Rotational Dynamics

- What happens to cause rotating objects speed up?
- What is the relationship between the forces that cause increased rotational speed and the angular acceleration of an object?

Objectives:

- Students will understand, know the relationship between, and be able to explain the concepts of angular acceleration, torque, and moment of inertia.
- Students will be able to use the rotational version of Newton's 2nd Law to solve problems.
- Students will be able to explain how the center of mass and the arrangement of that mass affects the angular acceleration of an object.

Mass is the characteristic of an object that resists acceleration; if a net force is applied, an object accelerates:

$$\Sigma F = ma$$

The larger the mass, the smaller the acceleration will be.

The larger the mass, the more resistance the object has to being accelerated.

$\tau = F_\perp \ell$

$$\boxed{\Sigma\tau = I\alpha}$$

Relationship between angular acceleration and net torque

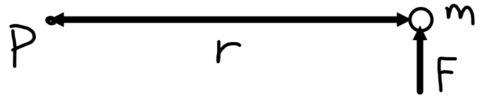
angular acceleration $\left(\frac{rad}{s^2}\right)$

Sum of torques

"Moment of inertia"

Newton's 2nd Law (rotating objects)

Consider a mass *m* constrained to remain *r* meters away from a center of rotation at point P.



If F is applied at *m*, *m* will accelerate upward.

$$\Sigma F = ma$$
$$F = ma$$

m experiences α about the axis of rotation at P

$$a = \alpha r$$
$$\therefore F = ma = m(\alpha r) = mr\alpha$$
$$\therefore F = mr\alpha$$

Forces must exert torques if there is to be rotation.

$$\tau = F \cdot r$$
$$= (mr\alpha)\, r$$
$$\tau = mr^2\alpha$$

$\Sigma\tau = (mr^2)\alpha$ Generalized for a sum of applied torques.

