

CONSERVATION OF ENERGY (COE)

We wish to develop this idea and make it more useful to use for problem solving

COE: energy can be neither created nor destroyed.

Objectives:

Students will understand conservation of energy and the connection between work and kinetic energy

Students will be able to explain how gravitational forces and elastic forces contribute to conservation of energy

Students will be able to use CLEE to solve conservation of energy problems.

The Work / KE Theorem says that net work changes KE:

$$W_{\text{net}} = \Delta KE$$

But some work can be thought of as a temporary storage (work due to "conservative forces"):

$$\begin{aligned} W_g &= F_g \cdot \Delta h \\ &= m \cdot g \cdot \Delta h \\ &= mgh - mgh_0 \end{aligned}$$

$$\begin{aligned} W_s &= \bar{F}_s \cdot \Delta x \\ &= \frac{1}{2} k \Delta x \cdot \Delta x \\ &= \frac{1}{2} k \Delta x^2 \\ &= \frac{1}{2} k x^2 - \frac{1}{2} k x_0^2 \end{aligned}$$

$$\begin{aligned} F_s &= kx \\ \bar{F}_s &= \frac{kx + kx_0}{2} \\ \bar{F}_s &= \frac{1}{2} k (x + x_0) \\ &= \frac{1}{2} k \Delta x \end{aligned}$$

So we can break GPE and EPE out of the Work / KE Theorem:

$$\begin{aligned} mgh - mgh_0 + \frac{1}{2} kx^2 - \frac{1}{2} kx_0^2 + W_{\text{NC}} &= \Delta KE = \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2 \\ \underbrace{\frac{1}{2} mv_0^2 + mgh_0 + \frac{1}{2} kx_0^2}_{\text{Snapshot of energy storage}} + \underbrace{W_{\text{NC}}}_{\substack{\text{Work done by forces other} \\ \text{than gravity/springs} \\ \text{over time}}} = \underbrace{\frac{1}{2} mv^2 + mgh + \frac{1}{2} kx^2}_{\text{Snapshot of energy storage}} \end{aligned}$$

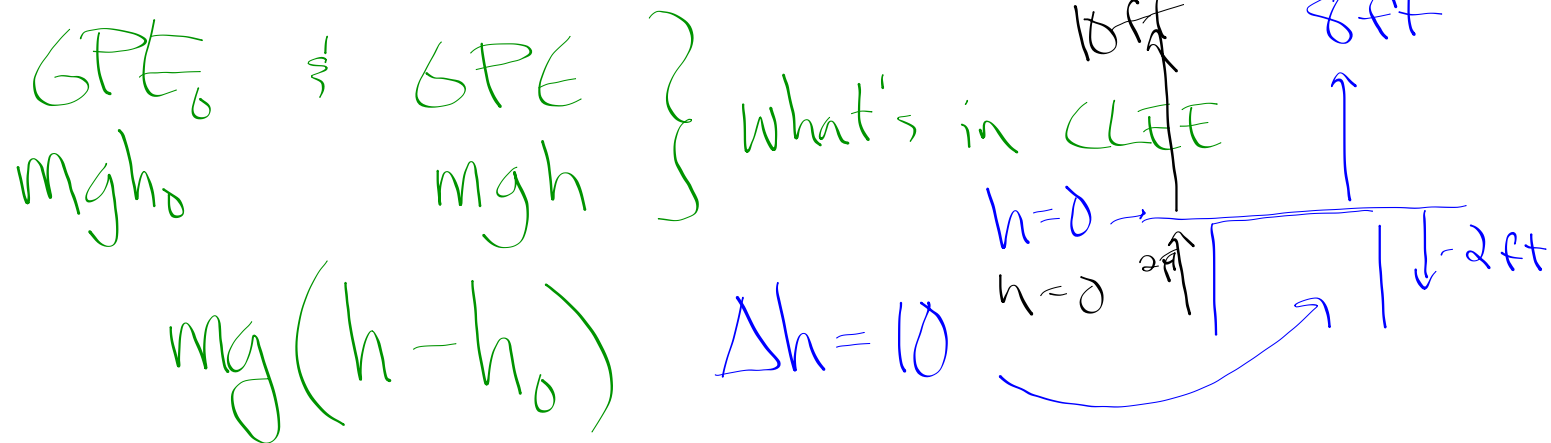
Gravitational Potential Energy:

There is no such thing as "negative energy"
 $mgh < 0$ ($GPE < 0$): the object has less
 GPE than it would at $h=0$

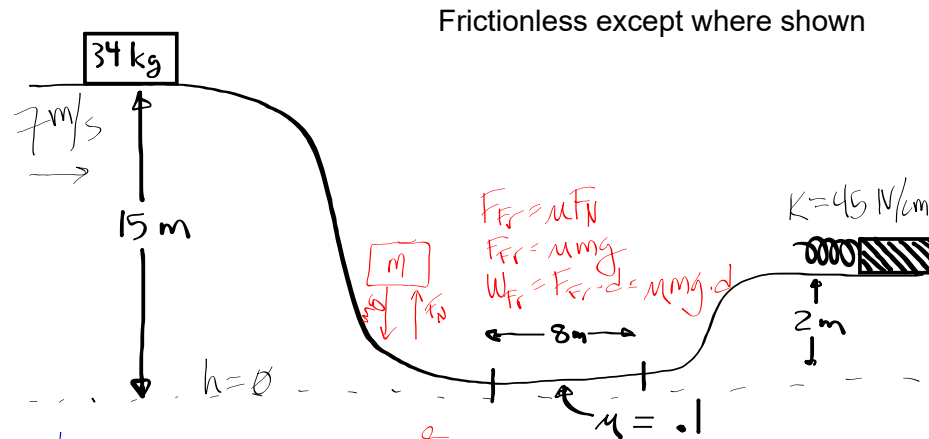
Pick your $h=0$ carefully!

set $h=0$ at or below the lowest spot
 an object will ever be located.

