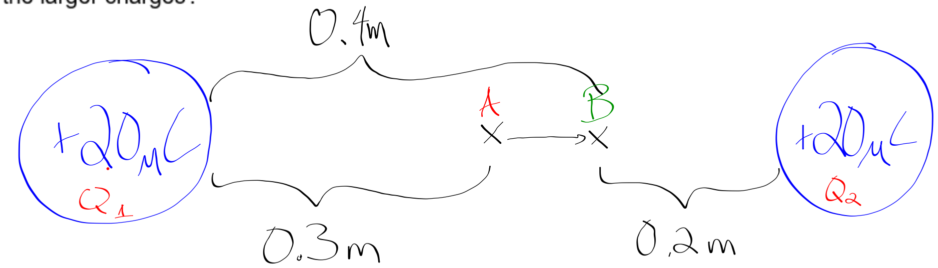
17. A +20 $\mu$C charge is placed 60 cm from an identical +20 $\mu$C charge. How much work would be required to move a +0.20 $\mu$C test charge from a point midway between the two larger charges to a point 10 cm closer to either of the larger charges?




$$\left.\begin{array}{l} V_{A1} = \frac{kQ_1}{r_1} = 9e9\,\frac{20e-6}{.3} = 600{,}000\,V \\ V_{A2} = \frac{kQ_2}{r_2} = 600{,}000\,V \end{array}\right\} 1.2e6\,V$$

$$\left.\begin{array}{l} V_{B1} = \frac{kQ_1}{r_1} = 9e9\,\frac{20e-6}{.4} = 450{,}000\,V \\ V_{B2} = \frac{kQ_2}{r_2} = 9e9\,\frac{20e-6}{0.2} = 900{,}000\,V \end{array}\right\} 1.35e6\,V$$

$$V_{BA} = V_B - V_A = 150{,}000\,V$$

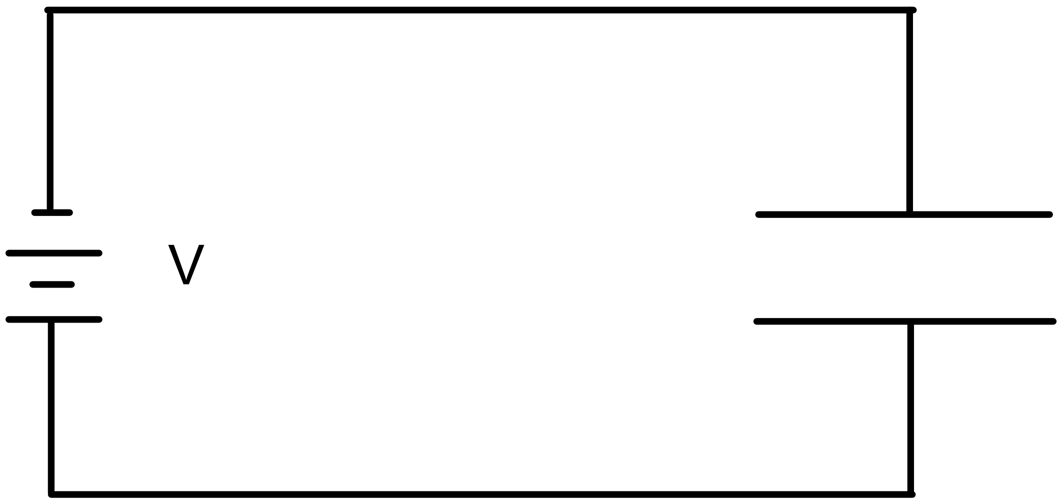
$$V_{BA} = \frac{W_{BA}}{q} \qquad 150{,}000\,V = \frac{W_{BA}}{0.2e-6}$$

