Rotational Motion

experience

When we ~~consider~~ things that rotate, we unconsciously place them within one of two categories:

- Viewing the rotation from **within** the rotating frame of reference.
  (riding a merry-go-round, going around a corner in a car, the spinning tube of a washing machine)
- Viewing the rotation from **outside** the rotating frame of reference.
  (a ball on a string, a spinning wheel, the hands of a clock)

experience

When viewed from within a rotating frame of reference, there appears to be a force "pulling" things outward, away from the center of motion. This "force" IS NOT REAL, but it has mistakenly been given a name anyway -- centrifugal.

This is wrong! This centrifugal force does not exist!

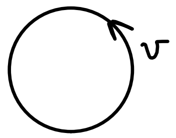
For an object to move in a circle, a centripetal force, directed inward, is necessary. If you have circular motion, you must have a net force directed inward, and this net force is called the CENTRIPETAL FORCE.

Note: the centripetal force is not a force in and of itself -- it is just the name we give the net force -- the sum of all real forces acting on an object that are directed into the center of motion.

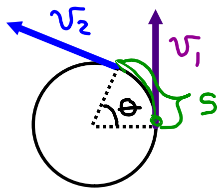
Deriving an expression for the centripetal acceleration and centripetal force required for uniform circular motion.

Uniform circular motion (UCM): motion in a circle at constant speed and at a constant radius.

Assume mass *m* travels at speed v_s with a radius *r*.



IF WE WISH TO FIND THE $\vec{a}_c$ CENTRIPETAL ACCELERATION $\vec{a}_c$, WE NEED TO DETERMINE $\Delta v / \Delta t$

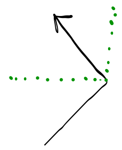


IN Δt, THE MASS WILL HAVE TRAVELLED THE DISTANCE $v_s \Delta t$

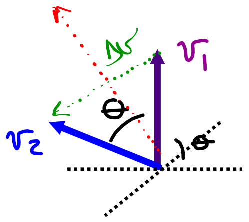
$v_s \Delta t = s =$ ARC LENGTH

$\theta = s/r$ (FROM EARLIER THIS YEAR)

$\theta = \boxed{\dfrac{(v_s \Delta t)}{r}}$

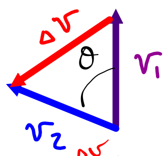


$\Delta v = v_2 - v_1$



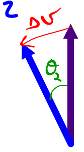
APPLY $\theta = s/r$ TO THIS AS WELL:

$\theta = \dfrac{s}{r} = \dfrac{\Delta v}{v_s}$



$\theta = \dfrac{\Delta v}{v_s} \Rightarrow \Delta v = \theta v_s$

$\Delta v = \left(\dfrac{v_s \Delta t}{r}\right) v_s$



DIVIDE BOTH SIDES BY Δt: $\dfrac{\Delta v}{\Delta t} = \dfrac{v_s^2}{r} = \vec{a}_c$

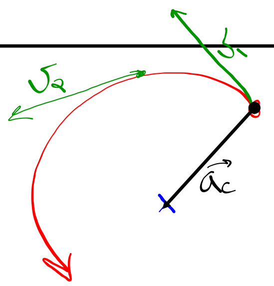
FOR UCM:

$\vec{a}_c$ = CENTRIPETAL ACCELERATION

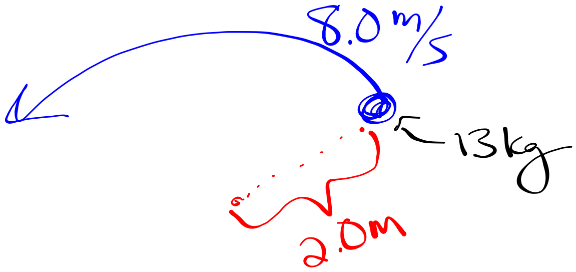
$\vec{a}_c = \dfrac{v^2}{r}$ (ALWAYS DIRECTED INWARD)

$\Sigma F_{\text{INTO/OUT OF CENTER}} = m\vec{a}_c = m\left(\dfrac{v^2}{r}\right) = \dfrac{mv^2}{r}$

$\Sigma F_{\text{INTO/OUT OF CENTER}} = \dfrac{mv^2}{r} =$ CENTRIPETAL FORCE



EXAMPLE 1: If a 13.0 kg ball on a 2.0-meter string is in uniform circular motion (UCM) with a speed of 8.0 m/sec, what acceleration must the ball need to experience?



$$a_c = \frac{v^2}{r} = \frac{8^2}{2} = \boxed{32\ \frac{m}{s^2}}$$

EXAMPLE 1: If a 13.0 kg ball on a 2.0-meter string is in uniform circular motion (UCM) with a speed of 8.0 m/sec, what acceleration must the ball need to experience?

