Compound Inequalities

Algebra I

Khan Academy Video of Compound Inequalities:
http://www.khanacademy.org/video/compound-inequalities?topic=solving-linear-inequalities

Compound Inequalities (From www.ck12.org)

Learning Objectives

- Write and graph compound inequalities on a number line.
- Solve compound inequalities with *"and."*
- Solve compound inequalities with *"or."*
- Solve compound inequalities using a graphing calculator (TI family).
- Solve real-world problems using compound inequalities.

Introduction

In this section, we'll solve compound inequalities—inequalities with more than one constraint on the possible values the solution can have.

There are two types of compound inequalities:

1. Inequalities joined by the word *"and,"* where the solution is a set of values greater than a number *and* less than another number. We can write these inequalities in the form "$x > a$ and $x < b$," but usually we just write "$a < x < b$." Possible values for x are ones that will make *both* inequalities true.
2. Inequalities joined by the word *"or,"* where the solution is a set of values greater than a number *or* less than another number. We write these inequalities in the form "$x > a$ or $x < b$." Possible values for x are ones that will make *at least one* of the inequalities true.

You might wonder why the variable x has to be *greater than* one number and/or *less than* the other number; why can't it be greater than both numbers, or less than both numbers? To see why, let's take an example.

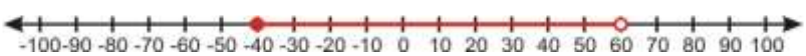
Consider the compound inequality "$x > 5$ and $x > 3$." Are there any numbers greater than 5 that are *not* greater than 3? No! Since 5 is greater than 3, everything greater than 5 is also greater than 3. If we say x is greater than both 5 and 3, that doesn't tell us any more than if we just said x is greater than 5. So this compound inequality isn't really compound; it's equivalent to the simple inequality $x > 5$. And that's what would happen no matter which two numbers we used; saying that x is greater than both numbers is just the same as saying that x is greater than the bigger number, and saying that x is less than both numbers is just the same as saying that x is less than the smaller number.

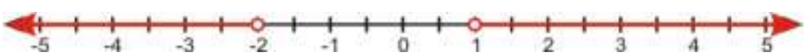
Compound inequalities with “or” work much the same way. Every number that’s greater than 3 *or* greater than 5 is also just plain greater than 3, and every number that’s greater than 3 is certainly greater than 3 *or* greater than 5—so if we say “$x > 5$ or $x > 3$,” that’s the same as saying just “$x > 3$.” Saying that x is greater than at least one of two numbers is just the same as saying that x is greater than the smaller number, and saying that x is less than at least one of two numbers is just the same as saying that x is less than the greater number.

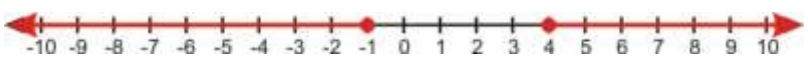
Write and Graph Compound Inequalities on a Number Line

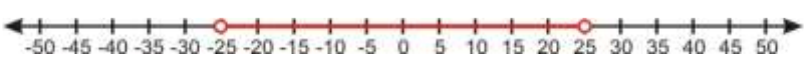
Example 1

Write the inequalities represented by the following number line graphs.

a) 

b) 

c) 

d) 

Solution

a) The solution graph shows that the solution is any value between -40 and 60, including -40 but not 60.

Any value in the solution set satisfies both $x \geq -40$ *and* $x < 60$.

This is usually written as $-40 \leq x < 60$.

b) The solution graph shows that the solution is any value greater than 1 (not including 1) or any value less than -2 (not including -2). You can see that there can be no values that can satisfy both these conditions at the same time. We write: $x > 1$ *or* $x < -2$.

c) The solution graph shows that the solution is any value greater than 4 (including 4) or any value less than -1 (including - 1). We write: $x \geq 4$ *or* $x \leq -1$.

d) The solution graph shows that the solution is any value that is both less than 25 (not including 25) and greater than -25 (not including -25). Any value in the solution set satisfies both $x > -25$ *and* $x < 25$.

This is usually written as $-25 < x < 25$.

