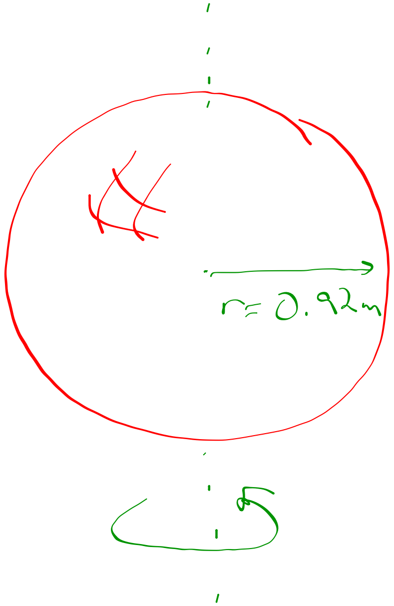
26. Calculate the moment of inertia of a 12.0-kg sphere of radius 0.205 m when the axis of rotation is through its center. The moment of inertia of a sphere that rotates about its center is *2/5mr²*. Here m is the mass of the sphere and r is the radius of the sphere.

$$I = \frac{2}{5}mr^2 = \frac{2}{5}(12)(0.205^2) = 0.202\ kg\cdot m^2$$

31. A 1.84-m-diameter sphere can be rotated about an axis through its center by a torque of 12.3 N·m which accelerates it uniformly from rest through a total of 180 revolutions in 15.0 seconds. What is the mass of the sphere?



$\theta_0 = \emptyset$

$\theta = 180 \text{ rev} = 1130.4 \text{ rad}$

$\omega_0 = \emptyset$

$\omega \cong 150 \frac{\text{rad}}{\text{s}}$

$\alpha = 10.05 \frac{\text{rad}}{\text{s}^2}$

$t = 15 \text{ s}$

$\theta = \cancel{\theta_0} + \cancel{\omega_0 t} + \frac{1}{2}\alpha t^2$

$\alpha = \frac{2\theta}{t^2} = \frac{2(1130.4)}{15^2}$

$\alpha = 10.05 \text{ rad}/\text{s}^2$

$\Sigma\tau = I\alpha$

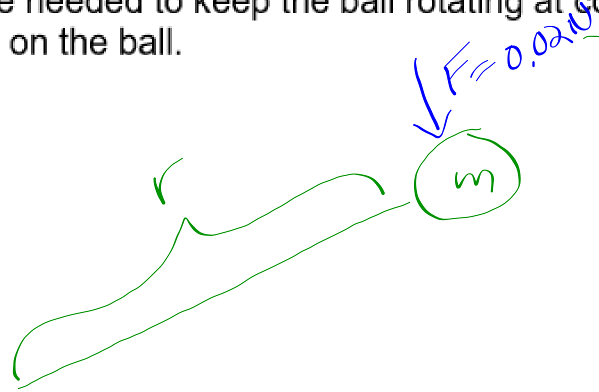
$I = \frac{\Sigma\tau}{\alpha} = \frac{12.3}{10.05} = 1.22 \text{ kg}\cdot\text{m}^2$

$I_{\text{sphee}} = \frac{2}{5}mr^2$

$m = \frac{5I}{2r^2} = \frac{5(1.22)}{2(0.92^2)} = 3.6 \text{ kg}$

28. A small 12.4-kg ball on the end of a light rod is rotated in a horizontal circle of radius 2.20 m. Calculate:
    a) the moment of inertia of the system about the axis of rotation.
    b) the torque needed to keep the ball rotating at constant angular velocity if air resistance exerts a force of 0.0200 N on the ball.

$\rightarrow \alpha = 0$



(a) $I_{point} = mr^2$
$= (12.4)(2.2)^2$
$= 60\ kg \cdot m^2$

(b) $\Sigma \tau = I\cancel{\alpha} = 0$

$\tau_{air} + \tau_{app} = 0$

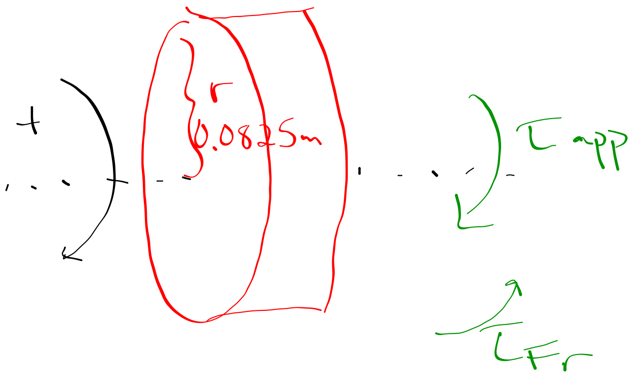
$\tau_{app} = -(F_{air})\ell = -(0.02)(2.2)$

