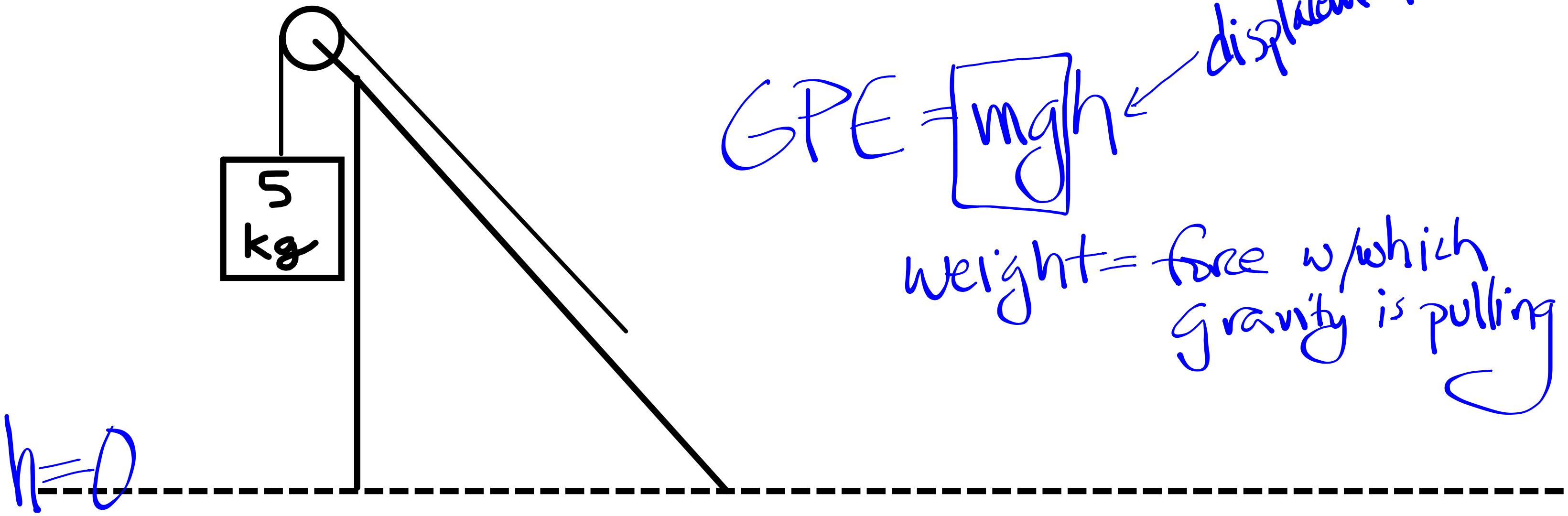
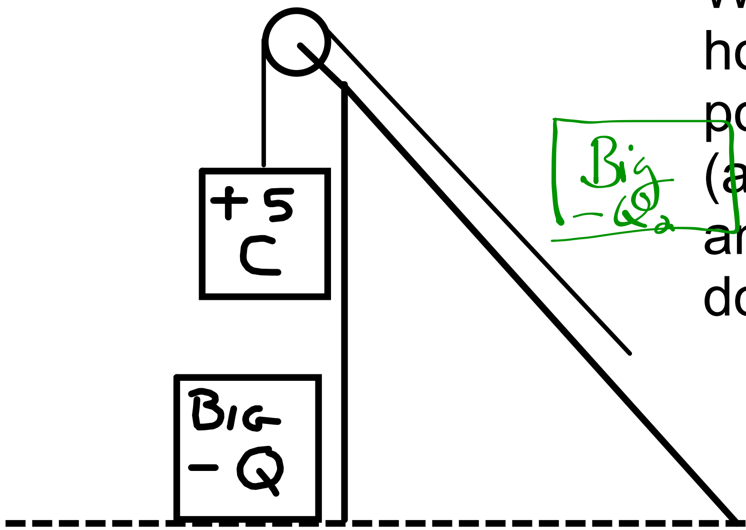


GPE = $\boxed{mg}h$ ← displacement

weight = force w/which gravity is pulling

This mass, due to its position relative to the earth, and the fact that all mass attracts all other mass, has GPE.

If the string is cut, GPE gets converted to KE and ultimately the falling mass can do work.



