Test:

$$\Sigma \tau = I\alpha$$

$$\Sigma F = ma$$

$$\alpha = \frac{a_t}{r}$$

yes

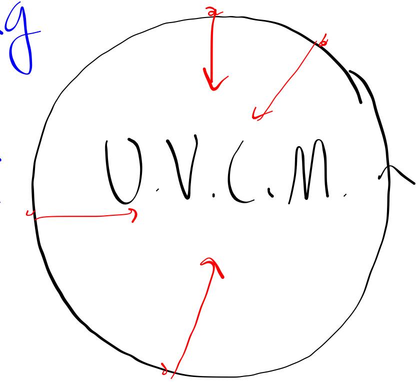
$$\cancel{\alpha = \frac{m_2 g r_2 - m_1 g r_1}{I + m_1 r_1^2 + m_2 r_2^2}}$$

NO

$F_g = mg$

$F_c = \frac{mv^2}{r}$

Yes



~~$\Sigma F = F_c$~~

~~top: $F_a + mg = F_c$~~

~~side: $F_a = F_c$~~

~~bottom: $F_a - mg = F_c$~~

No

Test → based on homework/examples

and

LABS (Atwood Lab)

(different scenario/easier math)

39. A centrifuge rotor has a moment of inertia of 4.00×10^{-2} kg·m^2. How much energy is required to bring it from rest to 10,000 rpm?

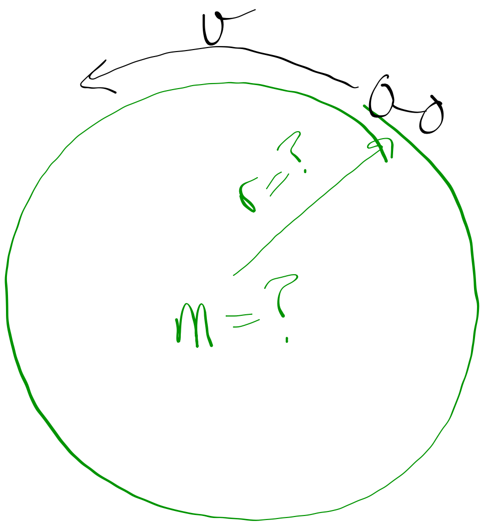
$10{,}000\ \text{rpm} = 1047\ \frac{\text{rad}}{\text{s}}$

$W_{NC} = KE_{rot}$

$W_{NC} = \frac{1}{2} I \omega^2 = \frac{1}{2}(4e{-2})(1047^2)$

$= 2.19e4\ \text{J}$

10. An astronaut, standing on a new planet, finds that a 35-kg dog weighs 1400 N. She further notes that the period of a satellite just skimming the surface of the planet (having an orbit equal to the radius of the planet) is 150 minutes. What is the radius of the planet? [8.21 x 10^7 m]



$$W = m \cdot g \qquad F_g = \frac{GMm}{r^2}$$

$$1400 = 35\,g$$

$$g = 40\ m/s^2$$

$$a_c = \frac{v^2}{r} = g$$

$$v = \frac{2\pi r}{T} = \boxed{\frac{2\pi r}{(9{,}000\ s)}}$$

$$\frac{\left(\frac{2\pi r}{9{,}000}\right)^2}{r} = 40$$

$$\frac{2^2\pi^2 r}{9000^2} = 40$$

$$r = 8.21e7\ m$$

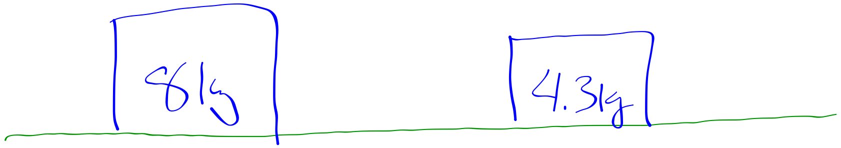
41. When an object has symmetry, its moment of inertia often can be expressed as a simple formula. For instance, the moment of inertia for a hoop rotated about its center is mr^2. For a uniform disk rotating about its center, the moment of inertia is $1/2mr^2$. A uniform sphere rotated about its center is $2/5mr^2$. However as you know, most objects do not enjoy the benefit of symmetry. As a result, if we can even come up with a formula for their moments of inertia, the formulas might not be all that simple. Often, the moments of inertia for these objects are determined experimentally by applying a known torque to the object, measuring the angular acceleration it experiences, and calculating its moment of inertia using $\sum\tau = I\alpha$. If this calculated moment of inertia is set equal to mr^2 (the basic formula for the simplest of objects – a point mass), the r that satisfies this equation is called the object's *radius of gyration*. With all of that explanation behind us now, we are finally ready for this problem. A merry-go-round has a mass of 1560 kg and a *radius of gyration* of 18.5 m. How much work is required to accelerate it from rest to a rotation rate of one revolution in 7.10 seconds? (Hint: Think about CLEE and how a change in KE relates to work).