$= -0.044\ N \cdot m$

29. A grinding wheel is a uniform cylinder of radius 8.25 cm and mass 0.880 kg. Calculate:
    a) its moment of inertia.
    b) the torque needed to accelerate it from rest to 1200 rpm in 4.00 seconds if a frictional torque of 0.0145 N·m is also acting.

(a) $I_{disk} = \frac{1}{2}mr^2 = \frac{1}{2}(0.88)(0.0825)^2$

$= 3e^{-3}\ kg\cdot m^2$



$\theta_0 = \emptyset$

$\theta =$

$\omega_0 = \emptyset$

$\omega = 1200\ rpm = 125.6\ \frac{rad}{s}$

$\alpha =$

$t = 4$

$\Sigma\tau = I\alpha$

$\tau_{app} - \tau_{Fr} = I\alpha$

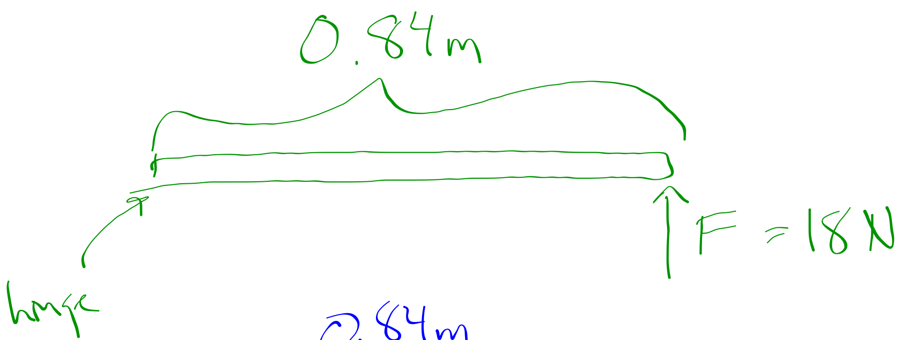
$\tau_{app} = I\alpha + \tau_{Fr}$

$= (3e^{-3})(31.4) + 0.0145$

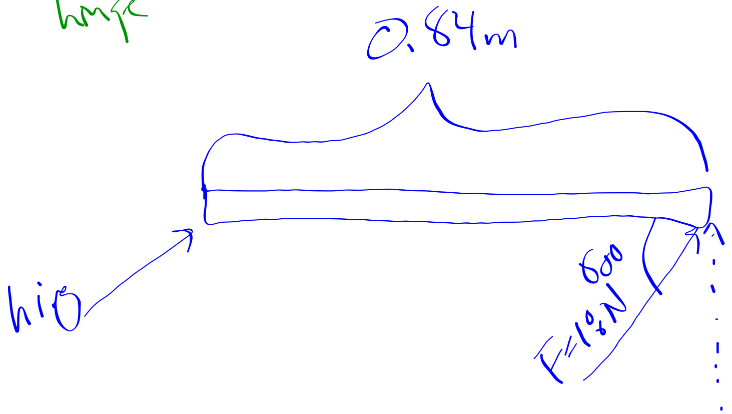
$= \boxed{0.109\ N\cdot m}$

$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{125.6}{4} = 31.4\ \frac{rad}{s^2}$

24. A person exerts a force of 18 N on the end of a door 84 cm wide. What is the magnitude of the torque if the force is exerted
   a) perpendicular to the face of the door?
   b) at a 60° angle to the face of the door?



