a component directed into the center of motion. Therefore we want to use its x-component when summing all forces into or out of the COM.

N

mg

Why is the purple arrow (the normal force) as long as it is? Because its y-component MUST be equal and opposite to gravity, mg.

N

mg

θ

F_x

$\Sigma F = ma$

$\Sigma F = m\left(\frac{v^2}{r}\right)$

$F_x = \frac{mv^2}{r}$

ALSO,

$\tan\theta = \frac{F_x}{mg}$

$\therefore F_x = mg\tan\theta$

F_x, the red arrow and the horizontal component of the normal force, is what provides the centripetal force necessary to get the car around the corner.

$F_x = \frac{mv^2}{r}$

$\cancel{m}g\tan\theta = \frac{\cancel{m}v^2}{r}$ (MASS DOES NOT MATTER)

$\tan\theta = \frac{v^2}{rg}$

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

$\theta = \tan^{-1}\left(\frac{25^2}{110(9.8)}\right) = \boxed{30.1°}$

EXAMPLE 6: On an icy day, how fast can a car travel around a 200.0-ft. corner that is only banked at 12.0 degrees?

$$g \tan \theta = \frac{v^2}{r}$$

$$v = \sqrt{r \cdot g \cdot \tan \theta} = \sqrt{(200)(32.2)(\tan 12)}$$

$$37 \, ft/s \approx 25 \, mph$$