Using the Kinematic Equations

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CONCEPT 1 Using the Kinematic Equations

Students will learn how to solve all types of problems using the kinematic equations.

In this Concept, you will learn how to solve all types of problems using the kinematic equations known as the BIG 4 – relationships that are valid if and only if the acceleration of all objects remain constant.

Key Equations

Averages

$$v_{avg} = \frac{\Delta x}{\Delta t}$$
$$a_{avg} = \frac{\Delta v}{\Delta t}$$

The Big Four (the last three we will use most of the time)

$\mathbf{v}_{average} = (\mathbf{v}_0 + \mathbf{v})/2$

$\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0\mathbf{t} + (1/2)\mathbf{a}\mathbf{t}^2$

$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}\mathbf{t}$

$\mathbf{v}^2 = \mathbf{v}_0{}^2 + 2\mathbf{a}\,(\mathbf{x}-\mathbf{x}_0)$

Guidance

- When beginning a one dimensional problem, define a positive direction. The other direction is then taken to be negative. Traditionally, "positive" is taken to mean "up" for vertical problems and "to the right" for horizontal problems; however, any definition of direction used consistency throughout the problem will yield the right answer.
- Gravity near the Earth pulls an object toward the surface of the Earth with an acceleration of 9.8 $\mathrm{m/s}^2 (\approx 10\ \mathrm{m/s}^2)$. Generally in class, we will assume gravity is 9.8 m/s^2. If we are utilizing the English system of units, be sure to make the acceleration of gravity 32.2 ft/s^2. In the absence of air resistance, all objects will fall with the same acceleration. Air resistance can cause low-mass, large area objects to accelerate more slowly.
- The BigFour equations define the graphs of position and velocity as a function of time. When there is no acceleration (constant velocity), position increases linearly with time – distance equals rate times time. Under constant acceleration, velocity increases linearly with time but distance does so at a quadratic rate. The slopes of the position and velocity graphs will give instantaneous velocity and acceleration, respectively.

The following examples are from the CK-12 website and have not been modified to reflect the exact method you have been shown by Mr. K in class. Mr. K does not suggest you waste time by changes x's to h's for height. Just always use x (unless it is a two-dimensional problem – then you might also need to use y's, however we haven't arrived at that point yet so don't worry about that right now). Also, instead of calling the acceleration of gravity "g", just keep it "a" and know that it is the acceleration of gravity, which should be obvious from the problem. Finally, in homework always assume g = 9.8 m/s^2 – we won't round it to 10 m/s^2 as the examples do below.

In the other assigned reading linked to the website, Mr. K illustrates his method. That being said, the methonds followed in these examples are similar,serve as a good reference, and should be something you can follow.

Example Problem 1

While driving through Napa you observe a hot air balloon in the sky with tourists on board. One of the passengers accidentally drops a wine bottle and you note that it takes 2.3 seconds for it to reach the ground. (a) How high is the balloon? (b) What was the wine bottle#8217;s velocity just before it hit the ground?

Question ***a****:* $h = ?\,[m]$

Given: $t = 2.3\ s$

$g = 10\ m/s^2$

$v_i = 0\ m/s$

Equation: $\Delta x = v_i t + \frac{1}{2}at^2$ or $h = v_i t + \frac{1}{2}gt^2$

Plug n#8217; Chug: $h = 0 + \frac{1}{2}(10\ m/s^2)(2.3\ s)^2 = 26.5\ m$

Answer:

$$\boxed{\mathbf{26.5\ m}}$$

Question ***b****:* $v_f = ?\,[m/s]$

Given: (same as above)

Equation: $v_f = v_i + at$

Plug n#8217; Chug: $v_f = v_i + at = 0 + (10\ m/s^2)(2.3\ s) = 23\ m/s$

Answer:

$$\boxed{\mathbf{23\ m/s}}$$

Example Problem 2

The second tallest building in the world is the Petronas Tower in Malaysia. If you were to drop a penny from the roof which is 378.6 m (1242 ft) high, how long would it take to reach the ground? You may neglect air friction.

Question: $t = ?\,[s]$

Given: $h = 378.6\ m$

$g = 10\ m/s^2$

$v_i = 0\ m/s$

Equation: $\Delta x = v_i t + \frac{1}{2}at^2$ or $h = v_i t + \frac{1}{2}gt^2$

Plug n#8217; Chug: since $v_i = 0$, the equation simplifies to $h = \frac{1}{2}gt^2$ rearranging for the unknown variable, t, yields

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(378.6\ m)}{10.0\ m/s^2}} = 8.70\ s$

Answer:

$$\boxed{8.70\ \mathrm{s}}$$

Watch this Explanation

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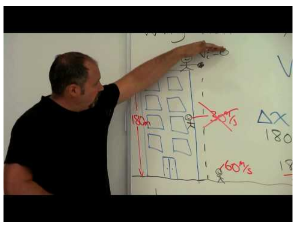

