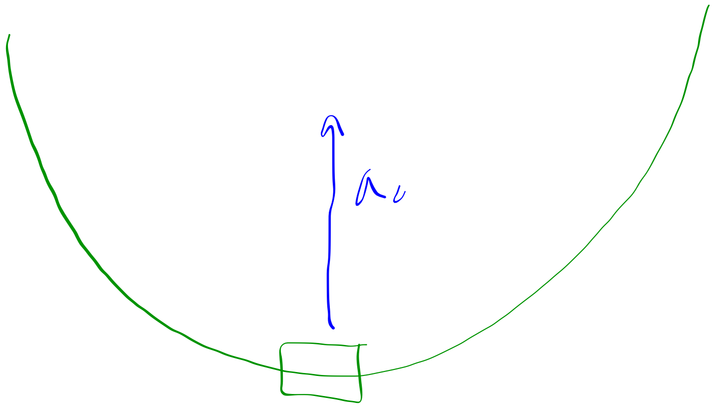
2. A jet plane traveling 1800 km/h (500 m/s) pulls out of a dive by moving in a circle arc of radius 3.00 km. What is the plane's acceleration in *g*'s? (One "*g*" is 9.8 m/s^2, the acceleration we normally experience at the surface of home-sweet-home – earth).



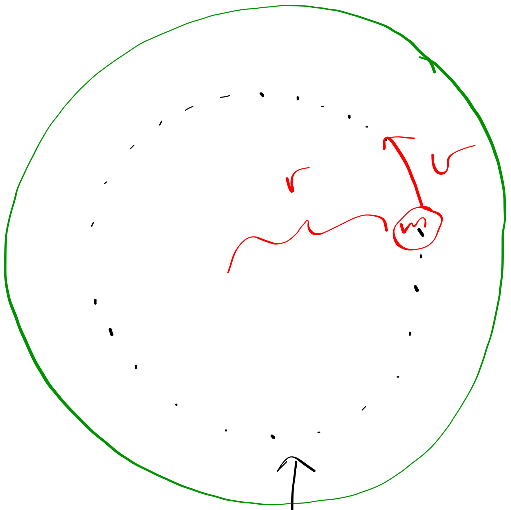
$$a_c = \frac{v^2}{r}$$ ✓

(in m/s^2)

$$1g = 9.8\ m/s^2$$

9. A coin is placed 18.0 cm from the axis of a rotating turntable of variable speed. When the speed of the turntable is slowly increased, the coin remains fixed on the turntable until a rate of 58 rpm (rotations-per-minute) is reached, at which point the coin slides off. What is the coefficient of static friction between the coin and the turntable?

0.18 m



$C = 2\pi r$

$$F_c = \frac{mv^2}{r} = F_{fr} = \mu m g$$

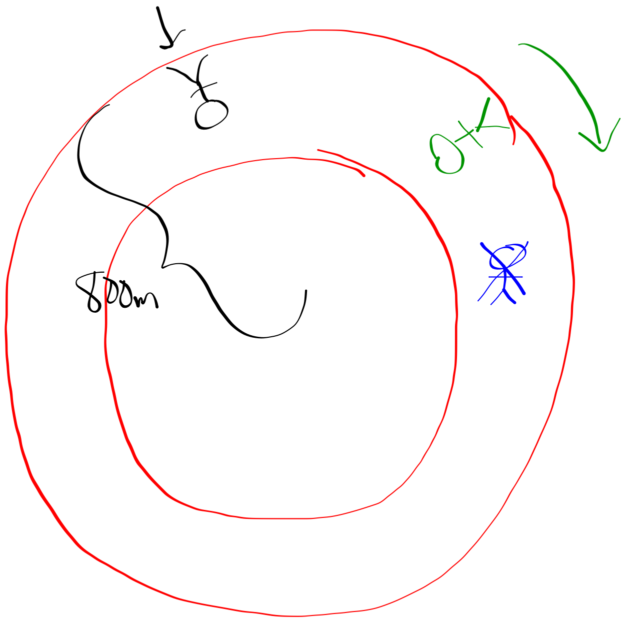
$$\mu = \frac{v^2}{r \cdot g}$$

$$58\,\frac{\cancel{rot}}{\cancel{min}} \cdot \frac{1\,\cancel{min}}{60\,s} \cdot \frac{2\pi r\,m}{1\,\cancel{rot}} = \_\_\_\_\ m/s$$

15. A projected space station consists of a circular tube having a diameter of 1.6 km which is set rotating about its center (like a tubular bicycle tire, or a giant hollow donut).

