Test:

$\Sigma \tau = I\alpha$

$\Sigma F = ma$

$\alpha = \frac{a_t}{r}$

$\tau = F_\perp \ell$

yes

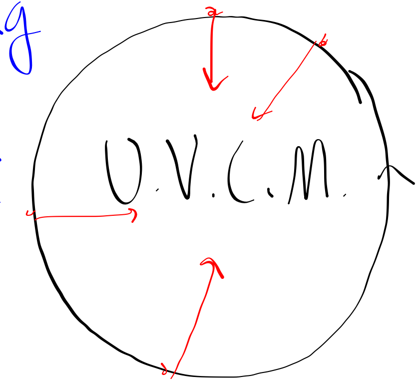
$\cancel{\alpha = \frac{m_2 g r_2 - m_1 g r_1}{I + m_1 r_1^2 + m_2 r_2^2}}$

NO

$F_g = mg$

$F_c = \frac{mv^2}{r}$

Yes



~~$\Sigma F = F_c$~~

~~top: $F_a + mg = F_c$~~

~~side: $F_a = F_c$~~

~~bottom: $F_a - mg = F_c$~~

No

Test → based on homework/examples

and

LABS (Atwood Lab)

(different scenarios/easier math)

39. A centrifuge rotor has a moment of inertia of 4.00×10^{-2} kg·m^2. How much energy is required to bring it from rest to 10,000 rpm?

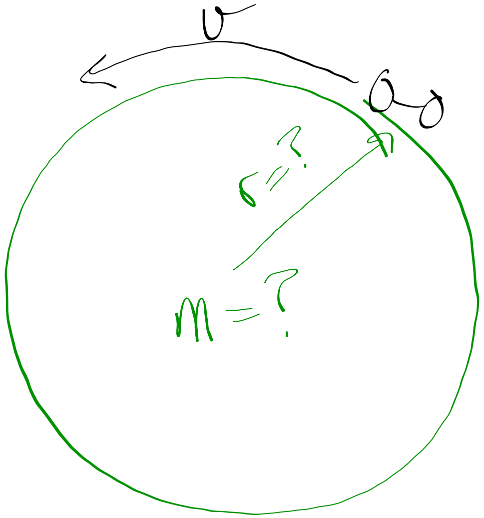
$10{,}000\ \text{rpm} = 1047\ \frac{\text{rad}}{\text{s}}$

$W_{NC} = KE_{rot}$

$W_{NC} = \frac{1}{2}I\omega^2 = \frac{1}{2}(4e{-2})(1047^2)$

$= 2.19e4\ J$

10. An astronaut, standing on a new planet, finds that a 35-kg dog weighs 1400 N. She further notes that the period of a satellite just skimming the surface of the planet (having an orbit equal to the radius of the planet) is 150 minutes. What is the radius of the planet? [8.21 x 10^7 m]



$$W = m \cdot g \qquad F_g = \frac{GMm}{r^2}$$

$$1400 = 35g$$

$$g = 40\ m/s^2$$

$$a_c = \frac{v^2}{r} = g$$

$$v = \frac{2\pi r}{T} = \boxed{\frac{2\pi r}{(9{,}000\ s)}}$$

$$\frac{\left(\frac{2\pi r}{9{,}000}\right)^2}{r} = 40$$

$$\frac{2^2\pi^2 r}{9000^2} = 40$$

$$r = 8.21e7\ m$$

41. When an object has symmetry, its moment of inertia often can be expressed as a simple formula. For instance, the moment of inertia for a hoop rotated about its center is mr^2. For a uniform disk rotating about its center, the moment of inertia is $1/2mr^2$. A uniform sphere rotated about its center is $2/5mr^2$. However as you know, most objects do not enjoy the benefit of symmetry. As a result, if we can even come up with a formula for their moments of inertia, the formulas might not be all that simple. Often, the moments of inertia for these objects are determined experimentally by applying a known torque to the object, measuring the angular acceleration it experiences, and calculating its moment of inertia using $\sum\tau=I\alpha$. If this calculated moment of inertia is set equal to mr^2 (the basic formula for the simplest of objects – a point mass), the r that satisfies this equation is called the object's *radius of gyration*. With all of that explanation behind us now, we are finally ready for this problem. A merry-go-round has a mass of 1560 kg and a *radius of gyration* of 18.5 m. How much work is required to accelerate it from rest to a rotation rate of $\underbrace{\text{one revolution in 7.10 seconds?}}$ (Hint: Think about CLEE and how a change in KE relates to work).

