Uniform Circular Motion

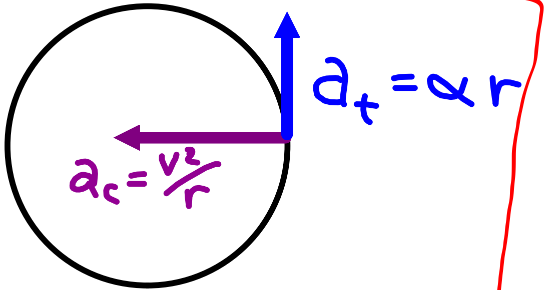
- Speed is constant
- The centripetal acceleration is the only acceleration
- a_c is directed radially inward

Now for the case when circular motion is uniformly **accelerated**:

- Speed is changing
- There are two separate lineal accelerations

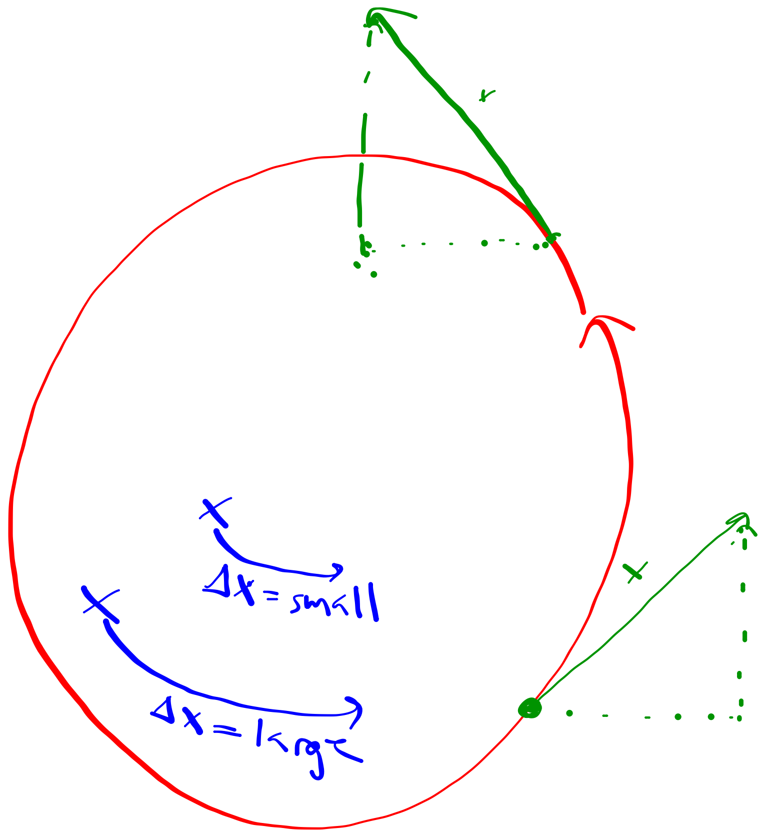
CENTRIPETAL ACCELERATION (a_c)

- Directed inward
- $a_c = v^2/r$
- Responsible for changing the direction



TANGENTIAL ACCELERATION (a_t)

- Directed in the direction of instantaneous travel
- $a_t = \alpha r$
- Responsible for increasing / decreasing the angular velocity



Δx = small
Δx = large

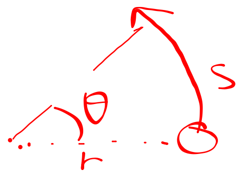
Linear Quantities vs. Angular Quantities

Linear Displacement (meters) x	Angular Displacement (radians) θ
Linear Velocity (m/sec) v	Angular Velocity (radians/sec) ω
Linear Acceleration (m/s²) a	Angular Acceleration (radians/s²) α
When **linear** acceleration is constant: $x = x_0 + v_0 t + \frac{1}{2} a t^2$ $v = v_0 + at$ $v^2 = v_0^2 + 2a(x - x_0)$	When **angular** acceleration is constant: $\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$ $\omega = \omega_0 + \alpha t$ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Relating Linear Quantities to Angular Quantities

$$\boxed{\theta = s/r}$$

BY DEFINITION

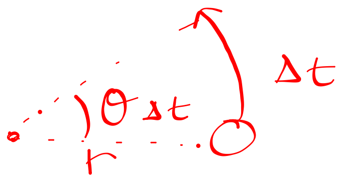


If an object is rotating for a given amount of time (Δt), an angular displacement ($\Delta\theta$) and linear dislacement (Δs) are realized.