$$\boxed{W_{BA} = 0.03\ J}$$

# How do you store electrical energy?

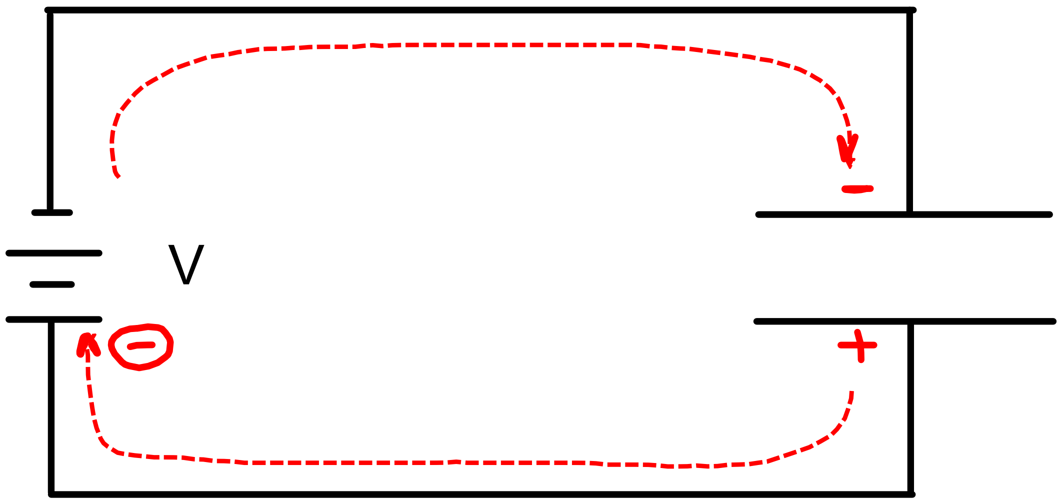
How do you store electrical energy? (electric charge)

With **capacitors**.

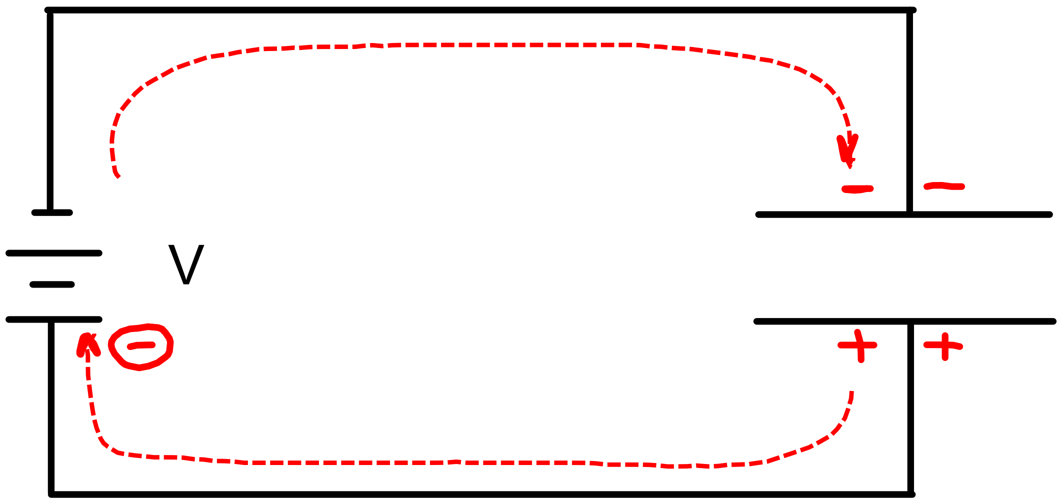



Two conducting, parallel plates separated by a gap

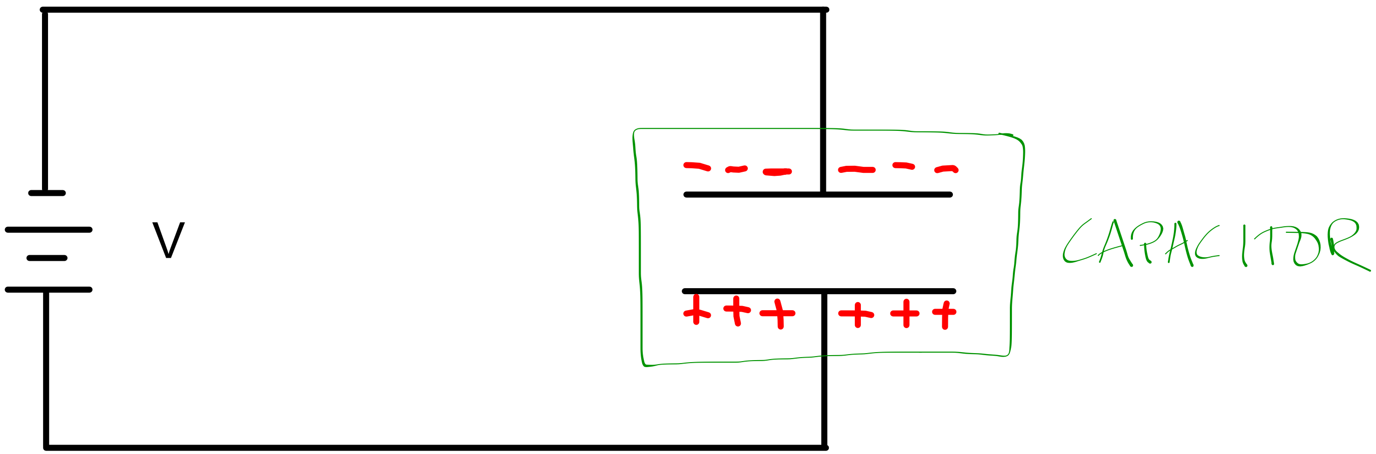
Consider this simple circuit -- two parallel conducting plates connected as shown to the opposite ends of a battery or other source of potential difference (voltage).