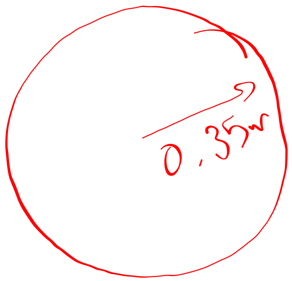
$\hookrightarrow \omega = \frac{2\pi \text{ rad}}{7.1 s} =$

$$I = mr^2 = (1560)(18.5^2) = 5.3e5 \text{ kg}\cdot\text{m}^2$$

$$W_{NC} = \frac{1}{2} I \omega^2$$

23. A 35-cm radius disk initially rotating about its center at 4 rad/sec accelerates with an angular acceleration of 6 rad/sec^2 for 13 seconds. How many revolutions does the disk make during the 13.0 seconds? What are the linear accelerations of a point at the edge of the disk 5.0 seconds after it starts accelerating in both the radial and tangential directions? [88.97 revolutions; a_r = 404.6 m/sec^2; a_t = 2.1 m/sec^2]

$\Delta\theta = ?$



$\theta_0 = 0$

$\theta =$

$\omega_0 = 4\ \frac{rad}{s}$

$\omega =$

$\alpha = 6\ \frac{rad}{s^2}$

$t = 13\,s$

$\Big\}\ \Delta\theta = ?$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\Delta\theta = (4)13 + \frac{1}{2}(6)(13^2)$

a) $\Delta\theta = 559\ rad \cdot \frac{1\ rot}{2\pi\ rad}$

$= 89$ rotations

b) $a_c = \frac{v^2}{r} = \omega^2 r = (34^2)(0.35)$

$404.6\ m/s^2$

$\omega = \omega_0 + \alpha t$

$= 4 + 6(5)$

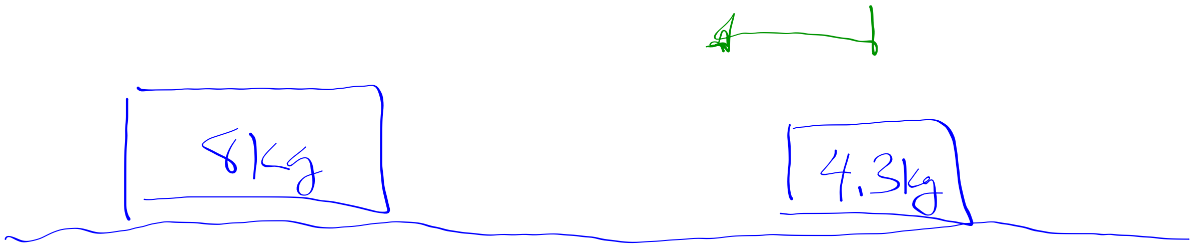
$= 34\ rad/s$

$a_t = \alpha \cdot r$

$= (6)(0.35)$

$= 2.1\ m/s^2$

11. Two masses are on a frictional, horizontal surface. If the 8-kg mass is brought close to a 4.3-kg mass on a surface with a coefficient of friction of .2, at what distance will the 4.3-kg mass begin to slide toward the 8-kg mass? [1.65×10^{-5} m]

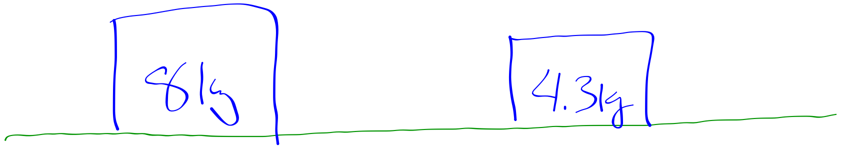


$\Sigma F = 0$

$F_g - F_{Fr} = 0$

$F_{Fr} = F_g$

11. Two masses are on a frictional, horizontal surface. If the 8-kg mass is brought close to a 4.3-kg mass on a surface with a coefficient of friction of .2, at what distance will the 4.3-kg mass begin to slide toward the 8-kg mass? [1.65 x 10^{-5} m]

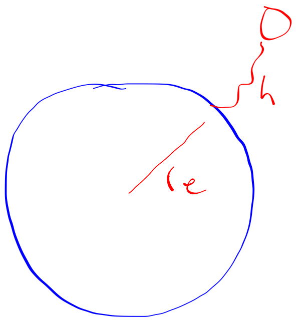


$$F_g \stackrel{+}{=} F_{Fr}$$

$$\frac{G m_1 \cancel{m_2}}{r^2} = \mu \cancel{m} g$$

$$r = \sqrt{\frac{G m_1}{\mu g}} = \sqrt{\frac{(6.67e{-11})8}{(0.2)9.8}} = 1.65e{-5}\,m$$

