A car travels at 5 m/s for 12 seconds. How far has it travelled?

60 m

A car starts from rest and accelerates at $2\ m/s^2$ for 12 seconds. How far has it travelled?

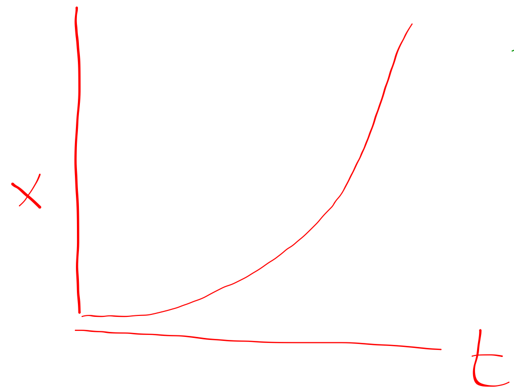
each second, it will be going 2 m/s faster than it was the second before.

120m

24m

144 m

210 m



A car starts at 7 m/s and accelerates at 2 m/s^2 for 12 seconds. How far has it travelled?

~~144m~~ ~~203,000m~~

~~149m~~

~~151m~~

Objectives:

1. Students will understand how position, velocity, and time are related under conditions of constant acceleration.
2. Students will see how to use the Big 4 equations to predict elements of the motion of different objects.

Quantity (all functions of time!)	Variable / Units	
Initial Position (position at t = 0)	x_0	$x(0)$
Final Position	x	$x(t)$
Initial Velocity (velocity at t = 0)	v_0	$v(0)$
Final Velocity	v	$v(t)$
Acceleration	a	
Initial time	t_0 (often Ø)	
Final time	t	
The time that elapses $\Delta t = t - t_0$	t (often)	

↑ often Ø

Deriving relations for uniformly accelerated motion:

We want a set of expressions that can be used to make predictions when an object accelerates.

Assumption: the acceleration is uniform (constant and unchanging in value)

EQ #1 $\quad \bar{v} = \dfrac{x - x_0}{t}$ From definition for average velocity

EQ #2 $\quad a = \dfrac{v - v_0}{t}$ From definition for average acceleration

Solve Eq. #2 for v (gives us Eq. #2a):

$$a \cdot t = v - v_0$$

$$v = v_0 + at$$

EQ #1 $\bar{v} = \frac{x - x_0}{t}$ $\bar{v}$ = AVERAGE VELOCITY

Solve for x (gives us Eq #3):

$$\bar{v} \cdot t = x - x_0$$

$$x = x_0 + \bar{v} \cdot t$$

Because velocity increases at a uniform rate (a is constant), then the average velocity is equivalent to the mathematical average of the initial and final velocities:

$$\bar{v} = \boxed{\frac{v + v_0}{2}} \quad \text{(EQ \#4)}$$

Substitute EQ. #4 into EQ. #3:

$$x = x_0 + \left(\frac{v + v_0}{2}\right) t \quad \text{(EQ. \#5)}$$

Substituting Eq. #2a into EQ. #5:

$$v = \boxed{v_0 + at}$$

$$x = x_0 + \left(\frac{v_0 + a \cdot t + v_0}{2}\right) t$$

$$x = x_0 + \left(\frac{2v_0 + at}{2}\right) t$$

$$x = x_0 + \left(\frac{2v_0 t + at^2}{2}\right)$$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2} a t^2}$$

With this expression, we can determine how far an accelerating object has travelled without needing to know anything about its final velocity!

Can we find an expression that will allow us to find an accelerating object's final velocity without needing to know the length of time that it accelerates?

$$x = x_0 + \left(\frac{v + v_0}{2}\right)t \quad (\text{EQ } \#5)$$

Next, we will solve Eq. #2 for t instead of a as we did before:

$$t = \boxed{\frac{v - v_0}{a}} \quad (\text{EQ } \#6)$$

Substituting Eq. #6 into Eq. #5, we obtain:

$$x = x_0 + \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right)$$

Solving for v^2 we obtain:

$$x = x_0 + \frac{v^2 - v_0^2}{2a}$$

$$2a \cdot x = 2a \cdot x_0 + v^2 - v_0^2$$

$$v^2 = v_0^2 + 2ax - 2ax_0$$

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$

Collectively, these four relationships will be called:

THE BIG 4

Kirsch	rest of us
a	a
Z	2

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\bar{v} = \boxed{\frac{v + v_0}{2} = \frac{x - x_0}{t}}$$

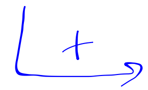
constant acceleration

Applying the Big 4 to problems:

1. Draw a picture.

2. Establish a reference frame. (Pick an origin and a positive direction.)

3. Inventory / assign variables (x_o, x, v_o, v, a, t)



4. Check units -- convert as needed (h/w only)

5. Check - is acceleration constant for the whole problem? (If not, break into separate problems.)

6. Use the Big 4 to solve for unknowns:

    -You may have to do this a couple of times

    -You may need to use the quadratic formula

    -Sometimes you have dual answers

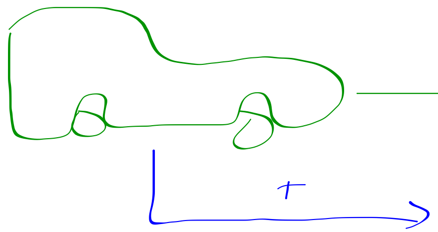
    -Not sure which equation to use? Go in order.

https://www.youtube.com/watch?v=ZM8ECpBuQYE&feature=youtu.be

A car starts at 7 m/s and accelerates at 2 m/s^2 for 12 seconds. How far has it travelled?

$\Delta x = x - x_0$

$228 - 0 = 228\ m$



$x_0 : 0\ m$

$x : 228\ m$

$v_0 : 7\ m/s$

$v :$

$a : 2\ m/s^2$

$(\Delta t)\ t : 12\ s$

$x = x_0 + v_0 t + \frac{1}{2} a t^2$

$x = 0 + (7)(12) + \frac{1}{2}(2)(12)^2$

$x = 228\ m$

Some typical "hidden" values in kinematics:

When an object goes up then falls down, its velocity is 0 at the top of its arc (why?). The velocity is also 0 just as an object reverses direction.

If two objects are present in the same question, it's two problems! Each object gets its own variable inventory (but the "x" values will relate to each other).

The variable *t* refers to a CHANGE in time, not the actual time on our imaginary stopwatch. All the Big 4 equations occur over time, not instantaneously. If a question asks about a time period that doesn't start at time 0, find the amount of time that passes during the question and use that for *t*.

Thoughts about homework:

Many of these are stinkers - try them and if you get stuck, just move on! (10-15 minutes at the **most** unless you enjoy the challenge.) Usually there aren't so many but there are typically one or two.

Homework isn't for practice - it's to identify the things you don't understand yet so you can ask questions later.

Make sure you end up with all the problems correctly solved (and easily identifiable) in your notes within a week after review.