Uniformly Accelerated Motion

(Constant acceleration)

A car travels at 5 m/s for 12 seconds. How far has it travelled?

$$v = \frac{\Delta x}{\Delta t}$$

$$5\frac{m}{s} = \frac{\Delta x}{12s}$$

$$\Delta x = 60m$$

$$v = \frac{\Delta x}{\Delta t} \quad \Delta t \to 0$$

$$a = \frac{\Delta v}{\Delta t} \quad \Delta t \to 0$$

*

A car starts from rest and accelerates at 2 m/s² for 12 seconds. How far has it travelled?

24?

~~288~~?

12?

$a = \frac{\Delta v}{\Delta t}$

$2\,\frac{m}{s^2} = \frac{\boxed{\Delta v}}{12s}$

$\Delta v = 24\,\frac{m}{s}$

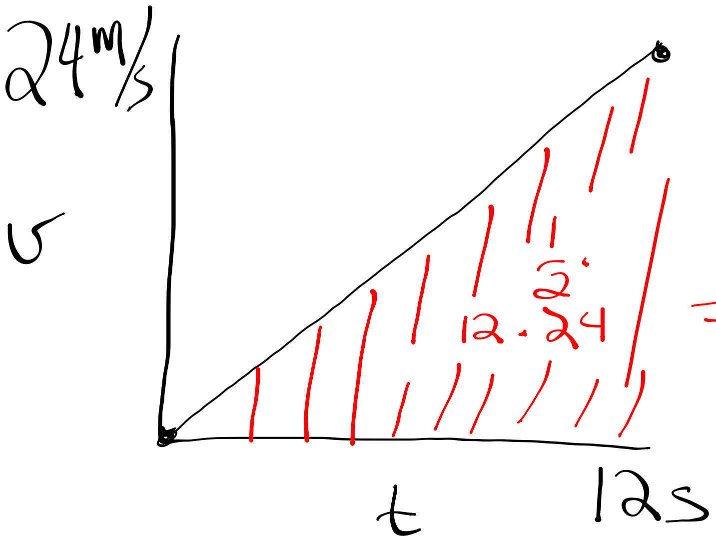
~~$\Delta v = \frac{\Delta x}{\Delta t}$~~

~~$24\,\frac{m}{s} = \frac{\Delta x}{12s}$~~

$\Delta x = \cancel{288}\,m$?

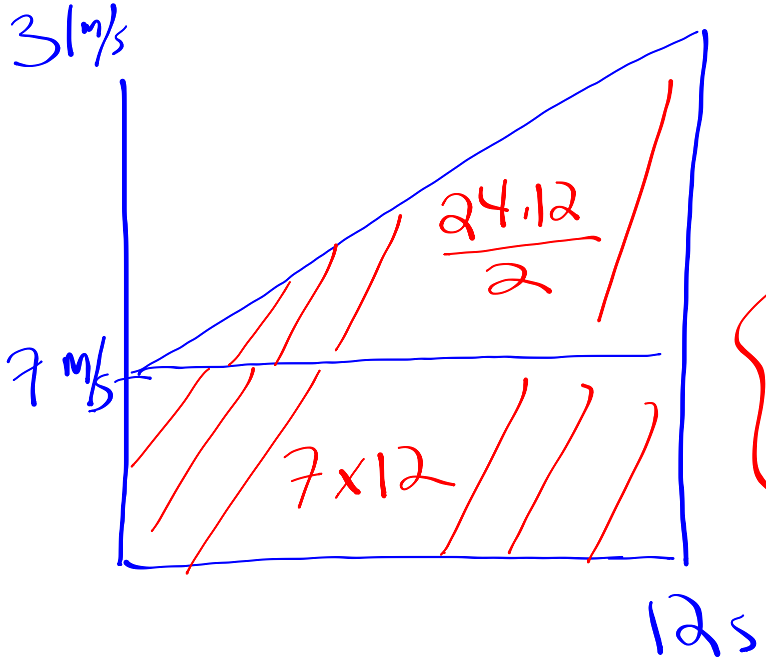
$2 \cdot 12 = 24$

$2\,\frac{m}{s^2} \cdot 12s \cdot 12s = \cancel{288}\,m$



= 144m

A car starts at 7 m/s and accelerates at 2 m/s^2 for 12 seconds. How far has it travelled?