Big $-Q_2$

We can see how this is similar, and how we might be able to talk about the potential energy associated with charge (and the KE the charge can acquire and therefore the work it can ultimately do).

$$\boxed{F} = K\frac{Qq}{r^2}$$

$$\boxed{d} = r$$

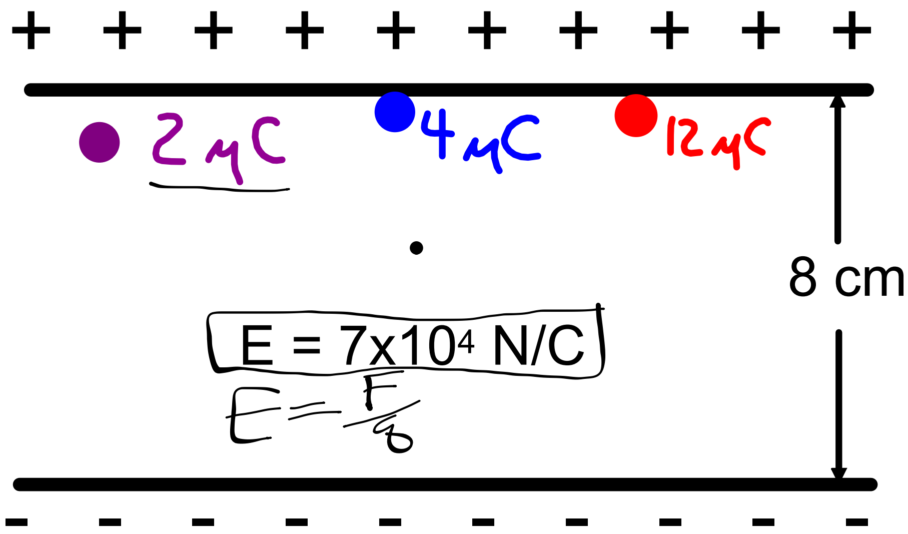
How do we properly refer to this situation?

What if the lower charge was positive? How should refer to this?

First, let's consider a new situation of charge distribution -- two parallel conducting plates with equal and opposite charge distributed upon them. What happens between these plates?

3D physic applet and demo of a uniform electric field between charged plates; the field becomes uniform when the plates are relatively large and close together.
http://www.falstad.com/vector3de/

WARM-UP: Assume three charges (as shown) are moved from the positive to the negative plate through the uniform electric field between the plates. How much work is done on each charge?



$$W_2 = F \cdot d$$

$$W = (0.14\,N)(0.08\,m)$$

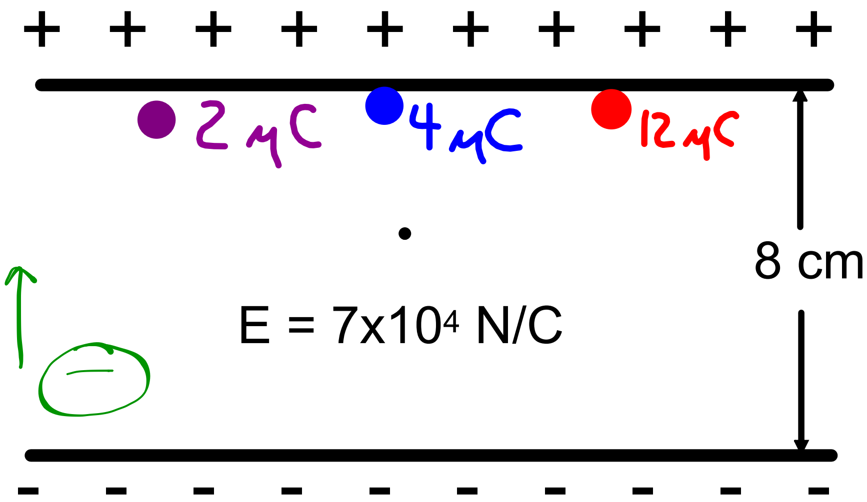
$$= \boxed{0.0112\ J}$$

$$F_2 = E \cdot q$$

$$= (7\times10^{4}\,N/C)(2\times10^{-6}\,C)$$

$$= 0.14\ N$$

WARM-UP: Assume three charges (as shown) are moved from the positive to the negative plate through the uniform electric field between the plates. How much work is done on each charge?



$$W_2 = F \cdot d = Eq \cdot d = (7\times10^4)(2\times10^{-6})(.08) = .0112\ J$$

$$W_4 = \text{"} = \text{"} = (7\times10^4)(4\times10^{-6})(.08) = .0224\ J$$

$$W_{12} = \text{"} = \text{"} = (7\times10^4)(12\times10^{-6})(.08) = .0672\ J$$

Each charge, upon reaching the negative plate, will have the calculated amount of energy in the form of KE.

