Rotational Kinetic Energy and Momentum

- Just like objects moving in straight lines, objects that rotate have kinetic energy
- Just like objects moving in straight lines, objects that rotate have momentum
- Straight line quantities are related to mass and velocity; rotational quantities are related to moment of inertia and angular velocity

$KE = \frac{1}{2}mv^2$

$p = mv$

Objectives:

- Students will understand and be able to give examples of rotational kinetic energy and momentum
- Students will understand the connection between linear KE/momentum and angular KE/momentum
- Students will be able to solve problems involving angular KE and momentum

TRANSLATION	ROTATION
x (METERS)	θ (RADIANS)
v (m/s)	ω (rad/s)
a (m/s^2)	α (rad/s^2)
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
m (kg)	I ($kg \cdot m^2$)
$\Sigma F = ma$	$\Sigma \tau = I\alpha$
$KE = \frac{1}{2}mv^2$	$KE_{ROT} = \frac{1}{2}I\omega^2$
$p = mv$	$L = I\omega$
$\Sigma F = \Delta p / \Delta t$	$\Sigma \tau = \Delta L / \Delta t$
WHEN $\Sigma F = 0$, THEN p IS CONSERVED	WHEN $\Sigma \tau = 0$, THEN L IS CONSERVED

THE TWO SYSTEMS ARE CONNECTED BY:

$\theta = s/r \quad \omega = v_T/r \quad \alpha = a_T/r$

NEW FOR TODAY

Let's consider rotational KE first:

Big Picture? We modify CLEE -- KE now has two terms, one for translation and one for rotation.

$$KE_o + GPE_o + EPE_o + W_{NC} = KE + GPE + EPE$$

$$\left(\underbrace{\tfrac{1}{2}mv_o^2}_{KE_{lin}} + \underbrace{\tfrac{1}{2}I\omega_o^2}_{KE_{rot}}\right) \qquad \left(\tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2\right)$$

Except for the new term, CLEE is used exactly as before.

Rolling objects are great examples of objects both translating and rotating.

EXAMPLE: What is the total KE of a rolling disk ($I = \underbrace{\tfrac{1}{2}mr^2}$) of mass m and radius r that is traveling at velocity v?

$$KE = KE_{lin} + KE_{rot}$$

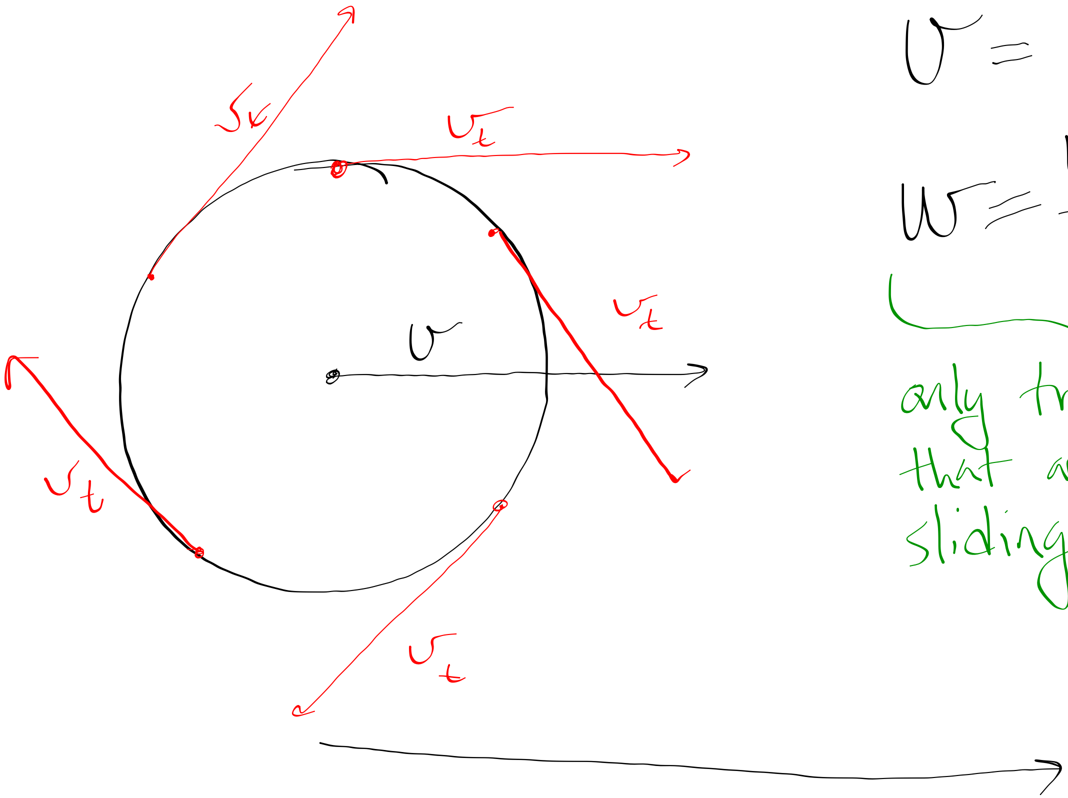
$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\omega = \frac{v}{r}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2$$

