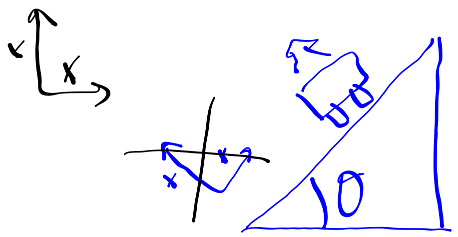
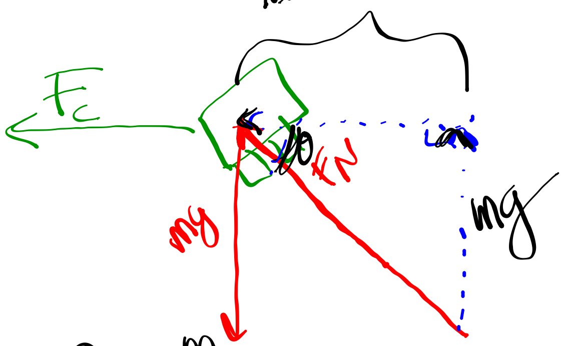
EXAMPLE 5: How much should a road with a 110 meter curve be banked (what is the angle) to accommodate cars travelling at 25 m/sec (so that friction is NOT required to get the car around the corner)?

$$F_M = \Sigma F_x = F_c$$




$$\tan\theta = \frac{mg}{F_c}$$

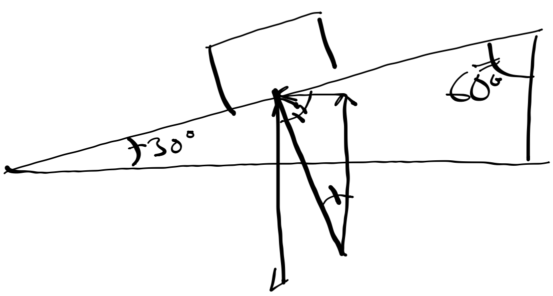
$$\tan\theta = \frac{mg}{\frac{mv^2}{r}} = mg\cdot\frac{r}{mv^2}$$

$$\tan\theta = \frac{r\cdot g}{v^2}$$

$$\theta = \tan^{-1}\frac{r\cdot g}{v^2} = \tan^{-1}\frac{(110)(9.8)}{25^2}$$

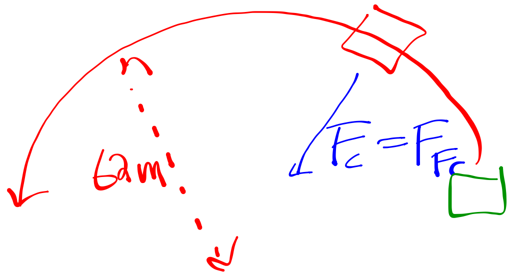
$$90-\theta \approx 60^\circ$$

$$\theta = 30^\circ$$

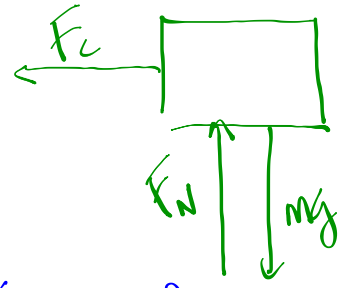


6. How large must the coefficient of friction be between the tires and the road if a car is to round a level curve of radius 62 m at a speed of 55 km/h?

