

Newton's Law of Universal Gravitation

$$F_{\text{GRAVITY}} = \frac{G m_1 m_2}{r^2}$$

$$G = \text{UNIVERSAL GRAVITATION CONSTANT}$$
$$= 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

m_1 = MASS OF 1ST OBJECT (kg)

m_2 = MASS OF 2ND OBJECT (kg)

r = DISTANCE BETWEEN OBJECTS (m)

EXAMPLE 1: What is the force of attraction due to gravity between an 18 kg mass and a 30 kg mass separated by 40 centimeters?

$$\begin{aligned} F_{\text{gravity}} &= \frac{G m_1 m_2}{r^2} \\ &= \frac{(6.67 \times 10^{-11}) (18) (30)}{(0.4)^2} \\ &= 2.25 \times 10^{-7} \text{ N} \end{aligned}$$

EXAMPLE 2: What is the acceleration of Earth's gravity on top of Mt. Everest? Mt. Everest has an elevation of 8848 meters above sea level. Assume the Earth's radius is 6.38×10^6 meters, and Earth's mass is 5.98×10^{24} kg.

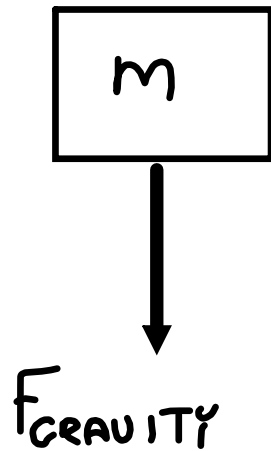
$$F_{\text{gravity}} = \frac{G m_1 m_2}{r^2} = m_2 g$$

$$\cancel{m_2} g = \frac{G m_1 \cancel{m_2}}{r^2}$$

$$g = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(6.38 \times 10^6 + 8848)^2} =$$

$$g = 9.77 \text{ m/s}^2$$

EXAMPLE 3: What is the acceleration of gravity at the surface of the sun? The mass of the sun is 2.0×10^{30} kg. The radius of the sun is 7.0×10^8 meters.

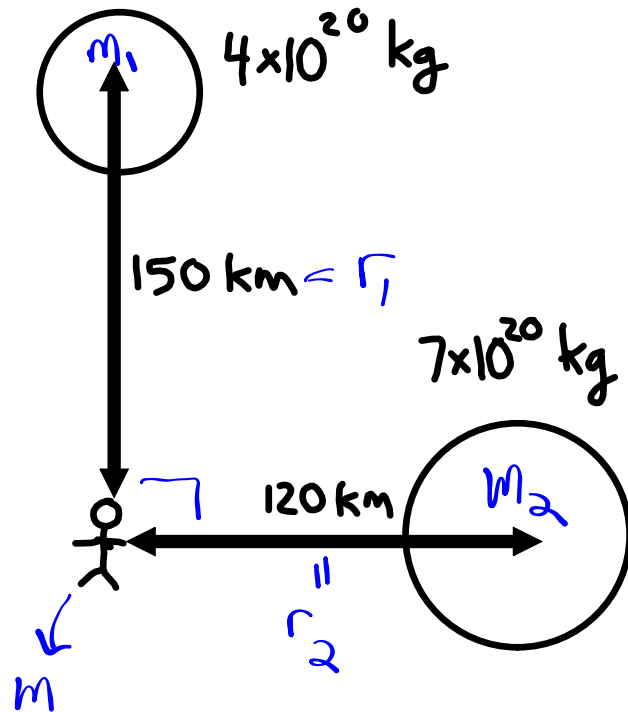


$$\downarrow a \quad \Sigma F = ma$$
$$F_{\text{GRAVITY}} = ma$$

$$G \frac{M_{\text{sun}} m}{r^2} = m a$$

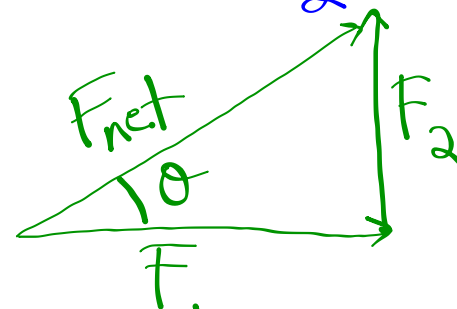
$$\frac{(6.67 \times 10^{-11})(2.0 \times 10^{30})}{(7.0 \times 10^8)^2} = \boxed{272.1 \text{ m/s}^2}$$

