Test over rotation:

THURS. 3/13

# Newton's Law of Universal Gravitation

$$F_{GRAVITY} = \frac{G\, m_1 m_2}{r^2}$$

G = UNIVERSAL GRAVITATION CONSTANT

$= 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$

$m_1$ = MASS OF 1ST OBJECT (kg)

$m_2$ = MASS OF 2ND OBJECT (kg)

r = DISTANCE BETWEEN OBJECTS (m)

EXAMPLE 1: What is the force of attraction due to gravity between an 18 kg mass and a 30 kg mass separated by 40 centimeters?

$$F_{gravity} = \frac{G m_1 m_2}{r^2}$$

$$= \frac{(6.67 \times 10^{-11})(18)(30)}{(.4)^2}$$

$$= 2.25 \times 10^{-7} \text{ N}$$

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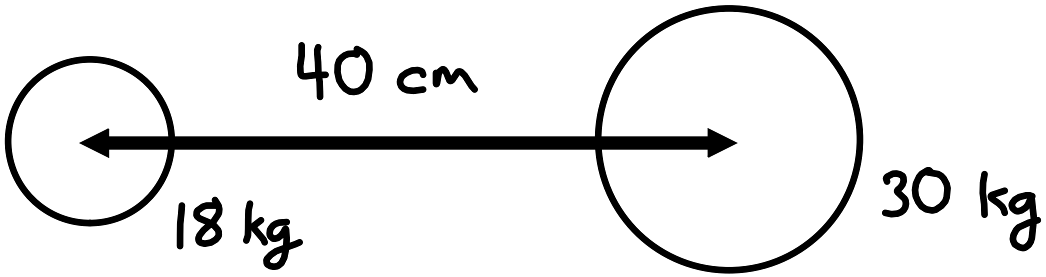



$$F = G\frac{m_1 m_2}{r^2} = 6.67\times10^{-11}\frac{(18)(30)}{(.4)^2}$$

$$= \boxed{2.25\times10^{-7}\,N \quad \text{ATTRACTION}}$$

PRETTY SMALL!

EXAMPLE 2: What is the acceleration of Earth's gravity on top of Mt. Everest? Mt. Everest has an elevation of 8848 meters above sea level. Assume the Earth's radius is 6.38x10$^6$ meters, and Earth's mass is 5.98x10$^{24}$ kg.

earth

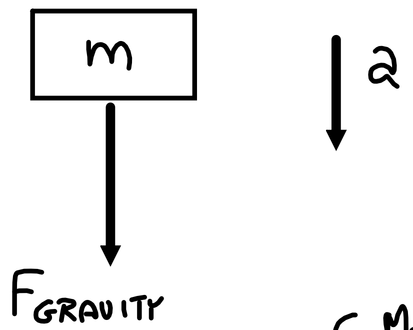
$$F_{gravity} = \frac{G m_1 m_2}{r^2} = m_2 g$$

$$\cancel{m_2} g = \frac{G m_1 \cancel{m_2}}{r^2}$$

$$g = \frac{(6.67\times10^{-11})(5.98\times10^{24})}{(6.38\times10^{6}+8848)^2} =$$

$$g = 9.77\ m/s^2$$

EXAMPLE 2: What is the acceleration of Earth's gravity on top of Mt. Everest? Mt. Everest has an elevation of 8848 meters above sea level. Assume the Earth's radius is $6.38\text{x}10^{6}$ meters, and Earth's mass is $5.98\text{x}10^{24}$ meters.




FOR AN OBJECT FALLING AT THE TOP OF MT. EVEREST:

$$\Sigma F = ma$$

$$F_{GRAVITY} = ma$$

$$G\frac{M_{EARTH}\cancel{m}}{r^2} = \cancel{m}a$$

$$r = r_{EARTH} + 8848\text{m}$$

$$a = \frac{(6.67\times10^{-11})(5.98\times10^{24})}{(6.38\times10^{6} + 8848)^2} = \boxed{9.767\ \text{m/s}^2}$$

AT THE EARTH'S SEA LEVEL ELEVATION (FOR COMPARISON):

$$a = \frac{GM_E}{r_E^2} = \frac{(6.67\times10^{-11})(5.98\times10^{24})}{(6.38\times10^{6})^2} = 9.794\ \text{m/s}^2$$

EXAMPLE 3: What is the acceleration of gravity at the surface of the sun? The mass of the sun is $2.0 \times 10^{30}$ kg. The radius of the sun is $7.0 \times 10^{8}$ meters.