$m = 13.0 \text{ kg}$

$r = 2.0 \text{ m}$

$v = 8.0 \text{ m/s}$

$$\vec{a}_c = \frac{v^2}{r} = \frac{8^2}{2} = \boxed{32 \text{ m/s}^2}$$

This acceleration is directed inward, toward the center of motion and is provided by the string. The REAL tensile force in the string provides the centripetal motion in this case.

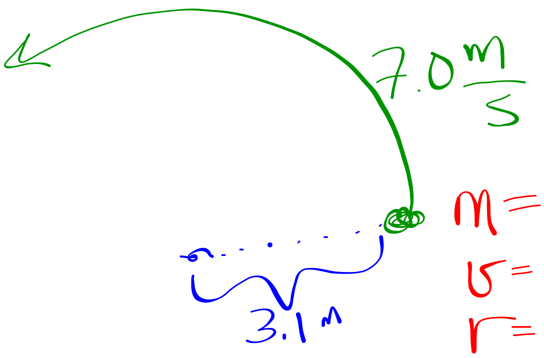
THE FORCE IN THE STRING:

$$\Sigma F_{IN/OUT} = ma$$

$$F = m\left(\frac{v^2}{r}\right)$$

$$F = \frac{(13)(8)^2}{2} = \boxed{416 \text{ N OF TENSION}}$$

EXAMPLE 2: A 350 gram model plane travels at 7.0 m/sec on the end of a 3.1 meter wire. Assume the path of the plane and the wire make a horizontal orbit. What is the tension in the string?



$m = 350\,g = .35\,kg$

$v = 7.0\,\frac{m}{s}$

$r = 3.1\,m$

1. Draw pic
2. Frame of ref (?)
3. Inventory
   - → conv.
   - → comp.
4. Subst./Solve
5. Interpret

$$\Sigma F = \frac{mv^2}{r} = \frac{(0.35)(7^2)}{3.1}$$

$$\boxed{= 5.53\,N}$$

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$m = .35 \text{ kg}$

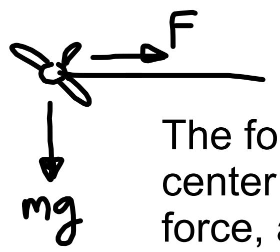
$r = 3.1 \text{ m}$

$v = 7.0 \text{ m/s}$

$\Sigma F = ma$

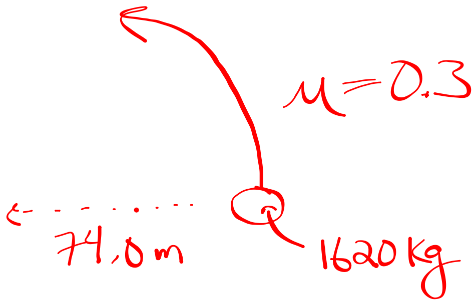
$\Sigma F = m\left(\frac{v^2}{r}\right)$

$F = \frac{(.35)(7.0)^2}{3.1} = \boxed{5.53 \text{ N}}$



The force of gravity is NOT directed into the center of motion, can not provide a centripetal force, and therefore is not included when forces are summed above.

EXAMPLE 3: How fast can a car round a corner having a radius of 74.0 meters? Assume the car's mass is 1620 kg, and assume the coefficient of friction between the car's tires and the road is 0.3.



$\mu = 0.3$

$$\Sigma F = F_{Fr} = \mu F_N$$
$$= \mu m g$$

$$F_{Fr} = F_c$$

$$\mu \cancel{m} g = \frac{\cancel{m} v^2}{r} = v = \sqrt{\mu g r}$$

$$v = \sqrt{(.3)(9.8)(74)}$$

$$v = 14.75 \frac{m}{s}$$

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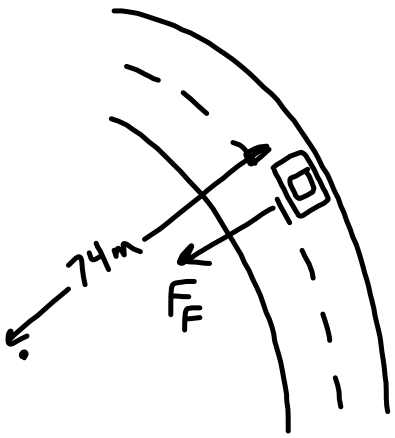


Friction is the only force that is acting into or out of the center of motion, so friction must be the force that supplies the centripetal force necessary to get the car around the corner. Without friction, there would be no force directed inward!