What does it mean if $h < 0$? (Hint - only Δh matters!)



EXAMPLE: How much is the spring shown below ($k=45 \text{ N/cm}$) deflected when the object, originally moving at 7 m/s , is brought to a stop against the spring?



$$\frac{1}{2}mv_0^2 + mgh_0 + \frac{1}{2}kx_0^2 + W_{fr} = \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2$$

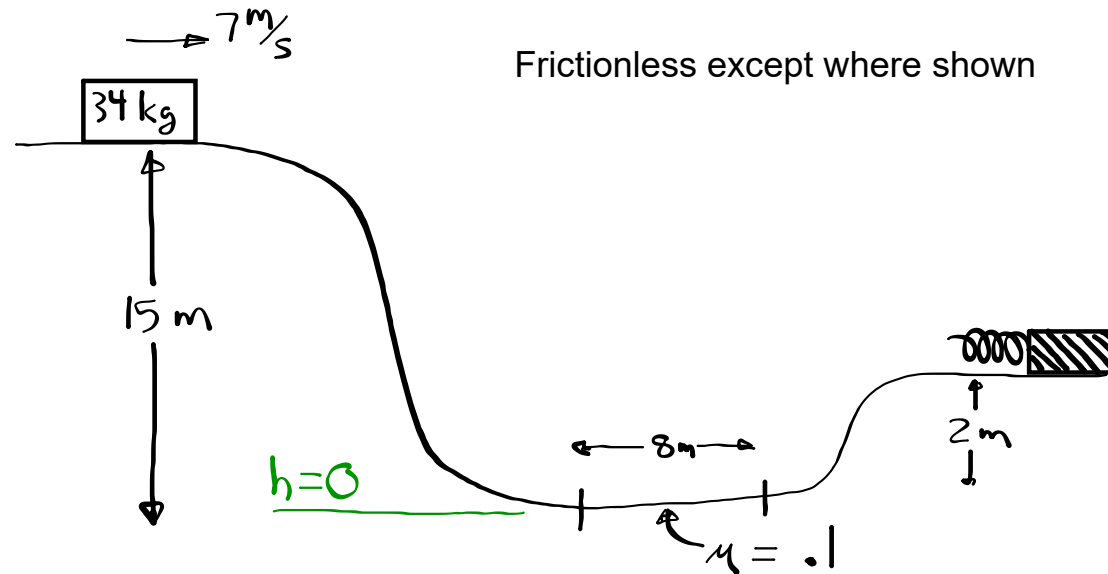
$$\frac{1}{2}(34)(7)^2 + (34)(9.8)(15) + - (0.1)(34)(9.8)(8) = (34)(9.8)(2) + \frac{1}{2}(45)x^2$$

$$X = \sqrt{\frac{\frac{1}{2}(34)(7^2) + (34)(9.8)(15) - (0.1)(34)(9.8)(8) - (34)(9.8)(2)}{\frac{1}{2}(45)}}$$

$$X = \sqrt{\frac{833 + 4998 - 266.56 - 666.4}{22.5}}$$

$$X = 14.75 \text{ cm}$$

EXAMPLE: How much is the spring shown below ($k=45 \text{ N/cm}$) deflected when the object, originally moving at 7 m/s , is brought to a stop against the spring?



$$KE_o + GPE_o + EPE_o + \sum W_{nc} = KE + GPE + EPE$$

$$\frac{1}{2}mv_o^2 + mgh_o + \frac{1}{2}kx_o^2 + W_{\text{FRICTION}} = \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2$$

$$\frac{1}{2}(34)(7)^2 + (34)(9.8)(15) + \frac{1}{2}(45)(0)^2 - (4mg) \cdot (d) = \frac{1}{2}(34)(0)^2 + (34)(9.8)(2) + \frac{1}{2}(45)x^2$$

$$\frac{1}{2}(34)(7)^2 + 34(9.8)(15) + 0 - (.1)(34)(9.8)(8) = 0 + 34(9.8)(2) + \frac{1}{2}(45)x^2$$

$$x = \boxed{14.75 \text{ cm}}$$

$$\frac{1}{2}mv_o^2 + mgh_o + \frac{1}{2}kx_o^2 + W_{NL} = \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2$$

↑
find v without
Big 4 or finding a

$$W_{app} + W_{Fr}$$

$$F_{app} \cdot \underset{\Delta x}{d} + F_{Fr} \cdot \underset{\Delta x}{d}$$

$$F_{app}(\Delta x) + F_{Fr}(\Delta x) \quad \leftarrow \text{find position!}$$