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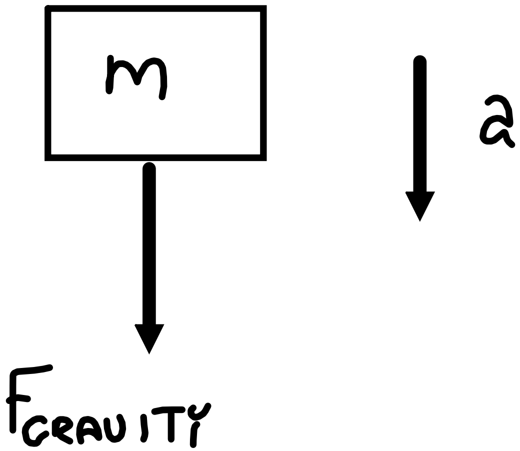


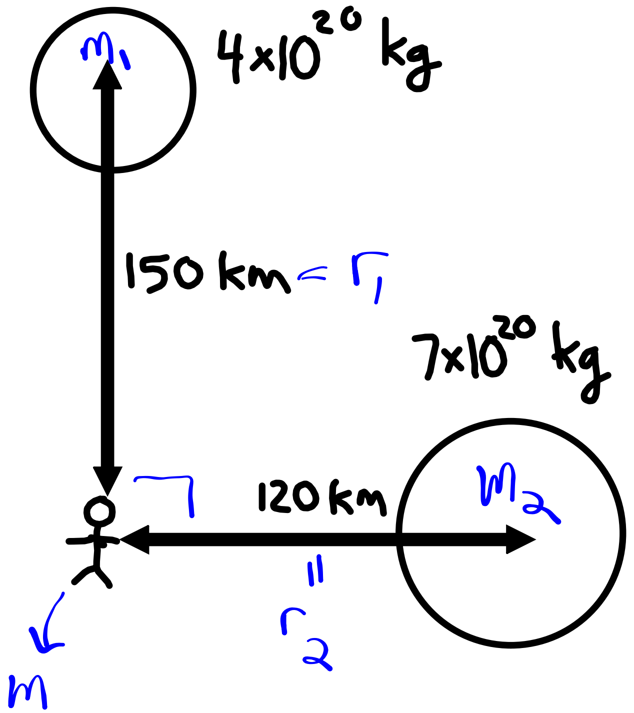

$$\Sigma F = ma$$

$$F_{GRAVITY} = ma$$

$$G \frac{M_{SUN} \cancel{m}}{r^2} = \cancel{m} a$$

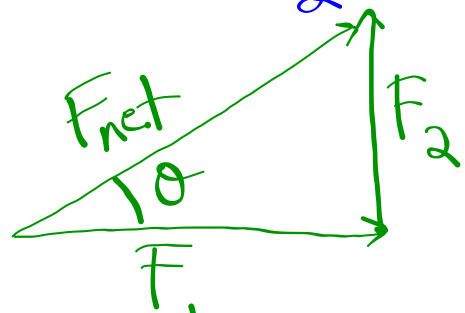
$$\frac{(6.67 \times 10^{-11})(2.0 \times 10^{30})}{(7.0 \times 10^{8})^2} = \boxed{272.1 \ m/s^2}$$

EXAMPLE 4: Determine the net force upon a 70-kg person located from two planets as shown below.




$$F_1 = \frac{G m_1 m}{r_1^2}$$

$$F_2 = \frac{G m_2 m}{r_2^2}$$




$$F_{net} = \sqrt{F_1^2 + F_2^2}$$

$$\theta = \tan^{-1} \frac{F_1}{F_2}$$

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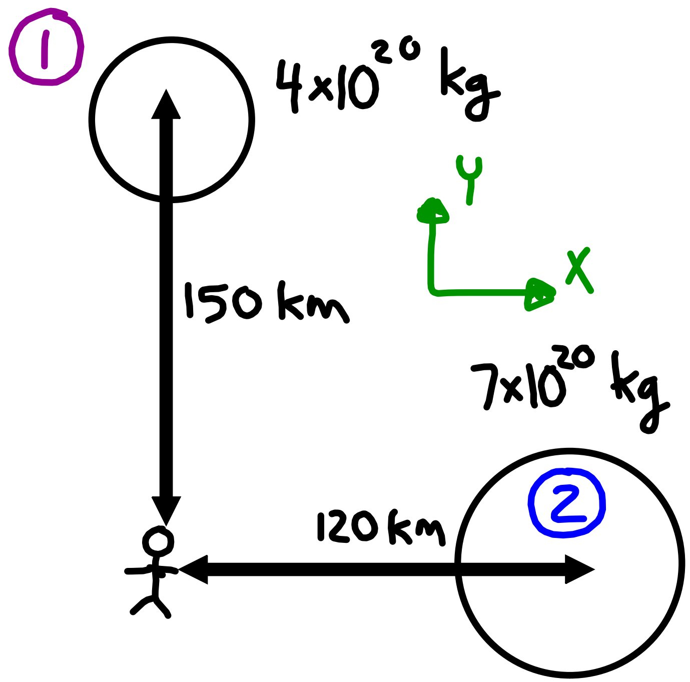
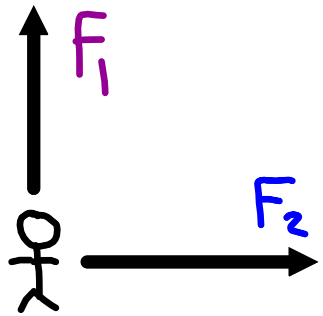
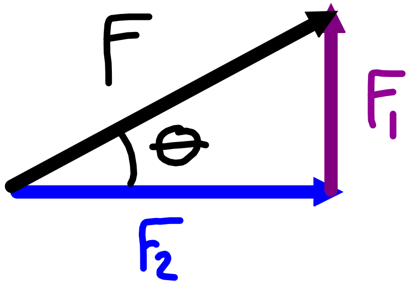



