Newton's Law of Universal Gravitation

$$F_{GRAVITY} = \frac{G\, m_1 m_2}{r^2}$$

size of a force between any two objects!

G = UNIVERSAL GRAVITATION CONSTANT

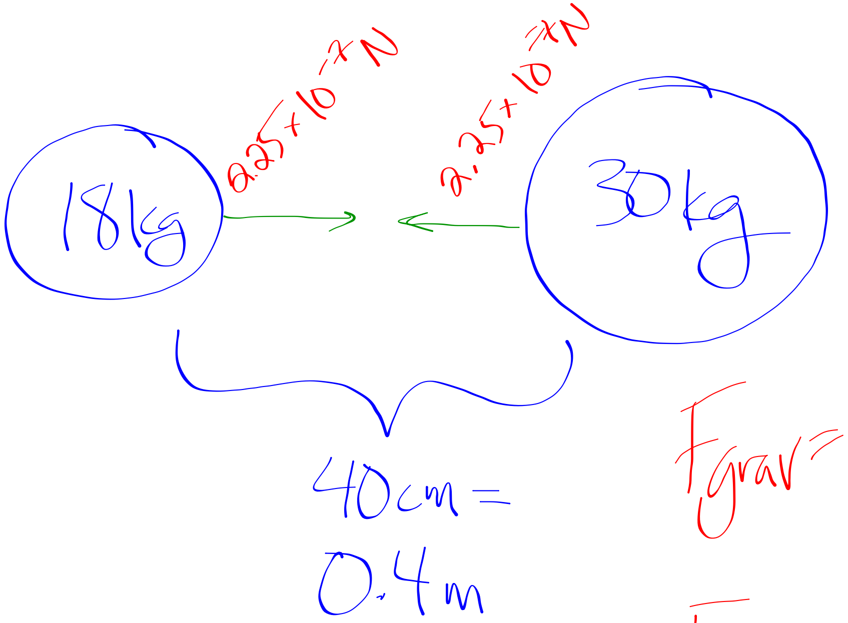
$= 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$

m_1 = MASS OF 1ST OBJECT (kg)

m_2 = MASS OF 2ND OBJECT (kg)

r = DISTANCE BETWEEN OBJECTS (m)

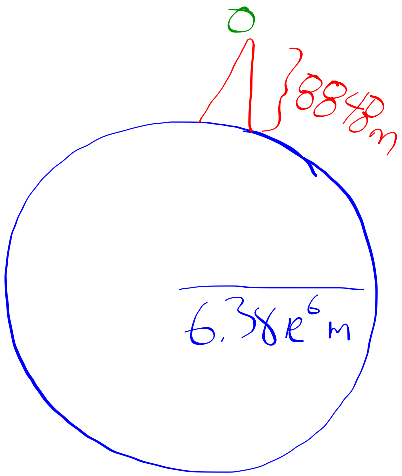
EXAMPLE 1: What is the force of attraction due to gravity between an 18 kg mass and a 30 kg mass separated by 40 centimeters?



$$F_{grav} = \frac{G m_1 m_2}{r^2} = \frac{(6.67e^{-11})(18)(30)}{0.4^2}$$

$$F_{grav} = 2.25 \times 10^{-7}\ N$$

EXAMPLE 2: What is the acceleration of Earth's gravity on top of Mt. Everest? Mt. Everest has an elevation of 8848 meters above sea level. Assume the Earth's radius is 6.38×10^6 meters, and Earth's mass is 5.98×10^{24} kg.



$$F_{grav} = \frac{G m_1 m_2}{r^2} = \frac{(6.67e^{-11})(5.98e^{24})\,m}{(6.38e^6 + 8848)^2}$$

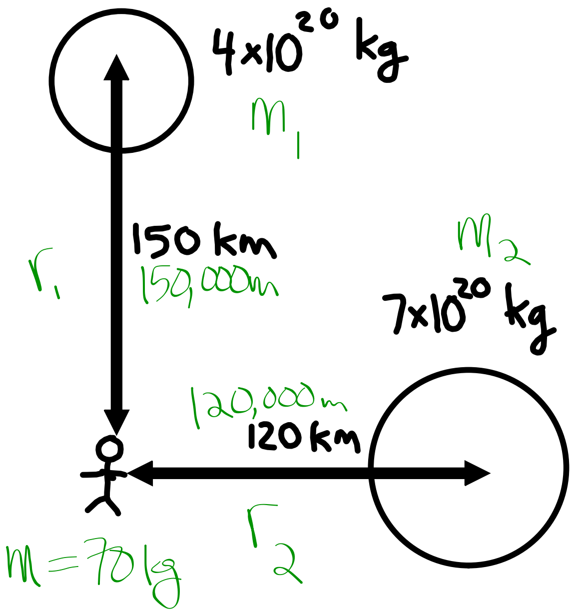
$$F_{grav} = 9.77\,m$$

$$\Sigma F = ma$$

$$F_{grav} = \cancel{m}a = 9.77\,\cancel{m}$$

$$a_g = 9.77\,m/s^2$$

EXAMPLE 4: Determine the net force upon a 70-kg person located from two planets as shown below.