a) On which part of the inside of the tube (the side closest to or furthest from the center) will people be able to walk? OUTER PART

b) What must be the rotation speed (in revolutions per day) if an effect equal to gravity at the surface of the earth (1 *g*) is to be felt? (Hint: When you are just standing on the ground here on earth, what is the size of the force that pushes up on you from the ground?)



$$F_N = \cancel{m}g = \frac{\cancel{m}v^2}{r}$$

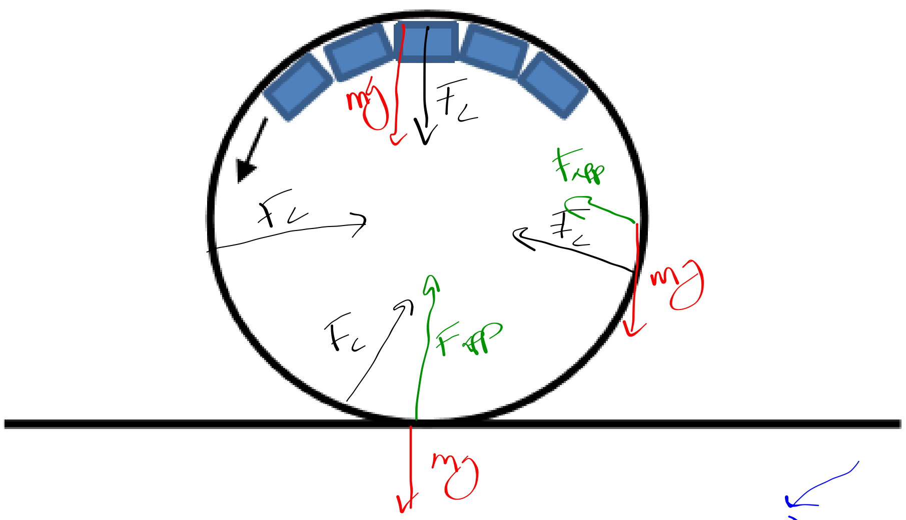
$r = 800m$

$g = 9.8$

$$v = \sqrt{r \cdot g} \quad (\text{in } m/s)$$

$$\frac{\cancel{m}}{\cancel{s}} \cdot \frac{3600\cancel{s}}{\cancel{hr}} \cdot \frac{\cancel{24 hr}}{day} \cdot \frac{1 \text{ rot}}{2\pi r \cancel{\text{ meters}}} = \frac{\text{rot}}{\text{day}}$$

12. What minimum speed must a roller coaster travel at when upside down at the top of a loop-de-loop on the track if the passengers are not to fall out? Assume a radius of curvature of 8.0 m. (And yes, assume these passengers not only have learned their physics but are entrusting their lives to it!! They are NOT wearing any seatbelts!)



minimum speed = $F_{app} = \emptyset$ at the top

$$F_c = \frac{mv^2}{r}$$

$$F_{app} = \frac{mv^2}{r} - mg$$

$$0 = \frac{mv^2}{r} - mg$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m}g$$

$$\boxed{v = \sqrt{r \cdot g}}$$

Rotational Kinematics - Objectives:

- Students will understand how the Big 4 apply to rotational motion
- Students will know the variables used to describe rotational motion
- Students will be able to relate translational motion to rotational motion
- Students will be able to use the rotational Big 4 to solve problems

Uniform Circular Motion

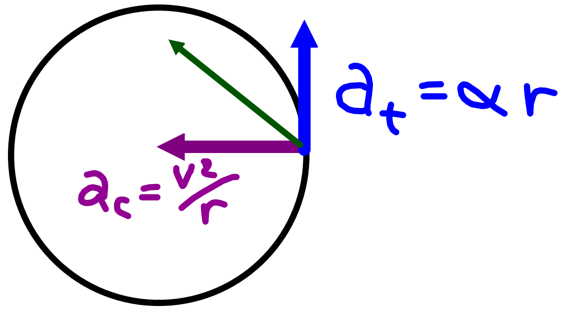
- Speed is constant
- The centripetal acceleration is the only acceleration
- a_c is directed radially inward

Now for the case when circular motion is uniformly **accelerated**:

- Speed is changing
- There are two separate lineal accelerations

CENTRIPETAL ACCELERATION (a_c)

