Newton's Law of Universal Gravitation

$$F_{GRAVITY} = \frac{G\, m_1 m_2}{r^2}$$

G = UNIVERSAL GRAVITATION CONSTANT

$= 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$

m_1 = MASS OF 1ST OBJECT (kg)

m_2 = MASS OF 2ND OBJECT (kg)

r = DISTANCE BETWEEN OBJECTS (m)

Objectives:

- Students will understand the law of universal gravitation
- Students will be able to explain what the universal gravitation constant means and implies
- Students will be able to use the formula for universal gravitation to solve problems

EXAMPLE 1: What is the force of attraction due to gravity between an 18 kg mass and a 30 kg mass separated by 40 centimeters?

$$= 0.4\,m$$

$$F_g = \frac{G m_1 m_2}{r^2} = \frac{(6.67e^{-11})(18)(30)}{0.4^2}$$

$$F_g = 2.25e^{-7}\ N$$

EXAMPLE 2:

a. What is the radius of the Earth? Assume the Earth's mass is 5.98×10^{24} kg.

$\Sigma F = ma$

$F_g = ma \rightarrow \frac{G m_e \cancel{m}}{r^2} = \cancel{m} a$

$r = \sqrt{\frac{G m_e}{a}} = \sqrt{\frac{(6.67e{-11})(5.98e24)}{9.8}}$

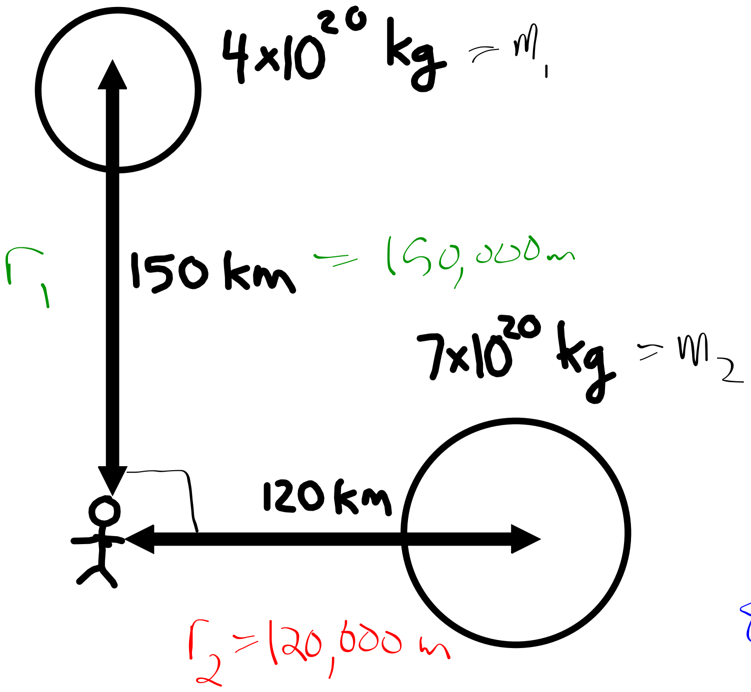
$r = 6.38e6$ m

b. What is the acceleration of Earth's gravity on top of Mt. Everest? Mt. Everest has an elevation of 8848 meters above sea level.

$$\frac{G m_e \cancel{m}}{r^2} = \cancel{m} a = \frac{(6.67e{-11})(5.98e24)}{(6.38e6 + 8848)^2}$$

$a = 9.77$ m/s^2

EXAMPLE 4: Determine the net force upon a 70-kg person located from two planets as shown below.

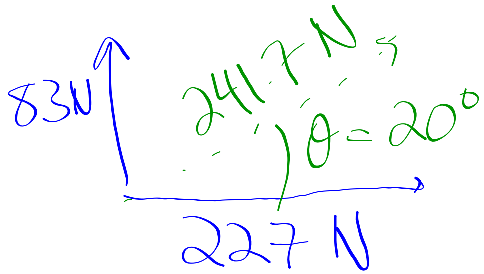


$$F_1 = \frac{G m_1 m}{r_1^2} = \frac{(6.67e^{-11})(4e20)(70)}{150{,}000^2}$$

$$F_1 = 83\text{ N}$$

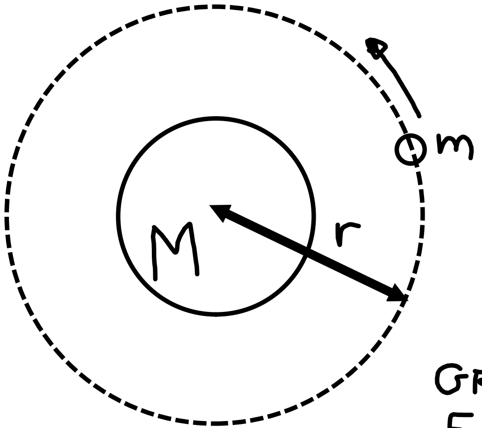
$$F_2 = \frac{G m_2 m}{r_2^2} = \frac{(6.67e^{-11})(7e20)(70)}{120{,}000^2}$$