$$F_{grav_1} = \frac{G m_1 m}{r_1^2} = \frac{(6.67e{-11})(4e20)(70)}{150{,}000^2}$$

$$= 83\ N$$

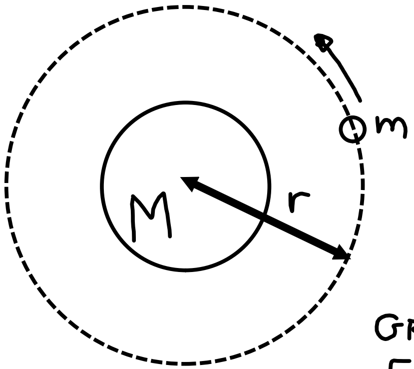
$$F_{grav_2} = \frac{G m_2 m}{r_2^2} = \frac{(6.67e^{-11})(7e20)(70)}{120{,}000^2}$$

$$= 227\ N$$

$F_{Net} = 242\ N$

$\theta = 20.1°$

SATELLITE MOTION



M = MASS OF OBJECT BEING ORBITED

m = MASS OF SATELLITE

r = RADIUS OF ORBIT

GRAVITY PROVIDES THE CENTRIPETAL FORCE NECESSARY FOR CIRCULAR MOTION

$F_{grav} = F_c$

$\Sigma F = ma$

$F_{GRAVITY} = m\left(\frac{v^2}{r}\right)$

$$\boxed{\frac{GMm}{r^2} = \frac{mv^2}{r}}$$

$$\boxed{v = \frac{2\pi r}{T}}$$

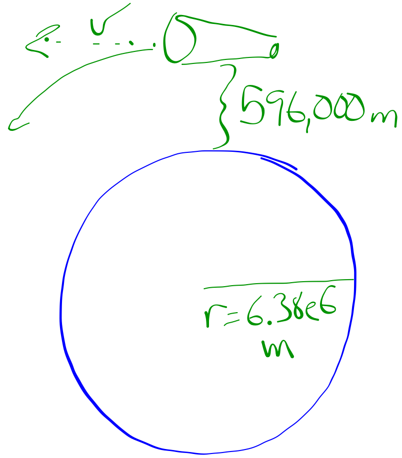
ALL RELATIONSHIPS STEM FROM THESE TWO.

T = THE PERIOD (THE TIME FOR ONE REVOLUTION)

convert (seconds)

$= 2\pi r$ meters

EXAMPLE 1: What is the orbital speed of the Hubble Space Telescope? The altitude of the HST is 596 km above the Earth. The radius of the Earth is 6.38×10^6 meters, and the Earth's mass is 5.98×10^{24} kg.



$$F_{grav} = F_c$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67e{-11})(5.98e24)}{(6.38e6)+596{,}000}}$$

$$\boxed{v = 7562 \text{ m/s}}$$

EXAMPLE 2: What is the period (in hours) of the Hubble Space Telescope?

$r = (6.38\times10^6 + 596000)$ $v = 7560\ \text{m/s}$

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi(6.38\times10^6+596000)}{7560}$$

(in seconds)

EXAMPLE 3: At what height above the Earth do geo-synchronous satellites orbit? The Earth's mass is 5.98×10^{24} kg and the Earth's radius is 6.38×10^{6} meters.

$$v = \frac{2\pi r}{T} = \frac{2\pi R}{1\ \text{DAY}}\left(\frac{1\ \text{DAY}}{24\ \text{HR}}\right)\left(\frac{1\ \text{HR}}{3600\ \text{SEC}}\right) = (7.272 \times 10^{-5})R\ \text{m/s}$$

SOLVE FOR R → THE DISTANCE FROM THE CENTER OF THE EARTH, AND THEN FIND THE HEIGHT.

$$\Sigma F = ma$$

$$\Sigma F = m\left(\frac{v^2}{r}\right)$$

$$G\frac{M\cancel{m}}{R^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{R}}$$

$$\frac{GM}{R} = (7.272 \times 10^{-5})^2 R^2$$

$$\frac{GM}{(7.272 \times 10^{-5})^2} = R^3$$

$$\sqrt[3]{\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(7.272 \times 10^{-5})^2}} = 4.224 \times 10^{7}\ \text{m}$$

$$R = r_{EARTH} + h$$

$$h = R - r_{EARTH}$$

$$= 4.224 \times 10^{7} - 6.38 \times 10^{6} = \boxed{3.586 \times 10^{7}\ \text{m}}$$

OR 3586 km