# Analyzing Circuits with Capacitors (and Resistors...):

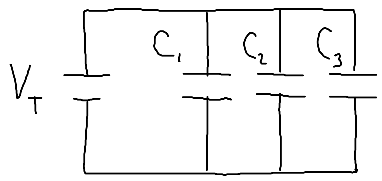
To analyze circuits with RESISTORS, use:

$$V = I R$$

To analyze circuits with CAPACITORS, use:

$$Q = C V$$

**Capacitors in Parallel -- finding the equivalent capacitance:**




When hooked up to the battery, each capacitor acquires a charge.

In parallel, the voltage across each capacitor is the same:

$$V_T = V_1 = V_2 = V_3$$

The total charge that leaves the battery gets distributed to all capacitors:

$$Q_T = Q_1 + Q_2 + Q_3 \quad \text{(parallel)}$$

BUT $Q = CV$

$$\therefore\ C_T V_T = C_1 V_1 + C_2 V_2 + C_3 V_3$$

$$C_T V_T = C_1 V_T + C_2 V_T + C_3 V_T$$

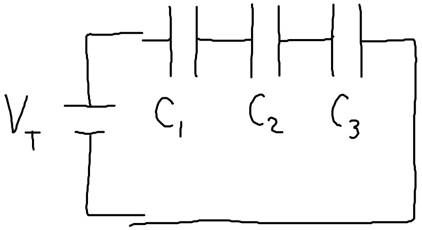
DIVIDING ALL TERMS BY $V_T$ YIELDS:

$$\boxed{C_T = C_1 + C_2 + C_3 + \ldots}$$

Capacitors in parallel are just added (think about the areas of the plates of each capacitor just merging)

The net effect of connecting capacitors in parallel is to increase the capacitance.

**Capacitors in Series -- finding the equivalent capacitance:**




When hooked up to the battery, each capacitor acquires **the same** charge.

In series, the charge on each capacitor is the same:

$$\boxed{Q_T = Q_1 = Q_2 = Q_3 \quad \text{in series}}$$

The total voltage applied to the circuit must get distributed to all capacitors:

$$V_T = V_1 + V_2 + V_3$$

But $Q = CV$ or $V = Q/C$

$$\therefore \; Q_T/C_T = Q_1/C_1 + Q_2/C_2 + Q_3/C_3$$

$$Q_T/C_T = Q_T/C_1 + Q_T/C_2 + Q_T/C_3$$

Dividing All Terms By $Q_T$ Yields:

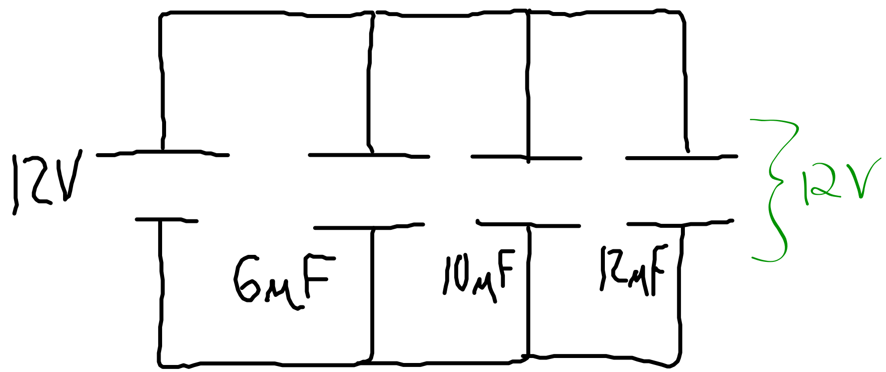
$$\boxed{1/C_T = 1/C_1 + 1/C_2 + 1/C_3 + \dots}$$

The net effect of connecting capacitors in series is to decrease the capacitance.