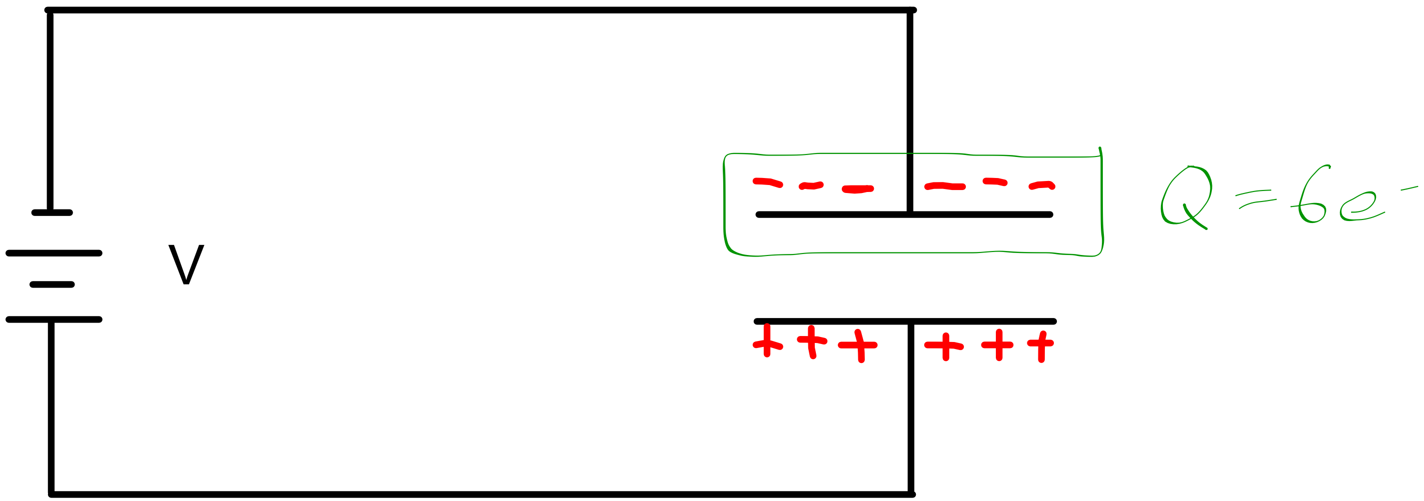

- The first electron that leaves the battery (it is "falling" through the existing potential difference the battery provides) travels until reaching the first plate.  Here, there is a dead-end; it can't go further.
- However, its presence pushes an electron on the second plate away and back to the battery, leaving a net positive charge on the second plate.
- The parallel plates are now partially charged.



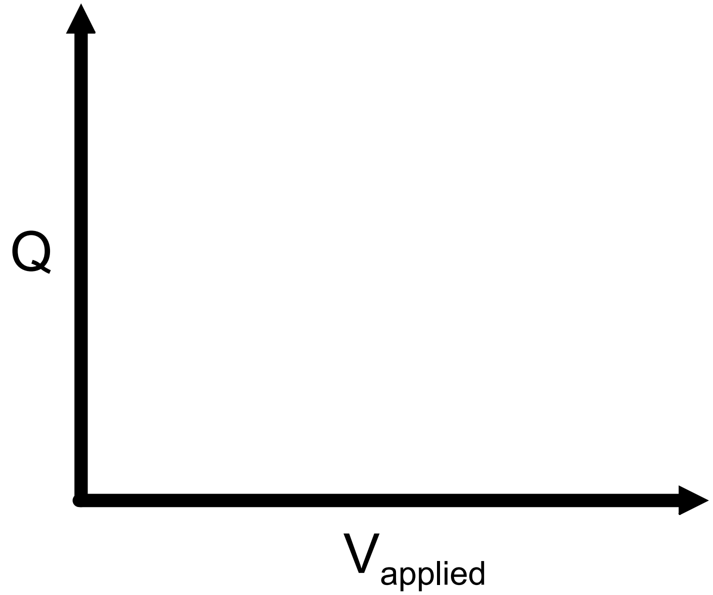

- The second charge leaving the battery has a more difficult time -- it has to fight the repulsion from the first charge that is currently on the first plate.
- If V is large enough, it will get placed on the plate and in turn, forces a second charge to leave and return to the battery from the second plate.
- Each subsequent charge has an increasingly difficult time in getting to the plate.



