Homework Review - 13.2

16 $8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$

56 30 24 = some big ol' #

33 4 stores

1 2 3 4
2 1 3 4
1 3 4 2
1 4 2 3

$${}_4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{1} = 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{24}$$

32 $\quad {}_6P_6 = \dfrac{6!}{(6-6)!} = \dfrac{6!}{0!} = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 30 \cdot 24 = 720$

Finding Probabilities using Combinations

A selection or arrangement of items where order doesn't matter

What is a combination?

ex. pocket change

\$.10, \$.05, \$.25 = 40¢

How does it differ from a permutation? In a permutation, order matters.

ex. $\boxed{\text{crud}}$ 3 letters

cru
cur
urc
dru
urd
udr
⋮

6 permutations

2 combinations

How can we find combinations?

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$

Count ... three people (Art, Bart, Cart)

$${}_nP_r = \frac{n!}{(n-r)!}$$

How many permutations of two? 6

How many combinations of two?

$${}_3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = 3\cdot 2\cdot 1 = 6$$

$\boxed{\begin{matrix}AB\\BA\end{matrix}}$ $\boxed{\begin{matrix}AC\\CA\end{matrix}}$ $\boxed{\begin{matrix}BC\\CB\end{matrix}}$ = 3 combinations

What's the Formula for Combinations?

$_nC_r = n! / ((n - r)!\, r!)$

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

Similar to permutations - but divide by all the different ways of rearranging ...

Example: A B C D E

$$_5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 5\cdot4\cdot3\cdot2\cdot1 = 240$$

$$_5C_5 = \frac{5!}{(5-5)!\,5!} = \frac{\cancel{5!}}{\cancel{0!}\,\cancel{5!}} = 1$$

$$_5P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5\cdot4\cdot\cancel{3!}}{\cancel{3!}} = 20$$

$$_5C_2 = \frac{5!}{(5-2)!\,2!} = \frac{5!}{3!\,2!} = \frac{5\cdot4\cdot\cancel{3!}}{\cancel{3!}\,2!} =$$

$$\frac{5\cdot\cancel{4}\,2}{\cancel{2}\cdot\cancel{1}} = 10$$

Evaluate the expression.

1. $_8C_4$ $= \frac{8!}{(8-4)!\,4!} = \frac{8!}{4!\,4!} = \frac{8\cdot7\cdot6\cdot5\cdot\cancel{4!}}{4!\,\cancel{4!}} = \frac{\cancel{8}\cdot7\cdot\cancel{6}\cdot5}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} = 70$

$\boxed{2.}$ $_5C_5 = 1$

$\boxed{3.}$ $_{12}C_0 = \frac{12!}{(12-0)!\,0!} = \frac{12!}{12!} = 1$

4. $_7C_1$ $\frac{7!}{(7-1)!\,1!} = \frac{7!}{6!} = \frac{7\cdot\cancel{6!}}{\cancel{6!}} = 7$

5. $_{15}C_{11}$ $= \frac{15!}{(15-11)!\,11!} = \frac{15!}{4!\,11!} = \frac{15\cdot14\cdot13\cdot12\cdot\cancel{11!}}{4!\,\cancel{11!}} = \frac{15\cdot\cancel{14}^{7}\cdot13\cdot\cancel{12}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} = 15\cdot7\cdot13 = \boxed{1365}$

6. $_{10}C_3$ $\frac{10!}{(10-3)!\,3!} = \frac{10!}{7!\,3!}$

$\frac{\cancel{10}^{5}\cdot\cancel{9}^{3}\cdot8\cdot\cancel{7!}}{\cancel{7!}\cdot\cancel{3}\cdot\cancel{2}} = 120$

$15 \cdot 8 = 120$

$\frac{10\cdot\cancel{9}^{3}\cdot\cancel{8}^{4}\cdot\cancel{7!}}{\cancel{7!}\,3!\,\cancel{2}\cdot1} = 120$

There are 15 students in Algebra. As a reward, Mr. Bregar will take 4 of them to play in a dumpster.

a) The first person he picks gets a free rotten egg. The second gets a turkey. The third gets a chicken. The fourth gets a potato.

Use permuations - why? ORDER MATTERS — it determines who gets which prize!

How many permutations of 4 students can he pick? $_{15}P_4 = \frac{15!}{(15-4)!} = 15 \cdot 14 \cdot 13 \cdot 12 = \underline{32760}$

b) There are no rotten eggs, turkeys, chickens, or potatoes.

Use combinations - why? ORDER DOESN'T MATTER - everyone's in the dumpster

How many combinations of 4 students can he pick?

$$_{15}C_4 = \frac{15!}{(15-4)!\,4!} = \frac{15 \cdot \overset{7}{\cancel{14}} \cdot 13 \cdot \cancel{12} \cdot \cancel{11!}}{\cancel{11!} \cdot \cancel{4 \cdot 3} \cdot \cancel{2}} = \boxed{1365}$$

Using combinations to find probabilities:

$$\frac{\text{\# of ways an event can occur}}{\text{\# of possible outcomes}}$$

Remember the probability formula...

Generally, the # of ways an event can occur and the sample space will be combinations (if order doesn't matter) or permutations (if order matters)

Example: Two pictures will get into the yearbook out of 14 possible students. What are the chances that it will be Kacey and Allie?

$$_{14}C_2 = \frac{14!}{(14-2)!\cdot 2!} = \frac{\cancel{14}^{7}\cdot 13\cdot \cancel{12!}}{\cancel{12!}\cdot \cancel{2!}} = 91 \text{ outcomes}$$

1 way to pick K & A $= \frac{1}{91}$

Open-Mike Night A coffee shop offers an open-mike night for poetry. Tonight, 15 people would like to read, but there is only enough time to have 7 people read.

a. Seven of the 15 people that would like to read are randomly chosen. How many combinations of 7 readers from the group of people that would like to read are possible?

$$_{15}C_7 = \frac{15!}{(15-7)!\,7!} = \frac{\cancel{15}\cdot\cancel{14}\cdot 13\cdot\cancel{12}\cdot 11\cdot\cancel{10}\cdot 9\cdot\cancel{8!}}{\cancel{8!}\,\cancel{7}\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} = 13\cdot 11\cdot 5\cdot 9 = \boxed{6435}$$

b. You and your friend are part of the group that would like to read. What is the probability that you and your friend are chosen? What is the probability that you are chosen first and your friend is chosen second? Which event is more likely to occur?

$\frac{1}{5}$

me friend

$$\frac{1287}{6435} = \boxed{\frac{1}{5}}$$

$$_{13}C_5 = \frac{13!}{(13-5)!\,5!} = \frac{13\cdot\cancel{12}\cdot 11\cdot\cancel{10}\cdot 9\cdot\cancel{8!}}{\cancel{8!}\,\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} = 1287$$

$$\frac{1}{15}\cdot\frac{1}{14} = \frac{1}{210}$$

$$\frac{_{13}P_5}{_{15}P_7} = \frac{\frac{13!}{\cancel{8!}}}{\frac{15!}{\cancel{8!}}} = \frac{13!}{15!} = \frac{1}{15\cdot 14} = \frac{1}{210}$$

Homework:

p. 858: 2-14 even, 15-20 all, 23, 24