For series:

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

EXAMPLE 1: What is the total charge delivered by the battery?




parallel

$$Q_T = Q_1 + Q_2 + Q_3$$

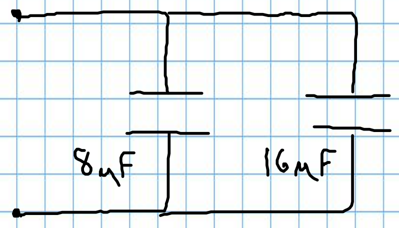
$$C_T = C_1 + C_2 + C_3$$

$$C_T = 28\,\mu F$$

$$Q = CV = (28e^{-6})(12)$$

$$= \boxed{3.36e^{-4}\ C}$$

EXAMPLE 2: What is the charge on the 8uF capacitor?

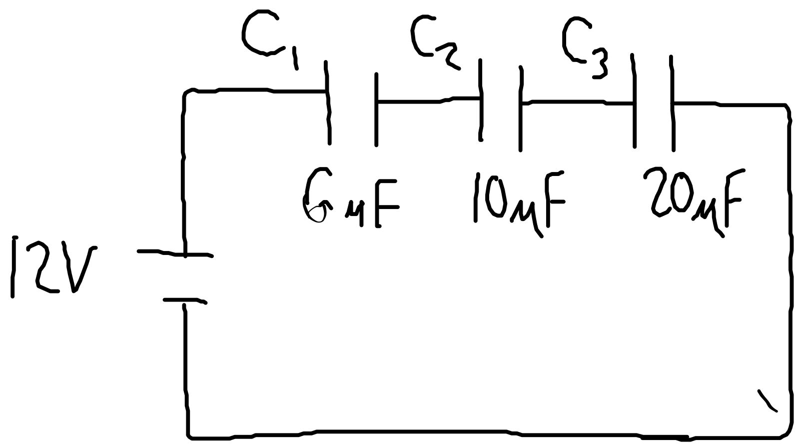



$Q_{16} = 10\mu C$

$V_T = V_8 = V_{16} = Q_{16}/C_{16} = \frac{10\mu C}{16\mu F} = .625V$

$Q_8 = C_8 V_8 = (8\times10^{-6})(.625) = \boxed{5.0\times10^{-6}\ C}$

EXAMLE 3: What is the charge delivered by the battery?




series

$V_T = V_1 + V_2 + V_3$

$Q_1 = Q_2 = Q_3$

$C_T = \dfrac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$

$C_T = 3.16e^{-6}$ F

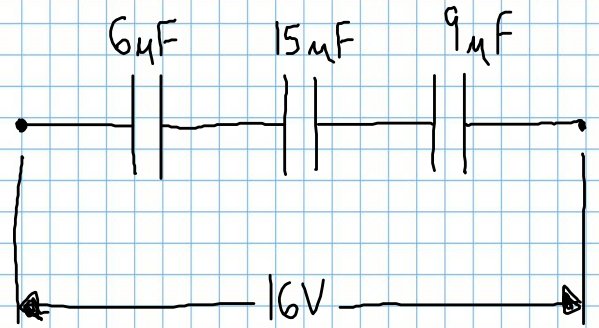
$\boxed{3.8e^{-6}\ C}$

$Q = CV$

$= (3.16e^{-6})12$

$Q_T = \boxed{3.79e^{-5}\ C}$

EXAMPLE 4: What is the potential difference across the 15uF capacitor?




$V_{15} = Q_{15}/C_{15}$

FIND $Q_{15}$:

$$\frac{1}{C_T} = \frac{1}{6\mu F} + \frac{1}{15\mu F} + \frac{1}{9\mu F}$$

$$C_T = 2.90\mu F$$

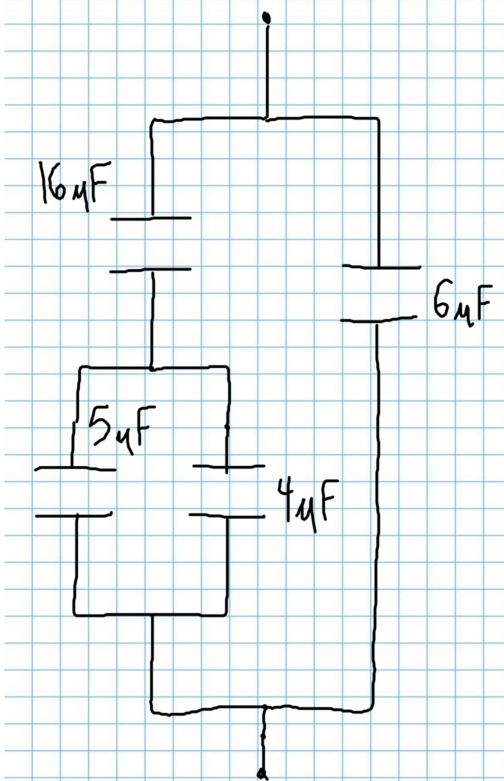
$$Q_6 = Q_{15} = Q_9 = Q_T = C_T V_T$$

$$= (2.9\times10^{-6})(16)$$

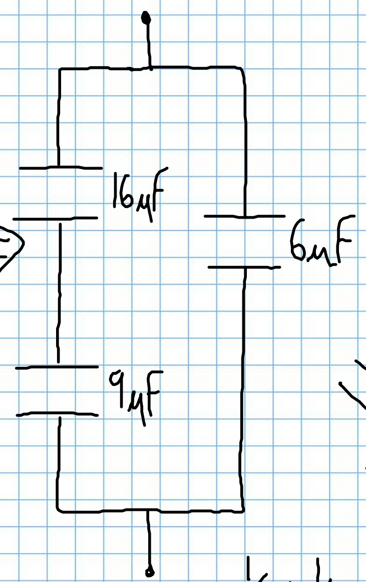
$$= 4.65\times10^{-5}C$$

$$V_{15} = \frac{Q_{15}}{C_{15}} = \frac{4.65\times10^{-5}}{15\times10^{-6}} = \boxed{3.10\ V}$$

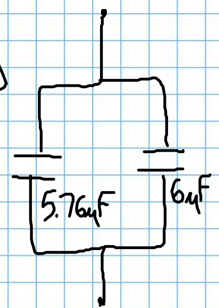
EXAMPLE 5: What is the equivalent total capacitance of this circuit?




$\Rightarrow$



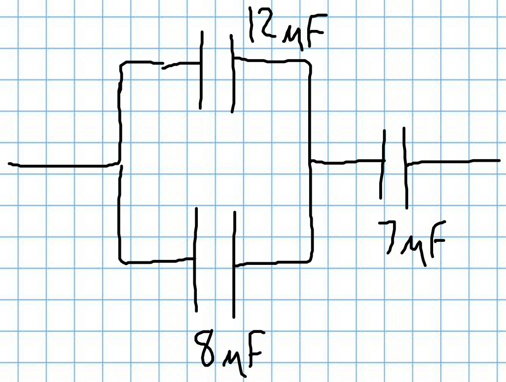

$\Rightarrow$




$\frac{1}{16} + \frac{1}{9} = \frac{1}{5.76}$

$C_T = 5.76 + 6 = \boxed{11.76\,\mu F}$

EXAMPLE 6: Find $Q_{12}$ and $V_7$ given that $Q_8$= 5uC.




$$\frac{Q_8}{C_8} = V_8 = \frac{5 \times 10^{-6}}{8 \times 10^{-6}} = .625\ V$$

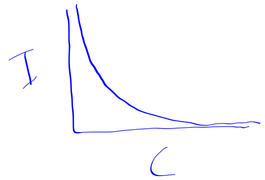
$$V_{12} = V_8 = .625\ V$$

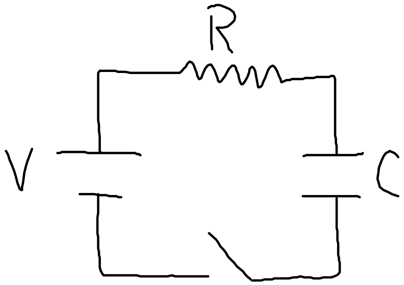
$$Q_{12} = C_{12} V_{12} = (12 \times 10^{-6})(.625) = 7.5 \times 10^{-6}\ C = \boxed{7.5\ \mu C}$$

$$Q_7 = Q_T = Q_8 + Q_{12} = 5\ \mu C + 7.5\ \mu C = 12.5\ \mu C$$

$$V_7 = Q_7 / C_7 = \frac{12.5\ \mu C}{7\ \mu F} = \boxed{1.79\ V}$$

## Resistors and Capacitors TOGETHER




When the switch is initially closed, there is no charge on C.