$55\ km/hr = 15.28\ m/s$




$\frac{mv^2}{r}$




$$F_c = F_{Fr}$$

$$F_N = \boxed{mg}$$

$$F_{Fr} = \mu F_N$$

$$= \mu mg$$

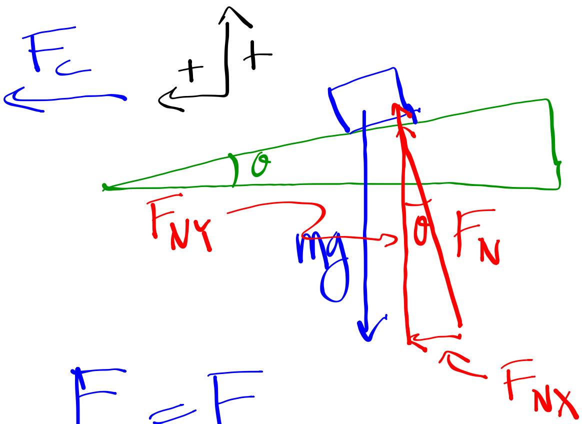
$$F_{Fr} = \frac{mv^2}{r} = \frac{m(15.28)^2}{62}$$

$$\frac{\mu \cancel{m} g}{g} = \frac{\cancel{m}(15.28)^2}{62 \cdot g}$$

$$\mu = \frac{15.28^2}{(62)(9.8)} = 0.384$$

16. What must a curve with a radius of 60.0 m be banked at (i.e. what is the banking angle) for a car traveling at 60 km/h? Remember, banked curves are designed so that for a given speed, NO fricition would be require to safely get around the corner. (Although not required, a more interesting, and difficult problem, would be to determine what the coefficient of static friction needs to be for a car not to skid when traveling at 90 km/h around this same curve, banked at the angle you determine in solving this problem).

Without friction




$F_c = F_{NX}$

since mg and $F_{Ny}$ are the only forces in the y-dimension, and $a_y = 0$ then

$F_{Ny} = -mg$

$$\tan\theta = \frac{o}{a} = \frac{F_{NX}}{F_{NY}} = \frac{F_c}{mg} = \frac{\frac{\cancel{m}v^2}{r}}{\cancel{m}g}$$

$$\tan\theta = \frac{v^2}{rg}$$

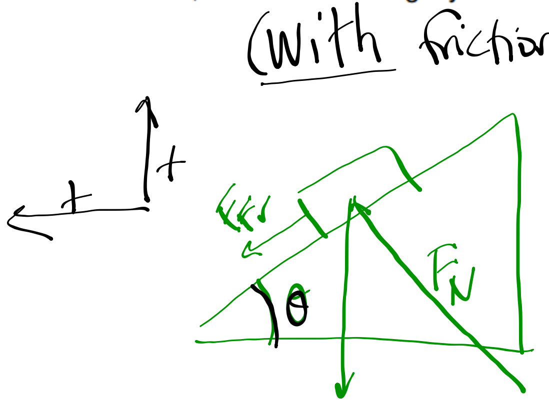
$$\theta = \tan^{-1}\frac{v^2}{r \cdot g} = \tan^{-1}\left(\frac{16.66^2 \, m/s}{60 \cdot 9.8}\right)$$

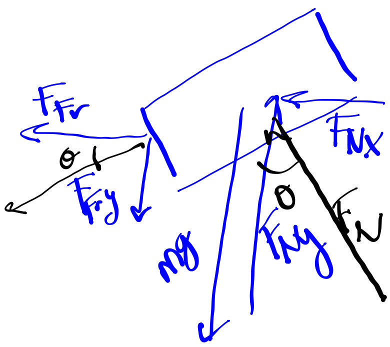
$$\theta = 25.3^\circ$$

16. What must a curve with a radius of 60.0 m be banked at (i.e. what is the banking angle) for a car traveling at 60 km/h? Remember, banked curves are designed so that for a given speed, NO fricition would be require to safely get around the corner. (Although not required, a more interesting, and difficult problem, would be to determine what the coefficient of static friction needs to be for a car not to skid when traveling at 90 km/h around this same curve, banked at the angle you determine in solving this problem).

