Two-Dimensional Collisions

Handled in the same manner as 1-D collisions with one exception: momentum is a vector.

For Elastic 2-D Collisions:

$\Delta P_x = 0 \qquad m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}' + m_2 v_{2x}'$

$\Delta P_y = 0 \qquad m_1 v_{1y} + m_2 v_{2y} = m_1 v_{1y}' + m_2 v_{2y}'$

$\Delta KE = 0 \qquad \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 (v_1')^2 + \frac{1}{2} m_2 (v_2')^2$

NOTE: KE IS A SCALAR → YOU CANNOT BREAK IT INTO X'S & Y'S

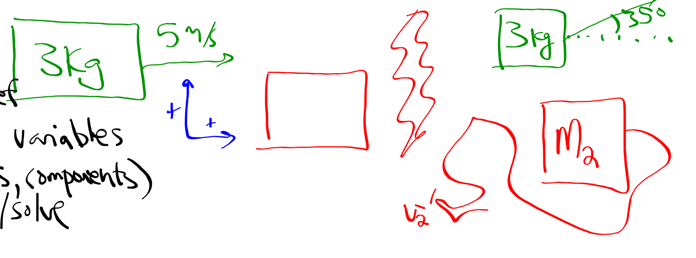
For 2-D Collisions not perfectly elastic:

$\Delta P_x = 0 \qquad m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}' + m_2 v_{2x}'$

$\Delta P_y = 0 \qquad m_1 v_{1y} + m_2 v_{2y} = m_1 v_{1y}' + m_2 v_{2y}'$

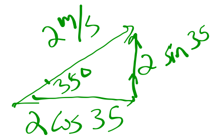
EXAMPLE: A 3-kg object travels at 5 m/s. It strikes (in an elastic collision) a 2nd motionless object. After the collision, the 1st object is observed to move at 2 m/s in a direction that makes an angle of 35° with its original direction. What is the final velocity and mass of the 2nd object?

1. Draw pic
2. Frame of ref
3. Identify variables (conversions, components)
4. Substitute/solve
5. Interpret



$m_1 = 3$ kg
$m_2 = ?$
$v_1 = 5\frac{m}{s}$
$v_2 = 0\frac{m}{s}$
$v_1' = 2$ m/s @ 35°
$v_2' = ?$

$v_{1x} = 5\frac{m}{s}$
$v_{1y} = 0\frac{m}{s}$
$v_{2x} = 0\frac{m}{s}$
$v_{2y} = 0\frac{m}{s}$
$v_{1x}' = 2\cos 35\frac{m}{s}$
$v_{1y}' = 2\sin 35\frac{m}{s}$
$v_{2x}' = ?$
$v_{2y}' = ?$



$\Delta p_x = 0$:

$$m_1 v_{1x} + \cancel{m_2 v_{2x}}^{\,0} = m_1 v_{1x}' + m_2 v_{2x}'$$

$$(3)(5) = (3)(2\cos 35) + m_2 v_{2x}'$$

$$\boxed{m_2 v_{2x}' = 10.1\ \frac{kg\cdot m}{s}} \rightarrow v_{2x}' = \frac{10.1}{m_2}$$

$\Delta p_y = 0$

$$\cancel{m_1 v_{1y}}^{\,0} + \cancel{m_2 v_{2y}}^{\,0} = m_1 v_{1y}' + m_2 v_{2y}'$$

$$0 = (3)(2\sin 35) + m_2 v_{2y}'$$

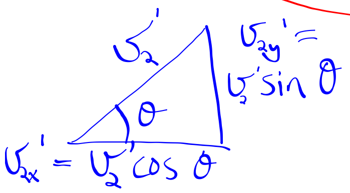
$$m_2 v_{2y}' = -3.44 \rightarrow v_{2y}' = \frac{-3.44}{m_2}$$

