Atwood Lab: Due Tuesday 3/11

($I = 10^{-3}\ kg \cdot m^2$ if you couldn't get it)

#1 work backwards to find ← τ or forces

→ Graded on process not #'s

→ Make your process clear

(formulas, substitution, etc.)

→ PREDICTION, TESTING, ERROR ANALYSIS

TRANSLATION	ROTATION
x (METERS)	$\theta = \frac{s}{r}$ θ (RADIANS)
v (m/s)	$\omega = \frac{v}{r}$ ω (rad/s)
a (m/s^2)	$\alpha = \frac{a_t}{r}$ α (rad/s^2)
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2} a t^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
m (kg)	I ($kg \cdot m^2$)
$\Sigma F = ma$	$\Sigma \tau = I\alpha$
$KE = \frac{1}{2} m v^2$	$KE_{ROT} = \frac{1}{2} I \omega^2$
$p = mv$	$L = I\omega$
$\Sigma F = \Delta p / \Delta t$	$\Sigma \tau = \Delta L / \Delta t$
WHEN $\Sigma F = 0$, THEN p IS CONSERVED	WHEN $\Sigma \tau = 0$, THEN L IS CONSERVED

THE TWO SYSTEMS ARE CONNECTED BY:

$\theta = s/r \qquad \omega = v/r \qquad \alpha = a_T/r$

NEW FOR TODAY

Let's consider rotational KE first:

Big Picture? We modify CLEE -- KE now has two terms, one for translation and one for rotation.

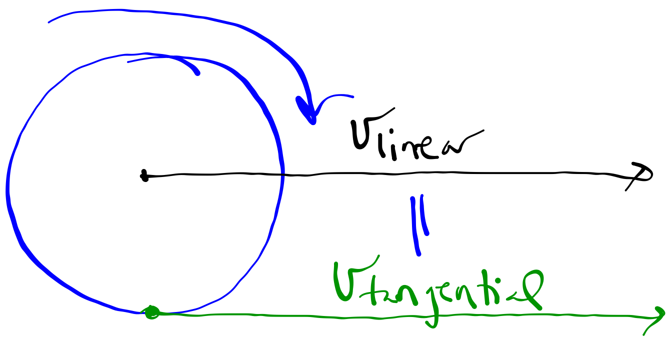
$$\underbrace{KE_o}_{\left(\frac{1}{2}mv_o^2 + \frac{1}{2}I\omega_o^2\right)} + GPE_o + EPE_o + W_{NC} = \underbrace{KE}_{\left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)} + GPE + EPE$$

Except for the new term, CLEE is used exactly as before.

Rolling objects are great examples of objects both translating and rotating.

EXAMPLE: What is the total KE of a rolling disk ($I = \frac{1}{2}mr^2$) of mass m and radius r that is traveling at velocity v?

$$KE_{total} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$



$$\omega = \frac{v_{linear}}{r}$$

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}m\cancel{r^2}\right)\left(\frac{v}{\cancel{r}}\right)^2$$

$$\frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$= \frac{3}{4}mv^2$$

EXAMPLE: What is the total KE of a rolling disk of mass m and radius r that is traveling at velocity v?

$$KE_{TOTAL} = KE_{TRANSLATION} + KE_{ROTATION}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$$

r's CANCEL

$$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$= \boxed{\frac{3}{4}mv^2}$$

Despite the box around "the answer", don't commit this expressing to memory. Rather what you want to try and remember is that KE has two parts and this is an example of how both parts can be considered using common variables.

EXAMPLE 2: If a ball ($I = 2/5mr^2$) moving at 5 m/sec heads up an incline, how high above the bottom of the incline will it get? Assume the ball's radius is 0.3 meters, and the ball's mass is 1.6 kg.