- Directed inward
- $a_c = v^2/r$
- Responsible for changing the direction

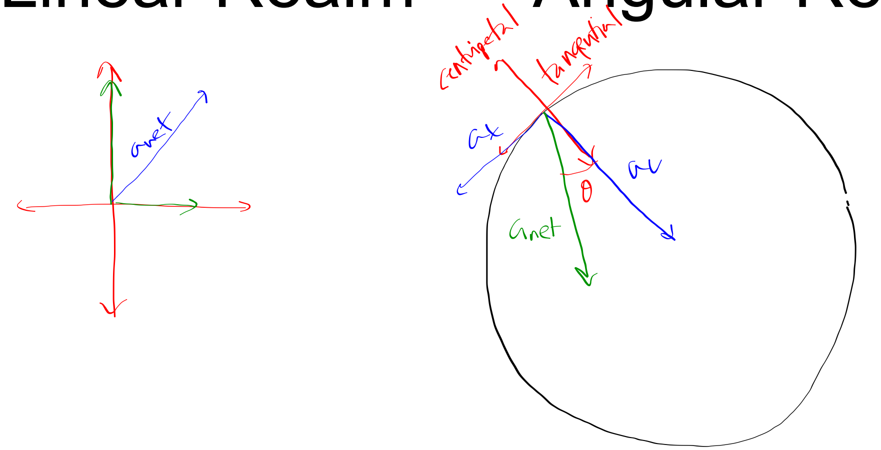


TANGENTIAL ACCELERATION (a_t)

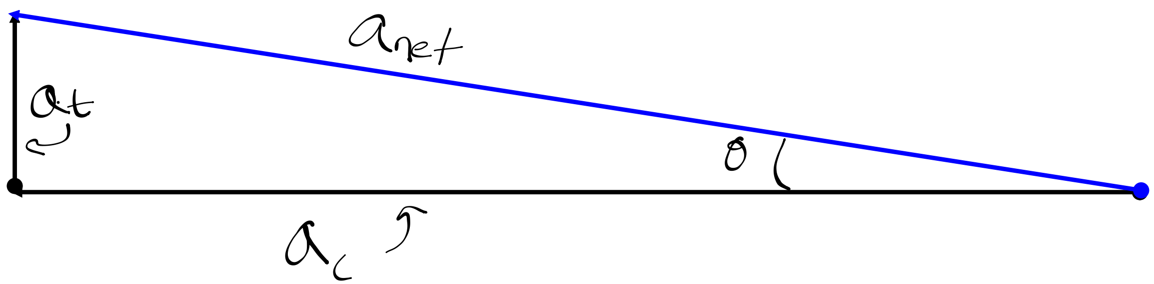
- Directed in the direction of instantaneous travel
- $a_t = \alpha r$
- Responsible for increasing / decreasing the angular velocity

Frame of Reference and Net Acceleration:

Linear Realm Angular Realm



$$\boxed{\begin{gathered} a_{net} = \sqrt{a_c^2 + a_t^2} \\ \theta = \tan^{-1}\left(\frac{a_t}{a_c}\right) \end{gathered}}$$



Linear Quantities vs. Angular Quantities

Linear	Angular
Linear Displacement (meters) Δx	Angular Displacement (radians) $\Delta \theta$
Linear Velocity (m/sec) v	Angular Velocity (radians/sec) ω (omega)
Linear Acceleration (m/s^2) a	Angular Acceleration (radians/s^2) α (alpha)
When **linear** acceleration is constant:	When **angular** acceleration is constant:
$x = x_0 + v_0 t + \frac{1}{2} a t^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$$\bar{v} = \frac{v_0 + v}{2} = \frac{\Delta x}{\Delta t}$$

$$\bar{\omega} = \frac{\omega_0 + \omega}{2} = \frac{\Delta \theta}{\Delta t}$$

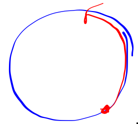
Relating Linear Quantities to Angular Quantities

$$\boxed{\theta = \frac{s}{r}}$$

BY DEFINITION

arc length

$s = \theta \cdot r$



$s \cong \Delta x$

If an object is rotating for a given amount of time (Δt), an angular displacement ($\Delta\theta$) and linear dislacement (Δs) are realized.

$$\Delta\theta = \frac{\Delta s}{r}$$

$$\frac{\Delta\theta}{\Delta t} = \frac{\Delta s}{\Delta t(r)} \quad \text{(DIVIDE BOTH SIDES BY } \Delta t\text{)}$$

$$\boxed{\omega = \frac{v}{r}}$$

linear velocity (like in $F_c = \frac{mv^2}{r}$ or $a_c = \frac{v^2}{r}$)

$v = \omega \cdot r$

If an object is also experiencing an angular acceleration (speeding up or slowing down) over some time period (Δt), there will be changes in the angular speed ($\Delta\omega$) and the linear speed (Δv).

$$\Delta\omega = \frac{\Delta v}{r}$$

$$\frac{\Delta\omega}{\Delta t} = \frac{\Delta v}{\Delta t(r)} \quad \text{(DIVIDING BOTH SIDES BY } \Delta t\text{)}$$

$$\boxed{\alpha = \frac{a_t}{r}}$$

one component of a rotating object's net acceleration

$a_t = \alpha \cdot r$

Linear Big 4:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\bar{v} = \frac{v_0 + v}{2} = \frac{\Delta x}{\Delta t}$$