= 228 m

Objectives:

1. Students will understand how position, velocity, and time are related under conditions of constant acceleration.
2. Students will see how to use the Big 4 equations to predict elements of the motion of different objects.

Quantity (all functions of time!)	Variable
Initial Position (position at t = 0)	x_0
Final Position	x
Initial Velocity (velocity at t = 0)	v_0
Final Velocity	v
Acceleration	a
Initial time	$\emptyset$
Final time	t
The time that elapses $\Delta t = t - \emptyset$	t

Deriving relations for uniformly accelerated motion:

We want a set of expressions that can be used to make predictions when an object accelerates.

Assumption: the acceleration is uniform (constant and unchanging in value)

EQ #1 $\bar{v} = \frac{x - x_0}{t}$ From definition for average velocity

EQ #2 $a = \frac{v - v_0}{t}$ From definition for average acceleration

Solve Eq. #2 for v (gives us Eq. #2a):

$$a \cdot t = v - v_0$$

$$v = v_0 + at$$

EQ #1 $\bar{v} = \frac{x - x_o}{t}$ $\bar{v}$ = AVERAGE VELOCITY

Solve for x (gives us Eq #3):

$$\bar{v} \cdot t = x - x_o$$

$$x = x_o + \bar{v} \cdot t$$

Because velocity increases at a uniform rate (a is constant), then the average velocity is equivalent to the mathematical average of the initial and final velocities:

$$\bar{v} = \boxed{\frac{v + v_o}{2}} \quad \text{(EQ. \#4)}$$

Substitute EQ. #4 into EQ. #3:

$$x = x_o + \left(\frac{v + v_o}{2}\right)t \quad \text{(EQ. \#5)}$$

Substituting Eq. #2a into EQ. #5:

$$v = \boxed{v_o + at}$$

$$x = x_o + \left(\frac{v_o + a \cdot t + v_o}{2}\right)t$$

$$x = x_o + \left(\frac{2v_o + at}{2}\right)t$$

$$x = x_o + \left(\frac{2v_o t + at^2}{2}\right)$$

$$\boxed{x = x_o + v_o t + \frac{1}{2}at^2}$$

With this expression, we can determine how far an accelerating object has travelled without needing to know anything about its final velocity!

Can we find an expression that will allow us to find an accelerating object's final velocity without needing to know the length of time that it accelerates?

$$x = x_0 + \left(\frac{v + v_0}{2}\right)t \quad (\text{EQ } \#5)$$

Next, we will solve Eq. #2 for t instead of a as we did before:

$$t = \boxed{\frac{v - v_0}{a}} \quad (\text{EQ } \#6)$$

Substituting Eq. #6 into Eq. #5, we obtain:

$$x = x_0 + \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right)$$

Solving for v^2 we obtain:

$$x = x_0 + \frac{v^2 - v_0^2}{2a}$$

$$2a \cdot x = 2a \cdot x_0 + v^2 - v_0^2$$

$$v^2 = v_0^2 + 2ax - 2ax_0$$

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$

Collectively, the four previously boxed relationships will be called:

THE BIG 4

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$v = v_0 + a t$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\bar{v} = \frac{v + v_0}{2}$$

ONLY VALID when $a = \text{constant}$

The starred relations are the ones we will end up using most frequently.

Applying the Big 4 to problems:

1. Establish a reference frame. Pick an origin and a positive direction

2. Draw a picture.

same time

3. Inventory / assign variables (x_o, x, v_o, v, a, t)

4. Check units -- convert as needed (to meters & seconds, or feet & seconds)

5. Make sure acceleration is constant for the entire duration of the problem (throughout t) (IF NOT, CHUNK)

6. Pick a relationship(s) and solve for the unknown(s)

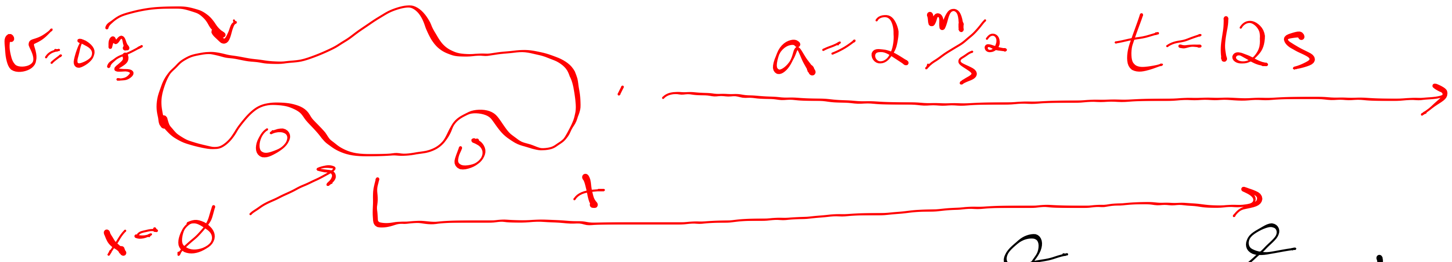
Use the Big-4

-You may have to do this a couple of times

-You may need to use the quadratic formula

-Sometimes you have dual answers

A car starts from rest and accelerates at 2 m/s² for 12 seconds. How far has it travelled?



$x_0 = 0\,m$

$x =$

$v_0 = 0\,\frac{m}{s}$

$v =$

$a = 2\,\frac{m}{s^2}$

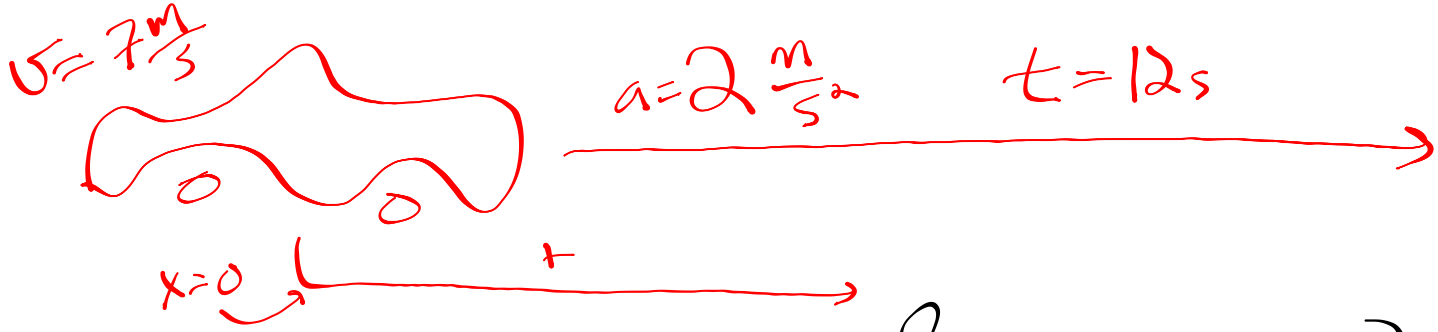
$t = 12\,s$

$$x = \cancel{x_0}^{0} + \cancel{v_0 t}^{0} + \frac{1}{2}at^2$$

$$x = \frac{1}{2}(2)(12^2)$$

$$= 144\,m$$

A car starts at 7 m/s and accelerates at 2 m/s^2 for 12 seconds. How far has it travelled?



$x_0 = 0\,m$

$x =$

$v_0 = 7\,\frac{m}{s}$

$v =$

$a = 2\,\frac{m}{s^2}$

$t = 12\,s$

$$x = \cancel{x_0}^{\,0} + v_0 t + \frac{1}{2} a t^2$$

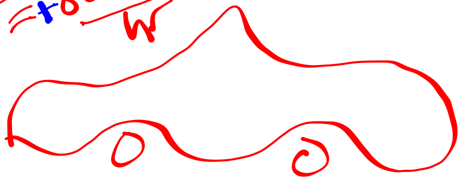
$$x = (7)(12) + \frac{1}{2}(2)(12^2)$$

$$= 84 + 144$$

$$\boxed{= 228\,m}$$

22. A car traveling 80 km/h decelerates at a constant 1.5 m/s^2. Calculate
   a) the distance it goes before it stops.
   b) the time it takes to stop.
   c) the distance it travels DURING the first and third seconds (not between those two times but during the 1st second of travel, and then during the 3rd second of travel).

$v = +80 \frac{km}{hr}$



$a = -1.5 \frac{m}{s^2}$

$x = 0$ $+$

$x_0 = 0\,m$

$x =$

$v_0 = 80 \frac{km}{hr} = 22.2 \frac{m}{s}$

$v = 0 \frac{m}{s}$

$a = -1.5 \frac{m}{s^2}$

$t =$

✓ (a) $v^2 = v_0^2 + 2a(x - x_0)$

✓ (b) $v = v_0 + at$