Example 2

Graph the following compound inequalities on a number line.

a) $-4 \leq x \leq 6$

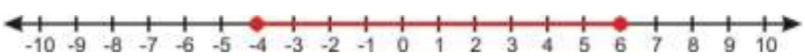
b) $x < 0$ or $x > 2$

c) $x \geq -8$ or $x \leq -20$

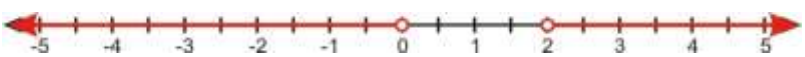
d) $-15 < x \leq 85$

Solution

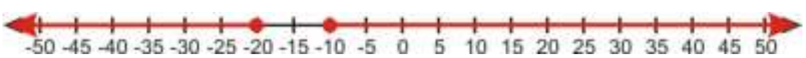
a) The solution is all numbers between -4 and 6, including both -4 and 6.



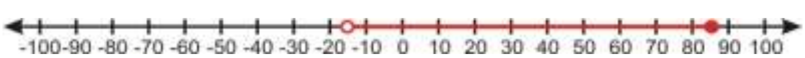
b) The solution is all numbers less than 0 or greater than 2, not including 0 or 2.



c) The solution is all numbers greater than or equal to -8 or less than or equal to -20.



d) The solution is all numbers between -15 and 85, not including -15 but including 85.



Solve a Compound Inequality With “and” or “or”

When we solve compound inequalities, we separate the inequalities and solve each of them separately. Then, we combine the solutions at the end.

Example 3

Solve the following compound inequalities and graph the solution set.

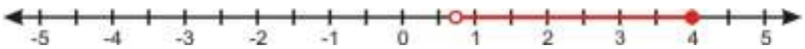
a) $-2 < 4x - 5 \leq 11$

b) $3x - 5 < x + 9 \leq 5x + 13$

Solution

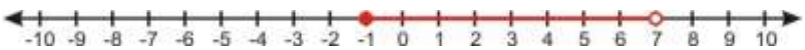
a) First we re-write the compound inequality as two separate inequalities with *and*. Then solve each inequality separately.

Answer: $\frac{3}{4} < x$ and $x \le 4$. This can be written as $\frac{3}{4} < x \le 4$.



b) Re-write the compound inequality as two separate inequalities with *and*. Then solve each inequality separately.

Answer: $x < 7$ and $x \geq -1$. This can be written as: $-1 \le x < 7$.



Example 4

Solve the following compound inequalities and graph the solution set.

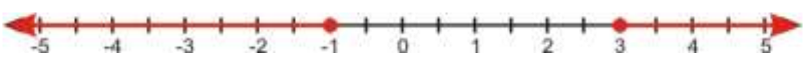
a) $9 - 2x \le 3$ or $3x + 10 \le 6 - x$

b) $\frac{x-2}{6} \le 2x - 4$ or $\frac{x-2}{6} > x + 5$

Solution

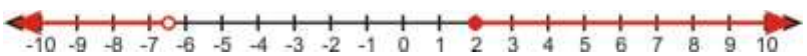
a) Solve each inequality separately:

Answer: $x \geq 3$ or $x \le -1$



b) Solve each inequality separately:

Answer: $x \geq 2$ or $x < -6.4$



The video at http://www.math-videos-online.com/solve-compound-inequality.html shows the process of solving and graphing compound inequalities in more detail. One thing you may notice in this video is that in the second problem, the two solutions joined with “or” overlap, and so the solution ends up being the set of all real numbers, or $(-\infty, \infty)$. This happens sometimes with compound inequalities that involve “or”; for example, if the solution to an inequality ended up being “$x < 5$ or $x > 1$,” the solution set would be all real numbers. This makes sense if you think about it: all real numbers are either a) less than 5, or b) greater than or equal to 5, and the ones that are greater than or equal to 5 are also greater than 1—so all real numbers are either a) less than 5 or b) greater than 1.

Compound inequalities with “and,” meanwhile, can turn out to have *no* solutions. For example, the inequality “$x < 3$ and $x > 4$” has no solutions: no number is both greater than 4 and less than 3. If we write it as $4 < x < 3$it’s even more obvious that it has no solutions; $4 < x < 3$implies that $4 < 3$, which is false