(with friction)

$F_{Fr} = \mu F_N$




$$\sum F_y = 0$$

$$-F_{Fr_y} + -mg + F_{Ny} = 0$$

$$-F_{Fr} \sin\theta - mg + F_N \cos\theta$$

$$-\mu F_N \sin\theta - mg + F_N \cos\theta = 0$$

$$\sum F_x = F_c = \frac{mv^2}{r}$$

$$F_{Frx} + F_{Nx} = \frac{mv^2}{r}$$

$$F_{Fr} \cos\theta + F_N \sin\theta = \frac{mv^2}{r}$$

$$\mu F_N \cos\theta + F_N \sin\theta = \frac{mv^2}{r}$$

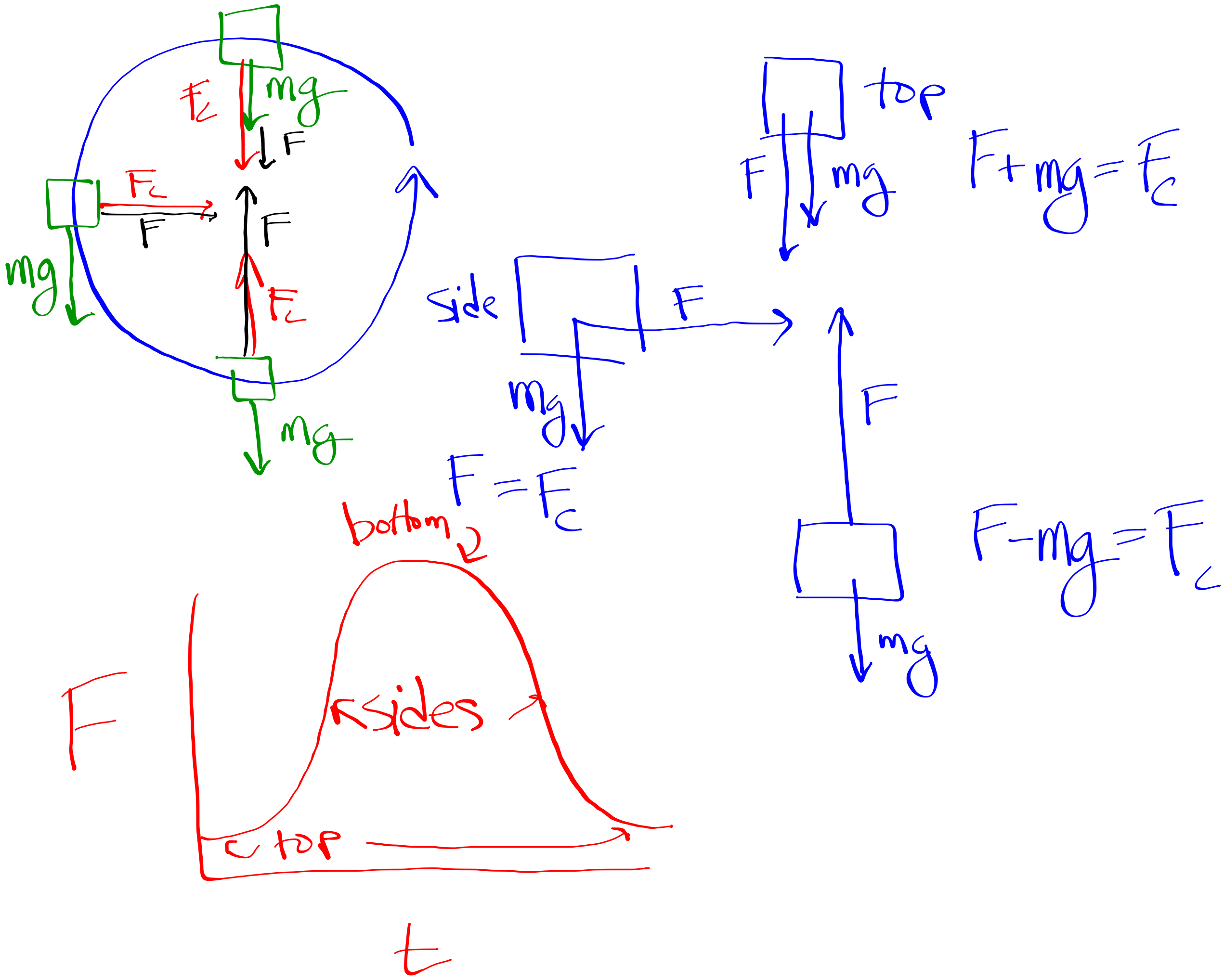
3. Calculate the centripetal acceleration of the earth in its orbit around the sun and the net force exerted on it. What exerts this force on the earth? Assume that the earth's orbit is a circle of radius $1.50 \times 10^{11}$ meters, and that the earth's mass is $5.98 \times 10^{24}$ kg.