9. If a satellite circles the Earth in 2 hours, what is the altitude of the satellite's orbit (how high is it above the Earth)? The mass of the Earth is 5.98 x 10^{24} kg, the radius of the Earth is 6.38 x 10^{6} meters. [1.68 x 10^{6} m]



$$F_g = F_c$$

$$\frac{GM\cancel{m}}{r^2} = \frac{\cancel{m}v^2}{\cancel{r}} = \left(\frac{2\pi r}{T}\right)^2$$

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$r^3 = \frac{GMT^2}{4\pi^2} = \frac{(6.67e{-11})(5.98e24)(7200)^2}{4\pi^2}$$

$$r = \sqrt[3]{\ldots} = 8e6$$

$$h = r - r_e = 8e6 - 6.38e6 =$$

$$= \boxed{1.6e6\,m}$$

31. If a spring (k = 340 N/cm) is compressed $\boxed{9\text{ cm}}$ by a disk on its side, what will the velocity of the rolling disk be when the spring is released? The disk has a mass of 1.9 kg and a radius of .3 meters. [9.83 m/sec]

$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$I_{disk} = \frac{1}{2}mr^2$

$\omega = \frac{v}{r}$

$k = 340\ N/\cancel{cm} \cdot \frac{100\ \cancel{cm}}{m} = \frac{34{,}000\ N}{m}$

$x = 9\ cm = 0.09\ m$

$$\frac{1}{2}\left(\frac{1}{2}m\cancel{r^2}\right)\left(\frac{v}{\cancel{r}}\right)^2$$

$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$\frac{1}{\cancel{2}} \cdot \frac{\cancel{4}\,2}{3}$$

$$\frac{3}{4}mv^2 = \frac{1}{2}kx_0^2$$

$$v = \sqrt{\frac{\frac{2}{3}kx_0^2}{m}} = \sqrt{\frac{\frac{2}{3}(34{,}000)(0.09)^2}{1.9}} = \boxed{9.83\ m/s}$$

44. A hollow cylinder (hoop) is rolling on a horizontal surface at a speed of 3.4 m/s when it reaches a 20° incline.

a) How far up the surface of the incline will it go? 3.45 m

b) How long will it be on the incline before it arrives back at the bottom?

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$a = \frac{-v_0^2}{2(x - x_0)} = \frac{-(3.4)^2}{2(3.45)} = -1.68 \ m/s^2$$

$$v = v_0 + at$$

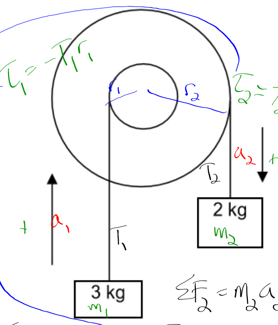
$$0 = 3.4 + (-1.68)t$$

$t = 2.02s$ (way up)

$2.02s$ (way down)

$\left.\right\} 4.04s$

26. A two-disk Atwood machine with radii of 15-cm and 38-cm, has a moment of inertia of 4 kg·m^2. What is the acceleration of the mass on the right? [.265 m/sec^2]



$\tau_1 = -T_1 r_1$ $\qquad \tau_2 = T_2 r_2$

$\Sigma F_1 = m_1 a_1$

$T_1 - m_1 g = m_1 a_1$

$T_1 = m_1 g + m_1 a_1$

$\Sigma F_2 = m_2 a_2$

$-T_2 + m_2 g = m_2 a_2$

$T_2 = m_2 g - m_2 a_2$

$\Sigma \tau = I\alpha$

$-T_1 r_1 + T_2 r_2 = I\alpha$

$-(m_1 g + m_1 a_1) r_1 + (m_2 g - m_2 a_2) r_2 = I\alpha$

$\alpha = \frac{a_t}{r} = \frac{a_1}{r_1} = \frac{a_2}{r_2}$

$a_1 = \frac{a_2 r_1}{r_2} \qquad \alpha = \frac{a_2}{r_2}$

$$-\left(m_1 g + m_1\left(\frac{a_2 r_1}{r_2}\right)\right) r_1 + (m_2 g - m_2 a_2) r_2 = I\left(\frac{a_2}{r_2}\right)$$

$$-m_1 g r_1 - \frac{m_1 a_2 r_1^2}{r_2} + m_2 g r_2 - m_2 a_2 r_2 = \frac{I a_2}{r_2}$$

$$-m_1 g r_1 r_2 - m_1 a_2 r_1^2 + m_2 g r_2^2 - m_2 a_2 r_2^2 = I a_2$$

$$I a_2 + m_1 a_2 r_1^2 + m_2 a_2 r_2^2 = -m_1 g r_1 r_2 + m_2 g r_2^2$$

$$a_2 (I + m_1 r_1^2 + m_2 r_2^2) = -m_1 g r_1 r_2 + m_2 g r_2^2$$

$r_1 = 14\text{cm} = 0.14\text{m}$
$r_2 = 38\text{cm} = 0.38\text{m}$
$m_1 = 3\text{kg}$
$m_2 = 2\text{kg}$
$I = 4\ \text{kg m}^2$