$$\frac{1}{2}mv_o^2 + \frac{1}{2}I\omega_o^2 + \cancel{mgh_o}^{0} + \cancel{\frac{1}{2}kx_o^2}^{0} + \cancel{W_{NC}}^{0} = \cancel{\frac{1}{2}mv^2}^{0} + \cancel{\frac{1}{2}I\omega^2}^{0} + mgh + \cancel{\frac{1}{2}kx^2}^{0}$$

$$\frac{1}{2}mv_o^2 + \frac{1}{2}I\omega_o^2 = mgh$$

$$\frac{1}{2}\cancel{m}v_o^2 + \frac{1}{2}\left(\frac{2}{5}\cancel{m}\cancel{r^2}\right)\left(\frac{v_o}{\cancel{r}}\right)^2 = \cancel{m}gh$$

$$\frac{5}{10}v_o^2 + \frac{2}{10}v_o^2 = gh$$

$$\frac{\frac{7}{10}v_o^2}{g} = h$$

$$h = 1.79\,m$$

EXAMPLE 2: If a ball ($I = 2/5mr^2$) moving at 5 m/sec heads up an incline, how high above the bottom of the incline will it get? Assume the ball's radius is 0.3 meters, and the ball's mass is 1.6 kg.

$$KE_o + GPE_o + EPE_o + W_{NC} = KE + GPE + EPE$$

$$\tfrac{1}{2}mv_o^2 + \tfrac{1}{2}I\omega_o^2 + 0 + 0 + 0 = 0 + mgh + 0$$

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\left(\tfrac{v}{r}\right)^2 = mgh$$

$$\tfrac{1}{2}\cancel{m}v^2 + \tfrac{2}{10}\cancel{m}v^2 = \cancel{m}gh$$

$$\tfrac{1}{2}v^2 + \tfrac{1}{5}v^2 = gh$$

$$\tfrac{7}{10}v^2 = gh$$

$$h = \frac{\tfrac{7}{10}v^2}{g} = \frac{\tfrac{7}{10}(5)^2}{9.8} = \boxed{1.79\,m}$$

Angular momentum is a conserved quantity when there are no outside torques acting on the system in question.

We can "cheat" and can conserve angular momentum as we did when assuming linear momentum is conserved during collisions if we minimize the time that passes.

When angular momentum is conserved:

IF $\Sigma\tau = \Delta L / \Delta t$, THEN

IF $\Sigma\tau = 0$, $\underline{\Delta L = 0}$

$\hookrightarrow$ & ANGULAR MOMENTUM IS CONSERVED

(I.E. - IT STAYS THE SAME)

$$I_o \omega_o = I_f \omega_f$$

Looking at this last equation, a system's angular momentum might change because its moment of inertia changes. How?

- The system's mass might change.
- The location of mass might change.

EXAMPLE 3: An ice skater spinning at 5 rad/sec has an I of 16 kg-m^2. After pulling her arms in, her I is 10 kg-m^2. What is her new angular velocity?

$$I_o \omega_o = I_f \omega_f$$

$$(16)(5) = (10)\omega_f$$

$$\omega_f = 8 \frac{rad}{s}$$

EXAMPLE 3: An ice skater spinning at 5 rad/sec has an I of 16 kg-m^2. After pulling her arms in, her I is 10 kg-m^2. What is her new angular velocity?

$$\Delta L = 0$$

$$\Delta L = I_f \omega_f - I_o \omega_o = 0$$

OR: $I_o \omega_o = I_f \omega_f$

$$(16)(5) = (10)\,\omega_f$$

$$\omega_f = \boxed{8 \text{ rad/sec}}$$

EXAMPLE 4: A metal ring ($I = \boxed{mr^2}$) spinning about its center has a radius of 0.5 meters and rotates at 6.5 rad/sec. If the ring's temperature increases so that its radius is now 0.55 meters, what is its new angular velocity?

$$I_o \omega_o = I_f \omega_f$$

$$\cancel{m} r_o^2 \omega_o = \cancel{m} r_f^2 \omega_f$$

$$\frac{r_o^2 \omega_o}{r_f^2} = \omega_f$$

EXAMPLE 4: A metal ring spinning about its center has a radius of 0.5 meters and rotates at 6.5 rad/sec. If the ring's temperature increases so that its radius is now 0.55 meters, what is its new angular velocity?

$$\Delta L = I_f \omega_f - I_o \omega_o = 0$$

$$I_o \omega_o = I_f \omega_f$$

RING IS A HOOP; $I = mr^2$

$$(\cancel{m} r_o^2)\omega_o = (\cancel{m} r_f^2)\omega_f$$

$$(.5)^2(6.5) = (.55)^2(\omega_f)$$

$$\omega_f = \boxed{5.37 \text{ rad/sec}}$$