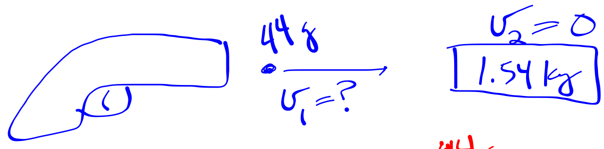
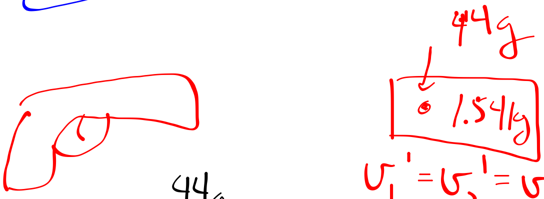
7. A 44-g bullet strikes and becomes embedded in a 1.54-kg block of wood placed on a horizontal surface just in front of the gun. If the coefficient of kinetic friction between the block and the surface is 0.28, and the impact drives the block a distance of 18.0 meters before it comes to rest, what was the muzzle speed of the bullet? (Hint: this is very, very similar to the ballistic pendulum problems that were highlighted in the reading).

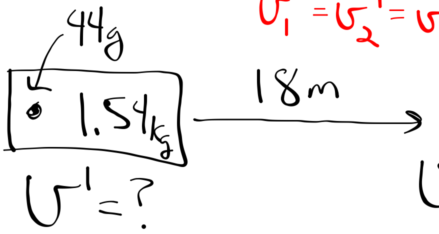


$m_1 = .044$ kg
$m_2 = 1.54$ kg
$v_1 = ?$
$v_2 = 0$
$v' = 9.94 \frac{m}{s}$

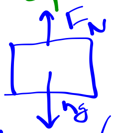
$W_{Fr} = F_{Fr} \cdot d$
$= \mu F_N \times d$



$v_1' = v_2' = v' = ?$ $9.94 \frac{m}{s}$



$$\frac{1}{2}m(v')^2 + W_{NC} = \frac{1}{2}mv^2$$



$$W_N = F_{Fr} \cdot d = \mu F_N \cdot d = \mu(mg) \cdot d$$

$$= (0.28)\left(1.54 + \left(44 \cdot \frac{1\,kg}{1000\,g}\right)\right)(9.8)(18)$$

$$= 78.24\,J$$

$$\frac{1}{2}m(v')^2 + W_{NC} = \cancel{\frac{1}{2}mv^2}^{\,0}$$

$$\frac{1}{2}(1.54 + 0.044)(v')^2 - 78.24 = 0$$

$$v' = \sqrt{\frac{(78.24)\,2}{(1.54 + 0.044)}} = 9.94 \frac{m}{s}$$

$m_1 = .044$ kg
$m_2 = 1.54$ kg
$v_1 = ?$
$v_2 = 0$
$v' = 9.94 \frac{m}{s}$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

$$v_1 = \frac{(m_1 + m_2) v' - m_2 v_2}{m_1}$$

$$v_1 = \frac{(1.584)(9.94) - 0}{0.044} = \boxed{357.84 \frac{m}{s}}$$

RECALL

$$\Sigma F = ma$$

$$\Updownarrow$$

$$\Sigma F = \frac{\Delta P}{\Delta t}$$

When:

$\Sigma F = 0$ (No outside forces exist to act on a system)

$\Sigma F = 0 = \frac{\Delta p}{\Delta t} = 0$

OR

$\Delta t \to 0$ (We can cheat -- because the time that passes is so small, the forces are unable to do any work because there is no displacement)

$\Sigma F = 0 = \Delta p$ **MOMENTUM IS CONSERVED**

$\Sigma F \neq 0$ (Then the concept of impulse is useful)

∴ momentum isn't constant.

$$\Sigma F = \Delta p / \Delta t$$

$$\underbrace{\Sigma F \Delta t}_{} = \Delta p$$

product of net force and change in time = change in momentum

This product of the net force and time is known as the **IMPULSE** imparted to the system whose momentum is changing.