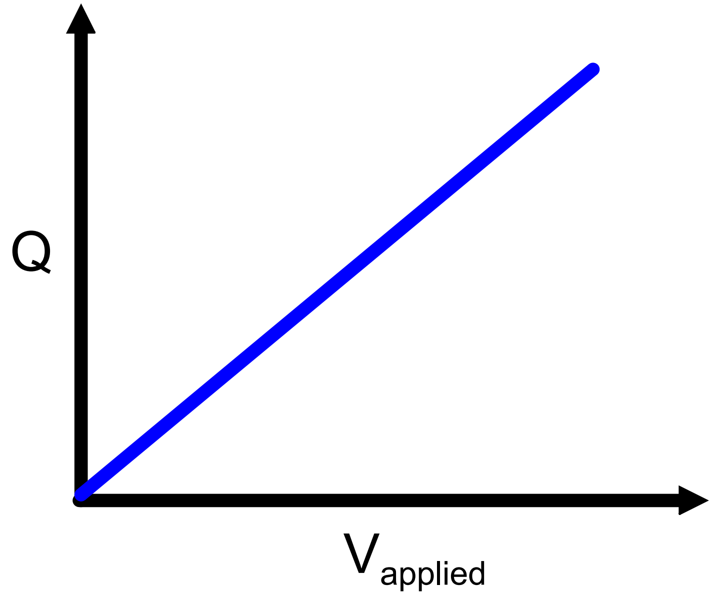

- For the V available, there is a maximum charge that can be placed upon the plates.
- In the figure, the parallel plates have their maximum charge stored upon them. The plates are said to be fully charged.
- The charge is held there by the applied electric potential difference V.
- If V were to be turned off, the charge collected on the plate would move and return to the battery.
- In effect, we have "stored" electrical charge and the energy associated with that charge, and will be able to reclaim it when V is turned off.




- The charge is referred to as Q. It is the charge stored on EACH plate.
- The parallel plates are said to have a total charge of Q on them, NOT 2Q.




If we plot the amount of charge that gets placed on either plate vs. the voltage applied across the plates . . .



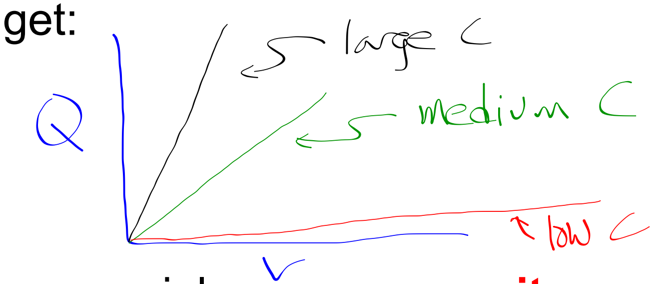

If we plot the amount of charge that gets placed on either plate vs. the voltage applied across the plates . . .

We find that the amount of charge on the plates is directly proportional to the voltage applied across the plates.

$$Q \propto V$$

If we place the proportionality sign with a constant (representing the slope of the previous graph), we get:

$$Q = CV$$




This constant, C, has been given a special name -- **capacitance**.

C = CAPACITANCE

UNITS OF C : $\frac{Q}{V} \Rightarrow \frac{\text{COULOMB}}{\text{VOLT}} = 1$ FARAD (F)

1 F IS HUGE.

MORE TYPICAL SIZES :

$\mu F \quad 1 \times 10^{-6}\,F$

$pF \quad 1 \times 10^{-12}\,F$

$nF \quad 1 \times 10^{-9}\,F$

The two parallel conducting plates are called a **capacitor**.

A **capacitor**: any two conducting materials electrically connected within a circuit and intended to store charge because of electrical isolation between the two conductors (i.e. there is a gap between the conductors preventing charge from flowing).