$$F_2 = 227\text{ N}$$



$$F_{Net} = 241.7\text{N} @ 20°$$

SATELLITE MOTION



M = MASS OF OBJECT BEING ORBITED

m = MASS OF SATELLITE

r = RADIUS OF ORBIT

GRAVITY PROVIDES THE CENTRIPETAL FORCE NECESSARY FOR CIRCULAR MOTION

$$\Sigma F = ma$$

$$F_{GRAVITY} = m\left(\frac{v^2}{r}\right)$$

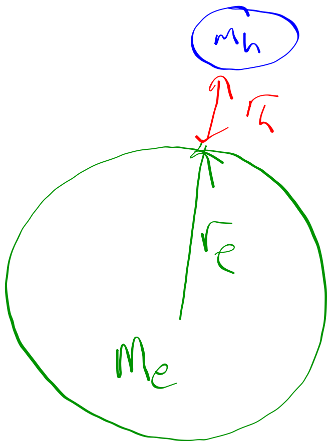
$$\boxed{\frac{GMm}{r^2} = \frac{mv^2}{r}}$$

$$\boxed{v = \frac{2\pi r}{T}}$$

ALL RELATIONSHIPS STEM FROM THESE TWO.

T = THE PERIOD (THE TIME FOR ONE REVOLUTION)

EXAMPLE 1: What is the orbital speed of the Hubble Space Telescope? The altitude of the HST is 596 km above the Earth. The radius of the Earth is 6.38×10^{6} meters, and the Earth's mass is 5.98×10^{24} kg.



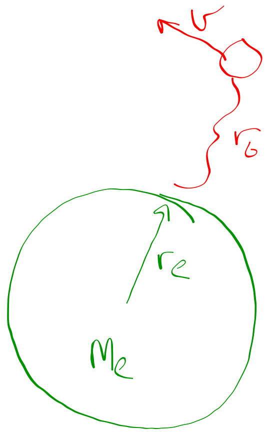
$$F_g = F_c$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67e{-11})(5.98e24)}{(6.38e6)+596{,}000}}$$

$$\boxed{v = 7561.5\ m/s}$$

EXAMPLE 3: At what height above the Earth do geo-synchronous satellites orbit? The Earth's mass is 5.98×10^{24} kg and the Earth's radius is 6.38×10^{6} meters.



$$F_g = F_c$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$T = 24\,hr = 86{,}400\,s$$

$$v = \frac{2\pi r}{T}$$

$$r = \frac{GM}{v^2}$$

$$r = \frac{GM}{\left(\frac{2\pi r}{T}\right)^2} = \frac{GMT^2}{4\pi^2 r^2}$$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{(6.67e{-11})(5.98e24)(86{,}400^2)}{4\pi^2}}$$

$$\cancel{r_e} + r_o = r = 4.2e7\ m - r_e \text{ (from earth's center)}$$

$$r_o = 4.2e7 - 6.38e6 = \boxed{3.6e7\,m}$$

$$\cancel{I = \frac{-m_1 g r_1^2 - m_1 a_1 r_1^2 + m_2 g r_1 r_2 - m_2 a_1 r_2^2}{a_1}}$$

$$\Sigma\tau = I\alpha$$

$$\alpha = \frac{\boxed{\Sigma\tau}}{I}$$

1e-3 kg·m²

1e-5 kg·m²

expressed in things we know!

m_1, m_2, r_1, r_2, g

YOUR GOAL: $\alpha = I, m_1, m_2, r_1, r_2, g$

(no other variables...)

$T_1 = m_1 g + m_1 a_1$

$T_2 = m_2 g - m_2 a_2$

$\tau_1 = T_1 r_1$

$\tau_2 = T_2 r_2$

$$\Sigma\tau = \underbrace{-\tau_1 + \tau_2}$$

$$\alpha = \frac{a_1}{r_1} = \frac{a_2}{r_2}$$

$$\alpha = \frac{\Sigma\tau}{I} \checkmark$$

$a_1 = \boxed{\alpha r_1}$

$a_2 = \boxed{\alpha r_2}$