$$F_1 = G\frac{(4\times10^{20})(70)}{(1.5\times10^5)^2} = 82.96\ N$$

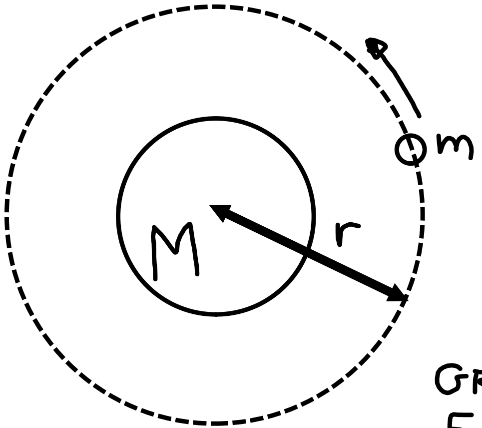
$$F_2 = G\frac{(7\times10^{20})(70)}{(1.2\times10^5)^2} = 226.9\ N$$




$$F = \sqrt{82.96^2 + 226.9^2} = \boxed{241.5\ N}$$

$$\theta = \tan^{-1}\left(\frac{82.96}{226.9}\right) = \boxed{20.08^\circ \text{ As shown}}$$

# SATELLITE MOTION




M = MASS OF OBJECT BEING ORBITED

m = MASS OF SATELLITE

r = RADIUS OF ORBIT

GRAVITY PROVIDES THE CENTRIPETAL FORCE NECESSARY FOR CIRCULAR MOTION

$$\Sigma F = ma$$

$$F_{GRAVITY} = m\left(\frac{v^2}{r}\right)$$

$$\boxed{\frac{GMm}{r^2} = \frac{mv^2}{r}}$$

$$\boxed{v = \frac{2\pi r}{T}}$$

ALL RELATIONSHIPS STEM FROM THESE TWO.

T = THE PERIOD (THE TIME FOR ONE REVOLUTION)

EXAMPLE 1: What is the orbital speed of the Hubble Space Telescope? The altitude of the HST is 596 km above the Earth. The radius of the Earth is $6.38 \times 10^6$ meters, and the Earth's mass is $5.98 \times 10^{24}$ kg.

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM\cancel{m}\cancel{r}}{\cancel{m}r\cancel{^2}}} = \sqrt{\frac{GM}{r}}$$

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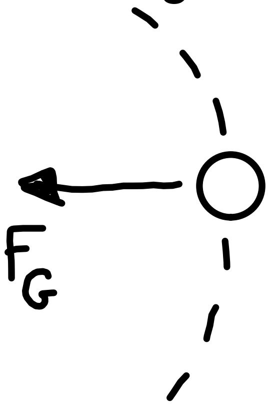


$$\Sigma F = ma$$

$$\Sigma F = m\left(\frac{v^2}{r}\right)$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{(6.38\times10^{6} + 596000)}} = \boxed{7560 \text{ m/s}}$$

EXAMPLE 2: What is the period (in hours) of the Hubble Space Telescope?

$r = (6.38 \times 10^6 + 596000)$ $\quad v = 7560 \text{ m/s}$

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi(6.38 \times 10^6 + 596000)}{7560}$$

EXAMPLE 2: What is the period (in hours) of the Hubble Space Telescope?

$$v = 7560 \, \text{m/s}$$

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.38\times10^6 + 596000)}{7560} = 5798 \text{ sec}$$

$$5798 \text{ sec}\left(\frac{1 \text{ hr}}{3600 \text{ sec}}\right) = \boxed{1.61 \text{ HR}}$$

EXAMPLE 3: At what height above the Earth do geo-synchronous satellites orbit? The Earth's mass is $5.98x10^{24}$ kg and the Earth's radius is $6.38x10^{6}$ meters.