$$v \cong 30{,}000 \text{ m/s}$$

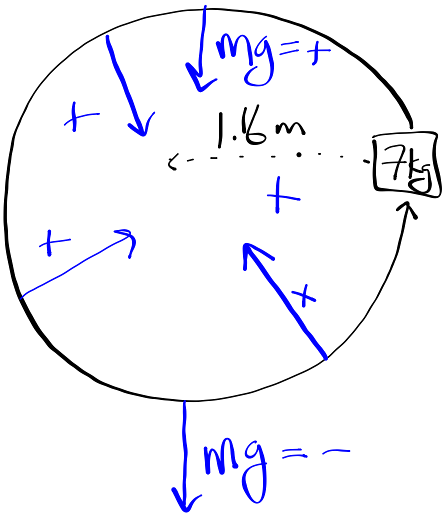
$$F_c = \frac{mv^2}{r} = \frac{(5.98 \times 10^{24})(30{,}000)^2}{1.5 \times 10^{11}}$$

# Vertical Circular Motion:

To maintain a constant centripetal (towards the center!) force, we must account for the fact that the direction of the gravitational force is always the same. Therefore, whatever other forces that are providing the centripetal force must be constantly changing in both direction AND magnitude.



EXAMPLE 1: A 7.0 kg mass at the end of a 116.0 cm string is spun in a vertical circle. The mass moves with a speed of 4.5 m/sec. What is the tension in the string when the mass is at the top of its travel? At the bottom?




$T_{top} = 53.6\ N$ 68.6 N

$T_{sides\ (average)} = 122.2\ N$

$T_{bottom} = 190.8\ N$ 68.6 N

$$F_c = \frac{mv^2}{r} = \frac{7(4.5^2)}{1.16} = 122.2\ N$$

$$mg = 7(9.8) = 68.6\ N$$

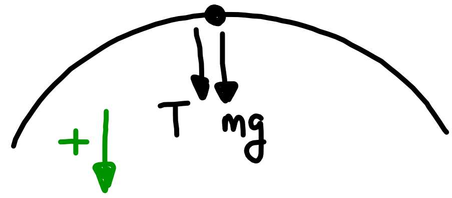
EXAMPLE 1: A 7.0 kg mass at the end of a 116.0 cm string is spun in a vertical circle. The mass moves with a speed of 4.5 m/sec. What is the tension in the string when the mass is at the top of its travel? At the bottom?

$m = 7.0 \text{ kg}$

$r = 1.16 \text{ m}$

$v = 4.5 \text{ m/s}$

AT THE TOP:




$$\Sigma F = ma$$

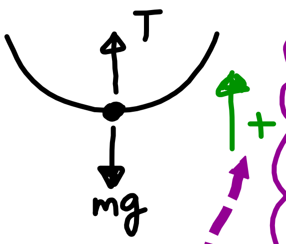
$$\Sigma F = m\left(\frac{v^2}{r}\right)$$

$$T + mg = \frac{mv^2}{r}$$

$$T = \frac{mv^2}{r} - mg$$

$$= \frac{(7)(4.5)^2}{1.16} - 7(9.8) = \boxed{53.6 \text{ N}}$$

AT BOTTOM:




The positive direction has been selected in each case to be in the direction of the necessary centripetal force -- always towards the center of motion. This way "$a_c$" can always be taken as positive.