As a result, effectively C has no resistance to current flow.

As C charges, the current decreases until it drops to zero when the capacitor has become fully charged.

The thing to understand about capacitors with resistors in DC circuits:

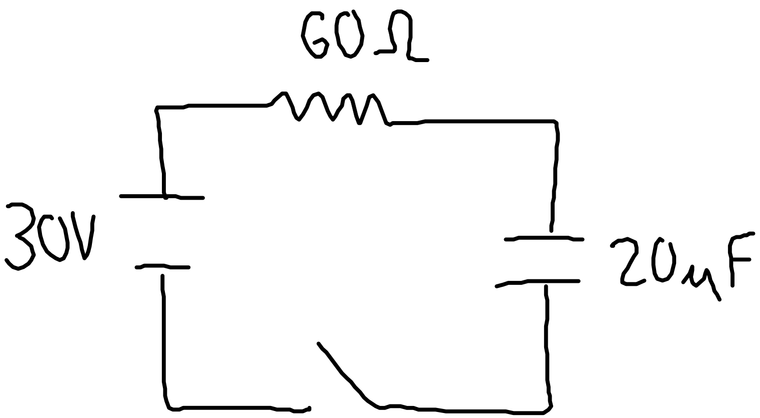
**Initially, immediately after the circuit is turned on:**
1. C has no charge
2. C has no resistance
3. Treat the circuit like C does not exist.

**After the capacitor has become fully charged:**
1. C has a voltage
2. C has infinite resistance
3. No current flows through C or anything in series with C

"STEADY STATE"

No current through any path that has a capacitor

EXAMPLE 7: What is the current through the resistor before the capacitor has any charge, and after the capacitor has become fully charged?




Start: $R_c = 0$

$V = IR$

$30 = I(60)$

$I = 0.5A$

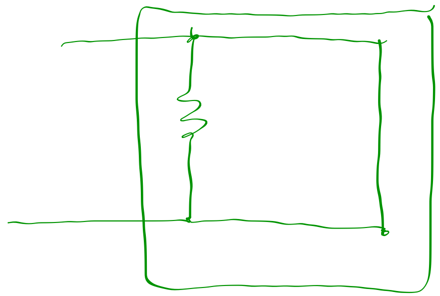
End: $R_c = \infty$

$I = 0A$

EXAMLE 8: Find I in the 12 Ohm resistor

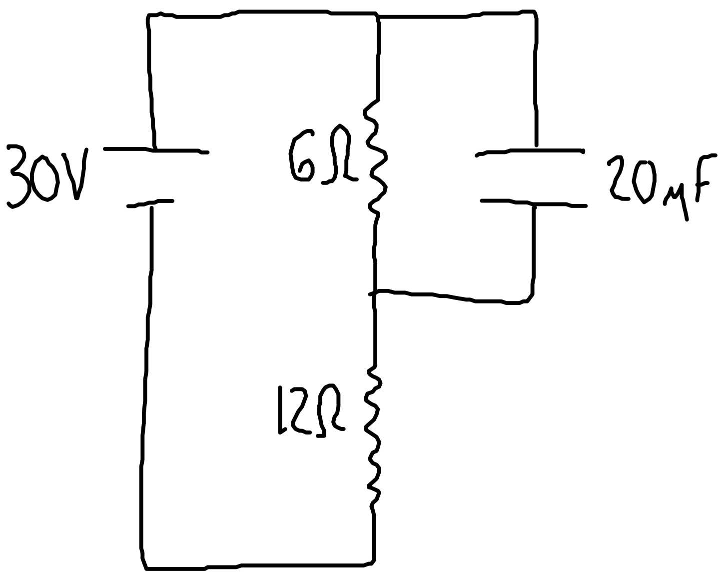
a) Initially

b) After a long time



$$\frac{1}{\frac{1}{R}+\frac{1}{0}} \longrightarrow 0$$

$R_{eq} = 0$




a) $V = IR$

$30 = I(12)$

$I = 2.5\,A$

b) $V = IR$

$30 = I(18)$

$I = 1.67\,A$