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$$v = \frac{2\pi r}{T} = \frac{2\pi R}{1\text{ DAY}}\left(\frac{1\text{ DAY}}{24\text{ HR}}\right)\left(\frac{1\text{ HR}}{3600\text{ SEC}}\right) = (7.272\times10^{-5})R \text{ m/s}$$

SOLVE FOR R → THE DISTANCE FROM THE CENTER OF THE EARTH, AND THEN FIND THE HEIGHT.

$$\Sigma F = ma$$

$$\Sigma F = m\left(\frac{v^2}{r}\right)$$

$$G\frac{M\cancel{m}}{R^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{R}}$$

$$\frac{GM}{R} = (7.272\times10^{-5})^2 R^2$$

$$\frac{GM}{(7.272\times10^{-5})^2} = R^3$$

$$\sqrt[3]{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{(7.272\times10^{-5})^2}} = 4.224\times10^{7}\text{ m}$$

$$R = r_{EARTH} + h$$

$$h = R - r_{EARTH}$$

$$= 4.224\times10^{7} - 6.38\times10^{6} = \boxed{3.586\times10^{7}\text{ m}}$$

OR 3586 km

# Atwood Lab:

#1: Either measure/calculate $\Sigma\tau, \alpha$
predict $I$

$\Sigma\tau = I\alpha$

Or use $10^{-3}\ kg \cdot m^2$ for $I$

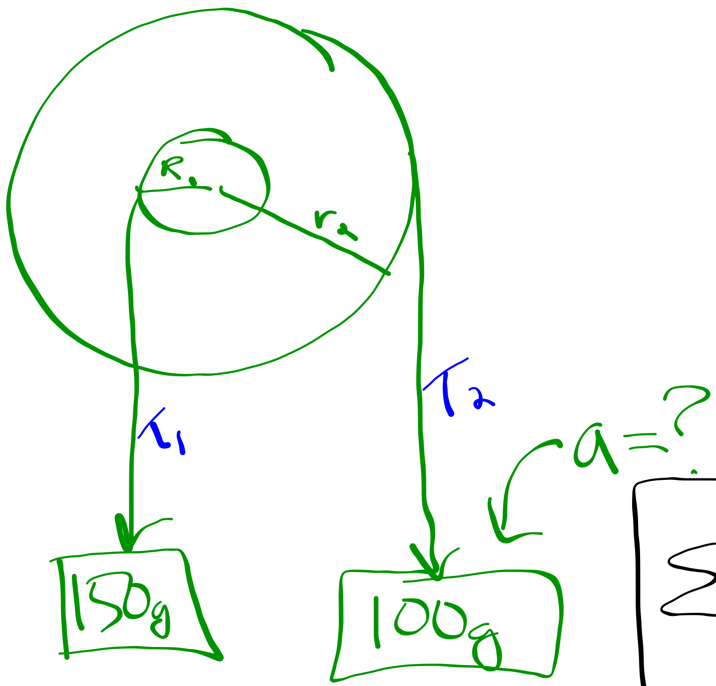
$\Sigma\tau$

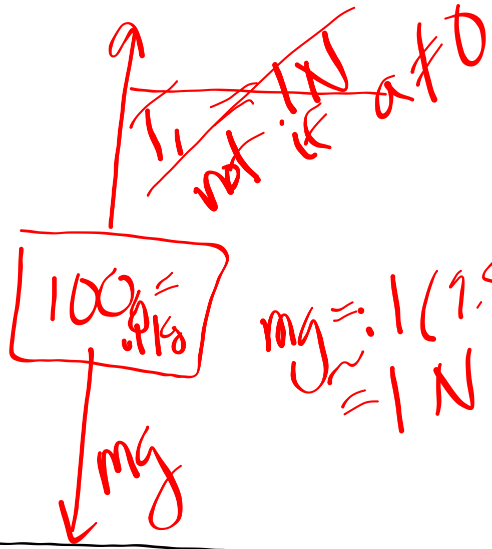
$F_1 = ?$
$F_2 = ?$ } spring scales

$r_1 =$
$r_2 =$ } known radii

$\theta = \#\ \text{rotations} \times 2\pi$
$t =$ } measurements

# #2: Predict:




$mg = .1(9.8)$
$= 1\ N$

$$\boxed{\Sigma\tau = (m_1 g)r_1 + (m_2 g)r_2 \quad \text{\underline{ONLY} if } a=0}$$

#3) <u>Test</u> to see if you really get $\alpha$

→ hang the masses

→ measure $\theta, t \rightarrow \alpha \rightarrow a_1, a_2$