$$F_F = \mu N$$

$$\Sigma F_{\text{IN/OUT OF C.O.M.}} = ma$$

$$\Sigma F = m\left(\frac{v^2}{r}\right)$$

$$\mu N = \frac{mv^2}{r}$$

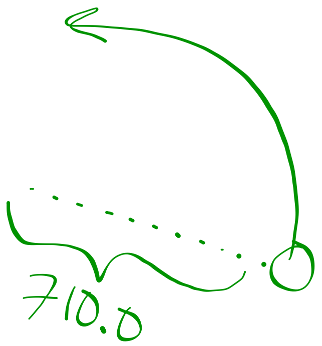
$$\mu \cancel{m} g = \frac{\cancel{m} v^2}{r} \qquad \text{(MASS DOES NOT MATTER!)}$$

$$\mu g = \frac{v^2}{r}$$

$$\sqrt{\mu g r} = v^2$$

$$\sqrt{(.3)(9.8)(74)} = \boxed{14.75 \text{ m/s}}$$

EXAMPLE 4: What minimum coefficient of friction is needed to prevent a car travelling at 60.0 mph from sliding off a curve having a radius of 710.0 feet?

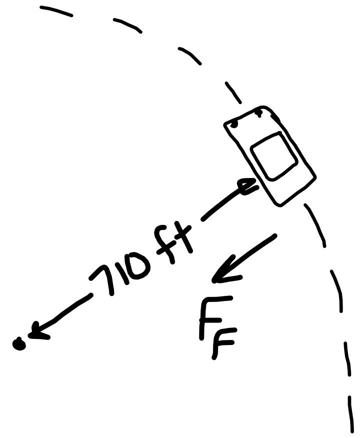


$v = 60\,\frac{\text{miles}}{\text{hr}} \rightarrow \underline{\quad}\,\frac{\text{ft}}{\text{s}}$

$\mu \cancel{m} g = \frac{\cancel{m} v^2}{r}$ $\qquad g = 32.2\,\frac{\text{ft}}{\text{s}}$

$$\boxed{\mu = \frac{v^2}{r \cdot g}}$$

EXAMPLE 4: What minimum coefficient of friction is needed to prevent a car travelling at 60.0 mph from sliding off a curve having a radius of 710.0 feet?



As before, the force of friction between the ground and the tires is the only force capable of providing a centripetal force because it is the only force directed into the COM.

$$v = 60\,\frac{\cancel{\text{miles}}}{\cancel{\text{HR}}}\left(\frac{5280\text{ ft}}{1\ \cancel{\text{mile}}}\right)\left(\frac{1\ \cancel{\text{HR}}}{3600\text{ sec}}\right) = 88\ \text{ft/sec}$$

$r = 710\text{ ft}$

$m = ?$

$$\Sigma F_{\text{IN/OUT OF COM}} = m(a)$$

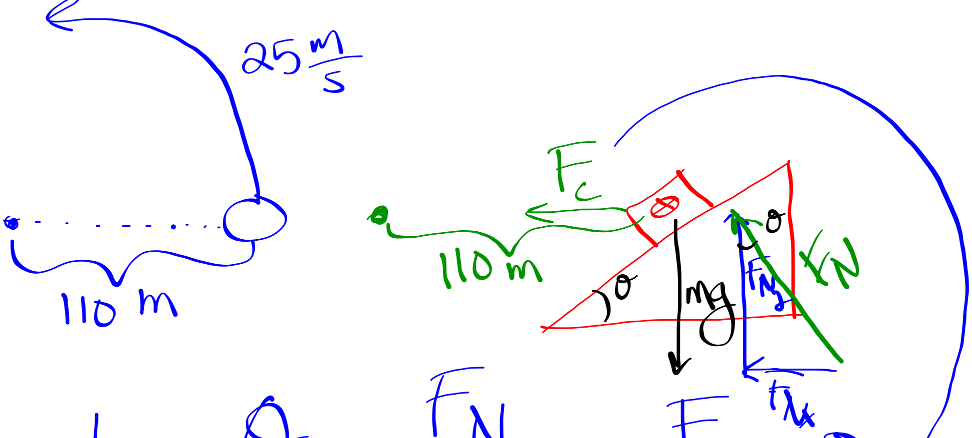
$$\Sigma F = m\left(\frac{v^2}{r}\right)$$

$$\mu N = \frac{mv^2}{r}$$

$$\mu \cancel{m} g = \frac{\cancel{m} v^2}{r}$$

$$\mu = \frac{v^2}{rg} = \frac{88^2}{(710)(32.2)} = \boxed{.34}$$

EXAMPLE 5: How much should a road with a 110 meter curve be banked (what is the angle) to accommodate cars travelling at 25 m/sec (so that friction is NOT required to get the car around the corner)?



