Collisions

For elastic collisions:

Momentum is conserved $(\Delta p = 0)$

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

Kinetic energy is conserved $(\Delta KE = 0)$

$$\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1(v_1')^2 + \tfrac{1}{2}m_2(v_2')^2$$

For inelastic collisions (any collision not perfectly elastic):

Momentum is conserved

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

Energy is still conserved, but KE is not; some of the original KE that exists before the collision leaves the system as sound, light, thermal, work, etc....

EXAMPLE #2: A 10-kg block moving at 9 m/s strikes an 18-kg block that is initially motionless. What are the final velocities of the two blocks if the collision is perfectly elastic?

EXAMPLE #2: A 10-kg block moving at 9 m/s strikes an 18-kg block that is initially motionless. What are the final velocities of the two blocks if the collision is perfectly elastic?

INITIALLY

$$\Delta p = 0 \qquad m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$
$$(10)(9) + (18)(0) = 10v_1' + 18v_2' \quad \boxed{1}$$

$$\Delta KE = 0 \qquad \cancel{\tfrac{1}{2}}m_1v_1^2 + \cancel{\tfrac{1}{2}}m_2v_2^2 = \cancel{\tfrac{1}{2}}m_1(v_1')^2 + \cancel{\tfrac{1}{2}}m_2(v_2')^2$$
$$10(9)^2 + 18(0)^2 = 10(v_1')^2 + 18(v_2')^2 \quad \boxed{2}$$

USE $\boxed{1}$ TO SOLVE FOR v_1' AND THEN SUBSTITUTE INTO $\boxed{2}$

$$90 = 10v_1' + 18v_2'$$
$$v_1' = \frac{90 - 18v_2'}{10}$$
$$10(9)^2 + 0 = 10\left(\frac{90-18v_2'}{10}\right)^2 + 18(v_2')^2$$
$$810 = \tfrac{1}{10}(90 - 18v_2')^2 + 18(v_2')^2$$
$$810 = 810 - 324v_2' + 50.4(v_2')^2$$
$$50.4(v_2')^2 - 324v_2' = 0$$
$$v_2' = 0 \text{ OR } \boxed{6.43 \text{ m/s IN + DIRECTION}}$$
$$v_1' = \boxed{-2.57 \text{ m/s}}$$
(FOUND BY SUBSTITUTING IN v_2')

EXAMPLE #3: A 3000-kg train car traveling at 0.5 m/s strikes and couples to a 4500-kg car moving at 0.2 m/s in the same direction.

1. What is the final velocity of both cars?
2. What percentage of the initial KE remains in the system after the collision?

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1) $\Delta p = 0 \quad m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$

$$3000(.5) + 4500(.2) = (3000 + 4500) v$$

$$v = \boxed{.32\,\text{m/s IN ORIGINAL DIRECTION}}$$

2) $\% KE = \dfrac{\text{FINAL KE}}{\text{INITIAL KE}} = \dfrac{\frac{1}{2}(7500)(.32)^2}{\frac{1}{2}(3000)(.5)^2 + \frac{1}{2}(4500)(.2)^2}$

$= .826$

$\hookrightarrow$ 82.6%

WHERE DID THE MISSING KE GO?

Two-Dimensional Collisions

Handled in the same manner as 1-D collisions with one exception: momentum is a vector.

For Elastic 2-D Collisions:

$\Delta P_x = 0 \qquad m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}' + m_2 v_{2x}'$

$\Delta P_y = 0 \qquad m_1 v_{1y} + m_2 v_{2y} = m_1 v_{1y}' + m_2 v_{2y}'$

$\Delta KE = 0 \qquad \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 (v_1')^2 + \frac{1}{2} m_2 (v_2')^2$

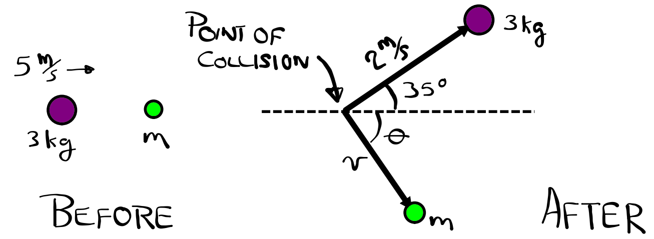
NOTE: KE IS A SCALAR → YOU CANNOT BREAK IT INTO X'S & Y'S

For 2-D Collisions not perfectly elastic:

$\Delta P_x = 0 \qquad m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}' + m_2 v_{2x}'$

$\Delta P_y = 0 \qquad m_1 v_{1y} + m_2 v_{2y} = m_1 v_{1y}' + m_2 v_{2y}'$

EXAMPLE: A 3-kg object travels at 5 m/s. It strikes (in an elastic collision) a 2nd motionless object. After the collision, the 1st object is observed to move at 2 m/s in a direction that makes an angle of 35^o with its original direction. What is the final velocity and mass of the 2nd object?



- Since P_y Before = 0, P_y After = 0

∴ θ is drawn below the horizontal, to right

- If elastic, then:

$\Delta P_x = 0$ $\quad m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}' + m_2 v_{2x}'$

$\boxed{1}$ $\quad 3(5) + m(0) = 3(2\cos 35) + m(v\cos\theta)$

$\Delta P_y = 0$ $\quad m_1 v_{1y} + m_2 v_{2y} = m_1 v_{1y}' + m_2 v_{2y}'$

$\boxed{2}$ $\quad 3(0) + m(0) = 3(2\sin 35) - m(v\sin\theta)$

$\Delta KE = 0$ $\quad \cancel{\tfrac{1}{2}} m_1 v_1^2 + \cancel{\tfrac{1}{2}} m_2 v_2^2 = \cancel{\tfrac{1}{2}} m_1 (v_1')^2 + \cancel{\tfrac{1}{2}} m_2 (v_2')^2$

$\boxed{3}$ $\quad 3(5)^2 + m(0)^2 = 3(2)^2 + m(v)^2$

$75 = 12 + mv^2$

$63 = mv^2$

Simplify $\boxed{2}$ $\quad 0 = 3(2\sin 35) - mv\sin\theta$

$0 = 3.441 - mv\sin\theta$

$mv\sin\theta = 3.441$

$\boxed{4}$ $\quad m = \dfrac{3.441}{v\sin\theta}$

Plug $\boxed{4}$ into $\boxed{1}$

$15 = 3(2\cos 35) + mv\cos\theta$

$15 = 4.915 + mv\cos\theta$

$15 = 4.915 + \left(\dfrac{3.441}{\cancel{v}\sin\theta}\right)\cancel{v}\cos\theta$

$15 = 4.915 + \dfrac{3.441}{\tan\theta}$ $\quad$ (Because $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$)

$\tan\theta = \left(\dfrac{3.441}{15 - 4.915}\right)$

$\theta = \tan^{-1}\left(\dfrac{3.441}{15-4.915}\right) \Rightarrow \theta = \boxed{18.84° \text{ As Shown}}$

From $\boxed{3}$: $\quad 63 = mv^2$

$63 = \left(\dfrac{3.441}{\cancel{v}\sin\theta}\right)v^{\cancel{2}}$

$63 = \left(\dfrac{3.441}{\sin 18.84}\right)v$ $\quad v = \boxed{5.9 \text{ m/s}}$

From $\boxed{2}$: $\quad m = \dfrac{3.441}{v\sin\theta} = \dfrac{3.441}{5.9(\sin 18.84)} = \boxed{1.8 \text{ kg}}$