What happens if we determine the work on a per-charge (coulomb) basis? For each case, determine the work done / charge being moved:

$$\frac{W_2}{q} = \frac{.0112\,J}{2\times10^{-6}\,C} = 5600\,J/C$$

$$\frac{W_4}{q} = \frac{.0224\,J}{4\times10^{-6}\,C} = 5600\,J/C$$

$$\frac{W_{12}}{q} = \frac{.0672\,J}{12\times10^{-6}\,C} = 5600\,J/C$$

1 Volt = 1 J/C

So let's call this quantity **W/q** something special -- **Electric Potential Difference**

$$\boxed{V_{ba} = \frac{W_{ba}}{q}}$$

Point A

Point B

the electrical potential difference between points A & B is the amount of work gained/lost between those points (per coulomb)

W_{ba} = the work required to move charge q from Point A to Point B (Joules)

- when work is (+), the E-field is resisting the motion and something else must do work on q to get it from A to B
- when work is (-), the E-field does the work and q acquires KE or energy in some other form

q = the charge that is being moved from Point A to Point B (coulombs)

- include the negative sign if q is negative

V_{ba} = the electric potential difference between Point A and Point B (**Volts**) (**1 V = 1J/C**)

When V is positive, a motionless negative charge would move from Point A to Point B by virtue of the electric field alone. (Negative charges will tend to move towards more positive voltages.) When V is negative, a motionless positive charge would move from Point A to Point B by virtue of the electric field alone. (Positive charges will tend to move towards more negative voltages.)

Some important points about Electric Potential:

- Electrical potential energy is like gravitational potential energy: it is RELATIVE (it's the difference between two points that matters, not the absolute amount of potential energy a charge has at a given location).
  - > A potential difference of 10 V implies that Point B has a potential that is 10 V higher than Point A.
- Some other charge (Q), must be present for there to be a potential difference.
- A "reference potential" (like h = 0) is a point infinitely far away -- this point's potential is 0 V.
- Electric potential difference is directly related to the kinetic energy a charge loses or gains when it moves through this difference if unaffected by any other forces except the electric field.
- We will always assume that charges start at Point A, and end at Point B. The notation V_{ba} reminds us that to find the difference, we subtract the potential at A from the potential at B (V_b-V_a)
- Because this is associated with energy, a scalar, the sign of V doesn't tell us anything about direction. Because electrostatic forces can be positive or negative, the sign we choose to use for work is arbitrary. Therefore, when working with electric potential, whenever either q or Q are negative, you must place the negative sign within the equations when doing computations in order to stay consistent with the convention.

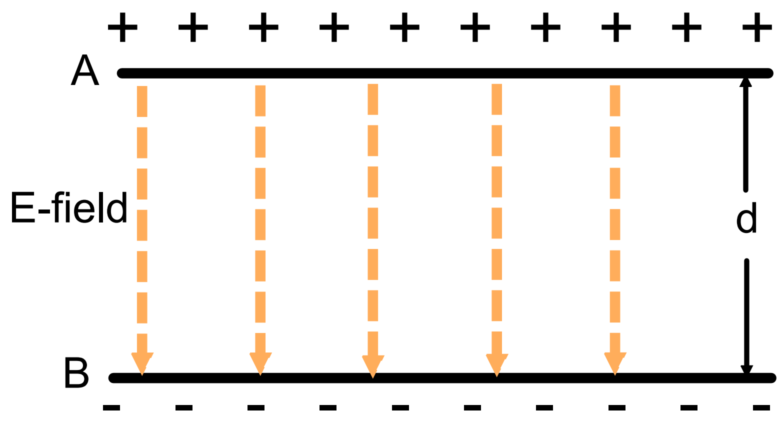
CASE 1: UNIFORM ELECTRIC FIELDS
(between charged plates)