$$\Sigma F = ma$$

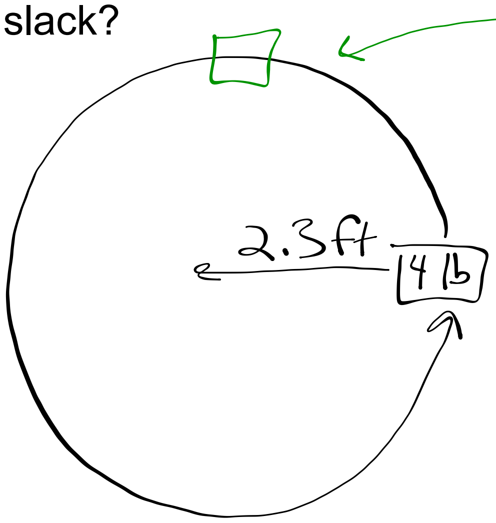
$$\Sigma F = m\left(\frac{v^2}{r}\right)$$

$$T - mg = m\left(\frac{v^2}{r}\right)$$

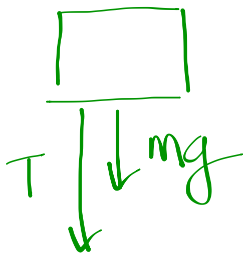
$$T = m\left(\frac{v^2}{r}\right) + mg$$

$$= \frac{(7)(4.5)^2}{1.16} + 7(9.8) = \boxed{190.8 \text{ N}}$$

EXAMPLE 2: A 4.0 lb object is swung in a vertical circle having a radius of 2.3 feet. At what speed will the string just begin to go slack? Where is the object along its circular path when the string first goes slack?




$T = 0$




$T + mg = F_c$

$mg = F_c$

$$\cancel{m}g = \frac{\cancel{m}v^2}{r}$$

$$v = \sqrt{rg} = \sqrt{(2.3)(32.2)}$$

$$v = 8.6\ ft/s$$

EXAMPLE 2: A 4.0 lb object is swung in a vertical circle having a radius of 2.3 feet. At what speed will the string just begin to go slack? Where is the object along its circular path when the string first goes slack?

The tension in the string is a minimum when the object is at the top of its travel. When at this point, the force of gravity is fully contributing to the necessary centripetal force, and therefore the tension within the string can be at a minimum. At all other locations, the string must provide more force necessary to compensate for the component of gravity that IS NOT contributing to the centripetal force.

THE STRING GOES SLACK WHEN $T = \emptyset$, SO

$$\Sigma F = ma$$

$$\Sigma F = m\left(\frac{v^2}{r}\right)$$

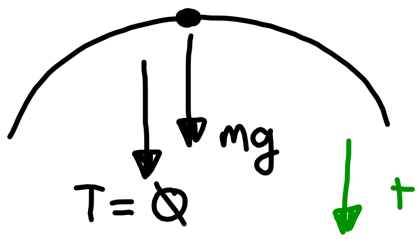
$$\cancel{m}g = \cancel{m}\left(\frac{v^2}{r}\right)$$

$$rg = v^2$$

$$\sqrt{rg} = v$$

$$v = \sqrt{(2.3)(32.2)} = 8.61\ \text{ft/s}$$

AT TOP



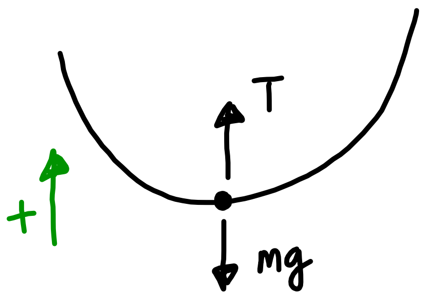

Note: the mass cancels here -- the solution is independent of mass. Therefore all objects would experience a slack string at this speed at the top of a path of this radius.

EXAMPLE 3: For the previous problem, if the string breaks at 110.0 lbs, at what speed will the string break?

EXAMPLE 3: For the previous problem, if the string breaks at 110.0 lbs, at what speed will the string break?