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}m\cancel{r^2}\right)\left(\frac{v}{\cancel{r}}\right)^2$$

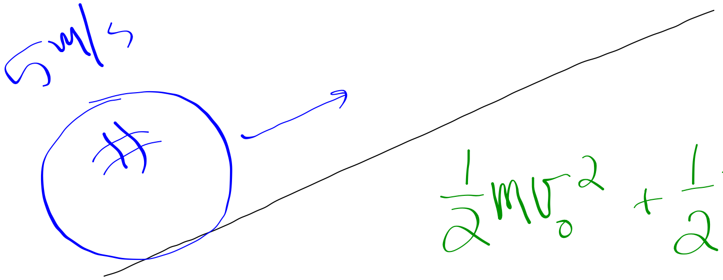
$$KE_{tot} = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \boxed{\frac{3}{4}mv^2}$$



$$v = v_t$$

$$\underbrace{\omega = \frac{v_t}{r} = \frac{v}{r}}_{\text{only true for objects that are rolling w/out sliding}}$$

EXAMPLE 2: If a ball ($I = 2/5mr^2$) moving at 5 m/sec heads up an incline, how high above the bottom of the incline will it get? Assume the ball's radius is 0.3 meters, and the ball's mass is 1.6 kg.



$$\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 + \cancel{mgh_0} + \cancel{\frac{1}{2}kx_0^2} + \cancel{W_{NC}} =$$

$$\cancel{\frac{1}{2}mv^2} + \cancel{\frac{1}{2}I\omega^2} + mgh + \cancel{\frac{1}{2}kx^2}$$

$$\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 = mgh$$

$$\underbrace{\frac{1}{2}}_{\frac{5}{10}}\cancel{m}v_0^2 + \underbrace{\frac{1}{2}\left(\frac{2}{5}\cancel{m}\cancel{r^2}\right)}_{\frac{2}{10}}\left(\frac{v_0}{\cancel{r}}\right)^2 = \cancel{m}gh$$

$$h = \frac{\frac{7}{10}v_0^2}{g} = \frac{\frac{7}{10}(5)^2}{9.8} = \boxed{1.8\text{ m}}$$

Angular momentum is a conserved quantity when there are no outside torques acting on the system in question.

We can "cheat" and can conserve angular momentum as we did when assuming linear momentum is conserved during collisions if we minimize the time that passes.

$p = m \cdot v$

$L = I \cdot \omega$

angular momentum

When angular momentum is conserved:

IF $\Sigma\tau = \Delta L / \Delta t$, THEN

IF $\Sigma\tau = 0$, $\Delta L = 0$

$\hookrightarrow$ & ANGULAR MOMENTUM IS CONSERVED

(I.E. - IT STAYS THE SAME)

$$I_o \omega_o = I_f \omega_f$$

Looking at this last equation, a system's angular momentum might change because its moment of inertia changes. How?

- The system's mass might change.
- The location of mass might change.

EXAMPLE 3: An ice skater spinning at 5 rad/sec has an I of 16 kg-m^2. After pulling her arms in, her I is 10 kg-m^2. What is her new angular velocity?

$$\Delta L = \emptyset$$

$$I_o \omega_o = I_f \omega_f$$

$$(16)(5) = (10)\omega_f$$

$$\omega_f = 8 \frac{rad}{s}$$

$\omega_o =$
$\omega =$
$I =$
$\alpha =$
$\Delta t =$
$\theta_o =$
$\theta =$

?

EXAMPLE 4: A metal ring spinning about its center has a radius of 0.5 meters and rotates at 6.5 rad/sec. If the ring's temperature increases so that its radius is now 0.55 meters, what is its new angular velocity?

$$\Delta L = I_f \omega_f - I_o \omega_o = 0$$

$$I_o \omega_o = I_f \omega_f$$

RING IS A HOOP; $I = mr^2$

$$(\cancel{m} r_o^2)\omega_o = (\cancel{m} r_f^2)\omega_f$$

$$(.5)^2(6.5) = (.55)^2(\omega_f)$$

$$\omega_f = \boxed{5.37 \text{ rad/sec}}$$