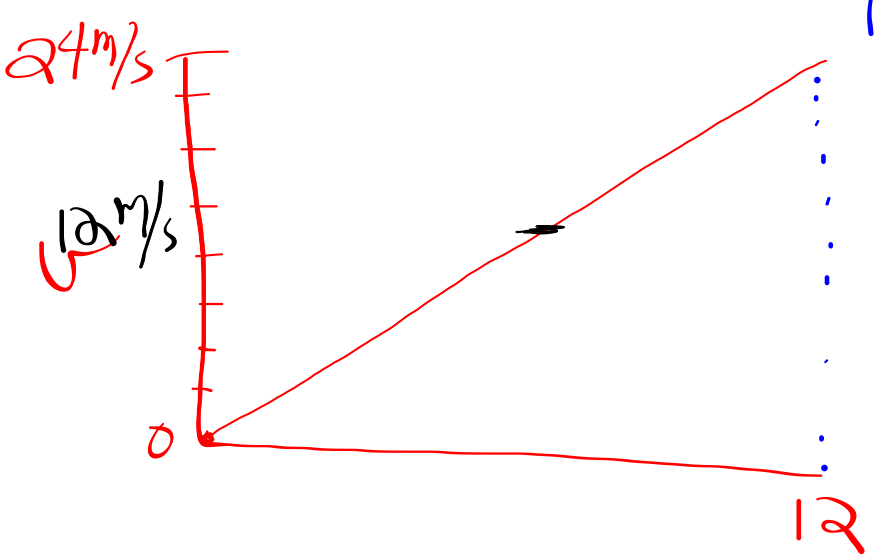
Uniformly Accelerated Motion (Constant a)

Example 1: A car travels at 5 m/s for 12 seconds. How far has it travelled?

60 m

EXAMPLE 2: A car starts from rest and accelerates at 2 m/s^2 for 12 seconds. How far has it travelled?

48m



$\bar{v} = 12\,m/s$

12s

$\Delta x = 144\,m$

EXAMPLE 3: A car starts at 7 m/s and accelerates at 2 m/s^2 for 12 seconds. How far has it travelled?

$2\, m/s^2 \cdot 12s =$

$24\, m/s + 7\, m/s =$

$31\, m/s \cdot 12s$

To make our future lives easier, lets adjust the variables we utilize for various quantities.

Quantity	Old Variable	New Variable
Initial Position (position at t = 0)	x_1	x_0
Final Position	x_2	x
Initial Velocity (velocity at t = 0)	v_1	v_0
Final Velocity	v_2	v
Acceleration	a	a constant
Initial time	t_1	t_0 0
Final time	t_2	t t
The time that elapses	t_2 - t_1	Δt t

Deriving relations for uniformly accelerated motion:

We want a set of expressions that can be used to make predictions when an object accelerates.

Assumption: the acceleration is uniform (constant and unchanging in value)

EQ #1 $\bar{v} = \dfrac{x - x_0}{t}$ From definition for average velocity

EQ #2 $a = \dfrac{v - v_0}{t}$ From definition for average acceleration

$a \cdot t = v - v_0$

Solve Eq. #2 for v (gives us Eq. #2a): $v = v_0 + at$

EQ #1 $\bar{v} = \frac{x - x_o}{t}$ $\bar{v}$ = AVERAGE VELOCITY

Solve for x (gives us Eq #3):

$$\bar{v} \cdot t = x - x_o$$
$$x = x_o + \bar{v} \cdot t$$

Because velocity increases at a uniform rate (a is constant), then the average velocity is equivalent to the mathematical average of the initial and final velocities:

$$\bar{v} = \boxed{\frac{v + v_o}{2}} \quad \text{(EQ. \#4)}$$

Substitute EQ. #4 into EQ. #3:

$$x = x_o + \left(\frac{v + v_o}{2}\right)t \quad \text{(EQ. \#5)}$$

Substituting Eq. #2a into EQ. #5:

$$v = \boxed{v_o + at}$$

$$x = x_o + \left(\frac{v_o + a \cdot t + v_o}{2}\right)t$$
$$x = x_o + \left(\frac{2v_o + at}{2}\right)t$$
$$x = x_o + \left(\frac{2v_o t + at^2}{2}\right)$$
$$\boxed{x = x_o + v_o t + \frac{1}{2}at^2}$$

With this expression, we can determine how far an accelerating object has travelled without needing to know anything about its final velocity!

Can we find an expression that will allow us to find an accelerating object's final velocity without needing to know the length of time that it accelerates?

$$x = x_0 + \left(\frac{v + v_0}{2}\right)t \quad (\text{EQ } \#5)$$

Next, we will solve Eq. #2 for t instead of a as we did before:

$$t = \boxed{\frac{v - v_0}{a}} \quad (\text{EQ } \#6)$$

Substituting Eq. #6 into Eq. #5, we obtain:

$$x = x_0 + \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right)$$

Solving for v^2 we obtain:

$$x = x_0 + \frac{v^2 - v_0^2}{2a}$$

$$2a \cdot x = 2a \cdot x_0 + v^2 - v_0^2$$

$$v^2 = v_0^2 + 2ax - 2ax_0$$

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$

Collectively, the four previously boxed relationships will be called:

THE BIG 4

TRUE ONLY WHEN a IS CONSTANT

a a
2 2
Z Z

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2 \quad \bigstar \longrightarrow \text{no } v!$$

$$v = v_0 + a t \quad \bigstar \longrightarrow \text{no } x, x_0!$$

$$v^2 = v_0^2 + 2a(x - x_0) \quad \bigstar \longrightarrow \text{no } t$$

$$\bar{v} = \frac{v + v_0}{2} = \frac{\Delta x}{\Delta t}$$

$x_0 \quad x \quad v_0 \quad v \quad a \quad t$

The starred relations are the ones we will end up using most frequently.

Applying the Big 4 to problems:

1. Establish a reference frame. Pick an origin and a positive direction

2. Draw a picture.

3. Inventory / assign variables (x_o, x, v_o, v, a, t)

4. Check units -- convert as needed (to meters & seconds, or feet & seconds)

5. Make sure acceleration is constant for the entire duration of the problem (throughout t)

6. Pick a relationship(s) and solve for the unknown(s)
(equation)

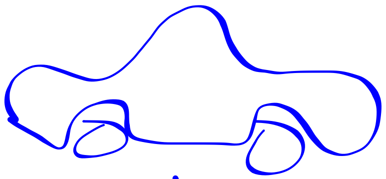
   -You may have to do this a couple of times

   -You may need to use the quadratic formula

   -Sometimes you have dual answers

   - You may need to work with variable expression

EXAMPLE 3: A car starts at 7 m/s and accelerates at 2 m/s² for 12 seconds. How far has it travelled?



$7\ m/s$ $\rightarrow$

$a = 2\ m/s^2$

$t = 12s$

$x = 0$ $\xrightarrow{+}$

$x_0 : 0$
$x : x$
$v_0 : 7\ \frac{m}{s}$
$v : v$
$a : 2\ m/s^2$
$t : 12s$

$$x = \cancel{x_0}^{0} + v_0 t + \frac{1}{2} a t^2$$

$$x = (7)(12) + \frac{1}{2}(2)(12^2)$$

$$x = 228m$$