$\hookrightarrow \omega = \frac{2\pi \text{ rad}}{7.1 s} =$

$I = mr^2 = (1560)(18.5^2) = 5.3e5 \text{ kg} \cdot \text{m}^2$

$W_{NC} = \frac{1}{2} I \omega^2$

11. Two masses are on a frictional, horizontal surface. If the 8-kg mass is brought close to a 4.3-kg mass on a surface with a coefficient of friction of .2, at what distance will the 4.3-kg mass begin to slide toward the 8-kg mass? [1.65 x 10^{-5} m]

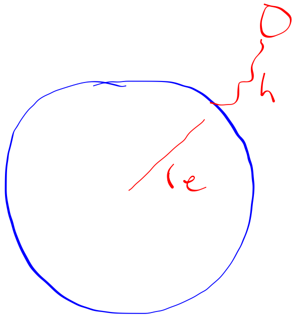


$$F_g = F_{Fr}$$

$$\frac{G m_1 \cancel{m_2}}{r^2} = \mu \cancel{m} g$$

$$r = \sqrt{\frac{G m_1}{\mu g}} = \sqrt{\frac{(6.67e{-11})8}{(0.2)9.8}} = 1.65e{-5}\,m$$

9. If a satellite circles the Earth in 2 hours, what is the altitude of the satellite's orbit (how high is it above the Earth)? The mass of the Earth is 5.98 x 10^{24} kg, the radius of the Earth is 6.38 x 10^{6} meters. [1.68 x 10^{6} m]



$$F_g = F_c$$

$$\frac{GM\cancel{m}}{r^2} = \frac{\cancel{m}v^2}{\cancel{r}} = \left(\frac{2\pi r}{T}\right)^2$$

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$r^3 = \frac{GMT^2}{4\pi^2} = \frac{(6.67e^{-11})(5.98e24)(7200)^2}{4\pi^2}$$

$$r = \sqrt[3]{\ldots} = 8e6$$

$$h = r - r_e = 8e6 - 6.38e6 =$$

$$= \boxed{1.6e6\,m}$$

31. If a spring (k = 340 N/cm) is compressed $\boxed{\text{9 cm}}$ by a disk on its side, what will the velocity of the rolling disk be when the spring is released? The disk has a mass of 1.9 kg and a radius of .3 meters. [9.83 m/sec]

$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$I_{disk} = \frac{1}{2}mr^2$

$\omega = \frac{v}{r}$

$k = 340\ N/cm \cdot \frac{100\ cm}{m} = \frac{34{,}000\ N}{m}$

$x = 9\ cm = 0.09\ m$

$$\frac{1}{2}\left(\frac{1}{2}m\cancel{r^2}\right)\left(\frac{v}{\cancel{r}}\right)^2$$

$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$\frac{1}{\cancel{2}} \cdot \frac{\cancel{4}\,2}{3}$$

$$\frac{3}{4}mv^2 = \frac{1}{2}kx_0^2$$

$$v = \sqrt{\frac{\frac{2}{3}kx_0^2}{m}} = \sqrt{\frac{\frac{2}{3}(34{,}000)(0.09)^2}{1.9}} = \boxed{9.83\ m/s}$$

44. A hollow cylinder (hoop) is rolling on a horizontal surface at a speed of 3.4 m/s when it reaches a 20° incline.

   a) How far up the surface of the incline will it go? 3.45 m

   b) How long will it be on the incline before it arrives back at the bottom?

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$a = \frac{-v_0^2}{2(x - x_0)} = \frac{-(3.4)^2}{2(3.45)} = -1.68 \text{ m/s}^2$$

$$v = v_0 + at$$

$$0 = 3.4 + (-1.68)t$$

$$\left.\begin{array}{ll} t = 2.02\text{s} & \text{(way up)} \\ \quad\;\; 2.02\text{s} & \text{(way down)} \end{array}\right\} 4.04\text{s}$$