The tension is a maximum when the object is at the bottom. When at this location, the string must not only provide the centripetal force required, but it also must counter the full force of gravity acting on the object. So set up the problem by looking at the forces acting when the object is at the bottom of it's travel.

AT BOTTOM:




$$m = \frac{4.0}{32.2} = .1242 \text{ slugs}$$

$$\Sigma F = ma$$

$$\Sigma F = m\left(\frac{v^2}{r}\right)$$

$$T - mg = m\frac{v^2}{r}$$

NO, m's DON'T CANCEL!

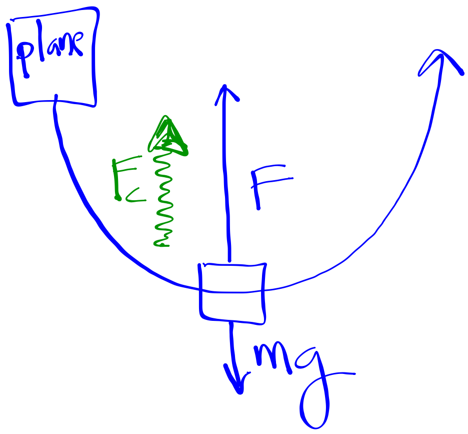
$$\sqrt{\frac{r(T-mg)}{m}} = v$$

$$\sqrt{\frac{(2.3)(110-4)}{.1242}} = \boxed{44.3 \text{ ft/s}}$$

EXAMPLE 4: An airplane finds itself at the bottom of a circular dive having a radius of 1390 meters. The plane is traveling at 245 m/s.

a) What is the apparent weight of a 70-kg person when at the bottom of the dive?

b) How many g's does the person experience at this moment?




$$F - mg = F_c = \frac{mv^2}{r}$$

$$F - (70)(9.8) = \frac{(70)(245)^2}{1390}$$

$$F = 3709\text{ N}$$

$$F - mg = \frac{mv^2}{r}$$

$$F = mg + \frac{mv^2}{r}$$

$$F = m\left(g + \frac{v^2}{r}\right) \Rightarrow \boxed{a_c}$$

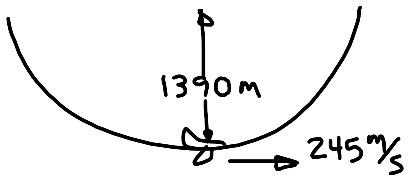
apparent weight

$$1g = 9.8\ \text{m/s}^2$$

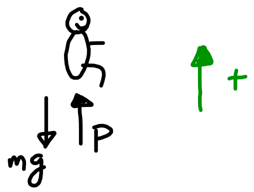
$$a_c = \frac{(9.8) + \left(\frac{245^2}{1390}\right)}{9.8\ \text{m/s}^2} = 5.4g$$

EXAMPLE 4: An airplane finds itself at the bottom of a circular dive having a radius of 1390 meters. The plane is traveling at 245 m/s.
a) What is the apparent weight of a 70-kg person when at the bottom of the dive?
b) How many g's does the person experience at this moment?

- A person's apparent weight is always equal to the force the ground (or in this case the seat from the plane) is exerting on the person. Usually, when you want the apparent weight, you will be solving for the normal force.
- It is the person's apparent weight we are after, so instead of drawing a FBD of the plane, we wish to draw a FBD of the person. Additionally, realize that the person moves as the plane does (if this were not true, the person would not be very happy).




AT BOTTOM:




a) $\Sigma F = ma$

$$\Sigma F = m\left(\frac{v^2}{r}\right)$$

$$P - mg = m\frac{v^2}{r}$$

$$P = \frac{mv^2}{r} + mg = \frac{70(245)^2}{1390} + 70(9.8) = \boxed{3708.6\text{ N}}$$

b) NORMALLY THIS PERSON WEIGHS

$$mg = 70(9.8) = 686\text{ N}$$

$$\frac{\text{ACTUAL WEIGHT}}{\text{NORMAL WEIGHT}} = \frac{3708.6}{686} = \boxed{5.41\text{ g's}}$$