$$\Delta\theta = \frac{\Delta s}{r}$$

$$\frac{\Delta\theta}{\Delta t} = \frac{\Delta s}{\Delta t(r)} \quad \text{(DIVIDE BOTH SIDES BY } \Delta t\text{)}$$

$$\boxed{\omega = \frac{v}{r}}$$

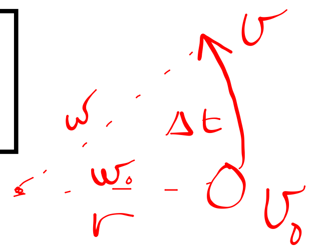


If an object is also experiencing an angular acceleration (speeding up or slowing down) over some time period (Δt), there will be changes in the angular speed ($\Delta\omega$) and the linear speed (Δv).

$$\Delta\omega = \frac{\Delta v}{r}$$

$$\frac{\Delta\omega}{\Delta t} = \frac{\Delta v}{\Delta t(r)} \quad \text{(DIVIDING BOTH SIDES BY } \Delta t\text{)}$$

$$\boxed{\alpha = \frac{a_t}{r}}$$



EXAMPLE 1: A typical compact disc records data starting at a radius of 25.0 mm and ending at a radius of 58.0 mm from its center. All disc players read information from the disc at a rate of 4500 mm/min.

a) What is t he initial angular velocity (in RPM) of the disc when it starts reading data?
b) What is the angular velocity (in RPM) of the disc when it finishes reading data at the outside radius?
c) If the CD plays continuously from the beginning to end, what is the angular acceleration (in rot/min^2) assuming a play time of 75.0 minutes?
d) What are a_t and a_c (in m/s^2) at a point when the data is being read at a radius of 50.0 mm?

1. Pic
2. Frame of reference +
3. Inventory variables
   → components
   → conversions

2π radians =
$360°$ =
1 Rotation/revolution

★ RADIANS
★ angular ⟷ linear

4. Substitute & solve
5. Interpret

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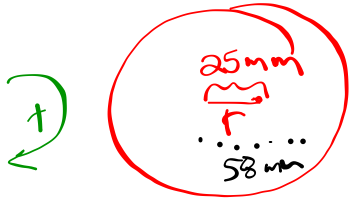

$r = 25\,mm = 0.025\,m$

$v = \frac{4500\,mm}{min} \cdot \frac{1\,m}{1000\,mm} \cdot \frac{1\,min}{60\,s}$

$= 0.075\,m/s$

$\theta_0 =$
$\theta =$
$\omega_0 =$
$\omega =$
$\alpha =$
$t =$

$\omega = \frac{v}{r}$

$\omega = \frac{0.075}{0.025}$

$\omega = 3\,\frac{radians}{s}$

$$3\,\frac{radians}{s} \cdot \frac{rot}{2\pi\,rad} \cdot \frac{60\,s}{min} = \boxed{28.7\,RPM}$$

b) What is the angular velocity (in RPM) of the disc when it finishes reading data at the outside radius?

same info except
$r = 58\,mm = 0.058\,m$

$\omega = \frac{0.075}{0.058} = 1.3\,\frac{rad}{s}$

$1.3\,\frac{rad}{s} = \boxed{12.4\,RPM}$

c) If the CD plays continuously from the beginning to end, what is the angular acceleration (in rot/min^2) assuming a play time of 75.0 minutes?

d) What are a_t and a_c (in m/s^2) at a point when the data is being read at a radius of 50.0 mm?