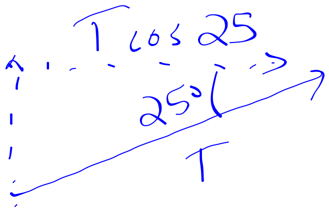
$$a_2 = \frac{-m_1 g r_1 r_2 + m_2 g r_2^2}{I + m_1 r_1^2 + m_2 r_2^2}$$

$$a_2 = \frac{-(3)(9.8)(0.14)(0.38) + (2)(9.8)(0.38^2)}{4 + (3)(0.14^2) + (2)(0.38)^2}$$

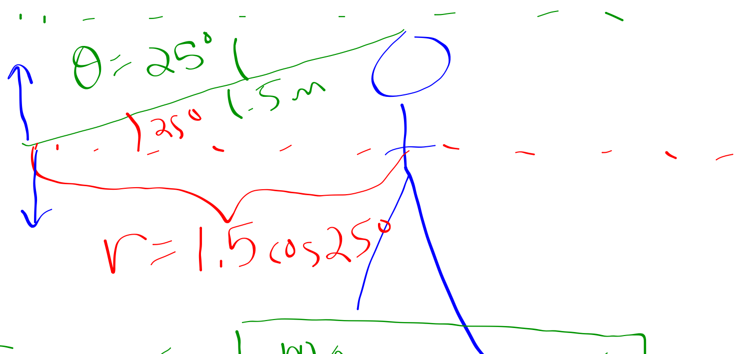
$$a_2 = \frac{-1.564 + 2.83}{4 + 6.0588 + 0.2888} = \frac{1.266}{4.3476}$$

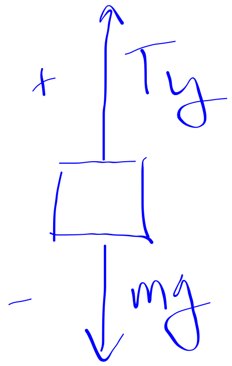
$$a_2 = 0.291\ \text{m/s}^2 \ \text{(downward)}$$

18. A boy swings a rock on a 1.4 meter string in a horizontal circle. If he swings it so that the string makes a 25° angle with the horizontal, how fast must it be moving? (Hint: the radius of the rock's orbit is NOT 1.4 meters).
[5.16 m/sec]



$T\sin 25 = T_y$





$$T_x = T\cos 25 = \boxed{\frac{mg}{\sin 25}\cos 25}$$

$$T_y = mg$$

$$T\sin 25 = mg$$

$$T = \frac{mg}{\sin 25}$$

$$F_c = T_x$$

$$\frac{\cancel{m}v^2}{r} = \frac{\cancel{m}g\cos 25}{\sin 25}$$

$$v = \sqrt{\frac{rg\cos 25}{\sin 25}}$$

31. If a $\boxed{\text{spring (k = 340 N/cm) is compressed 9 cm}}$ by a disk on its side, what will the velocity of the rolling disk be when the spring is released? The disk has a mass of 1.9 kg and a radius of .3 meters. [9.83 m/sec]

$F_s = Kx$ = ~~~ Newtons ✓

$$I_{disk} = \frac{1}{2}mr^2$$

$$\frac{1}{2}Kx_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad \omega = \frac{v}{r}$$

(for rolling objects)

$$\frac{1}{2}(34{,}000\ N/m)(0.09\ m)^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}m\cancel{r^2}\right)\left(\frac{v}{\cancel{r}}\right)^2$$

$$137.7 = \frac{3}{4}mv^2$$

$$v = \sqrt{\frac{\frac{4}{3}(137.7)}{m}} = 9.83\ m/s$$

34. A 2-kg toad sits on the edge of a 3-kg lazy susan (a disk), which has a radius of 0.34 m. If the system rotates initially at 4 rad/sec and the toad hops to a point 0.10 m from the center, what is the new angular velocity? [8.37 rad/sec]