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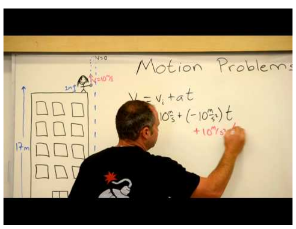

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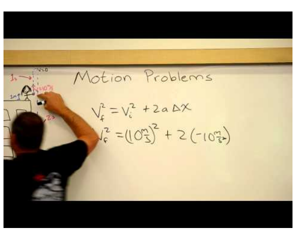

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Time for Practice

1. Sketchy LeBaron, a used car salesman, claims his car is able to go from 0 to 60 mi/hr in 3.5 seconds.
   a. What is the average acceleration of this car? Give your answer in $\mathrm{m/s^2}$. (Hint: you will have to perform a conversion.)
   b. How much distance does this car cover in these 3.5 seconds? Express your answer twice: in meters and in feet.
   c. What is the speed of the car in mi/hr after 2 seconds?
2. Michael Jordan had a vertical jump of about 48 inches.
   a. Convert this height into meters.

b. Assuming no air resistance, at what speed did he leave the ground?
c. What is his speed 3/4 of the way up?
d. What is his speed just before he hits the ground on the way down?

3. You are sitting on your bike at rest. Your brother comes running at you from behind at a speed of 2 m/s. At the exact moment he passes you, you start up on your bike with an acceleration of 2 $\mathrm{m/s}^2$.

a. Draw a picture of the situation, defining the starting positions, speeds, etc.
b. At what time *t* do you have the same speed as your brother?
c. At what time *t* do you pass your brother?
d. Draw another picture of the exact moment you catch your brother. Label the drawing with the positions and speeds at that moment.
e. Sketch a position vs. time graph for both you and your brother, labeling the important points (*i.e.,* starting point, when you catch him, etc.)
f. Sketch a speed vs. time graph for both you and your brother, labeling the important points (*i.e.,* starting point, when you catch him, etc.)

4. You are standing at the foot of the Bank of America building in San Francisco, which is 52 floors (237 m) high. You launch a ball straight up in the air from the edge of the foot of the building. The initial vertical speed is 70 m/s. (For this problem, you may ignore your own height, which is very small compared to the height of the building.)

a. How high up does the ball go?
b. How fast is the ball going right before it hits the top of the building?
c. For how many seconds total is the ball in the air?

5. Measure how high you can jump vertically on Earth. Then, figure out how high you would be able to jump on the Moon, where acceleration due to gravity is $1/6^{th}$ that of Earth. Assume you launch upwards with the same speed on the Moon as you do on the Earth.
6. A car is smashed into a wall during Weaverville#8217;s July 4^{th} Destruction Derby. The car is going 25 m/s just before it strikes the wall. It comes to a stop 0.8 seconds later. What is the average acceleration of the car during the collision?

7. A helicopter is traveling with a velocity of 12 m/s directly upward. Directly below the helicopter is a very large and very soft pillow. As it turns out, this is a good thing, because the helicopter is lifting a large man. When the man is 20 m above the pillow, he lets go of the rope.
   a. What is the speed of the man just before he lands on the pillow?
   b. How long is he in the air after he lets go?
   c. What is the greatest height reached by the man above the ground? (Hint: this should be greater than 20 m. Why?
   d. What is the distance between the helicopter and the man three seconds after he lets go of the rope?
8. You are speeding towards a brick wall at a speed of 55 MPH. The brick wall is only 100 feet away.
   a. What is your speed in m/s?
   b. What is the distance to the wall in meters?
   c. What is the minimum acceleration you should use to avoid hitting the wall?
9. What acceleration should you use to increase your speed from 10 m/s to 18 m/s over a distance of 55 m?
10. You drop a rock from the top of a cliff. The rock takes 3.5 seconds to reach the bottom.
    a. What is the initial speed of the rock?
    b. What is the magnitude (i.e., *numerical value*) of the acceleration of the rock at the moment it is dropped?
    c. What is the magnitude of the acceleration of the rock when it is half-way down the cliff?
    d. What is the height of the cliff?

Answers to Selected Problems

1. a. 7.7 m/s^2 b. 47 m, 150 feet c. 34 m/s
2. a. 1.22 m b. 4.9 m/s c. 2.46 m/s d. -4.9 m/s
3. b. 1 second c. at 2 seconds d. 4 m
4. a. 250 m b. 13 m/s, -13 m/s c. 14 s for round trip
5. Let#8217;s say we can jump 20 feet (6.1 m) in the air. ☺ Then, on the moon, we can jump 36.5 m straight up.
6. -31m/s^2
7. 23 m/s b. 3.6 seconds c. 28 m d. 45 m
8. 25 m/s b. 30 m c. 2.5 m/s^2
9. 2.04 m/s^2
10. a. $v_0 = 0$ b. 10 m/s^2 c. $-10\ \text{m/s}^2$ d. 60 m