EXAMPLE 4: Determine the net force upon a 70-kg person located from two planets as shown below.



$$F_1 = \frac{G m_1 m}{r_1^2}$$

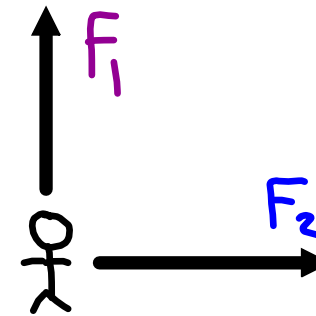
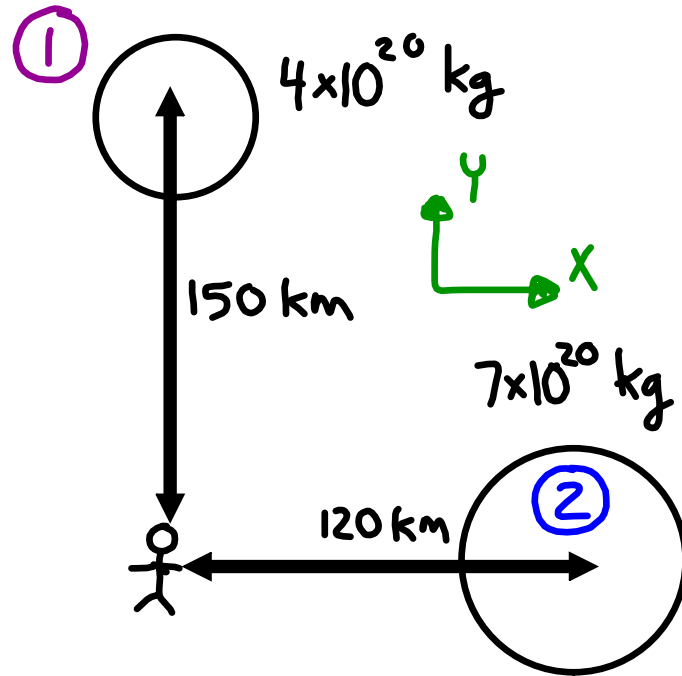
$$F_2 = \frac{G m_2 m}{r_2^2}$$



$$F_{\text{net}} = \sqrt{F_1^2 + F_2^2}$$

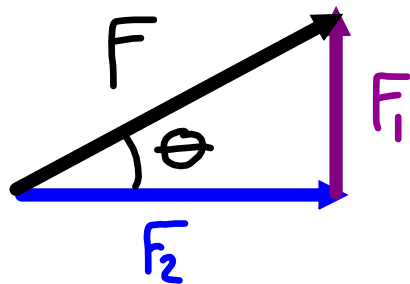
$$\theta = \tan^{-1} \frac{F_1}{F_2}$$

EXAMPLE 4: Determine the net force upon a 70-kg person located from two planets as shown below.



$$F_1 = G \frac{(4 \times 10^{20})(70)}{(1.5 \times 10^5)^2} = 82.96 \text{ N}$$

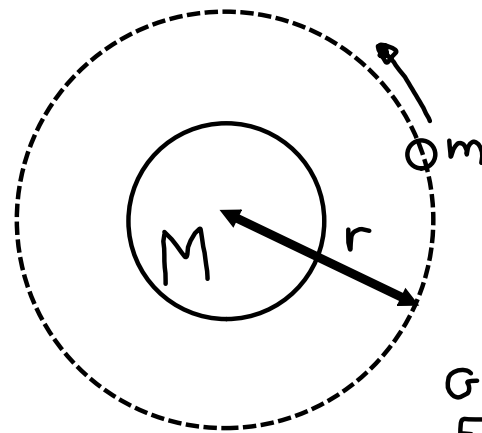
$$F_2 = G \frac{(7 \times 10^{20})(70)}{(1.2 \times 10^5)^2} = 226.9 \text{ N}$$



$$F = \sqrt{82.96^2 + 226.9^2} = \boxed{241.5 \text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{82.96}{226.9}\right) = \boxed{20.08^\circ \text{ As SHOWN}}$$