ANOTHER WAY TO THINK ABOUT THIS:

FIND THE ACTUAL ACCELERATION & CONVERT IT TO g's

USING: $1\text{ g} = 9.8\text{ m/s}^2$

$$\Sigma F = ma$$

$$3708.6 = 70a$$

$$\frac{3708.6}{70} = a = 52.98\text{ m/s}^2$$

$$52.98\,\frac{\text{m}}{\text{s}^2}\left(\frac{1\text{ g}}{9.8\text{ m/s}^2}\right) = \boxed{5.41\text{ g's}}$$

EXAMPLE 5: A 60.0 kg person is riding on a Ferris wheel having a diameter of 24.0 meters. The Ferris wheel turns at 1.6 RPM. What is the person's apparent weight when at the top and at the bottom?

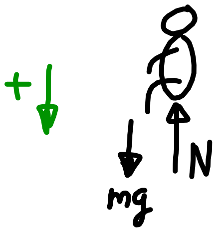
EXAMPLE 5: A 60.0 kg person is riding on a Ferris wheel having a diameter of 24.0 meters. The Ferris wheel turns at 1.6 RPM. What is the person's apparent weight when at the top and at the bottom?

First, find the lineal speed at which the person travels:

$$1.6\ \frac{\text{ROTATIONS}}{\text{MIN}}\left(\frac{2\pi r\ \text{m}}{1\ \text{ROTATION}}\right)\left(\frac{1\ \text{MIN}}{60\ \text{SEC}}\right) =$$

$$1.6\left(\frac{2\pi(12)}{1}\right)\left(\frac{1}{60}\right) = 2.01\ \text{m/s}$$

AT TOP:




$$\Sigma F = ma$$

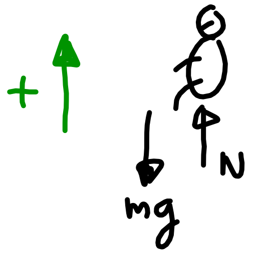
$$\Sigma F = m\frac{v^2}{r}$$

$$mg - N = m\frac{v^2}{r}$$

$$mg - \frac{mv^2}{r} = N$$

$$N = mg - \frac{mv^2}{r} = 60(9.8) - \frac{(60)(2.01)^2}{12} = \boxed{567.8\ \text{N}}$$

AT BOTTOM:




$$\Sigma F = m\frac{v^2}{r}$$

$$N - mg = m\frac{v^2}{r}$$

$$N = mg + m\frac{v^2}{r} = 60(9.8) + \frac{60(2.01)^2}{12} =$$

$$= \boxed{608.2\ \text{N}}$$

PERSON'S NORMAL WEIGHT

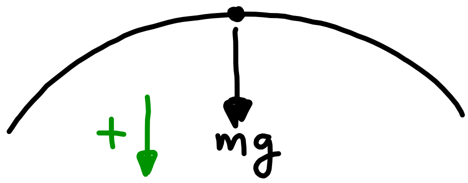
$$mg = 60(9.8) = \underline{588\ \text{N}}$$

THIS IS BETWEEN THESE TWO ANSWERS

EXAMPLE 6: A 590-gram rolling ball on a track attempts to do a loop-d-loop. If the loop has a radius of 80.0 centimeters, what minimum speed must the ball have?

EXAMPLE 6: A 590-gram rolling ball on a track attempts to do a loop-d-loop. If the loop has a radius of 80.0 centimeters, what minimum speed must the ball have?

The top is the critical point.




For the ball to just make it around, at the top no force is require from the track. Gravity provides all of the required centripetal forces.

$$\Sigma F = ma$$

$$\Sigma F = m\frac{v^2}{r}$$

$$\cancel{m}g = \cancel{m}\frac{v^2}{r}$$

AS BEFORE, MASS DOES NOT MATTER

$$v = \sqrt{gr} = \sqrt{(9.8)(.8)} = \boxed{2.8\ m/s}$$