$\Delta KE = 0$

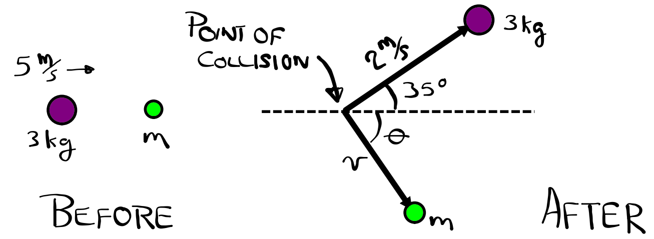
$$\frac{1}{2}m_1 v_1^2 + \cancel{\frac{1}{2}m_2 v_2^2} = \frac{1}{2}m_1(v_1')^2 + \frac{1}{2}m_2(v_2')^2$$

$$\cancel{\tfrac{1}{2}}(3)(5^2) = \cancel{\tfrac{1}{2}}(3)(2^2) + \cancel{\tfrac{1}{2}}m_2(v_2')^2$$

$$m_2(v_2')^2 = 63\ \frac{kg\cdot m^2}{s^2} \rightarrow (v_2')^2 = \frac{63}{m_2}$$



$$(v_2')^2 = (v_{2x}')^2 + (v_{2y}')^2$$



- SINCE P_y BEFORE = 0, P_y AFTER = 0

∴ θ IS DRAWN BELOW THE HORIZONTAL, TO RIGHT

- IF ELASTIC, THEN :

$\Delta P_x = 0 \qquad m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}' + m_2 v_{2x}'$

$\boxed{1} \qquad 3(5) + m(0) = 3(2\cos 35) + m(v\cos\theta)$

$\Delta P_y = 0 \qquad m_1 v_{1y} + m_2 v_{2y} = m_1 v_{1y}' + m_2 v_{2y}'$

$\boxed{2} \qquad 3(0) + m(0) = 3(2\sin 35) - m(v\sin\theta)$

$\Delta KE = 0 \qquad \cancel{\tfrac{1}{2}} m_1 v_1^2 + \cancel{\tfrac{1}{2}} m_2 v_2^2 = \cancel{\tfrac{1}{2}} m_1 (v_1')^2 + \cancel{\tfrac{1}{2}} m_2 (v_2')^2$

$\boxed{3} \qquad 3(5)^2 + m(0)^2 = 3(2)^2 + m(v)^2$

$75 = 12 + mv^2$

$63 = mv^2$

SIMPLIFY $\boxed{2}$ $\qquad 0 = 3(2\sin 35) - mv\sin\theta$

$0 = 3.441 - mv\sin\theta$

$mv\sin\theta = 3.441$

$\boxed{4} \qquad m = \dfrac{3.441}{v\sin\theta}$

PLUG $\boxed{4}$ INTO $\boxed{1}$

$15 = 3(2\cos 35) + mv\cos\theta$

$15 = 4.915 + mv\cos\theta$

$15 = 4.915 + \left(\dfrac{3.441}{\cancel{v}\sin\theta}\right)\cancel{v}\cos\theta$

$15 = 4.915 + \dfrac{3.441}{\tan\theta} \qquad \left(\text{BECAUSE } \dfrac{\sin\theta}{\cos\theta} = \tan\theta\right)$

$\tan\theta = \left(\dfrac{3.441}{15 - 4.915}\right)$

$\theta = \tan^{-1}\left(\dfrac{3.441}{15 - 4.915}\right) \Rightarrow \theta = \boxed{18.84^\circ \text{ AS SHOWN}}$

FROM $\boxed{3}$: $\qquad 63 = mv^2$

$63 = \left(\dfrac{3.441}{\cancel{v}\sin\theta}\right)v^{\cancel{2}}$

$63 = \left(\dfrac{3.441}{\sin 18.84}\right)v \qquad v = \boxed{5.9\ m/s}$

FROM $\boxed{2}$: $\qquad m = \dfrac{3.441}{v\sin\theta} = \dfrac{3.441}{5.9(\sin 18.84)} = \boxed{1.8\ kg}$