$\tau = F \cdot \ell$
$= (18)(0.84)$
$= 15.12\ N \cdot m$
(a)

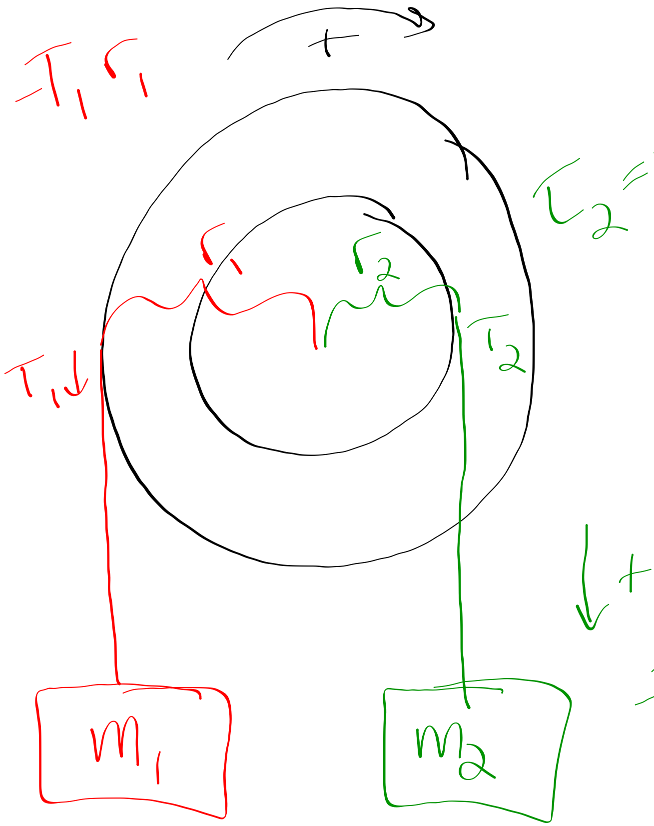


$F_\perp = 18 \cdot \sin 60°$
$= 15.59\ N$

(b) $\tau = F_\perp \cdot \ell = (15.59)(0.84) = 13.09\ N \cdot m$

$\tau_1 = -T_1 r_1$

$\tau_2 = T_2 r_2$



$\Sigma\tau = I\alpha$

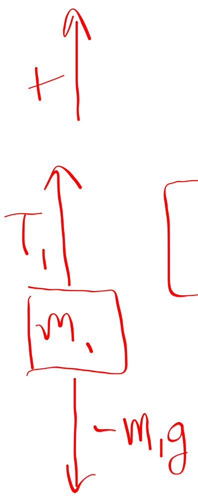
$I = \frac{\Sigma\tau}{\alpha}$

$I = \frac{-T_1 r_1 + T_2 r_2}{\alpha}$

$a_1 = a_{t1} = \alpha r_1 \rightarrow \alpha = \frac{a_1}{r_1}$

$a_2 = a_{t2} = \alpha r_2 \rightarrow \alpha = \frac{a_2}{r_2}$

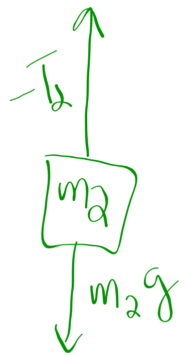
$\alpha = \frac{a_1}{r_1} = \frac{a_2}{r_2}$



$\Sigma F = ma$

$T_1 - m_1 g = m_1 a_1$

$T_1 = m_1 a_1 + m_1 g$



$\Sigma F = ma$

$-T_2 + m_2 g = m_2 a_2$

$T_2 = m_2 g - m_2 a_2$

$$\boxed{I = \frac{\Sigma\tau}{\alpha}}$$

$$\boxed{\alpha = \frac{a_1}{r_1} = \frac{a_2}{r_2}}$$

$$a_2 = \frac{a_1 r_2}{r_1}$$

$$I = \frac{-T_1 r_1 + T_2 r_2}{\alpha}$$

$$T_1 = m_1 a_1 + m_1 g$$

$$T_2 = m_2 g - m_2 a_2$$

$$I = \frac{-(m_1 a_1 + m_1 g) r_1 + (m_2 g - m_2 a_2) r_2}{\frac{a_1}{r_1}}$$

$$I = \frac{-(m_1 a_1 + m_1 g) r_1 + \left(m_2 g - m_2 \left(\frac{a_1 r_2}{r_1}\right)\right) r_2}{\frac{a_1}{r_1}}$$

$$I = \frac{-(m_1 a_1 + m_1 g) r_1^2 + \left(m_2 g - m_2 \left(\frac{a_1 r_2}{r_1}\right)\right) r_2 r_1}{a_1}$$

$$\boxed{I = \frac{-m_1 a_1 r_1^2 - m_1 g r_1^2 + m_2 g r_1 r_2 - m_2 a_1 r_2^2}{a_1}}$$

$m_1, m_2, r_1, r_2, \Delta x, t$

$$a_1 = \frac{2\Delta x}{t^2}$$