$$\tan\theta = \frac{F_{N_x}}{F_{N_y}} = \frac{F_c}{mg}$$

$$\theta = \tan^{-1}\frac{F_c}{mg}$$

$$= \tan^{-1}\frac{\frac{mv^2}{r}}{mg}$$

$$\theta = \tan^{-1}\left(\frac{\cancel{m}v^2}{r}\cdot\frac{1}{\cancel{m}g}\right)$$

$$\theta = \tan^{-1}\left(\frac{v^2}{r\cdot g}\right) = \tan^{-1}\left(\frac{25^2}{(110)(9.8)}\right)$$

$$\boxed{\theta = 30.1^\circ}$$

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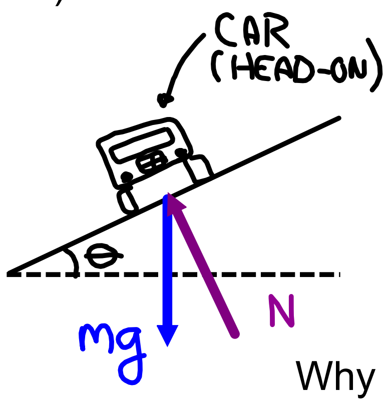
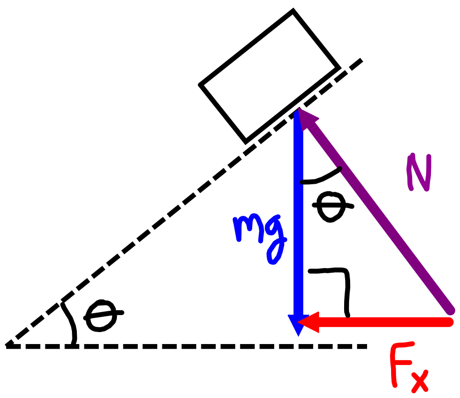


Of the two real forces acting on the car (there is no friction in this problem), only the normal force has a component directed into the center of motion. Therefore we want to use its x-component when summing all forces into or out of the COM.

Why is the purple arrow (the normal force) as long as it is? Because its y-component MUST be equal and opposite to gravity, mg.



F_x, the red arrow and the horizontal component of the normal force, is what provides the centripetal force necessary to get the car around the corner.

$$\Sigma F = ma$$

$$\Sigma F = m\left(\frac{v^2}{r}\right)$$

$$F_x = \frac{mv^2}{r}$$

ALSO,

$$\tan\theta = \frac{F_x}{mg}$$

$$\therefore F_x = mg\tan\theta$$

$$F_x = \frac{mv^2}{r}$$

$$\cancel{m}g\tan\theta = \frac{\cancel{m}v^2}{r} \quad \text{(MASS DOES NOT MATTER)}$$

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$\theta = \tan^{-1}\left(\frac{25^2}{110(9.8)}\right) = \boxed{30.1^\circ}$$

EXAMPLE 6: On an icy day, how fast can a car travel around a 200.0-ft. corner that is only banked at 12.0 degrees?

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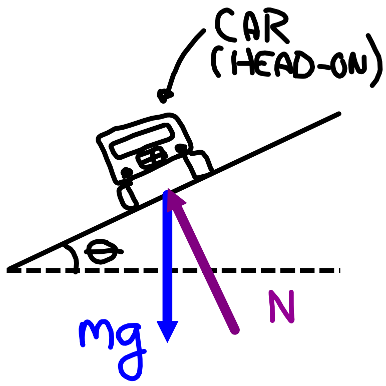

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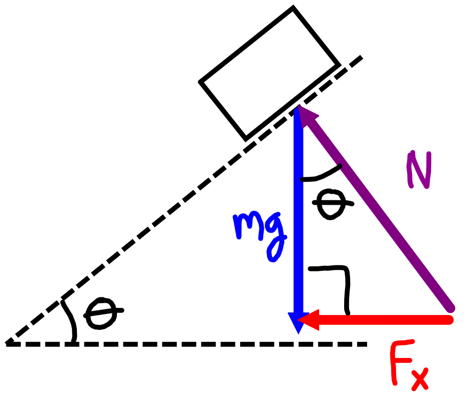

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ALSO,

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$$F_x = \frac{mv^2}{r}$$

$$\cancel{m}g\tan\theta = \frac{\cancel{m}v^2}{r} \quad \text{(MASS DOES NOT MATTER)}$$

$$rg\tan\theta = v^2$$

$$\sqrt{rg\tan\theta} = v$$

$$\sqrt{(200)(32.2)\tan 12} = \boxed{37 \text{ ft/sec}}$$