I, ω

$KE = \frac{1}{2} I \omega^2$ $\quad L = I\omega$

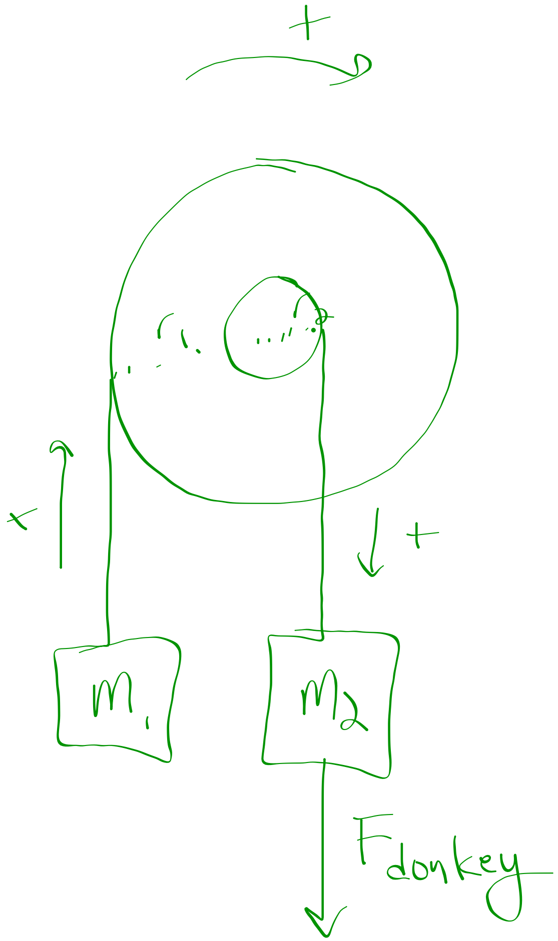
$\Sigma \tau = \emptyset$

L is conserved

$L_i = L_f$

$I_i \omega_i = I_f \omega_f$

$$\left(\underbrace{\frac{1}{2} m r^2}_{\text{disk}} + \underbrace{m r_0^2}_{\text{frog}}\right)\omega_i = \left(\frac{1}{2} m r^2 + m r^2\right)\omega_f$$

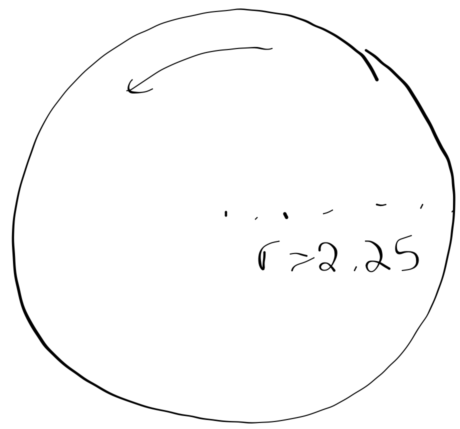


Find a_1 ....

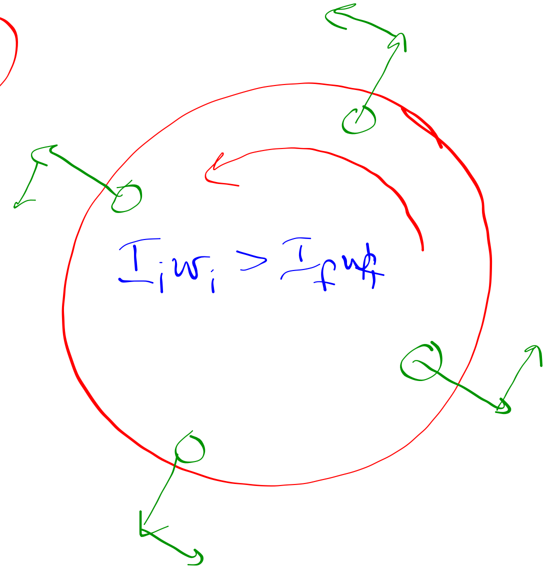
53. A 4.5-m-diameter merry-go-round is rotating freely with an angular velocity of 0.70 rad/s; its total moment of inertia is 1750 kg·m^2.

   a) Four people standing on the ground, each of 65-kg mass, suddenly step onto the edge of the merry-go-round. What will be the angular velocity of the merry-go-round now?
   b) What will be the angular velocity of the merry-go-round if the people were on it initially, and then jump off?

(a)



(b)



$$I_i w_i = I_f w_f$$

$$(1750)(0.7) = (1750 + 4(65)(2.25^2))\, w_f$$