Impulse is a vector quantity.

ΣF must be the average force if F isn't constant.

Units: [N x $\underbrace{\text{s], or [kg x (m/s}}_{\text{BECAUSE}}$)]

IMPULSE also equals the change in momentum.

$$\Sigma F \Delta t = \text{IMPULSE} = \Delta p$$

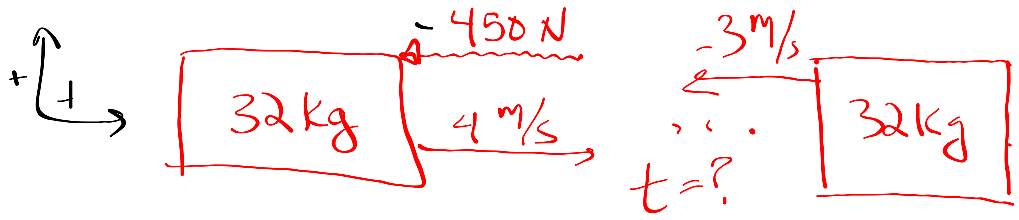
IMPULSE APPLIED BY OBJECT #1

CHANGE IN MOMENTUM OF OBJECT #2

The impulse of Object #1 creates a change in the momentum of Object #2 (for a collision between two objects).

When trying to determine the impulse delivered, looking at an object's change in momentum is usually more useful because velocities are easier to measure than either forces or short times.

EXAMPLE 1: How long must a 450 N force be applied to change the velocity of a 32-kg mass from 4 m/s to 3 m/s in the opposite direction? Assume a horizontal frictionless surface.



$m = 32\ kg$
$v = 4\ m/s$
$v' = -3\ m/s$
$\Sigma F = 450\ N$
$t = ?$

$$\Sigma F \cdot \Delta t = \Delta p$$

$$-450\ N \cdot \Delta t = mv' - mv$$

$$\Delta t = \frac{(32)(3) - (32)4}{-450}$$

$$= 0.498\ s$$

The 450 N force exerts an inpulse of

$450(.498) = 224.1\ N \cdot s$ IN THE + DIRECTION

The momentum of the 32-kg mass changes by

$(3)(32) - (-4)(32) = 224.1\ kg\ m/s$ IN THE + DIRECTION

Collisions

For elastic collisions:

Momentum is conserved $(\Delta p = 0)$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

Kinetic energy is conserved $(\Delta KE = 0)$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 (v_1')^2 + \frac{1}{2} m_2 (v_2')^2$$

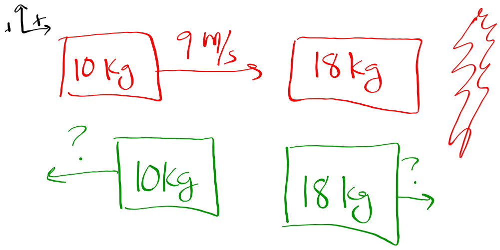
For inelastic collisions (any collision not perfectly elastic):

Momentum is conserved

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

Energy is still conserved, but KE is not; some of the original KE that exists before the collision leaves the system as sound, light, thermal, work, etc....

EXAMPLE #2: A 10-kg block moving at 9 m/s strikes an 18-kg block that is initially motionless. What are the final velocities of the two blocks if the collision is perfectly elastic?



$m_1 = 10\,kg$
$m_2 = 18\,kg$
$v_1 = 9\frac{m}{s}$
$v_2 = 0$
$v_1' = ?$
$v_2' = ?$

$\Delta p = 0$:

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$
$$(10)(9) + (18)(0) = 10v_1' + 18v_2'$$
$$90 = 10v_1' + 18v_2'$$
$$10v_1' = 90 - 18v_2'$$
$$v_1' = \boxed{\frac{90 - 18v_2'}{10}}$$

$\Delta KE = 0$:

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1(v_1')^2 + \frac{1}{2}m_2(v_2')^2$$
$$\frac{1}{2}(10)(9^2) + \frac{1}{2}(18)(0^2) = \frac{1}{2}10(v_1')^2 + \frac{1}{2}18(v_2')^2$$
$$(5)(81) + 0 = 5(v_1')^2 + 9(v_2')^2$$
$$405 = 5\left(\frac{90 - 18v_2'}{10}\right)^2 + 9(v_2')^2$$
$$405 = 5(9 - 1.8v_2')^2 + 9(v_2')^2$$
$$405 = 5\left(81 - 32.4v_2' + 3.24(v_2')^2\right) + 9(v_2')^2$$
$$\cancel{405} = \cancel{405} - 162v_2' + 16.2(v_2')^2 + 9(v_2')^2$$
$$25.2(v_2')^2 - 162v_2' = 0$$
$$v_2'(25.2v_2' - 162) = 0$$

$v_2' = 0$ or... $25.2v_2' = 162$

$$v_2' = 6.4\frac{m}{s}$$

$$v_1' = \boxed{\frac{90 - 18v_2'}{10}}$$
$$v_1' = \frac{90 - (18)(6.4)}{10}$$
$$= -2.52\frac{m}{s}$$

$v_1' = -2.52\frac{m}{s}$
$v_2' = 6.4\frac{m}{s}$

EXAMPLE #2: A 10-kg block moving at 9 m/s strikes an 18-kg block that is initially motionless. What are the final velocities of the two blocks if the collision is perfectly elastic?

$+ \rightarrow$

10 kg 18 kg INITIALLY

$\xrightarrow{}$ $9\ m/s$ $v = 0$

$\Delta p = 0 \qquad m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$

$$(10)(9) + (18)(0) = 10v_1' + 18v_2' \qquad \boxed{1}$$

$\Delta KE = 0 \qquad \cancel{\tfrac{1}{2}}m_1v_1^2 + \cancel{\tfrac{1}{2}}m_2v_2^2 = \cancel{\tfrac{1}{2}}m_1(v_1')^2 + \cancel{\tfrac{1}{2}}m_2(v_2')^2$

$$10(9)^2 + 18(0)^2 = 10(v_1')^2 + 18(v_2')^2 \qquad \boxed{2}$$

USE $\boxed{1}$ TO SOLVE FOR v_1' AND THEN SUBSTITUTE INTO $\boxed{2}$

$$90 = 10v_1' + 18v_2'$$

$$v_1' = \frac{90 - 18v_2'}{10}$$

$$10(9)^2 + 0 = 10\left(\frac{90 - 18v_2'}{10}\right)^2 + 18(v_2')^2$$

$$810 = \tfrac{1}{10}(90 - 18v_2')^2 + 18(v_2')^2$$

$$810 = 810 - 324v_2' + 50.4(v_2')^2$$

$$50.4(v_2')^2 - 324v_2' = 0$$

$v_2' = 0$ OR $\boxed{6.43\ m/s \text{ IN + DIRECTION}}$

$v_1' = \boxed{-2.57\ m/s}$ (FOUND BY SUBSTITUTING IN v_2')

EXAMPLE #3: A 3000-kg train car traveling at 0.5 m/s strikes and couples to a 4500-kg car moving at 0.2 m/s in the same direction.

1. What is the final velocity of both cars?
2. What percentage of the initial KE remains in the system after the collision?

1) $\Delta p = 0 \qquad m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$

$$3000(.5) + 4500(.2) = (3000+4500) v$$

$$v = \boxed{.32\ \tfrac{m}{s} \text{ IN ORIGINAL DIRECTION}}$$

2) $\% KE = \dfrac{\text{FINAL KE}}{\text{INITIAL KE}} = \dfrac{\frac{1}{2}(7500)(.32)^2}{\frac{1}{2}(3000)(.5)^2 + \frac{1}{2}(4500)(.2)^2}$

$= .826$

$\rightarrow$ 82.6%

WHERE DID THE MISSING KE GO?