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d) What are a_t and a_c (in m/s^2) at a point when the data is being read at a radius of 50.0 mm?

a) $\omega_0 = v/r = \frac{4500 \text{ mm/min}}{25 \text{ mm}} = 180 \frac{\text{rad}}{\text{min}} \Rightarrow \boxed{28.65 \text{ RPM}}$

b) $\omega = v/r = \frac{4500 \text{ mm/min}}{58 \text{ mm}} = 77.6 \frac{\text{rad}}{\text{min}} \Rightarrow \boxed{12.35 \text{ RPM}}$

c) $\theta_0 = 0$
$\theta = ?$
$\omega_0 = 180 \frac{\text{rad}}{\text{min}}$
$\omega = 77.6 \frac{\text{rad}}{\text{min}}$
$\alpha = ?$
$t = 75 \text{ min}$

$\omega = \omega_0 + \alpha t$

$77.6 = 180 + \alpha(75)$

$\alpha = -1.37 \text{ rad/min}^2$

$\Rightarrow \boxed{-.218 \text{ ROT/MIN}^2}$

d) $a_c = v^2/r = \frac{(4500 \text{ mm/min})^2}{50 \text{ mm}} = 405{,}000 \text{ mm/min}^2$

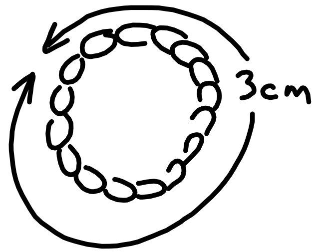
$= \boxed{.1125 \text{ m/s}^2 \text{ (INWARD)}}$

$a_t = \alpha r = (-1.37 \frac{\text{rad}}{\text{min}^2})(50 \text{ mm})$

$= -68.5 \frac{\text{mm}}{\text{min}^2}$

$= \boxed{-1.9 \times 10^{-5} \text{ m/s}^2}$

EXAMPLE 2: *Nannosquilla decemspinosa* is a small, legless crustacean living on the west coast of Panama. When stranded on the beach by high tide, it moves back to the water by doing sommersaults. If *nannosquilla* has a body length of 3.0 cm, takes this body length and curls it up as a wheel (having this circumference), rotates as a wheel at 70.0 RPM, and if it must travel 4.0 meters to return to the water, how long does it take it to get back into the water?



$2\pi r = 3.0\,\text{cm}$

$r = .4775\,\text{cm}$

$r = .004775\,\text{m}$

$\theta_0 = 0$

$\theta = 837.758\,\text{rad}$

$4.0\,\text{m}\left(\frac{2\pi\,\text{rad}}{.03\,\text{m}}\right) = 837.758\,\text{rad}$

$\omega_0 = 70\,\frac{\text{rot}}{\text{min}}\left(\frac{2\pi\,\text{rad}}{1\,\text{rot}}\right)\left(\frac{1\,\text{min}}{60\,\text{sec}}\right) = 7.33\,\text{rad/sec}$

$\omega = 7.33\,\text{rad/sec}$

$\alpha = 0$

$t = ?$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$837.158 = 0 + 7.33t$

$t = \boxed{114.29\,\text{sec}}$

EXAMPLE 3: A wheel with a diameter of 19.0 centimeters starts from rest and reaches a speed of 40.0 RPM after rotating through 46 radians.
a) Determine the wheel's constant angular acceleration.
b) How long did the above process take?

a) $\theta_0 = 0$

$\theta = 46$ rad

$\omega_0 = 0$

$\omega = 40 \frac{\cancel{\text{rot}}}{\cancel{\text{min}}}\left(\frac{1\ \cancel{\text{min}}}{60\ \text{sec}}\right)\left(\frac{2\pi}{1\ \cancel{\text{rot}}}\right) = 4.189$ rad/sec

$\alpha = ?$

$t = ?$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$4.189^2 = 0^2 + 2\alpha(46 - 0)$$

$$\boxed{\alpha = .191\ \text{rad/sec}^2}$$

b) $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$$46 = 0 + 0 + \tfrac{1}{2}(.191)t^2$$

$$t = \boxed{21.95\ \text{sec}}$$