- The two conductors are commonly flat plates, but they don't have to be.

What determines the capacitance of a capacitor?

http://phet.colorado.edu/en/simulation/capacitor-lab

# What determines the capacitance (C) of a capacitor?

Capacitance depends upon the structure of the actual capacitor.

For a parallel plate capacitor:

$$C = \boxed{K \frac{\varepsilon_0 A}{d}}$$

$Q = CV$

$$Q = \frac{K \cdot \varepsilon_0 \cdot A \cdot V}{d}$$

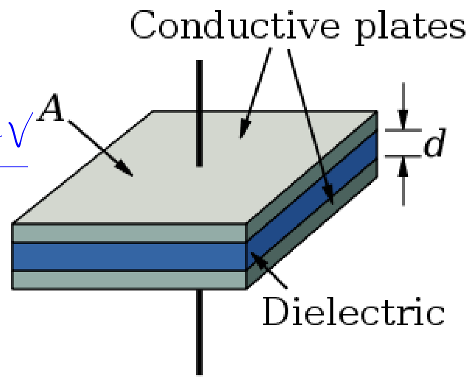



Image taken from: http://upload.wikimedia.org/wikipedia/commons/thumb/3/35/Parallel_plate_capacitor.svg/300px-Parallel_plate_capacitor.svg.png

A = AREA OF ONE PLATE ($m^2$)

d = DISTANCE SEPERATING EACH PLATE (m)

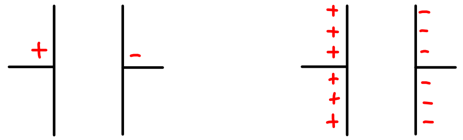
$\varepsilon_0$ = PERMITTIVITY OF FREE SPACE $= 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$ (A CONSTANT)

K = DIELECTRIC CONSTANT OF MATERIAL BETWEEN THE PLATES (UNITLESS)

Every capacitor with a different structure has a different capacitance. The value of the capacitance determines how much charge, and therefore energy, a capacitor can store.

## Storage of Electrical Energy

The energy stored will be equal to the work done to charge the plates of a capacitor. The more charge already present on the plates, the more work that has been done to charge it.




easy to add charge difficult to add charge

The work needed to add a small mount of charge $\Delta q$ will be:

$$\Delta W = \Delta q V$$

The average voltage as we look at this process of adding small amounts of charge over time is

$$V_{AVG} = \frac{V_{FINAL} - V_{INITIAL}}{2} = \frac{V_{FINAL} - 0}{2} = \frac{V}{2}$$

Total work that was done to charge the entire capacitor will equal the total charge present multiplied by the average electric potential difference (average voltage) that the charge moved through:

$$W_{ba} = q V_{ba}$$

$$W_{TOTAL} = Q V_{AVERAGE}$$

$$= Q \frac{V}{2}$$

$$W_{TOTAL} = \frac{1}{2} Q V$$

Since the total work done will equal the total energy stored,

$$E = \frac{1}{2} Q V \qquad \vec{E} = k\frac{Q}{r^2}$$

E = ENERGY STORED ON CHARGED CAPACITOR (J)

Q = CHARGE ON THE CAPACITOR (C)

V = FINAL VOLTAGE APPLIED ACROSS THE CAPACITOR (V)

# Alternative Expressions for the Energy Stored on a Capacitor:

$$Q = CV$$

$$E = \boxed{\tfrac{1}{2} QV}$$

$$E = \tfrac{1}{2}(CV)V = \boxed{\tfrac{1}{2} CV^2}$$

$$E = \tfrac{1}{2} Q\left(\frac{Q}{C}\right) = \boxed{\tfrac{1}{2}\frac{Q^2}{C}}$$

ALL 3 ARE EQUIVALENT - USE THE MOST CONVIENENT

# The impact of a dielectric:

Dielectrics increase the energy that can be stored on a capacitor, as well as the charge that can be placed on a capacitor.

Remember, dielectrics are insulating materials. Their presence lowers the effective E-field between the plates and therefore makes it possible for the external voltage source to place more charge on the plates than is possible if the gap between plates is just filled with air.

$$E = \frac{1}{2} QV = \frac{1}{2} CV^2$$

$$C = K \frac{\varepsilon_0 A}{d}$$

$$\therefore E = \frac{1}{2}\left(K \frac{\varepsilon_0 A}{d}\right) V^2$$

As you can see, the larger the dielectric constant, the larger the amount of energy that can be stored on a given capacitor.