$E = F/q$

$F = E \cdot q$

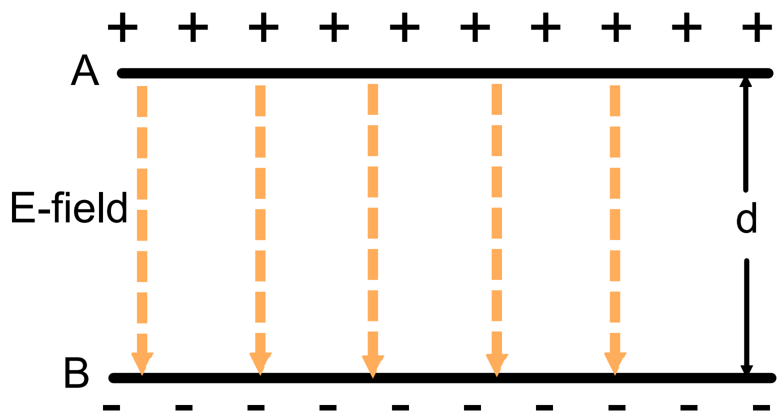
$W = F \cdot d$

$W = E \cdot q \cdot d$



- In words: the electric potential difference between two charged plates is the product of the electric field between the plates and the distance separating the plates.
- This assumes the E-field between the plates is uniform.

CASE 1: UNIFORM ELECTRIC FIELDS
(between charged plates)



$$V_{ba} = \frac{W_{ba}}{q} = \frac{F \cdot d}{q} = \frac{E\cancel{q} \cdot d}{\cancel{q}} = Ed$$

$$\boxed{V_{ba} = Ed}$$

In a uniform E-field: the electrical potential difference between pts. A & B is $\boxed{E \cdot d}$

- In words: the electric potential difference between two charged plates is the product of the electric field between the plates and the distance separating the plates.
- This assumes the E-field between the plates is uniform.

EXAMPLE 1: A positive charge acquires 18 J of kinetic energy as it moves between two charged plates. If the electric potential difference between the two plates is 35 V,

a) How much charge moved?

b) If the plates are 12 cm apart, what is the E-field between the plates?

c) Which plate has the higher electrical potential, the + or the - plate?

d) How much electrical potential enegy did the charge lose in moving as it did?

a) How much charge moved?

$$V = \frac{W}{q} \Rightarrow q = \frac{W}{V} = \frac{18\,J}{35\,J/C} = \boxed{.514\,C}$$

b) If the plates are 12 cm apart, what is the E-field between the plates?

$$V = Ed \Rightarrow E = V/d = 35/.12 = \boxed{291.67\ V/m}\ (= N/C)$$

c) Which plate has the higher electrical potential, the + or the - plate?

+

d) How much electrical potential enegy did the charge lose in moving as it did?