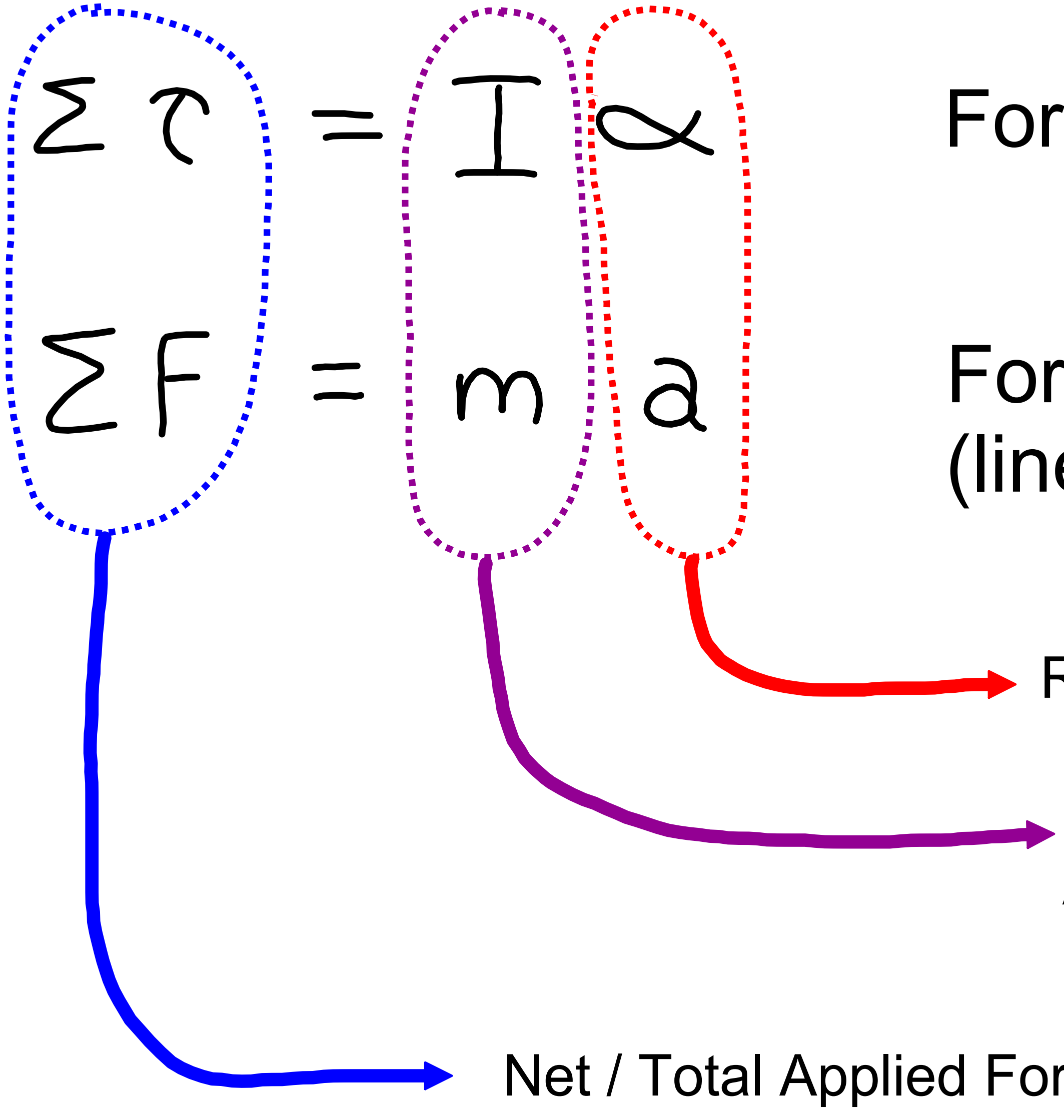
$$\boxed{\Sigma\tau = I\alpha}$$

I = mr^2 for a point mass, different for other objects

Newton's 2nd Law

$\Sigma \tau = I \alpha$ For rotating systems

$\Sigma F = m a$ For translating (linear) systems



Resulting acceleraton

Resistance to Acceleration

Net / Total Applied Force/Torque

MOMENT OF INERTIA

$$\Sigma \tau = I\alpha$$

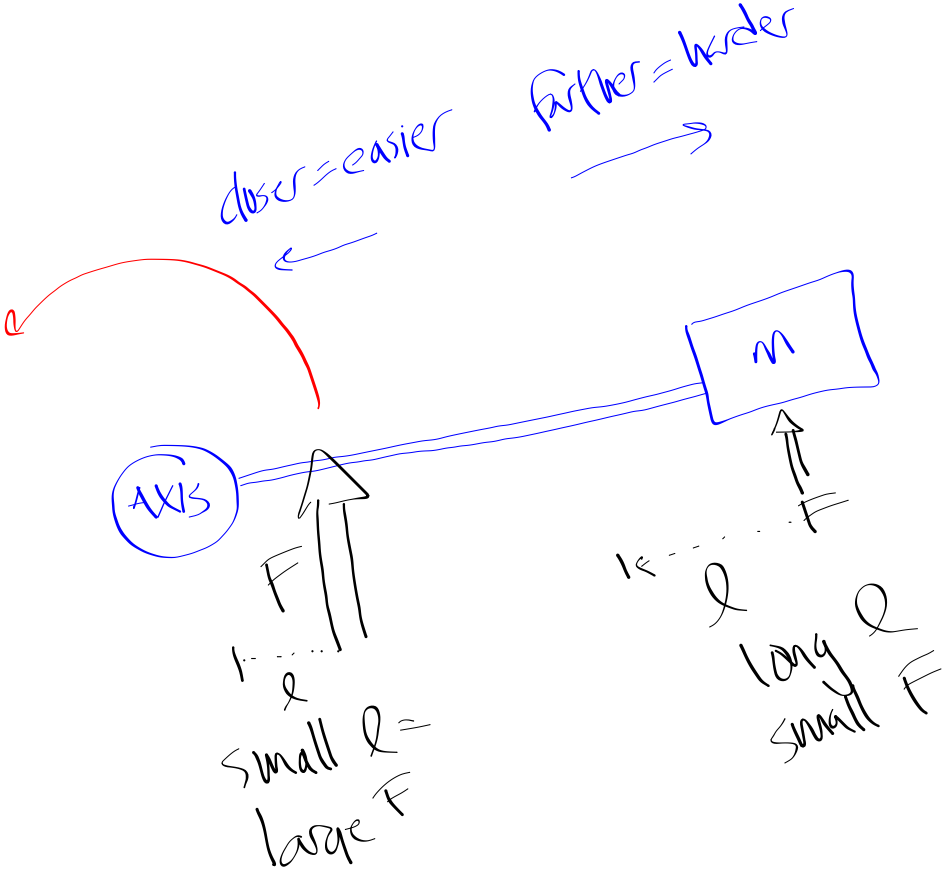
I = MOMENT OF INERTIA (like mass, but for rotating objects) $kg \cdot m^2$ $SLUG \cdot ft^2$

I is dependent upon:

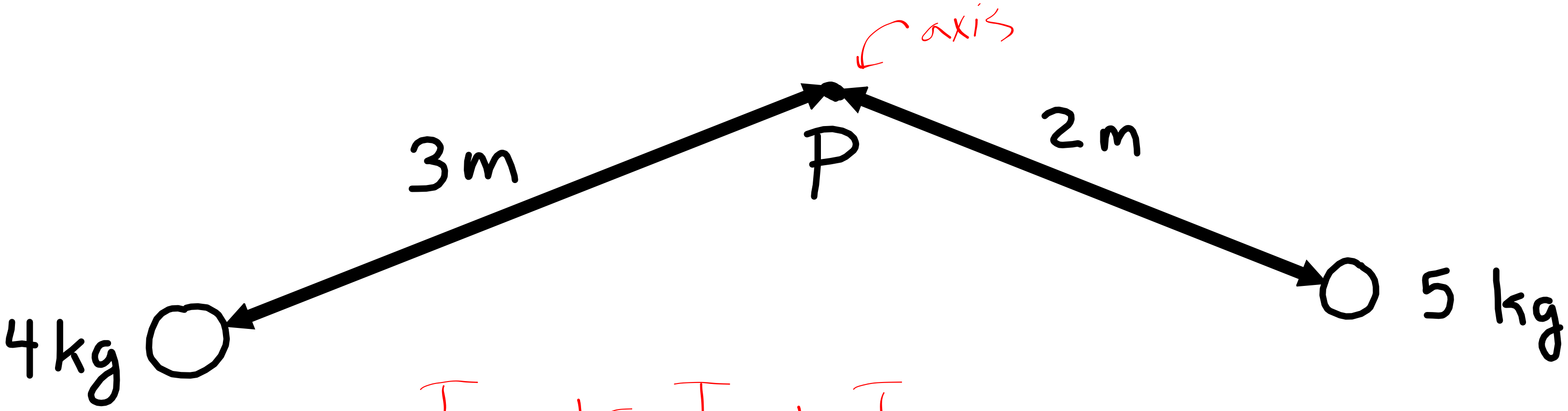
- The object's distribution of mass
- The location of the axis of rotation

$$I_{POINT\ MASS} = mr^2 \leftarrow \Sigma$$

(sum for multiple points)


closer = easier
farther = harder
m
AXIS
F
l
F
l
small l =
large F
long l
small F

Calculate the moment of inertia for this system if the axis of rotation is around P:



$I_{total} = I_4 + I_5$

$= (4)(3^2) + (5)(2^2)$

$= 36 + 20$

$I_{total} = \boxed{56 \text{ kg}\cdot\text{m}^2}$

IN SUMMARY:

$\mathbf{I}_{\text{system}}$ = the sum of all of the $\mathbf{I}$'s of all of the parts

$\mathbf{I}$ depends upon not just mass, but its distance from the axis of rotation

$\mathbf{I}$ depends upon the location of the axis of rotation

$\mathbf{I}$ is object dependent

$\mathbf{I}_{\text{point mass}} = mr^2$

$\mathbf{I}_{\text{hoop}} = mr^2$ (rotating about its center like a wheel)

$\mathbf{I}_{\text{disk}} = 1/2mr^2$ (rotating about its center like a wheel)

The moments of inertias you will need to know; all others will be provided, or you will solve for.

Many objects enjoy symmetry and uniformity and as a result, there moments of inertias can be expressed in terms of their masses, radii, lengths, and other basic parameters.

Note: you always must pay attention to where the axis of rotation is! These relations always apply to a specific location for the axis of rotation!

This link will take you to a table of moments of inertias for various objects: http://www.livephysics.com/physical-constants/mechanics-pc/moment-inertia-uniform-objects/

So what do you do to determine the moment of inertia of an object that isn't "nice" (i.e. one that doesn't enjoy symmetry or uniformity and therefore doesn't have a simple equation for its moment of inertia?)

measure

$$\Sigma\tau = I\alpha$$

measure

$$I = \frac{\Sigma\tau}{\alpha}$$

Accelerating Atwood Machines:

Phase 1: Find I

Since $\underset{\uparrow\ \text{find}}{\Sigma\tau} = I\underset{\uparrow\ \text{find}}{\alpha}$

then $I = \dfrac{\Sigma\tau}{\alpha}$ ✓

Tips: · All points on the Atwood machine will have identical $\Delta\theta$, ω, α

· Linear motion (s, v, a_t) of points on the edge of pulleys will be equal to the linear motion $(\Delta x, v, a)$ of strings attached to those pulleys

· Force acting on the edge of each pulley is not <u>necessarily</u> equal to the weight of each mass...

$$\alpha_1 = \alpha_2 = \alpha$$