Angular Big 4:

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\bar{\omega} = \frac{\omega_0 + \omega}{2} = \frac{\Delta\theta}{\Delta t}$$

$$F_c = \frac{mv^2}{r}$$

$$a_c = \frac{v^2}{r}$$

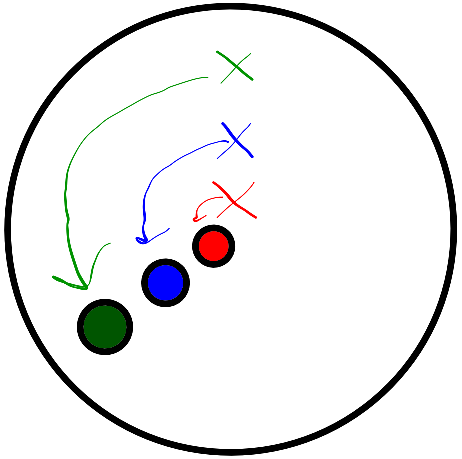
$$a_{net} = \sqrt{a_c^2 + a_t^2}$$

$$\theta = \tan^{-1}\frac{a_t}{a_c}$$

$$\theta = \frac{s}{r} \; ; \; s = \theta \cdot r$$

$$\omega = \frac{v}{r} \; ; \; v = \omega \cdot r$$

$$\alpha = \frac{a_t}{r} \; ; \; a_t = \alpha \cdot r$$



S = arc length

(Δx in linear big 4)

Different for objects at different distances from the center

θ = angle

Always the same

EXAMPLE 1: A typical compact disc records data starting at a radius of 25.0 mm (0.025m) and ending at a radius of 58.0 mm (0.058 m) from its center. All disc players read information from the disc at a rate of 4500 mm/min (0.075 m/s).

a) What is the initial angular velocity (ω_o) of the disc when it starts reading data? What is this velocity in RPM?

b) What is the angular velocity of the disc when it finishes reading data at the outside radius? What is this velocity in RPM?

c) If the CD plays continuously from the beginning to end, what is the angular acceleration assuming a play time of 75.0 minutes (4500 s)?

d) What are a_t and a_c (in m/s^2) at a point when the data is being read at a radius of 50.0 mm (0.050 m)?

a) $\omega_o = \frac{v}{r} = \frac{0.075\ m/s}{0.025\ m} = 3\ \frac{rad}{s}$

b) $\omega = \frac{v}{r} = \frac{0.075\ m/s}{0.058\ m} = 1.29\ \frac{rad}{s}$

c) $\theta = 0\ rad$

$\theta_o =$

$\omega_o = 3\ \frac{rad}{s}$

$\omega = 1.29\ \frac{rad}{s}$

$\alpha =$

$\Delta t = 4500\ s$

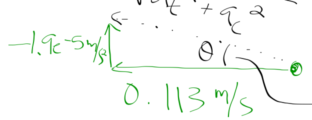
$\omega = \omega_o + \alpha t$

$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{1.29 - 3}{4500} = -3.8e{-4}\ \frac{rad}{s^2}$

d) $a_t = \alpha \cdot r = (-3.8e{-4})(0.05) = \boxed{-1.9e{-5}\ m/s^2}$

$a_c = \frac{v^2}{r} = \frac{(0.075)^2}{0.05} = \boxed{0.113\ m/s^2}$

$a_{net} = \sqrt{a_t^2 + a_c^2}$



$\theta \rightarrow \tan^{-1}\left(\frac{a_t}{a_c}\right)$

EXAMPLE 2: *Nannosquilla decemspinosa* is a small, legless crustacean living on the west coast of Panama. When stranded on the beach by high tide, it moves back to the water by doing sommersaults. If *nannosquilla* has a body length of 3.0 cm (0.03 m), takes this body length and curls it up as a wheel (having this circumference), rotates as a wheel at 70.0 RPM (7.33 rad/s), and if it must travel 4.0 meters to return to the water, how long does it take it to get back into the water? How long would it take if its angular acceleration were 0.56 rad/s^2?

EXAMPLE 3: A wheel with a diameter of 0.19 m starts from rest and reaches a speed of 40.0 RPM (4.19 rad/s) after rotating through 46 radians.

a) Determine the wheel's constant angular acceleration.

b) How long did the above process take?

c) What was the centripetal acceleration of a point 4.2 cm (0.042 m) from the center of the wheel?

d) After rotating through 22 radians, what was the instantaneous velocity of a point on the edge of the wheel?