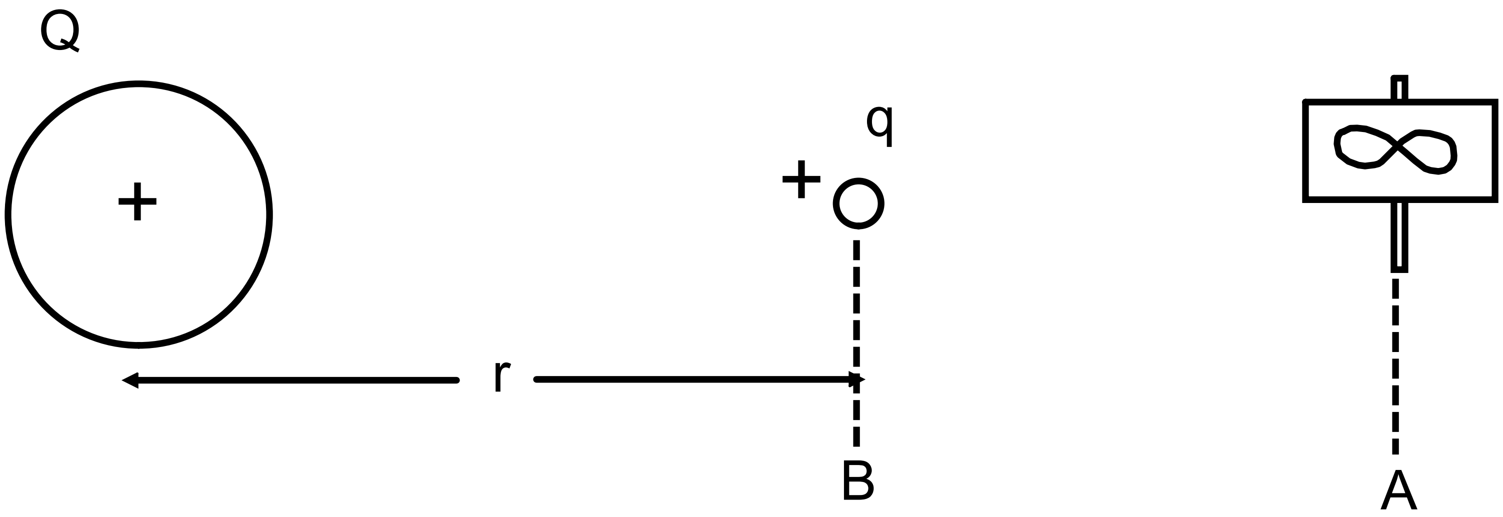
18 J LOST

OR $\Delta EPE = \boxed{-18\,J}$

$W_{ba} = \Delta EPE$

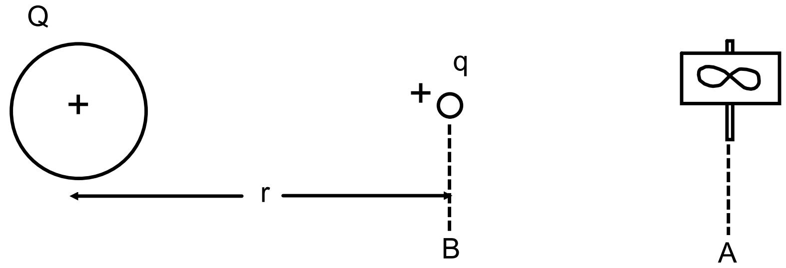
WHEN W_{ba} IS DEFINED AS THE WORK THAT MUST BE DONE TO MOVE q FROM A TO B

CASE 2: NON-UNIFORM ELECTRIC FIELDS
(point charges)



What is the electric potential difference between Point B (a point r meters from Q) and Point A (a point in this case infinitely far away from Q so that if q were there, it would have no electric potential energy as a result of Q)?

We need to know how much work it will take to bring q from Point A to Point B.



If you know calculus:

$$W_{ba} = \Sigma F\cdot d = \int_{\infty}^{r} F\cdot dr = \int_{\infty}^{r} \frac{kQq}{r^2}\,dr = -\frac{kQq}{r}\Bigg|_{\infty}^{r}$$

We will ignore the (-) in our final answer here for it is dependent upon the type of charges involved anyway.

$$= -\frac{kQq}{r} + \cancel{\frac{kQq}{\infty}}^{\,0} = \boxed{-\frac{kQq}{r}}$$

- Point A = a point in space infinitely far from Q that is not influenced by Q
- Point B = a point in space r meters from Q

$$V_{ba} = \frac{W_{ba}}{q} = \frac{\frac{kQ\cancel{q}}{r}}{\cancel{q}} = \frac{kQ}{r}$$

In a non-uniform E-field, an object r meters away from the source will have $\frac{kQ}{r}$ J/C of Elec. P.E.

GENERAL EXPRESSION:

$$\boxed{V_{r\infty} = \frac{kQ}{r}}$$

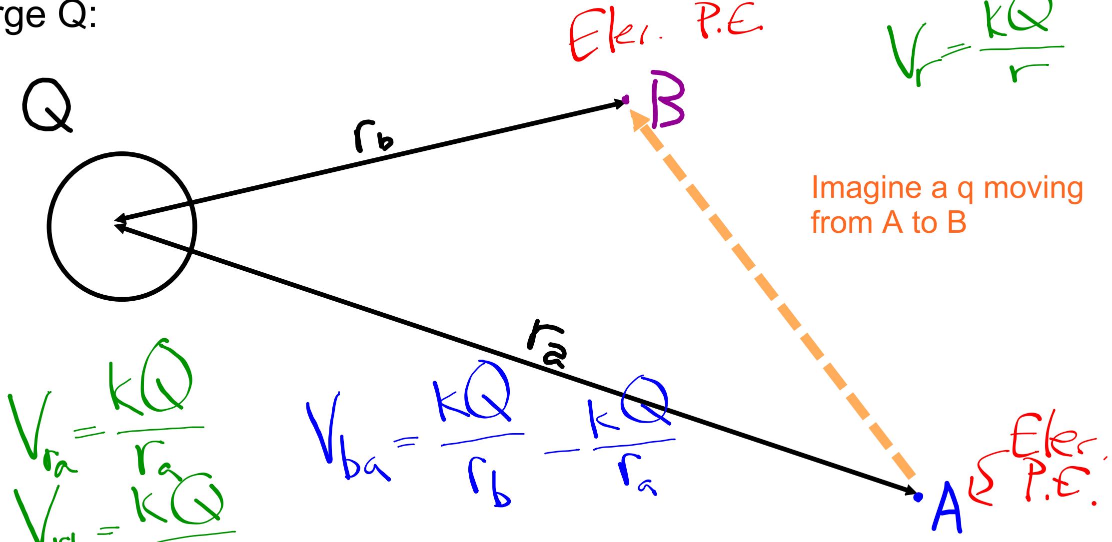
$$V_r = \frac{kQ}{r}$$

Gives the potential difference at a point in space that is r meters from the point charge Q RELATIVE TO A POINT INFINITELY FAR AWAY WHERE V = 0.

The point infinitely far away will serve as our common reference when referring to the electric potentials at points in space.

So to find the electric potential between TWO points in space near a charge Q:

$$V_r = \frac{kQ}{r}$$



$$V_{r_a} = \frac{kQ}{r_a}$$

$$V_{r_b} = \frac{kQ}{r_b}$$

$$V_{ba} = \frac{kQ}{r_b} - \frac{kQ}{r_a}$$

In order to determine the work needed to move q from A to B, or to determine the energy associated with this, we would first need to know the electric potential difference between these two points, V_{ba}.

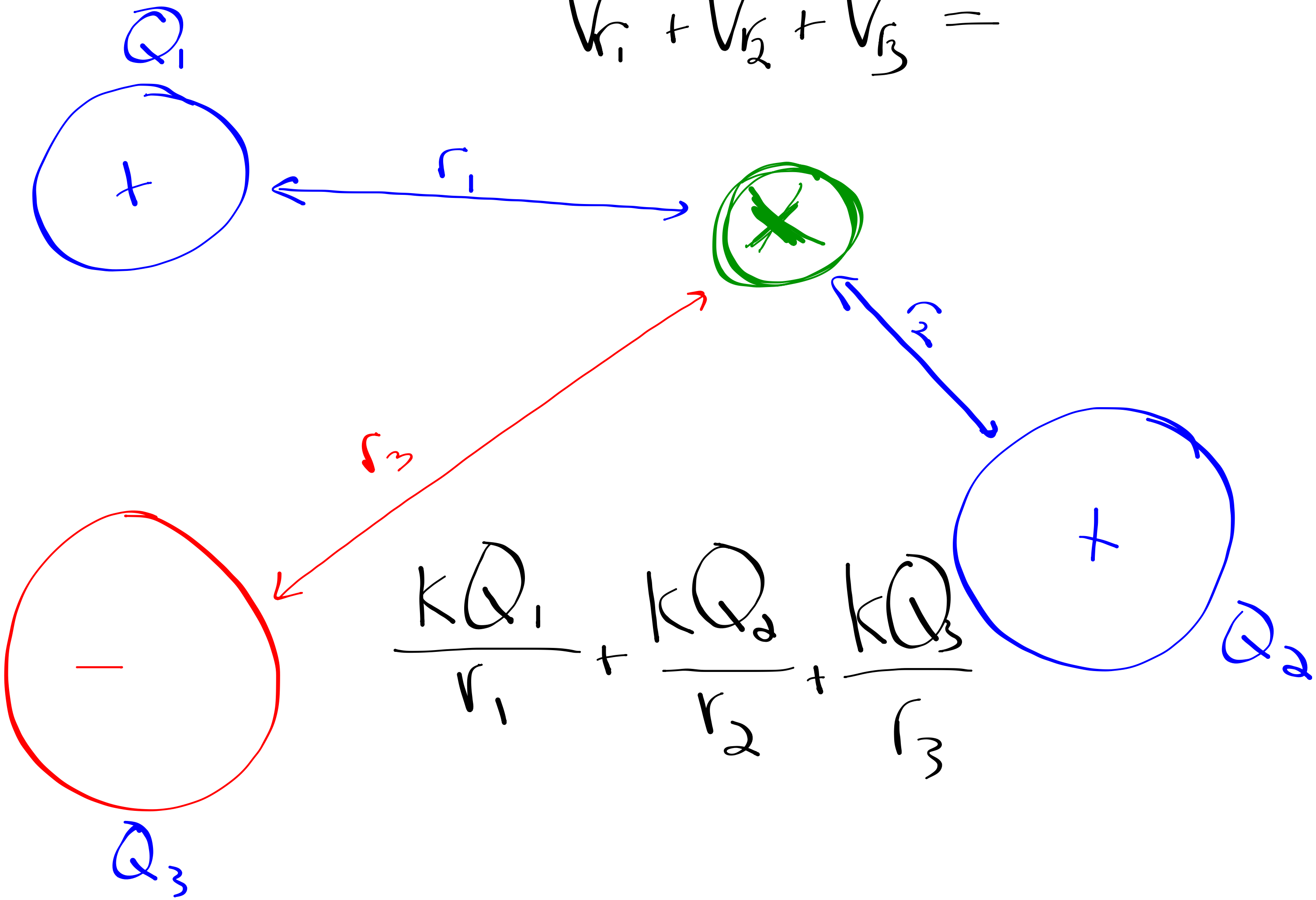
Using a point infinitely far away as our common reference point:

$$V_{ba} = (V_{b\infty} - V_{a\infty}) = \left(\frac{kQ}{r_b} - \frac{kQ}{r_a}\right)$$

We'll use this to help us define the energy associated with moving a q between two different points near a single Q.

Of course, this is best illustrated with examples . . .

$$V_{r_1} + V_{r_2} + V_{r_3} =$$



$$\frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} + \frac{kQ_3}{r_3}$$

EXAMPLE 2: How much work is required to bring a +3 *u*C charge from a point that is 1.5 m away from a +20 *u*C charge to a point that is only 0.5 m away from the same +20 *u*C charge?

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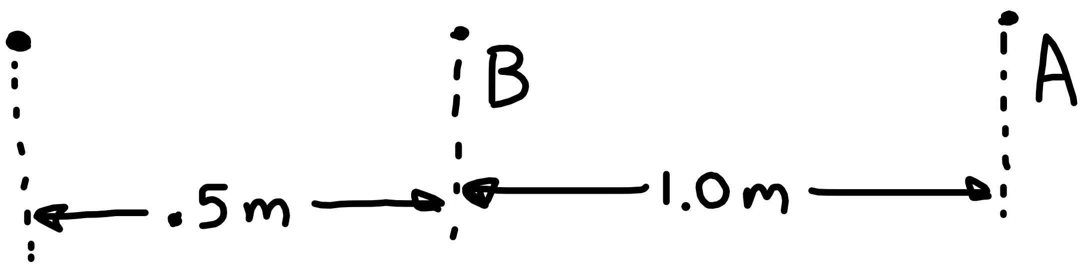
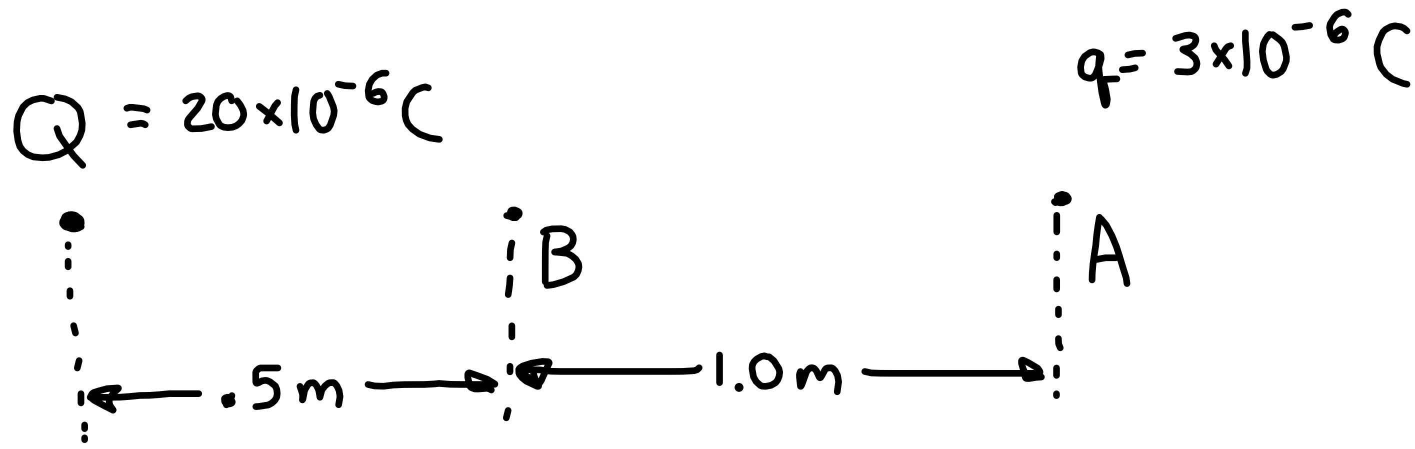
$q = 3\times10^{-6}\ C$