SATELLITE MOTION



M = MASS OF OBJECT BEING ORBITED

m = MASS OF SATELLITE

r = RADIUS OF ORBIT

GRAVITY PROVIDES THE CENTRIPETAL FORCE NECESSARY FOR CIRCULAR MOTION

$$\Sigma F = ma$$

$$F_{\text{GRAVITY}} = m \left(\frac{v^2}{r} \right)$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v = \frac{2\pi r}{T}$$

ALL RELATIONSHIPS
STEM FROM THESE
TWO.

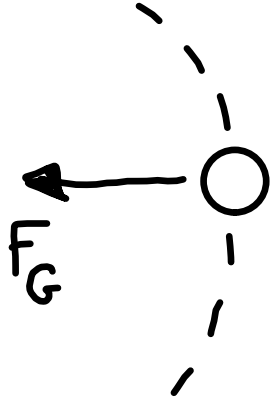
T = THE PERIOD (THE TIME FOR ONE REVOLUTION)

EXAMPLE 1: What is the orbital speed of the Hubble Space Telescope? The altitude of the HST is 596 km above the Earth. The radius of the Earth is 6.38×10^6 meters, and the Earth's mass is 5.98×10^{24} kg.

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM\cancel{m}r}{\cancel{m}r^2}} = \sqrt{\frac{GM}{r}}$$

EXAMPLE 1: What is the orbital speed of the Hubble Space Telescope? The altitude of the HST is 596 km above the Earth. The radius of the Earth is 6.38×10^6 meters, and the Earth's mass is 5.98×10^{24} kg.



$$\Sigma F = ma$$

$$\Sigma F = m\left(\frac{v^2}{r}\right)$$

$$G \frac{Mm}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(6.38 \times 10^6 + 596000)}} = \boxed{7560 \text{ m/s}}$$

EXAMPLE 2: What is the period (in hours) of the Hubble Space Telescope?

$$r = (6.38 \times 10^6 + 596000) \quad v = 7560 \text{ m/s}$$

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi (6.38 \times 10^6 + 596000)}{7560}$$

(in seconds)

EXAMPLE 2: What is the period (in hours) of the Hubble Space Telescope?

$$v = 7560 \text{ m/s}$$

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi (6.38 \times 10^6 + 596000)}{7560} = 5798 \text{ sec}$$

$$5798 \text{ sec} \left(\frac{1 \text{ hr}}{3600 \text{ sec}} \right) = \boxed{1.61 \text{ HR}}$$

EXAMPLE 3: At what height above the Earth do geo-synchronous satellites orbit? The Earth's mass is 5.98×10^{24} kg and the Earth's radius is 6.38×10^6 meters.

$$v = \frac{2\pi r}{T} = \frac{2\pi R}{1 \text{ DAY}} \left(\frac{1 \text{ DAY}}{24 \text{ HR}} \right) \left(\frac{1 \text{ HR}}{3600 \text{ sec}} \right) = (7.272 \times 10^{-5}) R \text{ } \frac{\text{m}}{\text{s}}$$

SOLVE FOR R → THE DISTANCE FROM THE CENTER OF THE EARTH, AND THEN FIND THE HEIGHT.

$$\Sigma F = ma$$

$$\Sigma F = m \left(\frac{v^2}{r} \right)$$

$$G \frac{Mm}{R^2} = m \frac{v^2}{R}$$

$$\frac{GM}{R} = (7.272 \times 10^{-5})^2 R^2$$

$$\frac{GM}{(7.272 \times 10^{-5})^2} = R^3$$

$$\sqrt[3]{\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(7.272 \times 10^{-5})^2}} = 4.224 \times 10^7 \text{ m}$$

$$R = r_{\text{EARTH}} + h$$

$$h = R - r_{\text{EARTH}}$$

$$= 4.224 \times 10^7 - 6.38 \times 10^6 = \boxed{3.586 \times 10^7 \text{ m}}$$

$$\text{OR } 3586 \text{ km}$$