$Q = 20\times10^{-6}\ C$



EXAMPLE 2: How much work is required to bring a +3 *u*C charge from a point that is 1.5 m away from a +20 *u*C charge to a point that is only 0.5 m away from the same +20 *u*C charge?



$W_{ba} = qV_{ba} \rightarrow$ NEED V_{ba}

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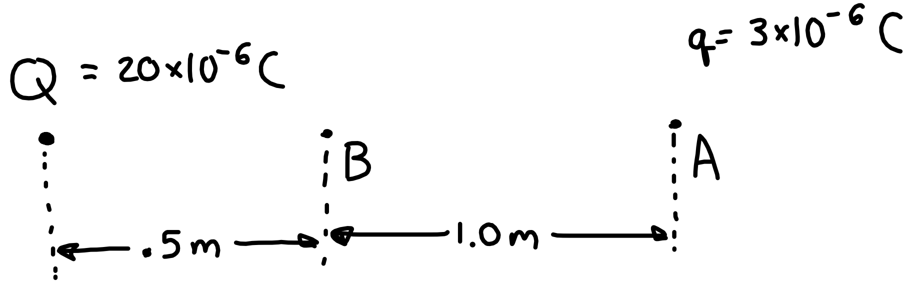

$$W_{ba} = qV_{ba}$$

$$V_{ba} = (V_{b\infty} - V_{a\infty}) = \frac{kQ}{r_b} - \frac{kQ}{r_a}$$

$$= \frac{(9\times10^9)(20\times10^{-6})}{.5} - \frac{(9\times10^9)(20\times10^{-6})}{1.5}$$

$$= 3.6\times10^{-5} - 1.2\times10^5$$

$$= 2.4\times10^5\ V$$

$$W_{ba} = qV_{ba} = (3\times10^{-6})(2.4\times10^5) = \boxed{.72\ J}$$

- This equals the increase in q's electric potential energy and also equals the work we must do on q to move it from Point A to Point B.
- If Q were not present, it wouldn't take any work to move q.
- The fact that W_{ba} is positive means that it will take work to move q.

EXAMPLE 3: How much work is required to bring a 7.2 *u*C charge from a point that is 0.5 m away from a -30 *u*C charge to a point that is 4.3 m away from the same -30 *u*C charge?

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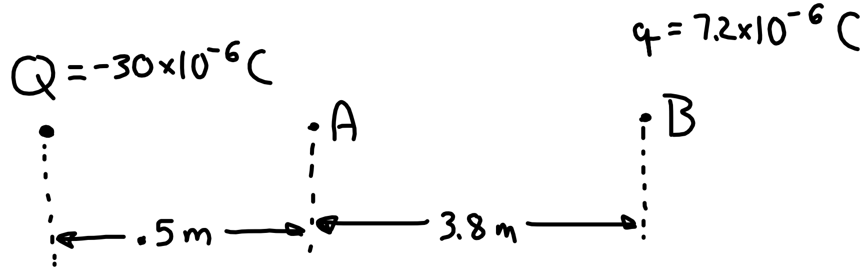

$$W_{ba} = qV_{ba}$$

$$V_{ba} = (V_{b\infty} - V_{a\infty}) = \frac{kQ}{r_b} - \frac{kQ}{r_a}$$

$$= \frac{(9\times10^{9})(-30\times10^{-6})}{4.3} - \frac{(9\times10^{9})(-30\times10^{-6})}{.5}$$

$$= -62790.7 - (-5.4\times10^{5})$$

$$= 477209.3$$

$$W_{ba} = qV_{ba} = (7.2\times10^{-6})(477209.3) = \boxed{3.44\ J}$$

- This equals the increase in q's electric potential energy and also equals the work we must do on q to move it from Point A to Point B.
- If Q were not present, it wouldn't take any work to move q.
- The fact that W_{ba} is positive means that it will take work to move q.
- NOTE: When either Q or q are negative, we put the negative into the equations used for the computations.